The hadronic contribution to $\Delta\alpha$

Massimo Passera
INFN Padova

International workshop on the high energy CEPC
Shanghai - October 26 2020

- $\Delta\alpha$ and the EW fit
- Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection
- The MUonE project

$\Delta\alpha$ and the EW fit

SM input parameters

$\Delta\alpha$

The fine-structure constant α is the most precise input of the SM:

α = 1/137.036 999 046 (27) [0.20 ppb] Science 360 (2018) 191 (Cs)

followed by

G_F = 1.1663787(6) 10^{-5} GeV^{-2} [0.5 ppm] PDG2020

M_Z = 91.1876(21) GeV [23 ppm] PDG2020

However, the effective $\alpha(q^2)$ entering the cross sections at q^2 energies is

$$\alpha(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)} \qquad \Delta\alpha(q^2) = \Delta\alpha_{\rm lep}(q^2) + \Delta\alpha^{(5)}_{\rm had}(q^2) + \Delta\alpha_{\rm top}(q^2)$$

At $q^2=M_Z^2$, the hadronic contribution induces an error of $O(10^{-4})$.
Six orders of magnitude in precision lost wrt to α!

- The hadronic contribution cannot be computed perturbatively because of the nonperturbative nature of strong interactions at low energy.

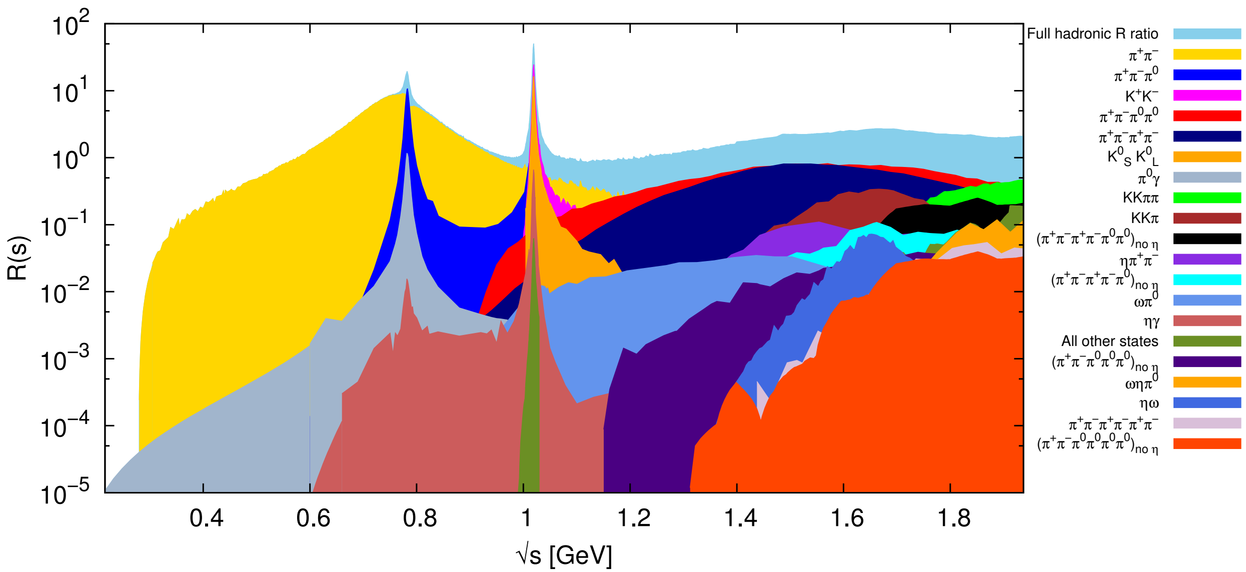
- Using analyticity and unitarity, at $q^2=M_Z^2$:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{M_Z^2}{4\alpha\pi^2}\, \mathrm{P}\int_{s_{th}}^{\infty} \mathrm{d}s\, \frac{\sigma_{\text{had}}(s)}{M_Z^2 - s}$$

Analysis	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)\times 10^4$	$\alpha^{-1}(M_Z^2)$
DHMZ10 [51]	275.59 ± 1.04	128.952 ± 0.014
HLMNT11 [40]	276.26 ± 1.38	128.944 ± 0.019
FJ17 [47]	277.38 ± 1.19	128.919 ± 0.022
DHMZ17 [54]	276.00 ± 0.94	128.947 ± 0.012
KNT18	276.11 ± 1.11	128.946 ± 0.015
DHMZ19 [37]	276.10 ± 1.00	128.946 ± 0.013
KNT19 [This work]	276.09 ± 1.12	128.946 ± 0.015

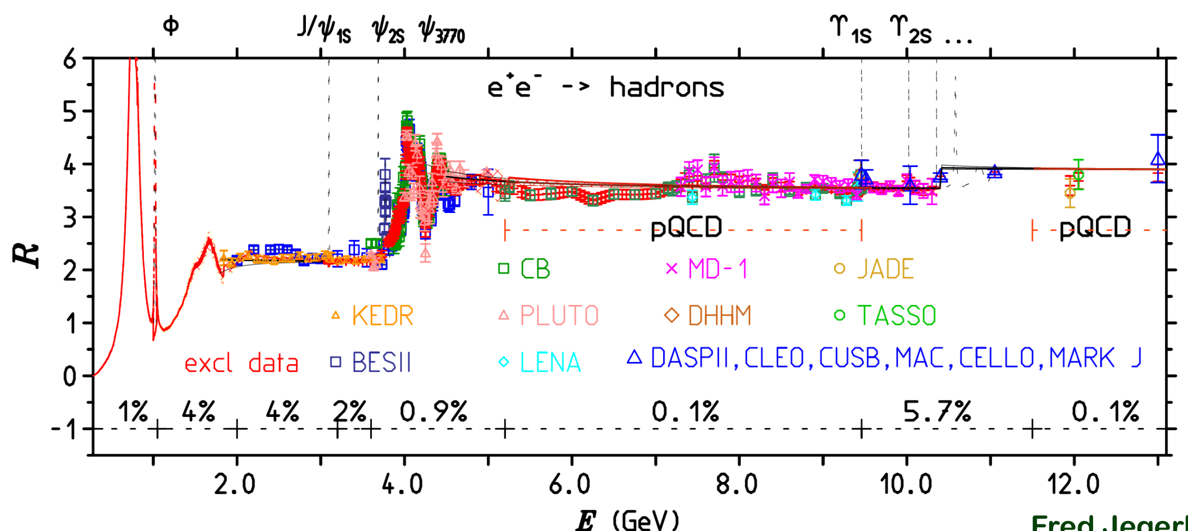
Keshavarzi, Nomura Teubner, PRD 2020

The low-energy hadronic cross section



Keshavarzi, Nomura Teubner, PRD 2018

$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi|\alpha(s)|^2}{3s}$$



Fred Jegerlehner 1905.05078

Impact of $\Delta\alpha$ on $\sin^2\theta_{eff}$

Present $\sin^2\theta_{eff}$ world average from all Z-pole measurements at lepton (LEP & SLC) and hadron (Tevatron and LHC) colliders:

$\sin^2\theta_{eff} = 0.23151\ (14)$ Erler and Schott, Prog. Part. Nucl. Phys. 2019

The EW fit prediction for $\sin^2\theta_{eff}$ (determined from all other parameters) is in good agreement:

$\sin^2\theta_{eff} = 0.23152\ (4)_{parametric}(4)_{th}$ Keshavarzi, Marciano, MP, Sirlin, PRD 2020 using Gfitter

The parametric error 4×10^{-5} on $\sin^2\theta_{eff}$ is dominated by the $\Delta\alpha_{had}^{(5)}(M_Z^2)$ uncertainty. If future colliders will reach a precision of ~< 1×10^{-5}, then $\Delta\alpha_{had}^{(5}(M_Z^2)$ will have to improve by a factor of >~ 3–4!

Improvements of $\Delta\alpha_{had}^{(5)}(M_Z^2)$?

How can we improve the precision of $\Delta\alpha_{had}^{(5)}(M_Z^2)$?

- **New low-energy data for $\sigma_{had}(s)$ (CMD-3, SND, KEDR, BESIII, Belle-2, …). Radiative Corrections to $\sigma_{had}(s)$ are crucial.**
- **Direct determination of $\Delta\alpha_{had}^{(5)}(M_Z^2)$ measuring the muon asymmetry $A^{\mu\mu}_{FB}(s)$ in the vicinity of the Z pole** Patrick Janot, JHEP 2016
- **Euclidean split method (Adler function). Needs spacelike offset $\Delta\alpha_{had}^{(5)}(-M_0^2)$ with $M_0\sim$ 2 GeV and pQCD.** Fred Jegerlehner, hep-ph/9901386
- **Future muon-electron scattering data at the MUonE experiment may help determine the spacelike offset $\Delta\alpha_{had}^{(5)}(-M_0^2)$** (see later)
- **Lattice QCD? Lots of work in progress for the hadronic vacuum polarization contribution to the muon g-2.**

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

Comparing the SM prediction with the measured muon g-2 value:

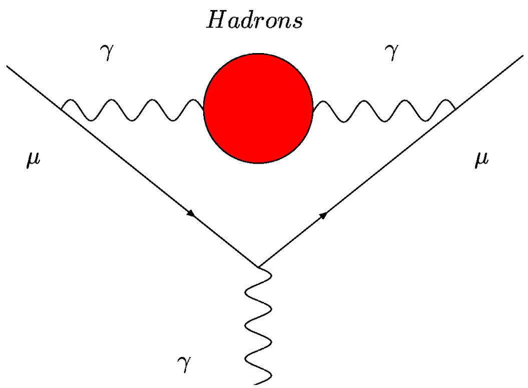
$a_\mu^{EXP} = 116592089\ (63) \times 10^{-11}$ BNL E821

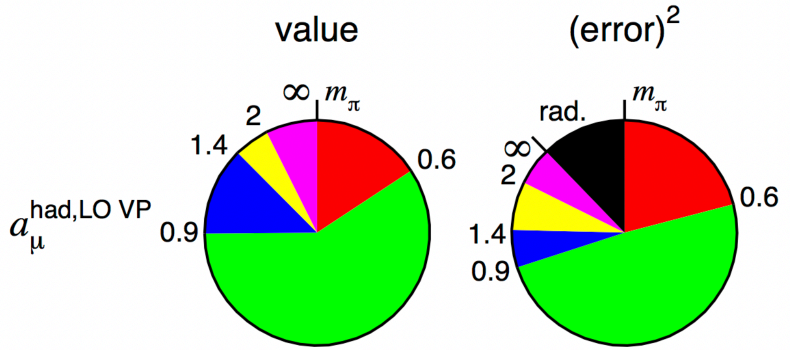
$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$ Muon g-2 TI

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 279\ (76) \times 10^{-11}$ 3.7 σ

White Paper of the Muon g-2 Theory Initiative:
arXiv:2006.04822

The muon g-2: Hadronic LO contribution





Keshavarzi, Nomura, Teubner 2018

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

a_μ^{HLO} = 6895 (33) x 10^{-11} — F. Jegerlehner, arXiv:1711.06089

= 6939 (40) x 10^{-11} — Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

= 6928 (24) x 10^{-11} — Keshavarzi, Nomura, Teubner, arXiv:1911.00367

= 6931 (40) x 10^{-11} — Muon g-2 TI WP: arXiv:2006.04822

- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Great progress in lattice QCD results. Recent BMW result with subpercent precision: a_μ^{HLO} = 7087(53)x10^{-11}. Tension with dispersive evaluations.** S. Borsanyi et al. 2002.12347.

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

a_μ^{HLO} →
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)}$ →
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

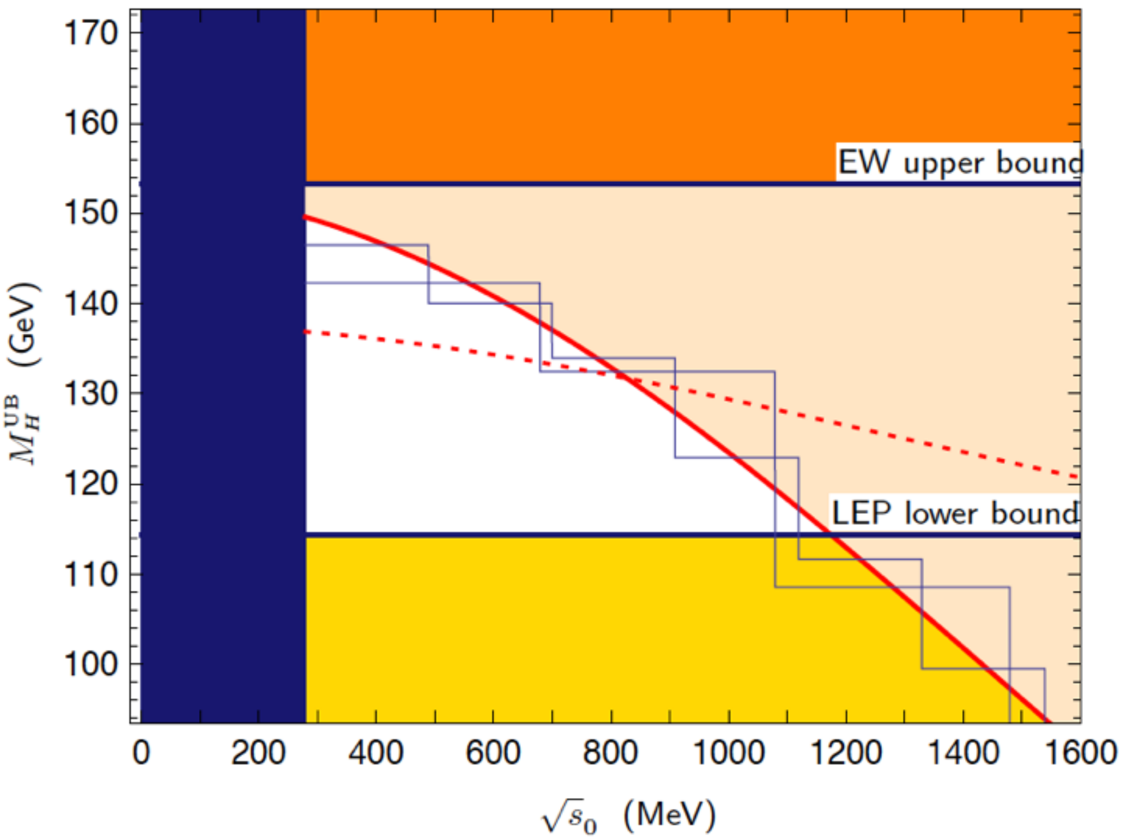
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ε>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$ →

The muon g-2: connection with the SM Higgs mass (2010)

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



Marciano, MP, Sirlin, 2008 & 2010

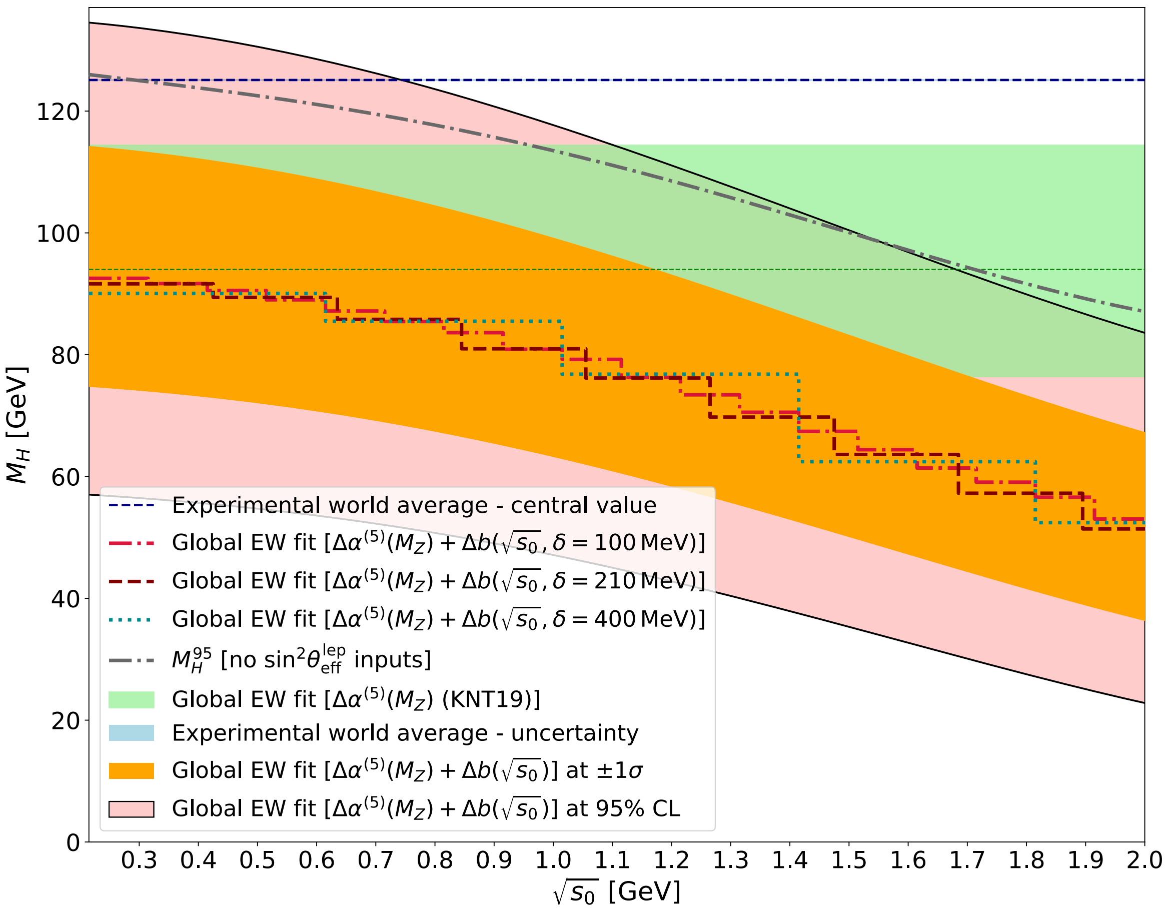
The muon g-2: connection with the SM Higgs mass (update)

Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to $\sigma(s)$, theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	···
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	···
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	···
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	···
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	···
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	···
R^0_l	20.767(25)	[108]	20.732(4)	···
R^0_c	0.1721(30)	[108]	0.17222(8)	···
R^0_b	0.21629(66)	[108]	0.21581(8)	···
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	···
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	···
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	···
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	···
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	···
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	···
A_c	0.670(27)	[108]	0.6679(2)	···
A_b	0.923(20)	[108]	0.93462(7)	···
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

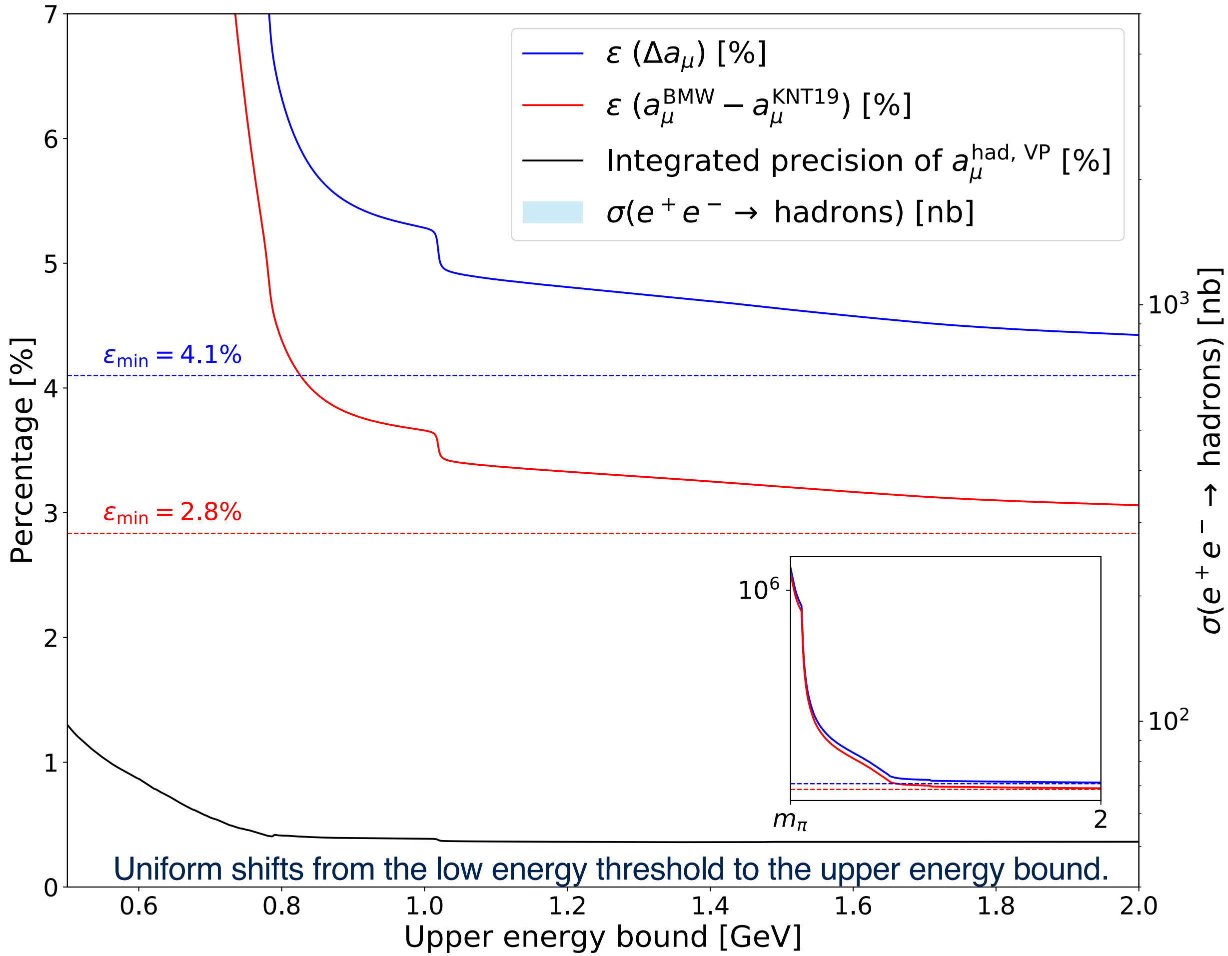
The muon g-2: connection with the SM Higgs mass (2020)



Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible,
but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts $\Delta\sigma(s)$?



Shifts below 1 GeV conflict with the quoted exp. precision of $\sigma(s)$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Recent related literature

- Crivellin, Hoferichter, Manzari and Montull, "Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits,'' PRL125 (2020) 9, 091801 [arXiv:2003.04886].

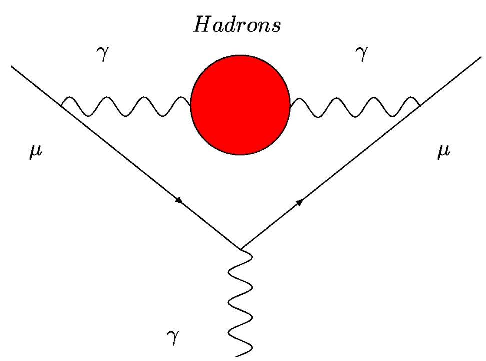
- Eduardo de Rafael, "On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,'' arXiv:2006.13880.

- Malaescu and Schott, "Impact of correlations between a_μ and α_{QED} on the EW fit", arXiv:2008.08107.

The MUonE project

Spacelike proposal for a_μ^{HLO}

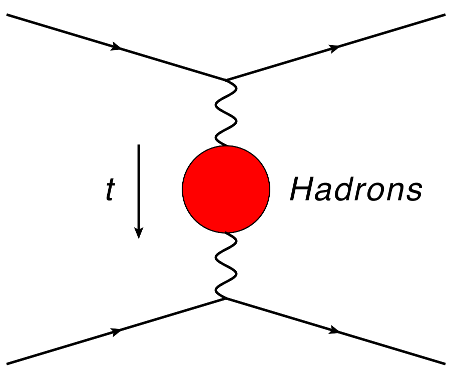
- **The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:**



$$a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\mathrm{had}}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2 (1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\, (1-x)\, \Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

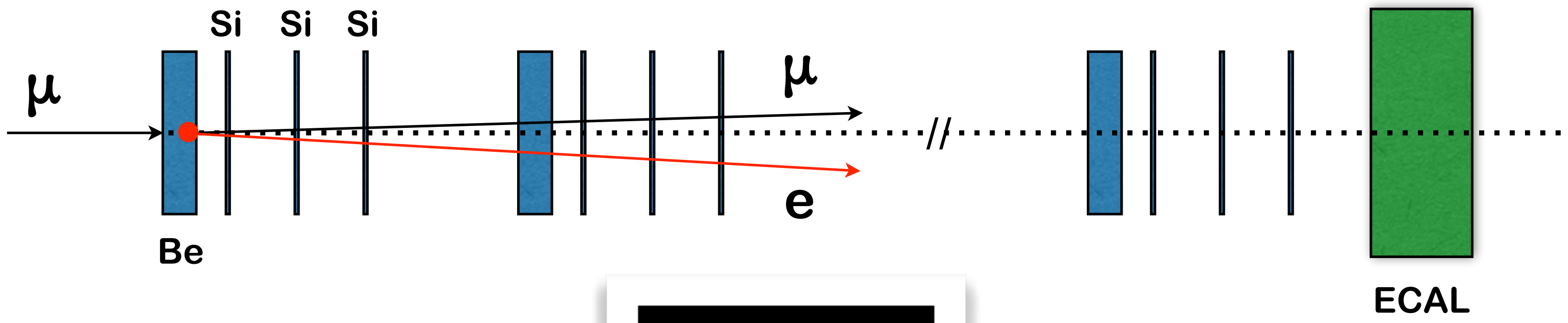
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.



Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

MUonE

- With 150 GeV muons, the high energy region inaccessible to MUonE contributes only 13% of the total a_μ^{HLO} integral. Recently it has been determined via lattice QCD Giusti&Simula and Marinković&Cardoso 2019
- Statistics: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2 years of data taking (2×10^7 s/yr) → $\mathscr{L}_{int} \sim 1.5 \times 10^7\, nb^{-1}$.
- With this $\mathscr{L}_{int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.
- Systematic effects must be known at ≲ 10ppm!
- Theory: To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!
- Interplay and complementarity with lattice determination of a_μ^{HLO}
- LoI submitted to CERN SPSC in 2019. Test run in 2021 recently approved. Full-statistics run hopefully in 2022–24.

Conclusions

- The present error of the hadronic contribution to $\Delta\alpha(M_Z^2)$ is not sufficient for the precision expected at future circular colliders.

- Possible improvements of $\Delta\alpha_{had}(M_Z^2)$ with new low-energy σ_{had} data, a direct determination via $A^{\mu\mu}_{FB}(s)$ near the Z pole, Euclidean split method with spacelike offset (MUonE may help determine it), lattice QCD?

- A connection exists between the muon g-2 and $\Delta\alpha_{had}(M_Z^2)$: is Δa_μ due to missed contributions in the hadronic $\sigma(s)$?

  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- MUonE will measure $\Delta\alpha_{had}(t<0)$ and will provide an independent spacelike determination of a_μ^{HLO} alternative to the dispersive and lattice ones. It may also help determine $\Delta\alpha(M_Z^2)$.