Top quark precision measurements in e^+e^- collisions

Andreas Maier

Measuring top quark properties

- Mass in well-defined scheme:
  - $\sigma(e^+e^- \to t\bar{t})$ at $\sqrt{s} \approx 350\,\mathrm{GeV}$: $\Delta m_t \approx 50\,\mathrm{MeV}$
  - $\sigma(e^+e^- \to t\bar{t}\gamma)$ at $\sqrt{s} \approx 380\,\mathrm{GeV}$: $\Delta m_t \approx 150\,\mathrm{MeV}$

    [Boronat, Fullana, Fuster, Gomis, Vos, Hoang, Widl, Mateu 2020]
- Width:
  - $\sigma(e^+e^- \to t\bar{t})$ at $\sqrt{s} \approx 350\,\mathrm{GeV}$: $\Delta\Gamma_t \approx 60\,\mathrm{MeV}$
- Yukawa coupling:
  - $\Gamma_{h\to gg}, \Gamma_{h\to\gamma\gamma}$: $\Delta\kappa_t < 0.01$

    [Boselli, Hunter, Mitov 2018]
  - $\sigma(e^+e^- \to t\bar{t}h)$ at $\sqrt{s} \geq 550\,\mathrm{GeV}$: $\Delta\kappa_t \approx 0.04$
  - $\sigma(e^+e^- \to t\bar{t}h)$ at $\sqrt{s} \approx 500\,\mathrm{GeV}$: $\Delta\kappa_t \approx 0.06$

    [arXiv:1409.7157, arXiv:1506.05992, arXiv:1807.02441]

    [Farell, Hoang 2005; Dawson, Reina 2017]
  - $\sigma(e^+e^- \to t\bar{t})$ at $\sqrt{s} \approx 350\,\mathrm{GeV}$: $\Delta\kappa_t \approx 0.25$

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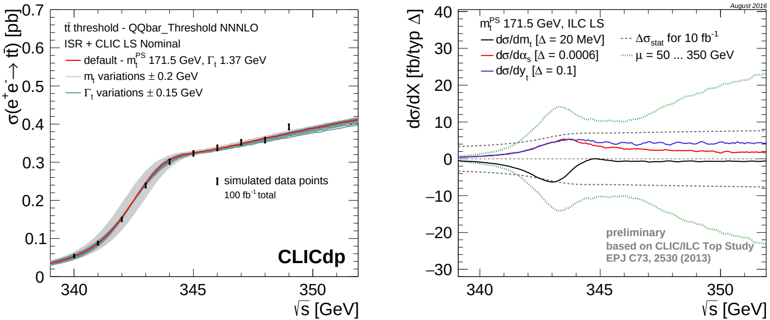
$\sigma(e^+e^- \to t\bar{t})$ near threshold

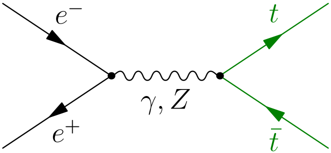
Threshold scan

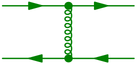


[Simon 2016]

Limited by theory error

$\sigma(e^+e^- \to t\bar{t})$ near threshold



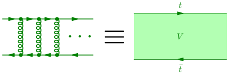
- Kinematics: $v \ll 1, \quad E_{\text{kin}} \sim m_t v^2, \quad |\mathbf{p}| \sim m_t v$
- Dominant interaction:
   $\Rightarrow$ Colour Coulomb potential $-\frac{C_F \alpha_s}{r}$
- $t\bar{t}$ “decays during bound state formation”:

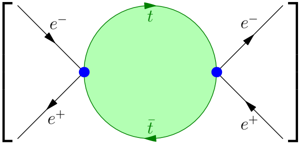
$$v \sim \alpha_s \Rightarrow E_{\text{kin}} \sim m_t \alpha_s^2 \sim -E_1$$
$$\alpha \sim \alpha_s^2 \Rightarrow \Gamma_{t\bar{t}} \sim m_t \alpha \sim -E_1$$

Potential non-relativistic effective field theory

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\mathcal{L}_{\text{PNREFT}} = \psi^\dagger \left(i\partial_0 + \frac{\partial^2 + i m_t \Gamma_t}{2m_t} \right) \psi + \mathcal{L}_{\text{anti-quark}}$$
$$- \int d^3\mathbf{r} \left[\psi^\dagger \psi\right](x+\mathbf{r}) \frac{C_F \alpha_s}{r} \left[\chi^\dagger \chi\right](x)$$
$$+ \{\text{NLO}\}$$

- Propagator: Coulomb Green Function 

- $\sigma_{\text{LO}} \propto \text{Im}$ 

- Higher orders suppressed by powers of $v \sim \alpha_s \sim \sqrt{\alpha} \sim y_t$

PNREFT at higher orders

Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{\mathrm{QCD}}$

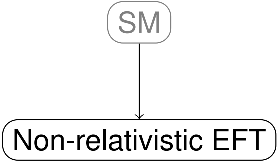
- hard modes: $k \sim m_t$
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

$\boxed{\text{SM}}$

PNREFT at higher orders

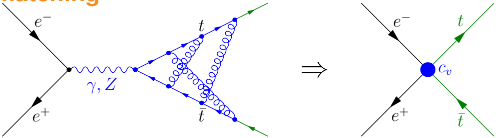
Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{\text{QCD}}$

- hard modes: $k \sim m_t \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

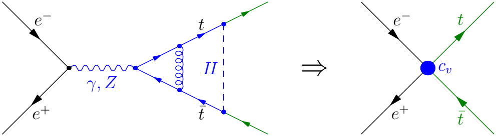


PNREFT at higher orders

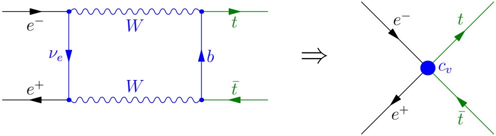
Hard matching



[Marquard, Piclum, Seidel, Steinhauser 2014]



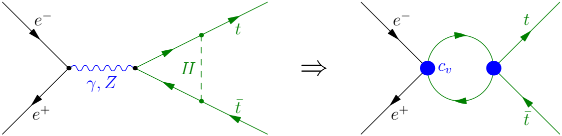
[Eiras, Steinhauser 2006]



[Grzadkowski, Kühn, Krawczyk, Stuart 1986]

[Guth, Kühn 1991]

[Hoang, Reißer 2004 & 2006]



PNREFT at higher orders

Scales: $m_t, m_W, m_Z, m_H \gg m_t v \gg m_t v^2 \gg \Lambda_{\mathrm{QCD}}$

- hard modes: $k \sim m_t \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_t v \rightarrow$ (non-local) potentials
- potential light particle modes $\rightarrow$ (non-local) potentials
- potential top quark modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

SM

↓

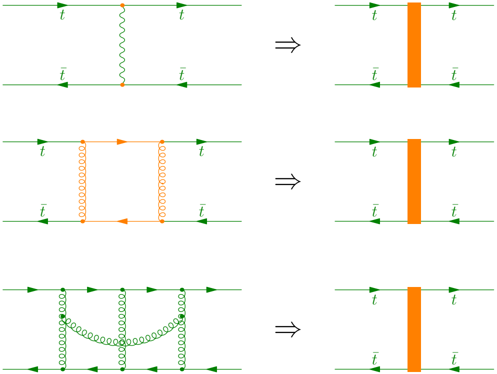
Non-relativistic EFT

↓

Potential non-relativistic EFT

PNREFT at higher orders

Soft matching



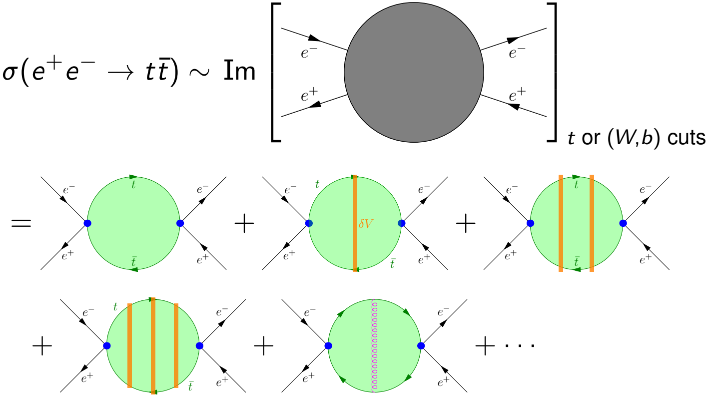
[Anzai, Kiyo, Sumino 2009]

[Smirnov, Smirnov, Steinhauser 2009]

[Lee, Smirnov, Smirnov, Steinhauser 2016]

$e^+e^- \to t\bar{t}$ at N^{3}LO PNREFT

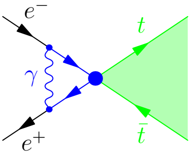
[Beneke et al.; many more people 2000-2015]



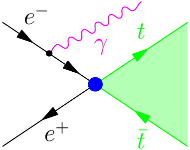
- $\sigma_{\text{LO}} \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k$
- N^{3}LO QCD + Higgs corrections $\sim \alpha_s^3 \sigma_{\text{LO}}, \alpha_s^2 v \sigma_{\text{LO}}, \alpha_s y_t^2 \sigma_{\text{LO}}, \ldots$
- N^{2}LO EW corrections $\sim \alpha \sigma_{\text{LO}}, \sqrt{\alpha} y_t \sigma_{\text{LO}}, \ldots$

Initial-state radiation

Photon corrections to initial state:



γ hard, hard-collinear



γ ultrasoft, (ultra)soft-collinear

↪ large logarithms $\log^2 \frac{m_t}{m_e}$, resummed into structure functions

[Fadin, Kuraev 1985; Fadin, Khoze 1987]

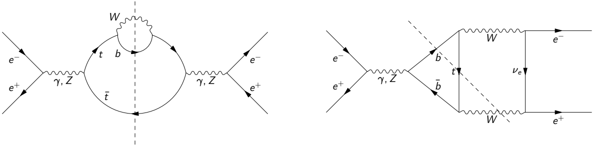
$$\sigma(s) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, \Gamma_{ee}^{LL}(x_1)\Gamma_{ee}^{LL}(x_2)\hat{\sigma}(x_1 x_2 s) + \sigma_{\text{const}}^{\text{ISR}}(s)$$

Non-resonant production

Unstable particle effective field theory [Beneke, Chapovsky, Signer, Zanderighi 2003-2004]

$$\sigma = \sigma_{\text{res}} + \sigma_{\text{non-res}}$$

$\sigma_{\text{non-res}}$: produce decay products (W, b, gluons) *without* intermediate $t\bar{t}$ resonance:



. . .

Starting at NLO (α/v), known at NNLO

[Beneke, Jantzen, Ruiz-Femenía 2010; Beneke, AM, Rauh, Ruiz-Femenía 2017]

Allows loose invariant mass cut: $\Delta M_t \gg \Gamma_t$.

QQbar_threshold

[Beneke, Kiyo, AM, Piclum 2016]

Public C++ library for $e^+e^- \to Q\bar{Q}$ near threshold:

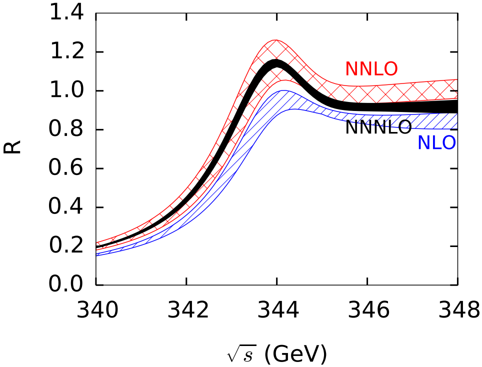
- N^3LO QCD + Higgs,
  N^2LO electroweak + nonresonant corrections
- Top and bottom quarks
- Includes Mathematica package
- Extensive options:
  - Mass schemes: PS, 1S, $\overline{\text{MS}}$, pole
  - Loose invariant Wb mass cut: $(m_t - m_{Wb})^2 \gg \Gamma_t m_t$
  - Coarse and fine-grained control over contributions
  - ...

```
Needs["QQbarThreshold‘"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
   TTbarXSection[
      sqrts, {80., 350.}, {171.5, 1.33},
      "N3LO"
   ],
   {sqrts, 340, 348}
]
```

```
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_xsection(
     344., {80., 350.}, {171.5, 1.33}, N3LO
  ) << ’\n’;
}
```

Top-pair production cross section

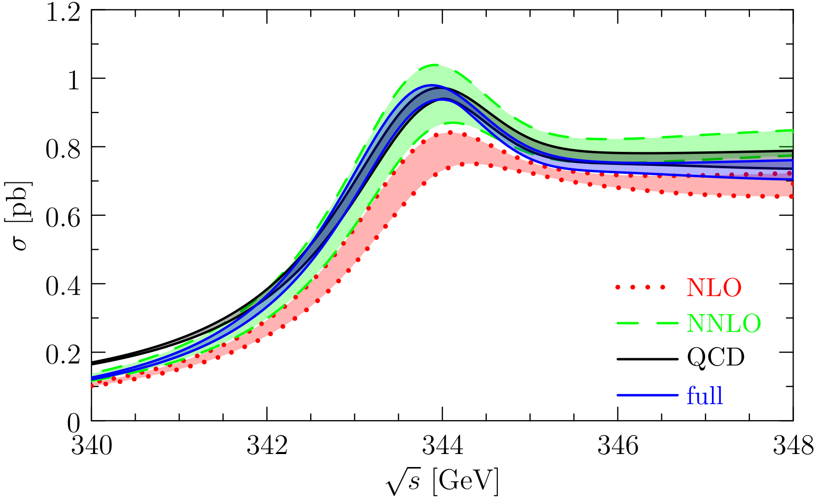
NNNLO QCD corrections [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]



- Apparent convergence at NNNLO, 3% scale uncertainty
- Similar convergence at NNLO + NNLL [Hoang, Stahlhofen 2013]

Top-pair production cross section

[Beneke, AM, Rauh, Ruiz-Femenía 2017]

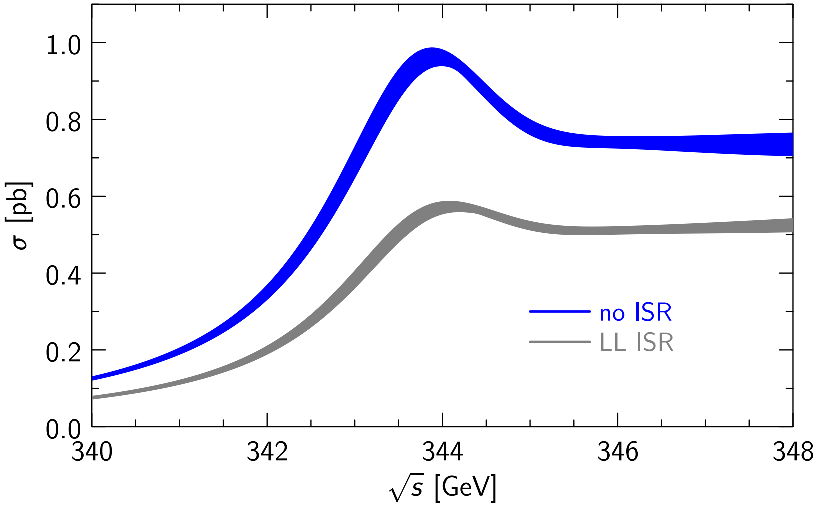


$m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$, $\Gamma_t = 1.33\,\mathrm{GeV}$, $m_H = 125\,\mathrm{GeV}$,

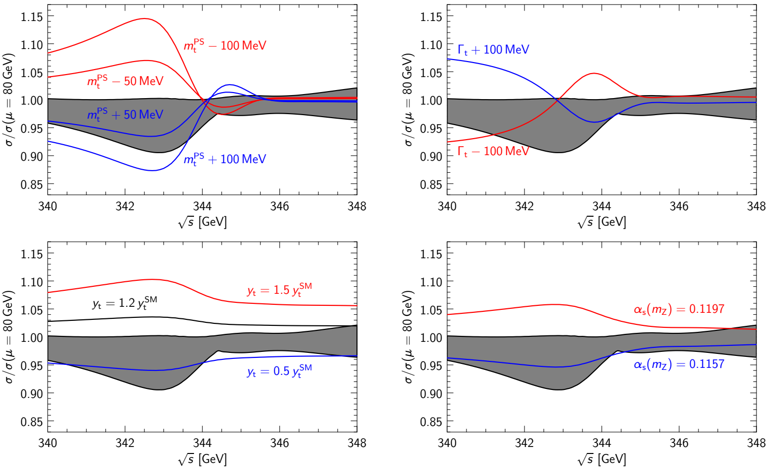
$\alpha_s(m_Z) = 0.1177$, $\alpha(m_Z) = 1/128.944$, m_W, m_Z

Top-pair production cross section

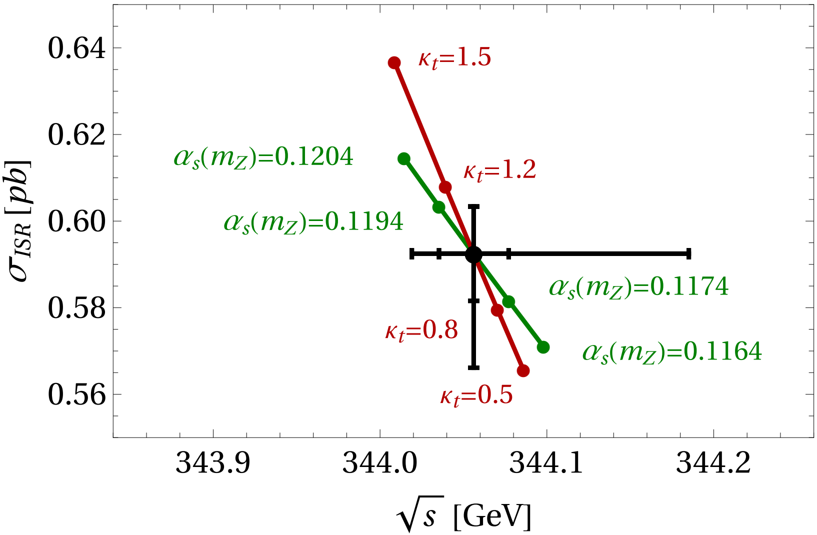
Initial-state radiation



Determination of top-quark properties



Peak position

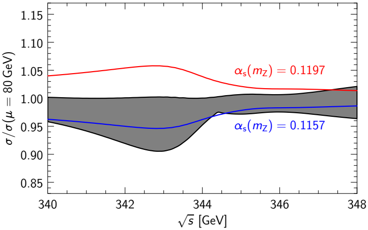


Conclusions

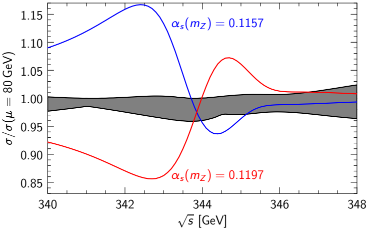
- Top pair threshold scan allows determination of
  - Mass with $\Delta m_t \approx 50\,\mathrm{MeV}$
  - Width with $\Delta\Gamma_t \approx 60\,\mathrm{MeV}$
  - Yukawa coupling with $\Delta\kappa_t \approx 0.25$
- Good knowledge of α_s required for Yukawa coupling
- Error dominated by theory: $\sim 3\%$ QCD scale uncertainty

Backup

PS vs. $\overline{\text{MS}}$ scheme

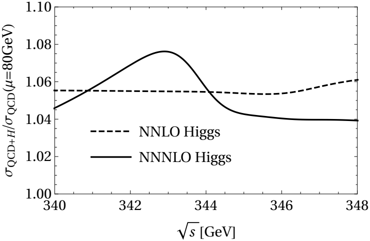


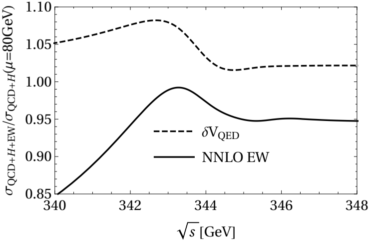
$$m_t^{\text{PS}}(20\,\text{GeV}) = 171.5\,\text{GeV}$$

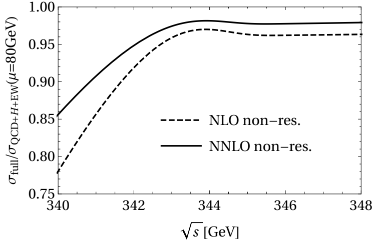


$$\bar{m}_t(\bar{m}_t) = 163.4\,\text{GeV}$$

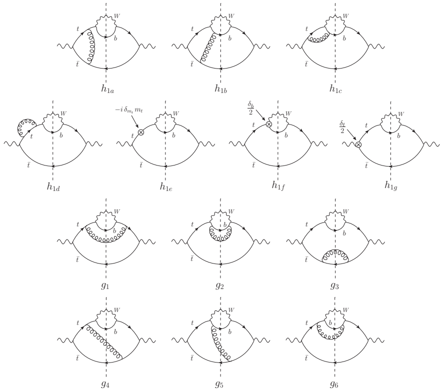
Size of single corrections





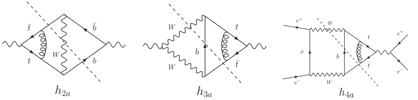


NNLO non-resonant production



“Squared” contribution

UV finite & endpoint divergent



“Interference” contribution

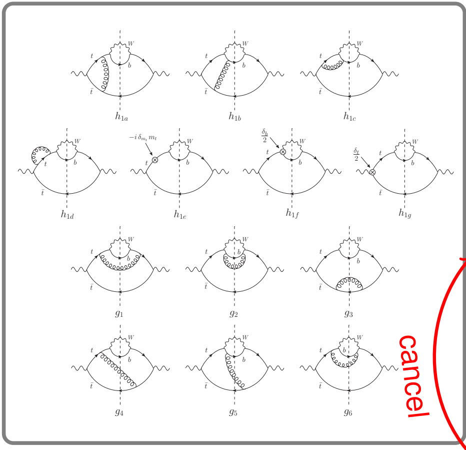
UV divergent & endpoint divergent

$\mathcal{O}(100)$ diagrams
calculated with
MadGraph5_aMC@NLO

“Automated” contribution

UV divergent & endpoint finite

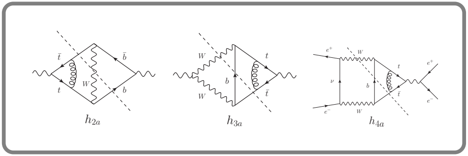
NNLO non-resonant production



“Squared” contribution

UV finite & endpoint divergent

cancel



“Interference” contribution

UV divergent & endpoint divergent

$\mathcal{O}(100)$ diagrams
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MadGraph5_aMC@NLO

“Automated” contribution

UV divergent & endpoint finite

NNLO non-resonant production

$$\sigma = \sigma_{\text{res}} + \sigma_{\text{sq}} + \sigma_{\text{int}} + \sigma_{\text{aut}}$$

d dim. phase space

4 dim. phase space

Split into separately finite pieces:

$$\sigma = \boxed{\sigma_{\text{res}} + \sigma_{\text{sq}} + \sigma_{\text{int}}^{\text{(EP div)}}} + \boxed{\sigma_{\text{int}}^{\text{(EP fin)}} + \sigma_{\text{aut}}}$$

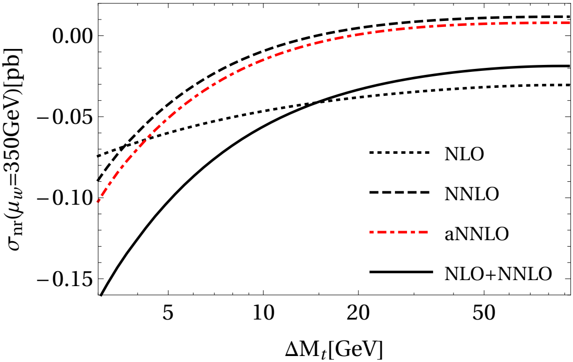
$$\int_y^1 dt\, g_{ia}(t) = \sum_n \frac{\hat{g}_{ia}^{(1,n)}(1-y)^{-n\epsilon}}{-n\epsilon} + \int_y^1 dt \left[g_{ia}(t) - \sum_n \frac{\hat{g}_{ia}^{(1,n)}}{(1-t)^{1+n\epsilon}} \right]$$

$t = \frac{p_t^2}{m_t^2}$, endpoint divergence for $t \to 1$

y: cut on invariant mass

NNLO non-resonant production

Effect of invariant mass cut



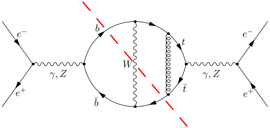
$$(m_t - \Delta M_t)^2 \leq p_t^2 \leq (m_t + \Delta M_t)^2$$

NLO: [Beneke, Jantzen, Ruiz-Femenía 2010]

aNNLO: $\Gamma_t \ll \Delta M_t \ll m_t$

[Jantzen,Ruiz-Femenía 2013; see also Hoang, Reißer, Ruiz-Femenía 2010]

Cancellation of endpoint divergences



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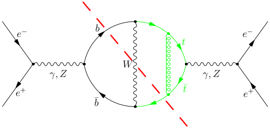

$\pitchfork$

$\pitchfork$

$\sigma_{\text{non-res}}$