Theory Status for Predictions for the $t\bar{t}$-Cross Section

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Doktoratskolleg Particles and Interactions

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FWF Der Wissenschaftsfonds.

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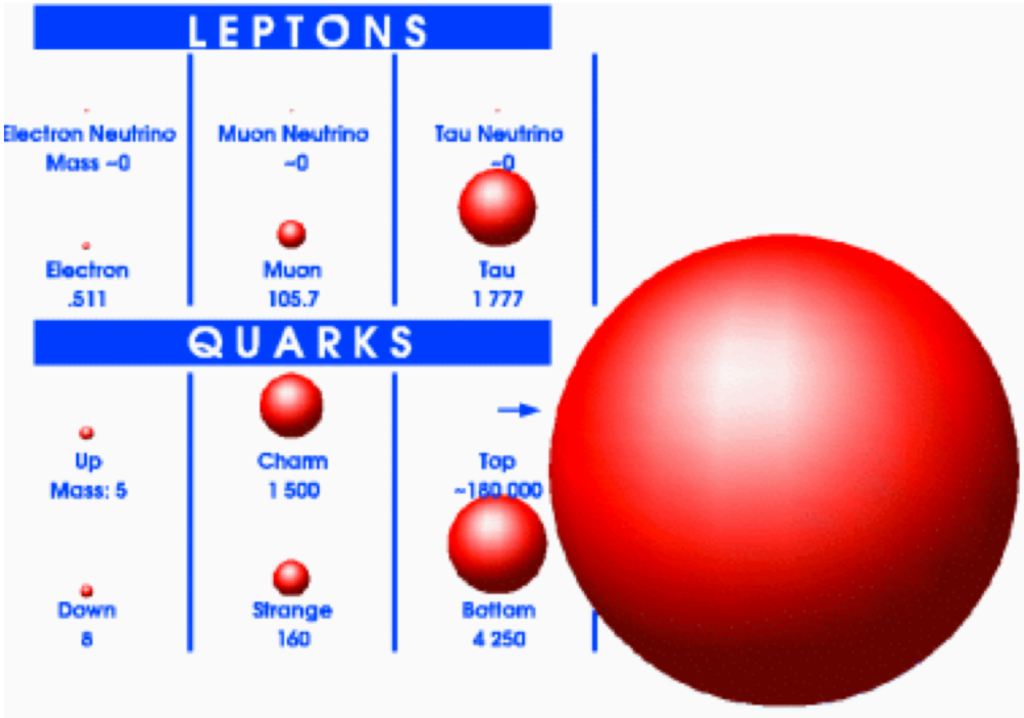
Outline

- Fixed-order calculations
- Strong coupling and mass schemes
- MC generators
- Top threshold physics
- Top mass from direct reconstruction measurements
- List of important issues I did not talk about in detail

.. not just the heaviest SM particle

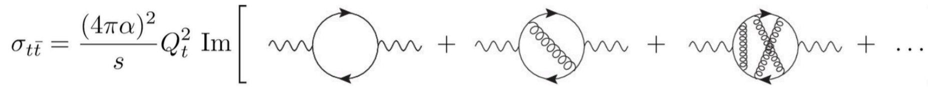


- Top quark: heaviest known particle
- Most sensitive to the mechanism of mass generation
- Peculiar role in the generation of flavor.
- Top might not be the SM-Top, but have a non-SM component.
- Top as calibration tool for new physics particles (SUSY and other exotics)
- Top production major background it new physics searches
- One of crucial motivations for New Physics

- Very special physics laboratory: $\Gamma_t \gg \Lambda_{QCD}$
  - Top treated a particle: p_T, spin, $\boldsymbol{\sigma}_{tot}$, $\boldsymbol{\sigma}$(single top), $\boldsymbol{\sigma}$(tt+X),.. $\rightarrow q \gg \Gamma_t$
  - Quantum state sensitive low-E QCD and unstable particle effects: m_t, endpoint regions $\rightarrow q \sim \Gamma_t$
  - Multiscale problem: p_T, m_t, Γ_t, Λ_{QCD}, . . . (depends on resolution of observable)

Status on FO Calculations

Stable Top :



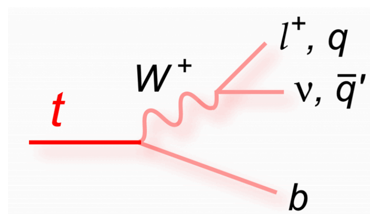
- Total inclusive cross section known to $O(\alpha_S^2)$ (FO)
- Total inclusive cross section known to $O(\alpha_S^3)$ (FO-Pade)
- NLO EW corrections (FO)

Kühn, Chetyrkin, Steinhauser, AHH,.. '96
Maier, Marquard.. '17
AHH, Mateu Zebarjad '08
Kiyo, Maier, Maierhofer, Marquard '09
Fleischer, Leike, Riemann, Wertenbach '03

- Full differential NNLO ttbar (subtractions)

Gao, Zhu'16
Chen, Dekkers, Heisler, Bernreuther '16

Top Decay (NWA):



- Total decay rate $O(\alpha_S^2)$ (FO)
- Fully differential $O(\alpha_S^2)$ (FO subtractions)

Charnecki etal '10
Gao, Li '12
Bruchseifer, Caola, Melnikov '13

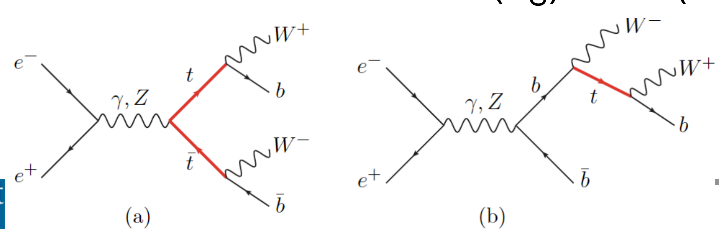
- NLO EW corrections

' 90s

Off-shell Production:

- Full off-shell $e^+e^- \to WWbb$ $O(\alpha_S)$/NLO (FO)

Guo, Ma, Zhang, Wang '08
MadGraph5@NLO, WHIZARD, ...
Standard now

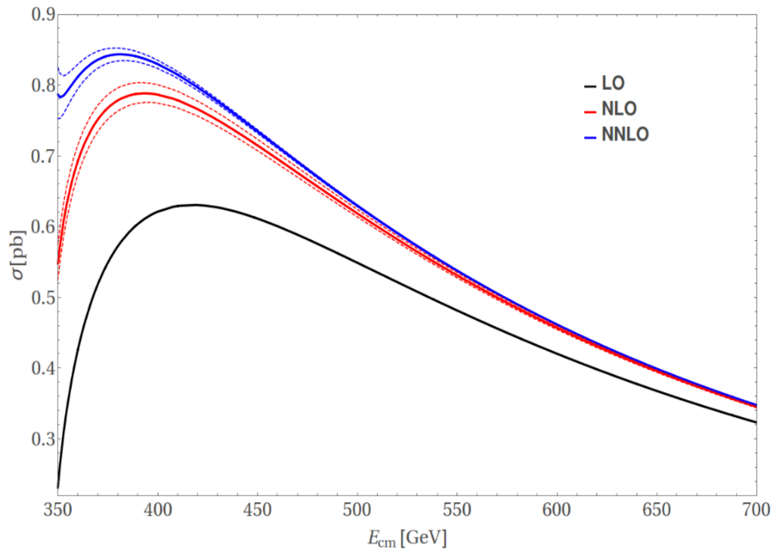


Status on FO Calculations

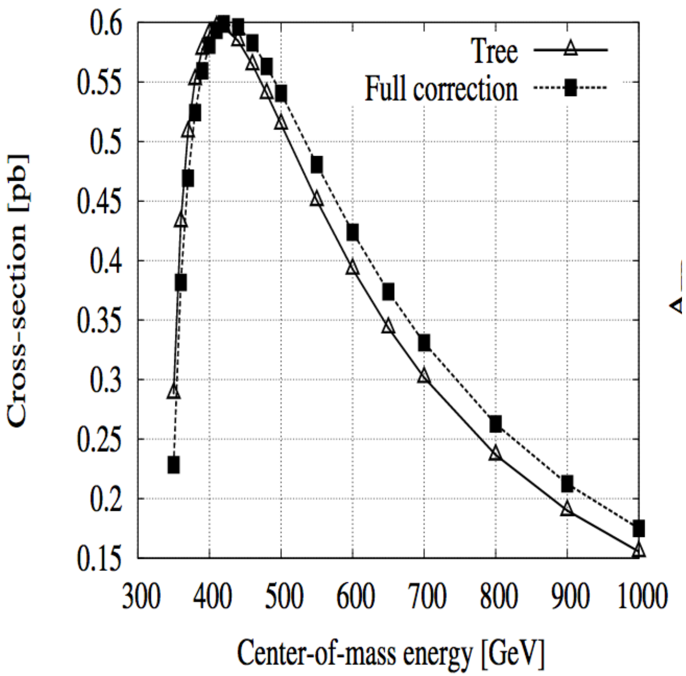
Is this good enough? **In general not!**

Example: total ttbar cross section

Chen, Dekkers, Heisler, Bernreuther '16



Fleischer, Leike, Riemann, Werthenbach '03



- Huge correction at threshold $E_{cm} \approx 2m_t$
- Coulomb corrections ~ $(\alpha_S/v)^n$
- Resummation mandatory (very well developed)

- EW Sudakov logs only for very large energies
- Fixed-order fine for FCC-ee

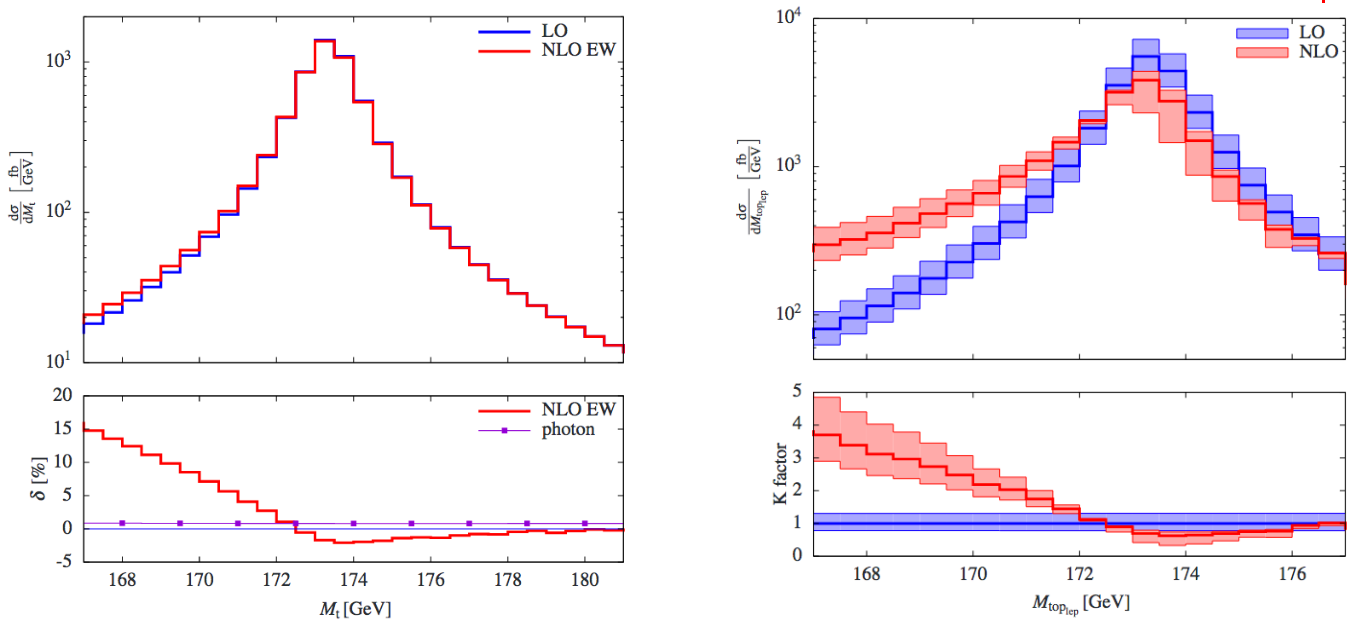
Status on FO Calculations

Is this good enough?   In general not!

Example: ‘reconstructed’ top invariant mass (full off-shell)

Pellen, Denner ’17

From an LHC paper (sorry for that..)



- Full off-shell calculation: Scale variation does not cover uncertainty.
- Fixed-order not sufficient, e.g. large QCD logs $\ln(m_t/\Gamma_t)$
- Resummation mandatory (not worked out yet, but possible using knowledge from flavor physics)

Strong Coupling

$$\frac{\mathrm{d}\alpha_s(R)}{\mathrm{d}\log R} = \beta(\alpha_s(R)) = -2\,\alpha_s(R)\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_s(R)}{4\pi}\right)^{n+1}$$

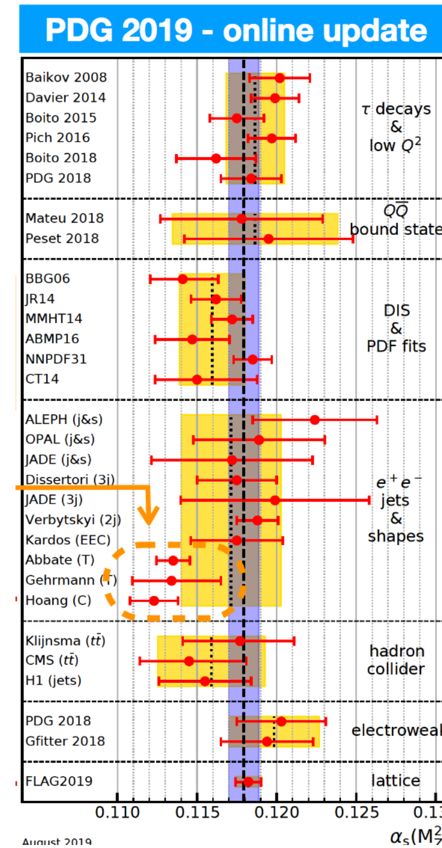
Baikov, Chetyrkin, Kühn '17

- Running known to 5 loops (β_4) : fully sufficient
- Uncertainty in $\alpha_S(M_Z)$: debated, under constant scrutiny, but always a limiting factor

example: MSbar-pole mass relation

$\delta\alpha_S$ = 0.001 gives 70 MeV uncertainty

$$m_t^{\mathrm{pole}} - \overline{m}_t(\mu) = \frac{4}{3}\left(\frac{\alpha_s(\mu)}{\pi}\right)\overline{m}_t(\mu) + \ldots$$



See: Pier, Andrii, Gabor, Zoltan

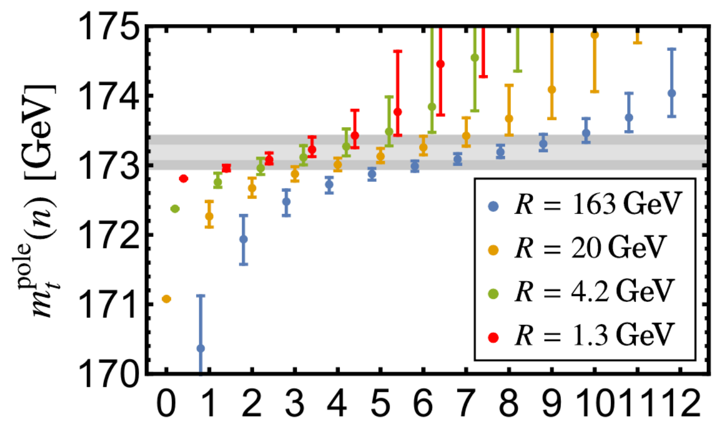
Improvement expected, but lots of hard work.

Consistency has actually higher priority at this time!!

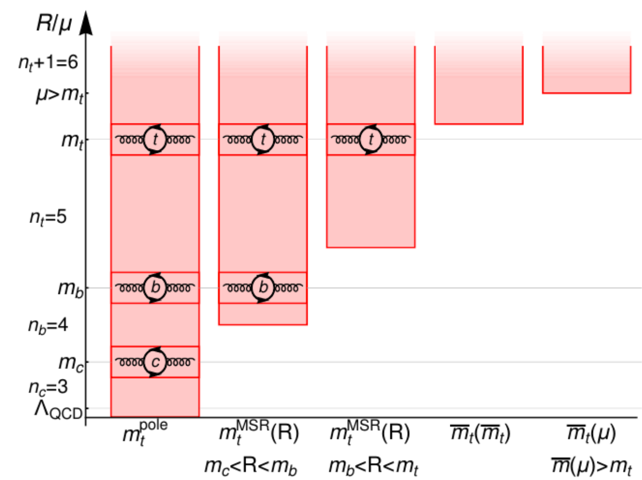
Recall: Measurements of QCD parameters more subtle than of physical observables.

Top Quark Mass Schemes

- High precision demands to take into account the properties of mass schemes and that one picks an adequate scheme
- Very well understood: $O(\alpha_S^4)$ results!   Marquard, Smirnov, Smirnov, Steinhauser'15



- Pole mass m_t^{pole} not adequate for almost all applications due to a renormalon ambiguity:
  - Δm_t^{pole} = 110 MeV   Beneke, Nason, etal '16
  - Δm_t^{pole} = 250 MeV   AHH, Lepenik, Preisser '17
- Pole ambiguity arises because IR effects absorbed into the mass
- Divergence pattern dependent on scale R that governs the dynamics of the mass dependence



- Ambiguity-free masses only absorb effects above their renormalization scale μ ("short-distance masses"): $m_t(\mu)$

Top Quark Mass Schemes

- Most popular short-distance mass schemes:

MSbar:

$$m_t^{\text{pole}} - \overline{m}_t(\mu) = \frac{4}{3}\left(\frac{\alpha_s(\mu)}{\pi}\right)\overline{m}_t(\mu) + \ldots$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\,\overline{m}_t(\mu) = -\overline{m}_t(\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right) + \ldots$$

Only meaningful for μ > m_t

Threshold masses:

kinetic	Bigi, Shifmann, Uraltsev '97	
1S	AHH, Ligeti, Manohar '98	
PS	Beneke '98	
RS	Pineda '01	

Constructed from ttbar threshold and B physics observables, renormalon study

MSR:

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = \frac{4}{3}\left(\frac{\alpha_s(R)}{\pi}\right) R + \ldots$$

AHH, Jain, Scimemi, Stewart '08

$$\frac{\mathrm{d}}{\mathrm{d}\ln R}\, m_t^{\text{MSR}}(R) = -\frac{4}{3} R\left(\frac{\alpha_s(R)}{\pi}\right) + \ldots$$

Derived from MSbar for R < m_t

Interpolates between pole and MSbar mass

MC Generators

- Fast machinery from LHC, just change initial state
- Less modeling for color neutralization processes needed
- NLO-matched MC generators standard.

Validation of NLO QCD for e^+e^- Collisions

Process	MG5_aMC σ^{LO}[fb]	MG5_aMC σ^{NLO}[fb]	MG5_aMC K	WHIZARD σ^{LO}[fb]	WHIZARD σ^{NLO}[fb]	WHIZARD K
$e^+e^- \to jj$	622.3(5)	639.3(1)	1.02733	622.73(4)	639.41(9)	1.02678
$e^+e^- \to jjj$	340.1(2)	317.3(8)	0.93297	342.4(5)	318.6(7)	0.9305
$e^+e^- \to jjjj$	104.7(1)	103.7(3)	0.99045	105.1(4)	103.0(6)	0.98003
$e^+e^- \to jjjjj$	22.11(6)	24.65(4)	1.11488	22.80(2)	24.35(15)	1.06798
$e^+e^- \to jjjjjj$	N/A	N/A	N/A	3.62(2)	0.0(0)	0.0
$e^+e^- \to b\bar{b}$	92.37(6)	94.89(1)	1.02728	92.32(1)	94.78(7)	1.02664
$e^+e^- \to b\bar{b}b\bar{b}$	$1.644(3) \cdot 10^{-1}$	$3.60(1) \cdot 10^{-1}$	2.1897	$1.64(2) \cdot 10^{-1}$	$3.67(4) \cdot 10^{-1}$	2.2378
$e^+e^- \to t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \to t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \to t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \to t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \to t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \to t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \to t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \to t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \to t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \to t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \to t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \to t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \to t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \to t\bar{t}W^{\pm}jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \to t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \to t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \to t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \to t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \to t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \to t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

Just pick what you need!

Not so fast..

MC Generators

- Multipurpose MC generators (Pythia, Herwig, Whizard, Sherpa) can simulate all aspects of particle production and decay at the observable level

How precise are they?

- The theoretical precision is tied to the precision of the parton showers, for a few very simple observable NLL, mostly LL or less.
- Tuned hadronization models compensate for the deficiency.
- In general we have observable precision > theoretical precision
- MCs are not very precise tools to extract QCD parameters or provide estimate of hadronization corrections to high-order perturbative analytical calculations
- NLO-matching does only improve the first hard gluon radiation. Does not improve observables governed by parton shower dynamics.

MC Generators

- NLL precise parton showers with full coherence and improved models are an important step that needs to be taken (many different aspects, work already ongoing).

e.g. second order kernel

double emssion

amplitude evolution (full coherence,

non-global logs, color reconnection)

Li, Skands '16

Höche Prestel' 14, '15

Forshaw, Holguin, Plätzer '19

Gieseke, Kirchgaesser, Plätzer,' Siodmok '19

Martinez, Forshaw, De Angelis, Plätzer, Seymour '18

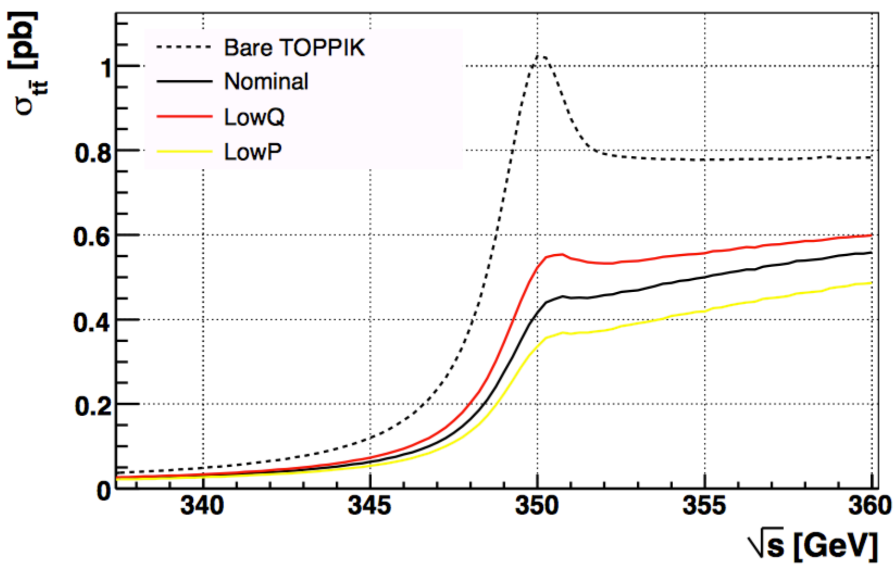
New generation of MCs needed! (Markow chain MCs will be gone eventually)

→ Definitely possible, community should support it more enthusiastically.

Top Threshold

Top pair total inclusive cross section:

$$\sigma(e+e- \rightarrow t\bar{t} + X) \text{ at } E_{cm} \approx 2m_t$$



Principle: m_t from $\sigma_{tt}(m_t)$

Advantages:

- count number of $t\bar{t}$ events
- color singlet state ← Crucial difference to top pairs at LHC
- background is non-resonant
- physics well understood (renormalons, summations)

➢ Top decay protects from non-pert effects

- Remnant of a topionium resonance (“postronium of QCD”): $R_{bind} = m_t\, \alpha_S \sim 30$ GeV
- Crucial to control e+e- luminosity spectrum
- Binding energy about twice the top quark width: $$E_{\text{bind}} \approx \frac{\alpha_s^2 m_t}{2} \approx 2\Gamma_t$$
- Can be calculated in pQCD (nonrelativistic expansion)
- Non-resonant effects very small, little background

Top Threshold

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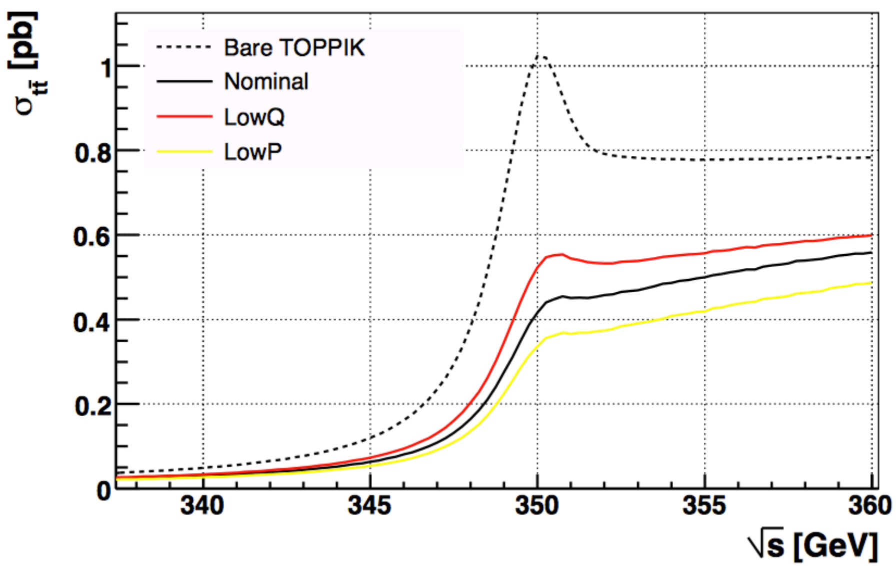

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Advantages:

- count number of $t\bar{t}$ events
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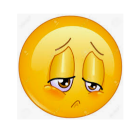
Crucial difference to top pairs at LHC

- Top decay protects from non-pert effects

- The only observable known where a threshold structure with resolution $\ll$ 1 GeV is generated by QCD dynamics at much larger scale: $R_{bind} = m_t\, \alpha_S \sim 30$ GeV
- Color single state protects from non-perturbative effects.

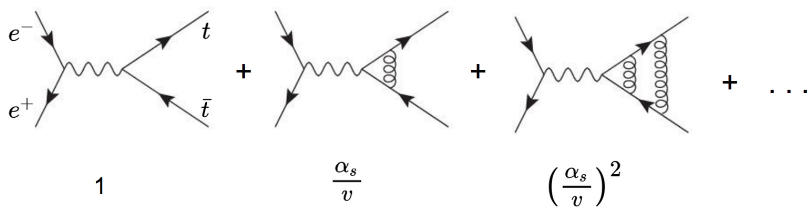
We could not be more lucky!

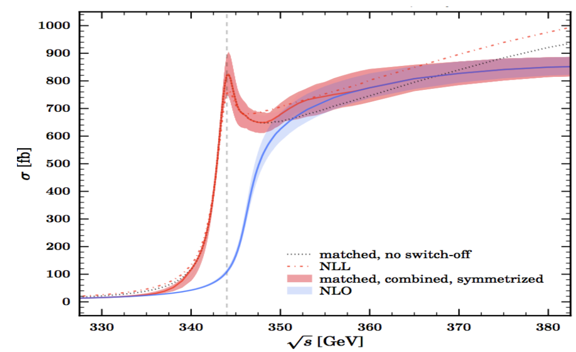
Unfortunately no such observable at the LHC !



Top Threshold

- Coulomb resummations
- Finite Width effects are leading order
- NRQCD effective field theory counting ($\alpha_S \sim v$)





- Total cross section at NNLO (FO in $\alpha_S \sim v$) — AHH, Beneke, Melnikov, Nagano, Ota, Penin, Pivovarov, Signer, Smirnov, Sumion, Teubner, Yakovlev, Yekhovsky '01
- Total cross section NNLO+NNLL (sum $\ln(\alpha_S) \sim \ln(v)$) — AHH, Stahlhofen, '13
- Total cross section NNNLO — Beneke, Kiyo, Marquard, Piclum, Steinhauser '13

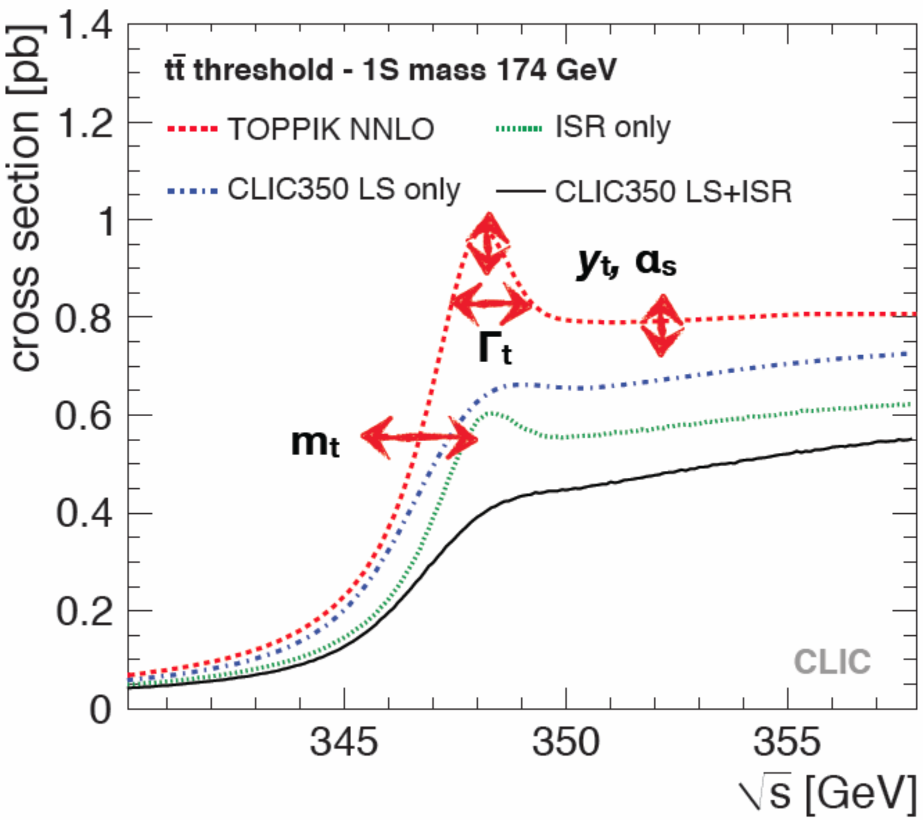
- Non-resonant EW effects NNLL — AHH, Reisser, Ruiz-Femenia '04, '10
- Non-resonant EW effects $\mathrm{NNNLO}_{\mathrm{partial}}$ — Beneke, Maier, Rauh, Ruiz-Femenia '17,

- Top p_t 3-momentum distribution NNLO — AHH, Teubner '00
- Full differential: NLO+LL — Chokoufe, AHH, Kilian, Reuter, Stahlhofen, Teubner, Weiss'17

Total cross section in very good shape.

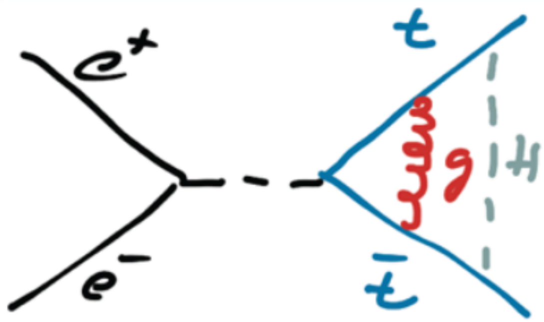
Top Threshold

Experimental Studies:



The cross-section around the threshold is affected by several properties of the top quark and by QCD

- Top mass, width, Yukawa coupling
- Strong coupling constant



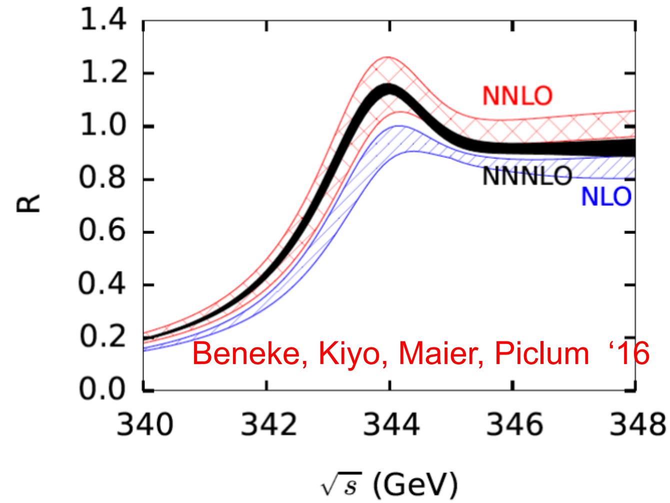
- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external α_s helps

Frank Simon

Top Threshold

Experimental Studies (total inclusive cross section):

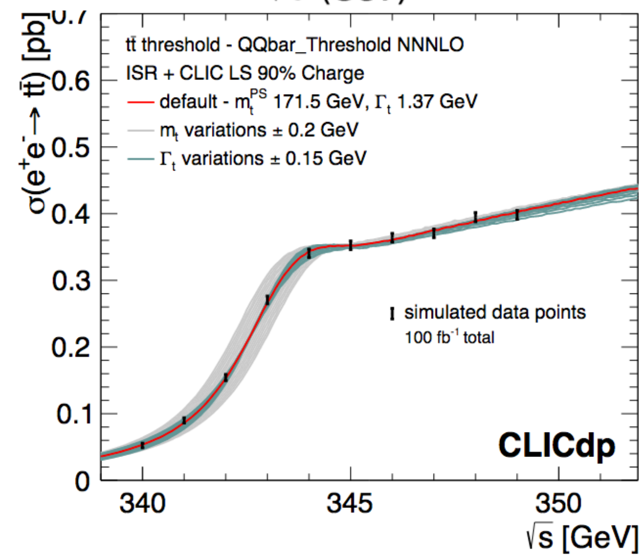
- Total cross section code NNNLO: QQbar_Threshold
- Dependence on the luminosity spectrum
- Use low-scale short-distance mass



- CLIC simulations study:

Abramowicz etal. '18

$\Delta m_t^{stat} \sim 40$ MeV, $\Delta m_t^{para} \sim 25 - 50$ MeV
$\Delta\Gamma_t^{stat} \sim 50$ MeV, $\Delta y_t^{para} \sim 50$ MeV
$\Delta y_t^{stat} \sim 10\%$

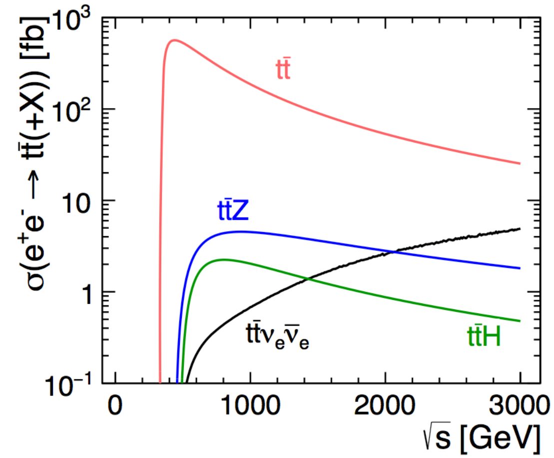


Very similar for all lepton colliders

Caveat: MC generators for the ttbar threshold do not yet exist!

Associated Top Threshold Physics (I)

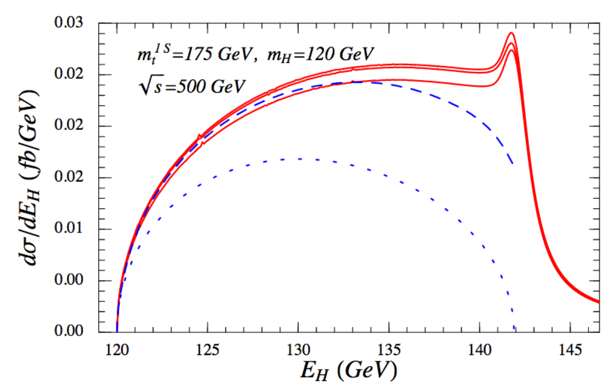
- A future e^+e^- collider with many associated ttbar thresholds
- Technology exists to extend ttbar threshold machinery to them, but much less event

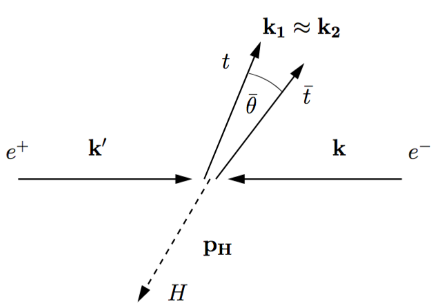


tt + H:

- NLO QCD — Dawson, Reina '17,
- NLO EW corrections — Dener, etal,, Belanger, etal. You, etal '03,
- NLL threshold — Farrell, AHH '05

- Kinematic threshold enhancement reaching far into the continuum region for associated tt production, enhances cross section





$\sqrt{s}$ [GeV]	m_H [GeV]	σ(Born) [fb]	$\sigma(\alpha_s)$ [fb]	σ(NLL) [fb]	$\frac{\sigma(\text{NLL})}{\sigma(\text{Born})}$	$\frac{\sigma(\text{NLL})}{\sigma(\alpha_s)}$	$\frac{\sigma(\text{NLL})_{\lvert\beta\rvert<0.2}}{\sigma(\alpha_s)_{\beta<0.2}}$
500	120	0.151	0.263	0.357(20)	2.362	1.359	1.78

Farrell, AHH '05

Associated Top Threshold Physics (II)

tt + γ:

Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

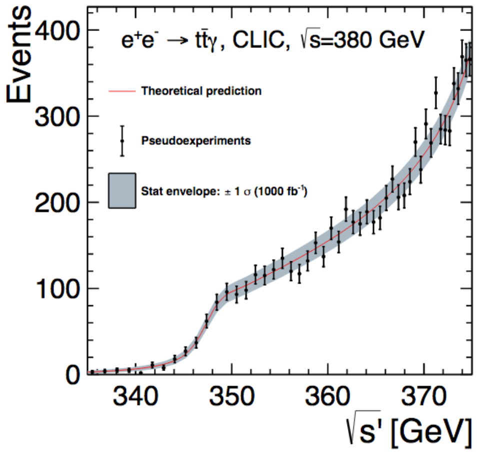
- Radiative return to the tt threshold allows for top threshold top mass measurements at higher energies.

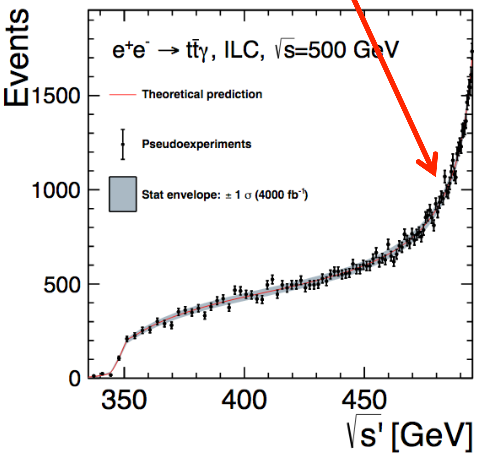
$$\frac{d\sigma_{t\bar{t}\gamma}}{d\cos\theta\, d\sqrt{s'}} = 2\,g(x,\theta)\sqrt{\frac{1-2x}{s}}\,\frac{\alpha_{\rm em}}{\pi}\,\sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}}\,,$$

$$s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

ISR enhancement





- Matched threshold (NNLL+NNLO)-continuum (NNNLO) cross section
- Realistic simulation experimental analysis
- Statistics dominated

cms energy	CLIC, $\sqrt{s}=380$ GeV		ILC, $\sqrt{s}=500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

Associated Top Threshold Physics

tt + γ:

Boronat, Fullana, Juster, Gomis, Vos, AHH, Widl, Mateu '19

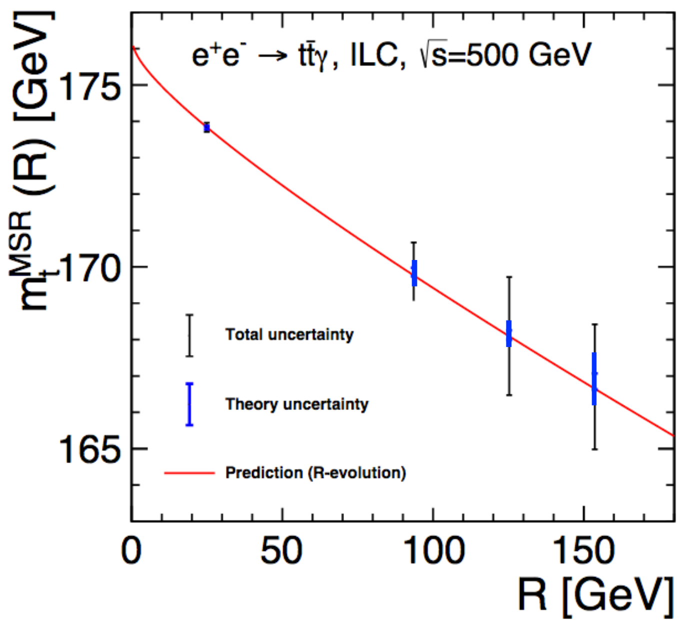
- Running MSR mass measurements

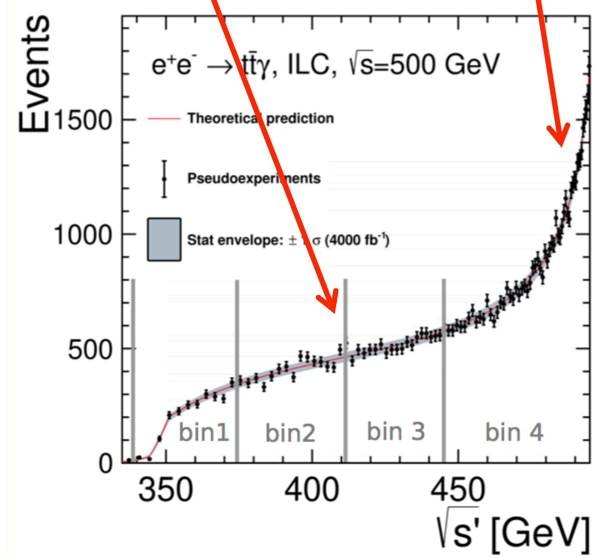
$$\frac{d\sigma_{t\bar{t}\gamma}}{d\cos\theta\, d\sqrt{s'}} = 2\, g(x,\theta)\sqrt{\frac{1-2x}{s}}\,\frac{\alpha_{\rm em}}{\pi}\,\sigma_{t\bar{t}}(s')$$

$$x = \frac{E_\gamma}{\sqrt{s}}\,, \qquad s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

Probes top mass sensitivity at scales $m_f v < m_t$

ISR enhancement



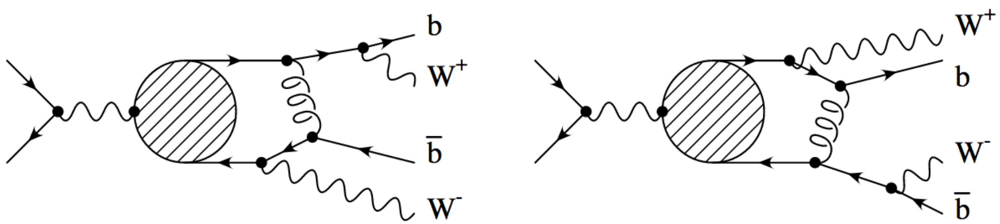


Top Threshold

Differential Cross Sections:

- Has not received much attention in the past, but important to correctly simulate experimental cuts
- Very (!) hard problem due to ultrasoft ($E \lesssim \Gamma_t$) gluon exchange between the top quarks and their decay products. They cancel in the fully inclusive cross section

Melnikov, Yakovlev '93



- Large (non-factorizable) effects possible due to selection cuts (size unknown!!)

  Effects increase the more restrictive cuts are.

  Small for generous (wide) cuts

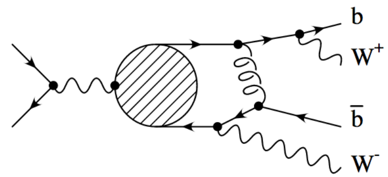
  Contribute at NLL/NLO order for differential cross section.

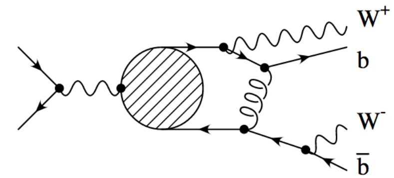
AHH, Reisser, Ruiz-Femenia '10

- Theoretically hard due to existence of Coulomb form factor that is defined in the non-relativsitic limit only (usual subtraction techniques known from NLO-revolution do not apply)

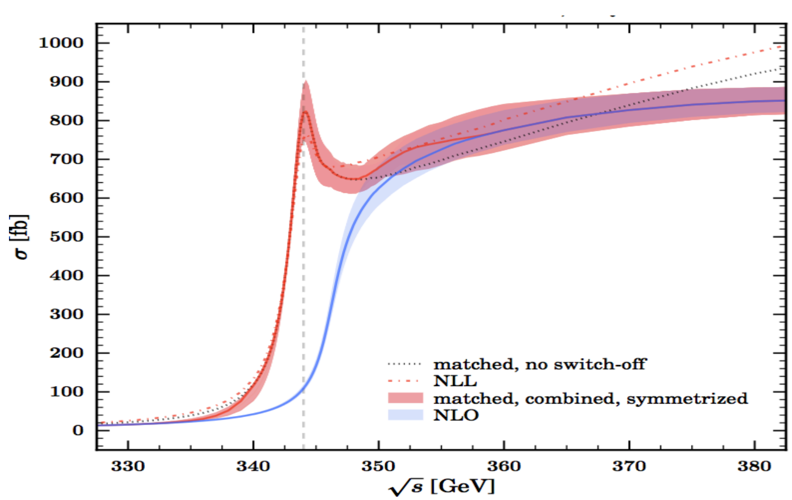
Top Threshold

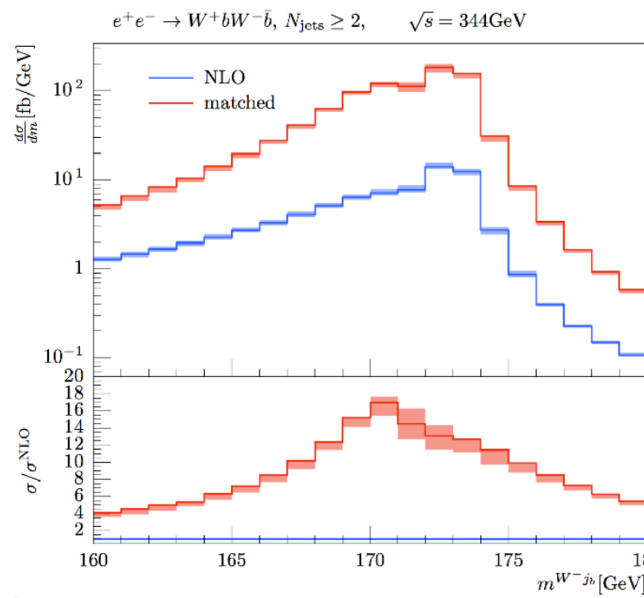
Differential Cross Sections:





- Whizard threshold implementation does NOT contain these effects !
  Therefore $\mathrm{NLO_{FO}}$ + $\mathrm{NLL_{threshold}}$ only for total cross section, $\mathrm{NLO_{FO}}$ + $\mathrm{LL_{threshold}}$ otherwise.

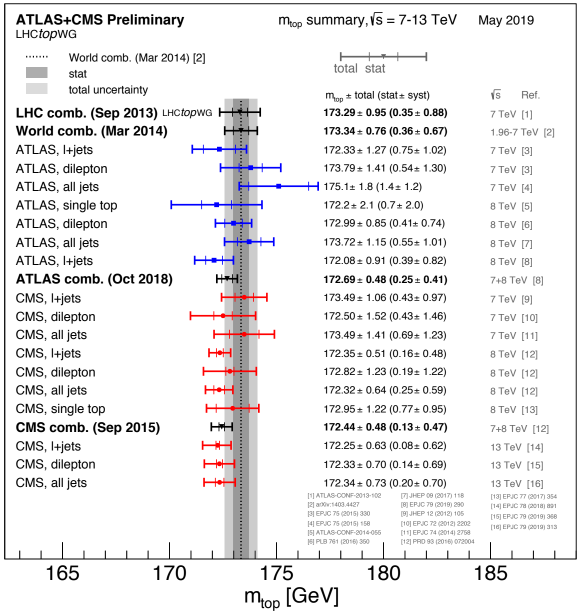




- Ultrasoft non-factorizable corrections still have to be added

Top Mass from Direct Reconstruction

- Direct mass measurements (template or matrix element fits) are the moste precise method to determine the top mass at the LHC
- Variables (M_{lb}, m^{reco}) cannot be described by FO computation and are described completely by parton shower and hadronization dynamics in Monte-Carlo generators.
- Because MC have limited (observable dependent) precision the measured top mass m_t^{MC} cannot be a priori assigned to a particular mass scheme.

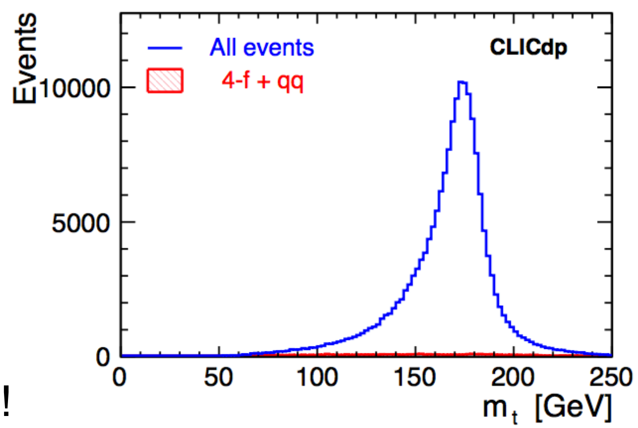


- The situation is not different at a lepton collider, but the systematic uncertainties are much smaller.

Abramowicz etal. '18

- CLIC simulation study: m_t^{reco} template fit E_{cm}= 380 GeV

$(\Delta m_t^{MC})^{stat} \sim$ 30 MeV $\quad (\Delta m_t^{MC})^{syst} \sim$ 50 MeV



Competitive with threshold measurements!

Worth to improve theory understanding of direkt method!

Top Mass from Direct Reconstruction

Why bother given that we have the top threshold?

- For lepton collision is it much easier to understand the MC top mass interpretation problem and we can use the consistency with the threshold mass measurements as a benchmark to improve the intrinsic precision of MC generators and make them into much more reliable tools.

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + \Delta_m^{\mathrm{pert}} + \Delta_m^{\mathrm{non-pert}} + \Delta_m^{\mathrm{MC}}$$

pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup

Was analyzed in
Plätzer, Samitz, AHH '18

Non-perturbative contribution:

- Effects of hadronization model
- May depend on parton shower setup

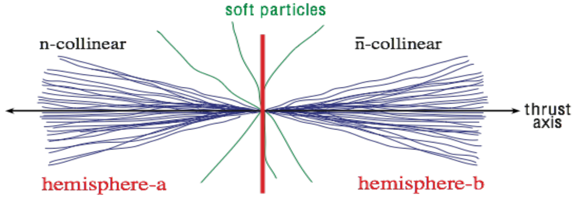
Monte Carlo shift:

- Contribution arising from systematic MC uncertainties not related to top
- E.g. b-jet modeling, finite width, ...

Top Mass from Direct Reconstruction

Plätzer, Samitz, AHH '18

- Analytic parton-level analysis of QCD factorization calculation (NLL') and the Herwig angular-ordered parton shower for the 2-jettiness τ_2 distribution for boosted top pair production in the NWA



1. Herwig shower is NLL precise for τ_2.
2. Definition of generator mass can be computed by comparison to NLL' QCD calculation.
3. Generator mass $m_t^{CB}(Q_0)$ depends on the shower cut Q_0=1.25 GeV.

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

$$m_t^{\text{MSR}}(Q_0) - m_t^{\text{CB}}(Q_0) = 120 \pm 70 \text{ MeV}$$

$$m_t^{\text{pole}} - m_t^{\text{CB}}(Q_0) = 480 \pm 260 \text{ MeV}$$

- First step of a general long-term project (work in progress, progress expected)
- Result shows that the question is very relevant also for LHC.

Top Mass from Direct Reconstruction

- By the time a lepton collider runs the theoretical aspect of direct top mass measurements will be understood to a degree comparable the expected experimental uncertainties. NLL precise parton showers and overall improvement of the precision of MC event generators are essential.

- We can use direct top mass measurements (in comparison with top threshold measurements) as a benchmark test for the precision of MC event generators.

- Conceptual progress on the top mass interpretation problem will be established first for e^+e^- collisions first and for pp collisions after that.

Topics Dropped and Final Statement

- Top Spin measurements at threshold (large QCD phases)
- B fragmentation from top (NNLO treatment)
- Boosted top physics (boosted heavy quark effective theory, fat jets)
- Groomed top jets (equivalence of LC and LHC)
- Resummation of logarithms $\ln(m_t/\Gamma_t)$

There are still many interesting unresolved problems to work on to sharpen the theoretical tools for future lepton colliders.

Development of a new generation of more precise Monte-Carlo generators must receive high priority in the community as being theory work that is valuable by itself (such as loop calculations).