2-loop (EW) calculations at the LHC and prospects for e^+e^- colliders

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Outline

Technical complexities of NNLO calculations

integrals, amplitudes, subtraction

Recent devolpment for NNLO QCD calculations and prospects for e^+e^- colliders

what have we learnt? How much can we profit? what could be achieved in the near future?
what needs substantial new work/resources/support?

Examples of processes of interests: $e^+e^- \to$ jets/ff, Higgs couplings and ZH, WW

Conclusions and remarks

I would like to thank F. Caola and A. Vicini for many useful discussions

NLO (automation) and beyond

One-loop corrections can be considered a solved problem

NLO revolution + automation

for more see Pellen's talk

1loop amplitudes can be computed in a fully automated way for in principle any input theory

Several publicly available tools implement 1 loop corrections in the full SM:
Gosam [Chiesa, Greiner, Heinrich, Jahn, Jones, Kerner, Luisoni, Mastrolia, Ossola, Peraro, Schlenk, Scyboz, Tramontano] Madgraph [Alwall, Frixione, Frederix, Hirshi, Maltoni, Mattelaer, Pagani, Shao, Steltzer, Torrielli, Zaro], Recola [Actis, Denner, Hofer, Lang, Scharf, Uccirati], OpenLoops [FB, Lang, Lindert, Pozzorini, Maierhoefer, Zhang, Zoller], NLOX [Honeywell, Quackenbush, Reina, Reuschle]

NLO subtraction:
Dipole [Catani, Seymour hep-ph/9605323][Catani, Dittmaier, Seymour, Trocsanyi hep-ph/0201036] FKS [Frixione, Kunst, Signer hep-ph/9512328]

Multipurpose NLO Monte Carlo generators: e.g. Sherpa [Bothmann, Gleisberg, Krause, Hoeche, Napoletano, Krauss, Kuttimalai, Liebschner,Schonherr, Schumann, Siegert, Winter], Madgraph_@MCNLO, Powheg [Alioli, Nason, Oleari, Re]

NNLO revolution + automation? Not quite yet

From 1 to higher loops the complexity scales very fast. Although a notion of automation not applicable, we are witnessing constant and fast progress

Amplitudes, Integrals, Subtraction: new mathematical properties/structure, order of magnitude increase in complexity

As for NNLO QCD parton level Monte Carlo generators:

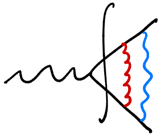
Matrix [Grazzini, Kallweit, Rathlev, Wiesemann] MCFM [Campbell, Ellis, Williams] Geneva [Alioli, Bauer, Berggren, Hornig, Tackmann, Vermilion, Walsh, Zuberi]
NNLOJET [Chen, Cruz-Martinez, Currie, Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Höfer, Huss, Majer, Mo, Morgan, Niehues, Pires, Walker, Whitehead]

Only Matrix and MCFM publicy available

Technical aspects (complexities) of higher-order calculations

Computing loop integrals

analytic → numerical

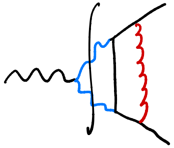


Harmonic Polylogarithms

$$H(a,\vec{b},x) = \int^x \mathrm{d}t \frac{H(\vec{b},t)}{a-t}$$

$$a,\vec{b} \in \{-1,0,1\}$$

very well known and understood

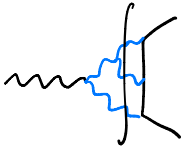


Goncharov Polylogarithms

$$G(a,\vec{c},x) = \int^x \mathrm{d}t \frac{G(\vec{c},t)}{t-a}$$

$$a,\vec{c} \in \mathbb{C}$$

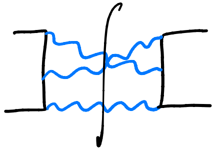
can be computed efficiently with several numerical codes



Elliptic functions (elliptic polylogarithms)

$$K(x) = \int_0^1 \mathrm{d}t \frac{1}{\sqrt{(1-t^2)(1-xt)}}$$

Fast and continuous progress. Active field of research with combined efforts from physics and math communities



Elliptic integrals, several energy scales involved (s,t,MZ,MW), non-planar topologies....

At the boundaries of (fair to say beyond?) our current knowledge

Need to be investigated for precision studies at e^-e^+ colliders

see talk by Mandal

Technical aspects (complexities) of higher-order calculations

Computing loop integrals is not the whole story. At the end of the day we need SCATTERING AMPLITUDES

Deriving "manageable" expressions for scattering amplitudes involving many energy scales (such as EW corrections) can be extremely challenging

i) Reduction to Master Integrals (MIs), aka IBP reduction

Considerable progress over the last few years. Publicly available computer programs more and more efficient, e.g.
LiteRed [Lee], Fire [Smirnov], Kira [Maierhöfer, Usovitsch, Uwer], Reduze [von Manteuffel, Studerus]

New avenues to cope with increasing complexity: construct algebraic identities using finite fields[von Manteuffel, Schabinger 1406.4513], IBP reduction via module intersections [Boehm,Georgoudis,Larsen,Schoenemann, Zhang 1805.01873] or construction of MIs coefficients using finite fields as in Kira2.0 [Klappert, Lange, Maierhöfer, Usovitsch 2008.06494]

ii) Good choice of a MI basis for the problem at hand

e.g. Henn canonical basis [Henn 1304.1806] extremely effective for multiloop QCD calculation. How about multiloop EW?

quasi-finite basis of MIs [von Manteuffel, Panzer, Schabinger 1411.7392]. Suited for numerical integration

all these aspects talk to each other

iii) Dealing with the rapidly growing algebraic complexity of the problem

Scattering amplitudes with many scales may involve hugely complicated rational functions.
Reconstruction of multivariate rational functions using finite fields [Peraro 1905.08019]

Technical aspects (complexities) of higher-order calculations

Computing loop integrals is not the whole story. At the end of the day we need SCATTERING AMPLITUDES

iv) Taking care of γ^5

multiloop calculation in d-dimensions involving vector-axial currents (typical of EW corrections) need a consistent treatment of γ^5

Larin's prescription [Larin hep-ph/9302240] or t'Hooft-Veltman

Altough consistent, these can make the practical calculation quite harder

v) UV renormalisation at 2-loop EW

Scattering amplitudes need to be renormalised for meaningful physical predictions

The issue of mass renormalisation in the presence of unstable particles has to be addressed:

@1loop EW: general and gauge invariant treatment in the so-called complex mass scheme (CMS) [Denner,Dittmaier hep-ph/0605312]

@2loop QCDxEW: recent results for mixed corrections to the off-shell Drell-Yan process, contribution from closed fermionic loops [Dittmaier, Schmidt, Schwarz 2009.02229] and extension of the CMS to this perturbative order

@2loop EW: a complete and systematic treatment at NNLO-EW not systematically explored (to the best of my knowledge)

Technical aspects (complexities) of higher-order calculations

Another crucial aspect of fixed order theory prediction: subtraction of IR singularities

Several frameworks developed for pp colliders at NNLO QCD:

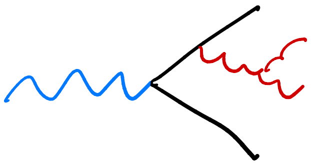
q_T [Catani, Grazzini], N-jettiness [Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams], Antenna [Gehrmann, Gehrmann-De Ridder, Glover], Sector-improved residues [Czakon, Mitov], CoLoRFulNNLO [Del Duca, Duhr, Kardos, Somogyi, Szőr, Trócsányi, Tulipánt], Projection-to-born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi], Nested soft-collinear [Caola, Melnikov, Roentsch]

Application of currently known methods to e^+e^-: state-of-the-art and beyond

if we restrict ourselves to QCD corrections, obviously easier than pp colliders.
Things change with higher-order EW corrections

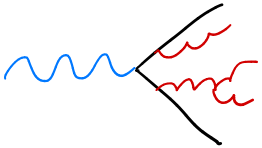
NNLO QCD

well established and
under control



N3LO QCD

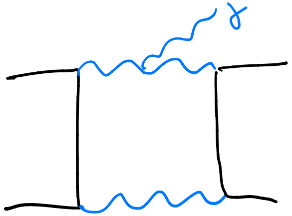
state-of-the-art



NNLO EW

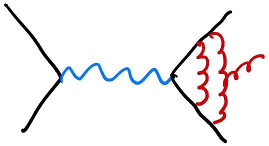
can borrow a lot from hadron collider studies

some further technical complications



$e^+e^- \to \gamma^*/Z \to$ jets, where do we stand

Given the presence of jets in the final state, QCD effects are very large and as such must be included.

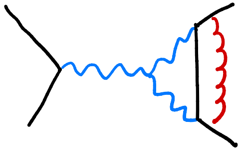


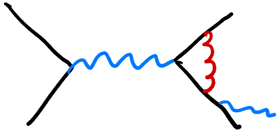
$e^+e^- \to 2$ jets at NNLO QCD and
$e^+e^- \to 3$ jets known at NNLO QCD [Garland, Gehrmann, Glover, Koukoutsakis, Remiddi hep-ph/0206067] [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 0710.0346]

Well studied and technically understood.
Clearly must be taken into account for high-precision analyses of e^+e^- to jets

More recently: two loop mixed QCDxEW corrections to the Neutral current Drell-Yan process at the resonance

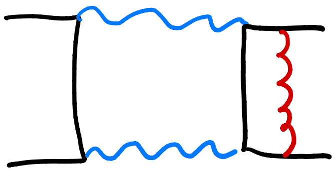
Inclusive [Bonciani, FB, Rana, Vicini 2007.06518] and
fully esclusive calculations [FB, Caola, Delto, Jaquier, Melnikov, Röntsch 2005.10221]





Relevant for studies at the Z resonance

Progress in the calculation of the complete QCDxEW corrections to the DY process. This will be a crucial ingredient also for e^+e^- colliders



[Bonciani, Di Vita, Mastrolia, Schubert 1604.08581]

[Mehedi Hasan, Schubert 2004.14908]

[Heller, von Manteuffel, Schabinger 1907.00491]

results expressed in terms of Goncharov polylogarithms,
thus fast and reliable evaluation

however, 2-loop
amplitudes not available yet

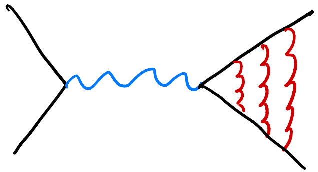
$e^+e^- \to \gamma^*/Z \to$ jets, where we can go next

What we can achieve in the "near" future

- $e^+e^- \to 2$ jets with complete QCDxEW corrections (including off-shell effects)

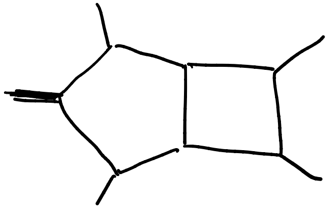
- $e^+e^- \to 2$ jets at N3LO QCD

  3-loop QCD form factors known [Baikov, Chetyrkin, Smirnov, Steinhauser 0902.3519] [Gehrmann, Glover, Huber, Ikizlerli, Studerus 1004.3653]
  Subtraction of IR singularities studied in differential $H \to bb$ @ 3loops [Mondini, Schiavi, Williams 1904.08960]



- $e^+e^- \to 4$ jets at NNLO QCD

  very challenging (maybe further in the future)
  However, we are witnessing constant progress in the evaluation of 2-loop 5-point amplitudes and first results on 2-loop 5-point integrals with one off-shell leg became recently available
  [Abreu, Ita, Moriello, Tschernow 2005.04195] [Canko, Papadopolous, Syrrakos 2009.13917]



- $e^+e^- \to Z \to bb$ (including b-mass effects)

  interest in forward-backward asymmetry tension in $Z \to bb$ from LEP: NNLO QCD reduces from 2.9σ to 2.6σ [Bernreuther, Chen, Dekkers, Gehrmann, Heisler 1611.07942]

  3-loop QCD form factors with massive quarks:
  available in larce N_c limit for vector, axial-vector, scalar and pseudo-scalar currents [Lee, A. Smirnov, V. Smirnov, Steinhauser 1804.07310]

  2-loop QCDxEW form factors with massive quarks:
  not yet available. Quite cumbersome and complicated (several energy scales: s, M_Z/M_W, m_{top}, m_b)

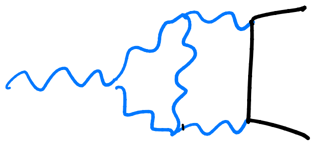
For very recent studies on the issue see 2003.13941 and 2010.08604

$e^+e^- \to \gamma^*/Z \to ff$, higher-order EW

EW corrections are undoubtely crucial for precision measurements at lepton colliders

Complete 2-loop EW corrections to Z-boson production and decay [Dubovyk, Freitas, Gluza, Riemann, Usovitsch,1804.10236]
complemented with detailed study of EW pseudo-observables [Dubovyk, Freitas, Gluza, Riemann, Usovitsch,1906.0881]

crucial ingredient for future studies



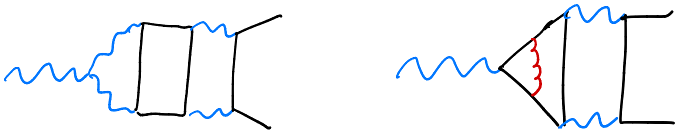
From the technical point of view:

computation of 2-loop integrals using numerical techniques
(mostly sector-decomposition and Mellin-Barnes)

full line-shape prediction at NNLO EW

What we will need in the future for such a high-precision collider

3-loop EW and QCDxEW corrections to Z production and decay

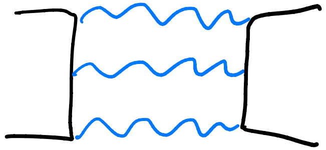


numerical approaches are the only way with current technologies

there is room for improvement on numerical methods

very demanding from the point of view of computing resources

Complete 2-loop EW corrections to $e+e- \to l^+l^-$



Highly-complicated multi-loop multi-scale process

It might be at the edge for current numerical approaches

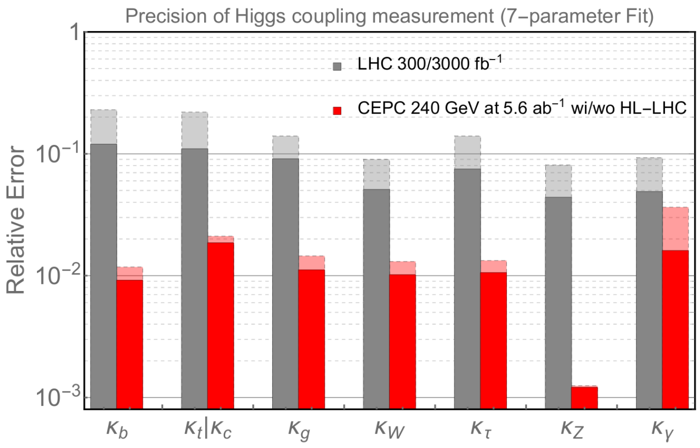
Need for improvements and most importantly, resources

Higgs at e+e- colliders: couplings and ZH

Lepton colliders as Higgs factories

At CEPC measurements of Higgs couplings will reach a 1% level accuracy

Astounding precision in decays to fermions of third generation a to gg.



Property	Estimated Precision
m_H	5.9 MeV
Γ_H	3.1%
$\sigma(ZH)$	0.5%
$\sigma(\nu\bar{\nu}H)$	3.2%

Decay mode	$\sigma(ZH) \times$ BR	BR
$H \to b\bar{b}$	0.27%	0.56%
$H \to c\bar{c}$	3.3%	3.3%
$H \to gg$	1.3%	1.4%
$H \to WW^*$	1.0%	1.1%
$H \to ZZ^*$	5.1%	5.1%
$H \to \gamma\gamma$	6.8%	6.9%
$H \to Z\gamma$	15%	15%
$H \to \tau^+\tau^-$	0.8%	1.0%
$H \to \mu^+\mu^-$	17%	17%
$H \to \mathrm{inv}$	–	$< 0.30\%$

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State-of-the-art:

H→bb at N3LO QCD at the fully differential level [Mondini, Schiavi, Williams 1904.08960]

$$\Gamma^{\rm N3LO}_{H\to b\bar{b}} = y_b^2(A_b + \alpha_s B_b + \alpha_s^2 C_b + \alpha_s^3 D_b) + y_b y_t(\alpha_s^2 C_{bt} + \alpha_s^3 D_{bt}) + \alpha_s^3\, y_t^2 D_t$$

more recently: Top-induced contributions to H→bb/cc with massive b,c [Mondini, Schubert, Williams 2006.03563]

Two loop QCD corrections in the full theory, i.e. including $C_{bt}y_by_t$ [Primo, Sasso, Somogyi, Tramontano 1812.07811]

Higgs at e+e- colliders: couplings and ZH

Lepton colliders as Higgs factories

For the foreseen accuracy, computation of higher orders for H decays will be needed

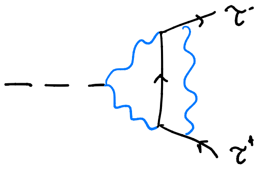
N4LO QCD: massless 4/5 loop integrals for H → bb/gg

inclusive O(α_s^4) corrections to H → bb [Baikov, Chetyrkin, Kuhn hep-ph/0511063]

considering the inclusion of top mass effects at N3LO QCD

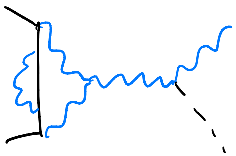
NNLO EW: massive 2-loop vertex correction

not available, but within reach

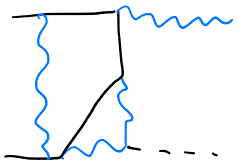


This program will depend also on advances on NNLO EW corrections to $e^+e^- \to ZH$

2 loop corrections to $e^+e^- \to ZH$



Form-factor type corrections



Could be approached with known techniques

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$H \to \mu^+\mu^-$	17%	17%
$H \to \text{inv}$	–	$< 0.30\%$

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2 loop corrections to $e^+e^- \to Z(l^+l^-)H$

Extremely challenging.
Beyond current technologies and methods.
It will require a huge conceptual and computing effort

$e^+e^- \to WW$

WW production at threshold: pivotal element of future studies at CEPC.

For a more detailed discussion see talk by Schwinn

Best theory prediction beyond NLO EW:

Dominant NNLO EW corrections for WW at threshold via unstable-particle effective theory [Actis, Beneke, Falgari, Schwinn 0807.0102]

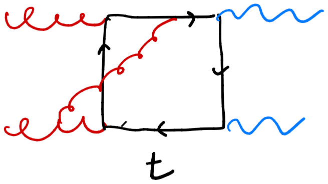
Looking ahead:

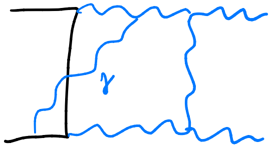
Extension to full NNLO EW, i.e. $e^+e^- \to 4f$ is extremely challenging: 6-point 2-loop amplitudes with internal masses.

With current technologies and computing resources most likely not within the timescale of a future collider

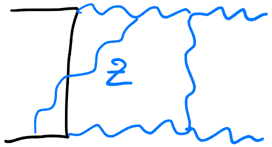
However we can move step by step and profit from progress and technologies develpoed for NNLO QCD calculations

Example: $gg \to VV$ at NLO QCD with heavy-quark mass dependence [Agarwal, von Manteuffel 1912.08794] [Davies, Mishima, Steinhauser, Wellmann 2002.05558]





Potentially common features and structures



new

Huge advance in the evaluation of two loop 4-point functions with numerical techniques. However computing is hugely demanding, resort to GPUs

Conclusions and remarks

At future e+e- colliders such as CEPC an unprecedented level of precision will be reached

Such high precision measurements pose a serious challenge to theoretical community:

In order to fully exploit the potential of the machine and the collected high-quality data,
the frontier of higher-order calculations will inevitably be pushede

We are constantly learning from pp colliders and the set of skills developed will be a crucial starting point for future colliders.

However: for the expected accuracy goal, 2loop and higher EW corrections are mandatory and the current technology might not be enough.
We need more!

Practical examples (in order of complexity, according to my own taste):

N3LO EW $1 \to 2$ (form factors), NNLO EW to $e^+e^- \to ff$, NNLO EW to $e+e- \to ZH$ and NNLO EW to $e+e- \to WW \to 4f$

Timescale of these projects potentially within reach of data collection from the new collider.

However: some are extremely challenging processes, which need

- new ideas, i.e. bright minds
- large collaborations will be required to carry on such long term projects
- cutting edge computing resources (modern CPU, GPU clusters)

Support