State of the art in Higgs/electroweak higher-order computations

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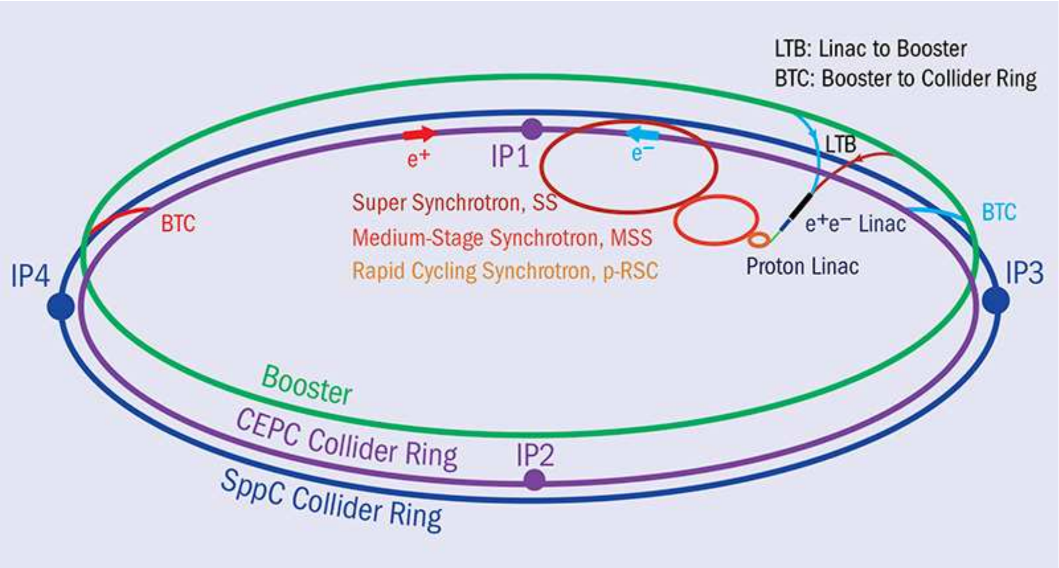

- Higgs decays
- Higgs production (HZ, $\nu\nu H$)
- Precision input parameters
- Technical considerations

based on arXiv:1906.05379

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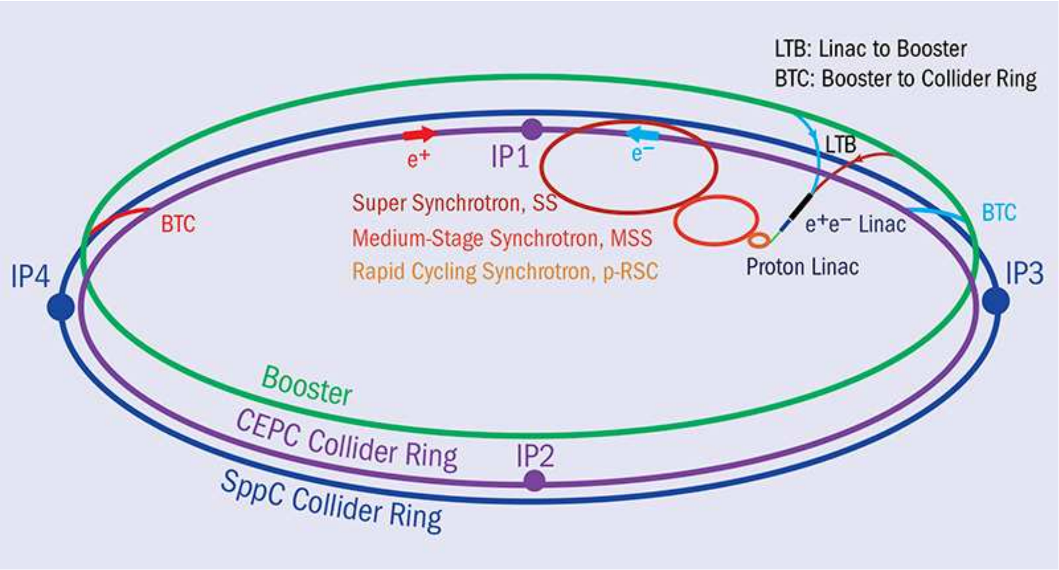

- Higgs decays
- Higgs production (HZ, $\nu\nu H$)
- Precision input parameters
- Technical considerations

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Need for theory input

- Comparison of **pseudo-observables** with SM to **probe new physics**
  ⟶ multi-loop corrections in full SM

- Extraction of **pseudo-observables** from **real observables**
  ⟶ detector acceptance, backgrounds

- “Other” eletroweak parameters (**“input” parameters**)
  ⟶ $m_{\rm t}$, $\alpha_{\rm s}$, etc. extracted from other processes

1906.05379:

“Theoretical uncertainties for electroweak and Higgs-boson measurements at FCC-ee”

Theory predictions for Higgs precision observables

Measurable properties of $h(125)$:

- **Spin, CP:** already underway at LHC
- **Mass:** direct LHC measurement more precision than SM/MSSM prediction
- **BRs, couplings:** currently $\mathcal{O}(20\%)$, improvement will greatly enhance sensitivity to higher new-physics scales Englert et al. ’14

Target precision of future e^+e^- colliders:

	CEPC	FCC-ee
hbb	1.0%	0.4%
hcc	1.9%	0.7%
$h\tau\tau$	1.2%	0.5%
$h\mu\mu$	5%	6%
hWW	1.1%	0.2%
hZZ	0.25%	0.15%
$h\gamma\gamma$	1.6%	1.5%
hgg	1.2%	0.8%

SM predictions for Higgs decays

Review: Lepage, Mackenzie, Peskin '14, see also LHC HXSWG '13

hbb:

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
- $\mathcal{O}(\alpha)$ QED+EW — Dabelstein, Hollik '92; Kniehl '92
- leading $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha\alpha_s)$ for large m_t — Kwiatkowski, Steinhauser '94, Butenschoen, Fugel, Kniehl '07
  $\longrightarrow$ Use for error estimate

Current theory error: $\Delta_{th} < 0.4\%$

With full 2-loop: $\Delta_{th} \sim 0.2\%$

Parametric error:

$$\left.\begin{array}{l} \delta m_b = 0.030 \text{ GeV} \\ \delta\alpha_s = 0.001 \end{array}\right\} \rightarrow \Delta_{par} \approx 0.8\%$$

$$\left.\begin{array}{l} \delta m_b = 0.005 \text{ GeV} \\ \delta\alpha_s = 0.0001 \end{array}\right\} \rightarrow \Delta_{par} \approx 0.3\%$$

SM predictions for Higgs decays

Review: Lepage, Mackenzie, Peskin '14, see also LHC HXSWG '13

hbb:

- $\mathcal{O}(\alpha_s^4)$ QCD corrections — Baikov, Chetyrkin, Kühn '05
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  $\longrightarrow$ Use for error estimate

Differential (exclusive) predictions:

- $\mathcal{O}(\alpha_s^3)$ QCD — Mondini, Schiavi, Williams '19; Mondini, Schubert, Williams '20
- $\mathcal{O}(\alpha_s^2)$ and NNLL resummation, matched to parton shower — Alioli et al. '20

SM predictions for Higgs decays

h$\tau\tau$:

With full 2-loop (no QCD): $\Delta_{\mathrm{th}} < 0.1\%$

Parametric error negligible

hWW*/hZZ*:

- complete $\mathcal{O}(\alpha) + \mathcal{O}(\alpha_{\mathrm{s}})$ for $h \to 4f$ Bredenstein, Denner, Dittmaier, Weber ’06
- leading $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ and $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2)$ for large m_{t} Djouadi, Gambino, Kniehl ’97
  Kniehl, Spira ’95; Kniehl, Steinhauser ’95
  → Small (0.2%) effect Kniehl, Veretin ’12

Theory error: $\Delta_{\mathrm{th,EW}} < 0.3\%$, $\Delta_{\mathrm{th,QCD}} < 0.5\%$

With NNLO final-state QCD corrections: $\Delta_{\mathrm{th,QCD}} < 0.1\%$

Parametric error:

$\delta M_{\mathrm{H}} \sim 10$ MeV $\quad\to \Delta_{\mathrm{par}} \approx 0.1\%$

Note: Distributions affected by corrections → implementation into MC tools

SM predictions for Higgs decays

hgg:

- $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ (in large m_t-limit) QCD corrections Baikov, Chetyrkin '06
  Schreck, Steinhauser '07
- $\mathcal{O}(\alpha)$ EW Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04

Theory error (dominated by QCD): $\Delta_{th} \approx 3\%$
With $\mathcal{O}(\alpha_s^4)$ in large m_t-limit (4-loop massless QCD diags.): $\Delta_{th} \approx 1\%$

Parametric error: $\delta\alpha_s = 0.001 \rightarrow \Delta_{par} \approx 3\%$
$\delta\alpha_s = 0.0001 \rightarrow \Delta_{par} \approx 0.3\%$

h$\gamma\gamma$:

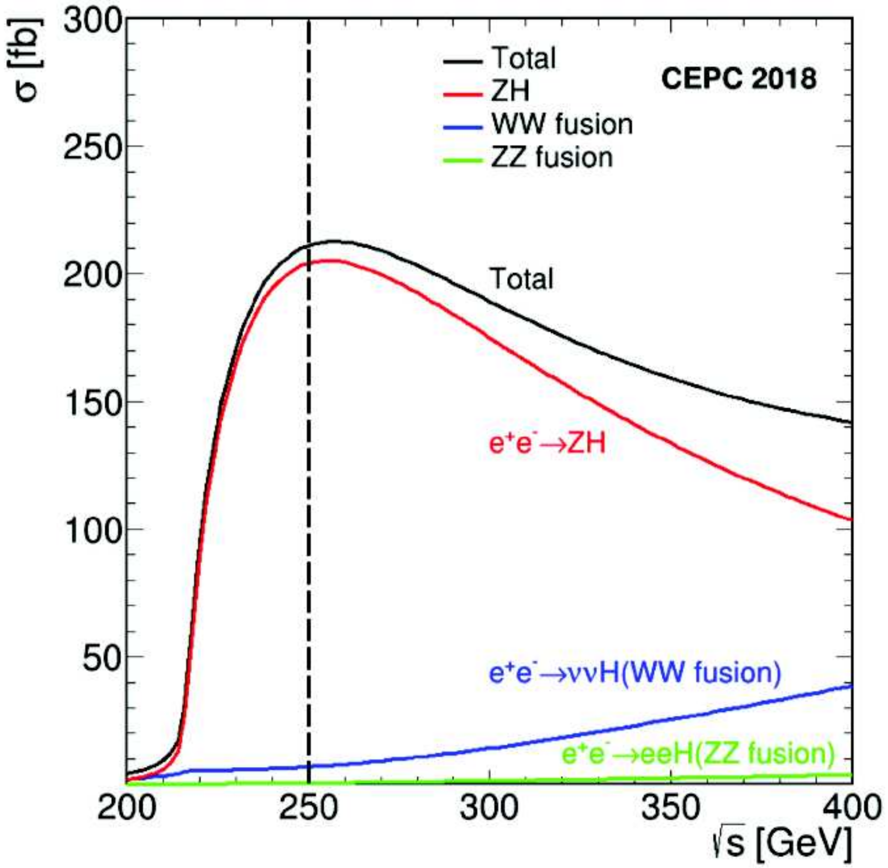
- $\mathcal{O}(\alpha_s^2)$ QCD corrections Zheng, Wu '90; Djouadi, Spira, v.d.Bij, Zerwas '91
  Dawson, Kauffman '93; Maierhöfer, Marquard '12
- $\mathcal{O}(\alpha)$ EW Aglietti, Bonciani, Degrassi, Vicini '04; Degrassi, Maltoni '04
  Actis, Passarino, Sturm, Uccirati '08

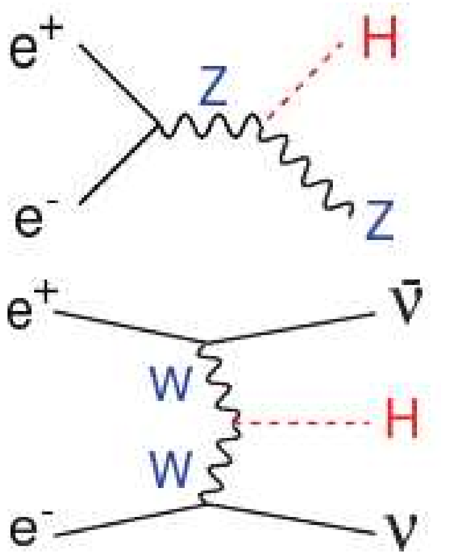
Theory error: $\Delta_{th} < 1\%$

Parametric error negligible

SM predictions for Higgs production

- hZ production: dominant at $\sqrt{s} \sim 240$ GeV
- WW fusion: sub-dominant but useful for constraining h width Han, Liu, Sayre ’13

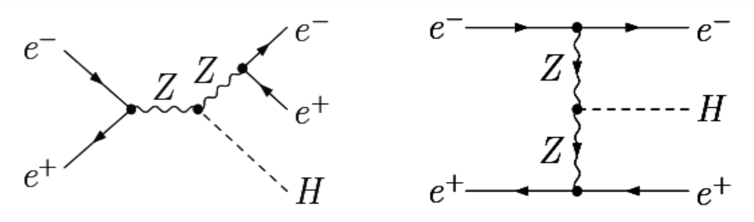




SM predictions for Higgs production

hZ production:

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl ’92; Denner, Küblbeck, Mertig, Böhm ’92; Consoli, Lo Presti, Maiani ’83; Jegerlehner ’86; Akhundov, Bardin, Riemann ’86
- Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available — Boudjema et al. ’04; Denner, Dittmaier, Roth, Weber ’03



- Can be combined with h.o. ISR QED radiation — Greco et al. ’17
- $\mathcal{O}(\alpha\alpha_s)$ corrections — Gong et al. ’16; Chen, Feng, Jia, Sang ’18

$\mathcal{O}(\alpha) \sim 5\text{–}10\%$ (large ISR effect)

$\mathcal{O}(\alpha\alpha_s) \sim 1.3\%$

$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma^{\rm exp.}_{\rm NNLO}$ (fb)
240	252.0	228.6	231.5	231.5
250	252.0	227.9	230.8	230.8

SM predictions for Higgs production

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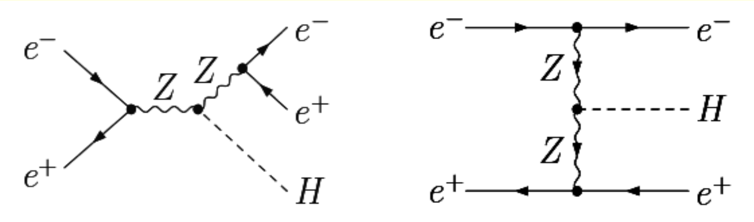

- Can be combined with h.o. ISR QED radiation — Greco et al. '17
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Theory error: $\Delta_{th} \sim O(1\%)$

With full 2-loop corrections for $ee \to HZ$: $\Delta_{th} \lesssim O(0.3\%)$

Parametric error: negligible if $\delta M_H < 100$ MeV

SM predictions for Higgs production

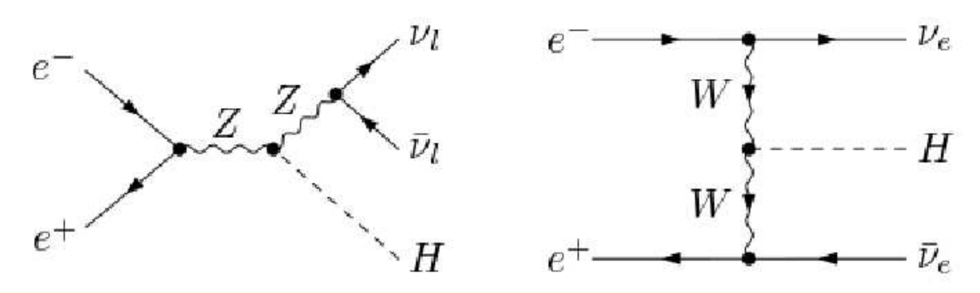
WW fusion:

- $\mathcal{O}(\alpha)$ corrections with h.o. ISR

Belanger et al. '02; Denner, Dittmaier, Roth, Weber '03

Theory error: $\Delta_{\mathrm{th}} \sim \mathcal{O}(1\%)$?

Parametric error: negligible



Full $\mathcal{O}(\alpha^2)$ calculation for 2→3 process is very challenging

$\longrightarrow$ Contributions with closed fermion loops maybe feasible

Input parameters

- $m_b,\ m_c$: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\text{MS}}} \sim 30$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 25$ MeV LHC HXSWG '16

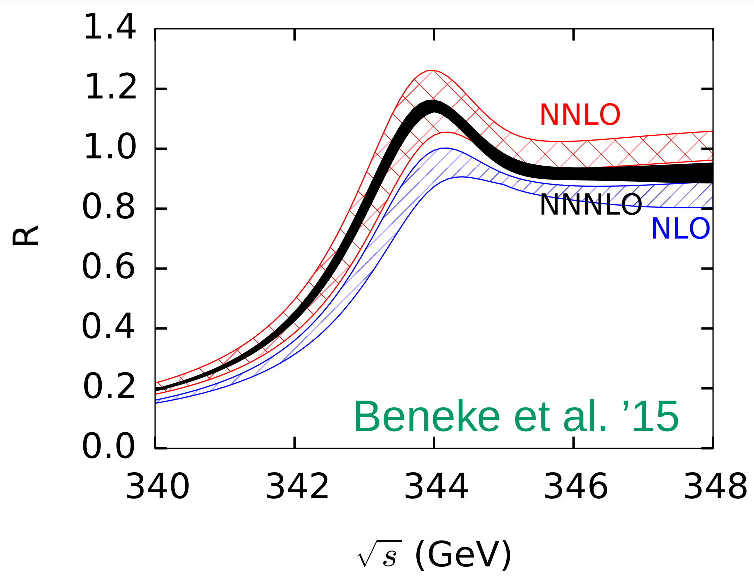
  $\longrightarrow$ estimated improvements $\delta m_b^{\overline{\text{MS}}} \sim 13$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 7$ MeV
  Lepage, Mackenzie, Peskin '14

- m_{t}: Current status $\delta m_{\text{t}} \sim 0.4$ GeV at LHC PDG '18
  $\longrightarrow$ Additional theory uncertainties? Butenschoen et al. '16
  Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \rightarrow t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:

$$
\begin{aligned}
\delta m_{\text{t}}^{\overline{\text{MS}}} = & [\quad]_{\text{exp}} \\
& \oplus [50\text{ MeV}]_{\text{QCD}} \\
& \oplus [10\text{ MeV}]_{\text{mass def.}} \\
& \oplus [70\text{ MeV}]_{\alpha_s} \\
& > 100\text{ MeV}
\end{aligned}
$$



Beneke et al. '15

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  Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:	**future:**	
$\delta m_{\text{t}}^{\overline{\text{MS}}} = [\quad]_{\text{exp}}$	$[20\ \text{MeV}]_{\text{exp}}$	
$\oplus\ [50\ \text{MeV}]_{\text{QCD}}$	$\oplus\ [30\ \text{MeV}]_{\text{QCD}}$	(h.o. resummation)
$\oplus\ [10\ \text{MeV}]_{\text{mass def.}}$	$\oplus\ [10\ \text{MeV}]_{\text{mass def.}}$	
$\oplus\ [70\ \text{MeV}]_{\alpha_{\text{s}}}$	$\oplus\ [15\ \text{MeV}]_{\alpha_{\text{s}}}$	$(\delta\alpha_{\text{s}} \lesssim 0.0002)$
> 100 MeV	$\lesssim 50$ MeV	

Input parameters

- $m_b,\ m_c$: From quarkonia spectra using Lattice QCD
  $\delta m_b^{\overline{\text{MS}}} \sim 30$ MeV, $\delta m_b^{\overline{\text{MS}}} \sim 25$ MeV LHC HXSWG '16

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  Lepage, Mackenzie, Peskin '14

- m_t: Current status $\delta m_\text{t} \sim 0.4$ GeV at LHC PDG '18
  $\longrightarrow$ Additional theory uncertainties? Butenschoen et al. '16
  Ferrario Ravasio, Nason, Oleari '18

  From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV: $\delta m_\text{t} \sim 50$ MeV

- M_H: from kinematic constraint fits $HZ(\ell\ell)$, $H(b\bar{b})Z$
  $\longrightarrow \delta M_\text{H} \sim 10...20$ MeV
  $\longrightarrow$ theory errors subdominant

Input parameters

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD:

    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

    $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

    $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

    $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

    $\longrightarrow$ Difficulty in evaluating systematics

  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$

    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

    $\longrightarrow$ Subject to sizeable non-pertubative power corrections

    $\longrightarrow$ Systematic uncertainties in power corrections?

  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18

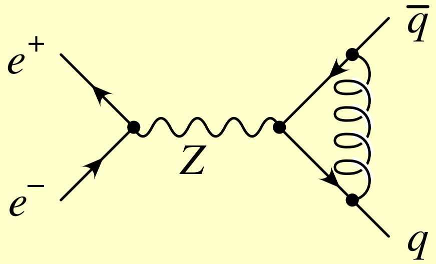
    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation

    Pich '14; Boito et al. '15,18

Input parameters

- α_s: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^{\ell}$):

    $\alpha_s = 0.120 \pm 0.003$ PDG '18



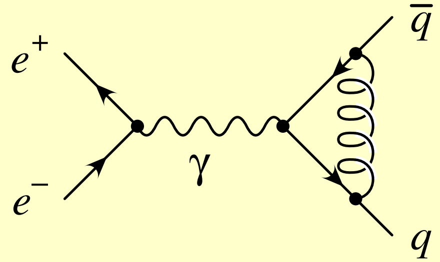
$\longrightarrow$ No (negligible) non-perturbative QCD effects

FCC: $\delta R_\ell \sim 0.001$

$\Rightarrow$ $\delta\alpha_s < 0.0002$ (subj. to theory error)

Caviat: R_ℓ could be affected by new physics

  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$



e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$

Kühn, Steinhauser, Teubner '07

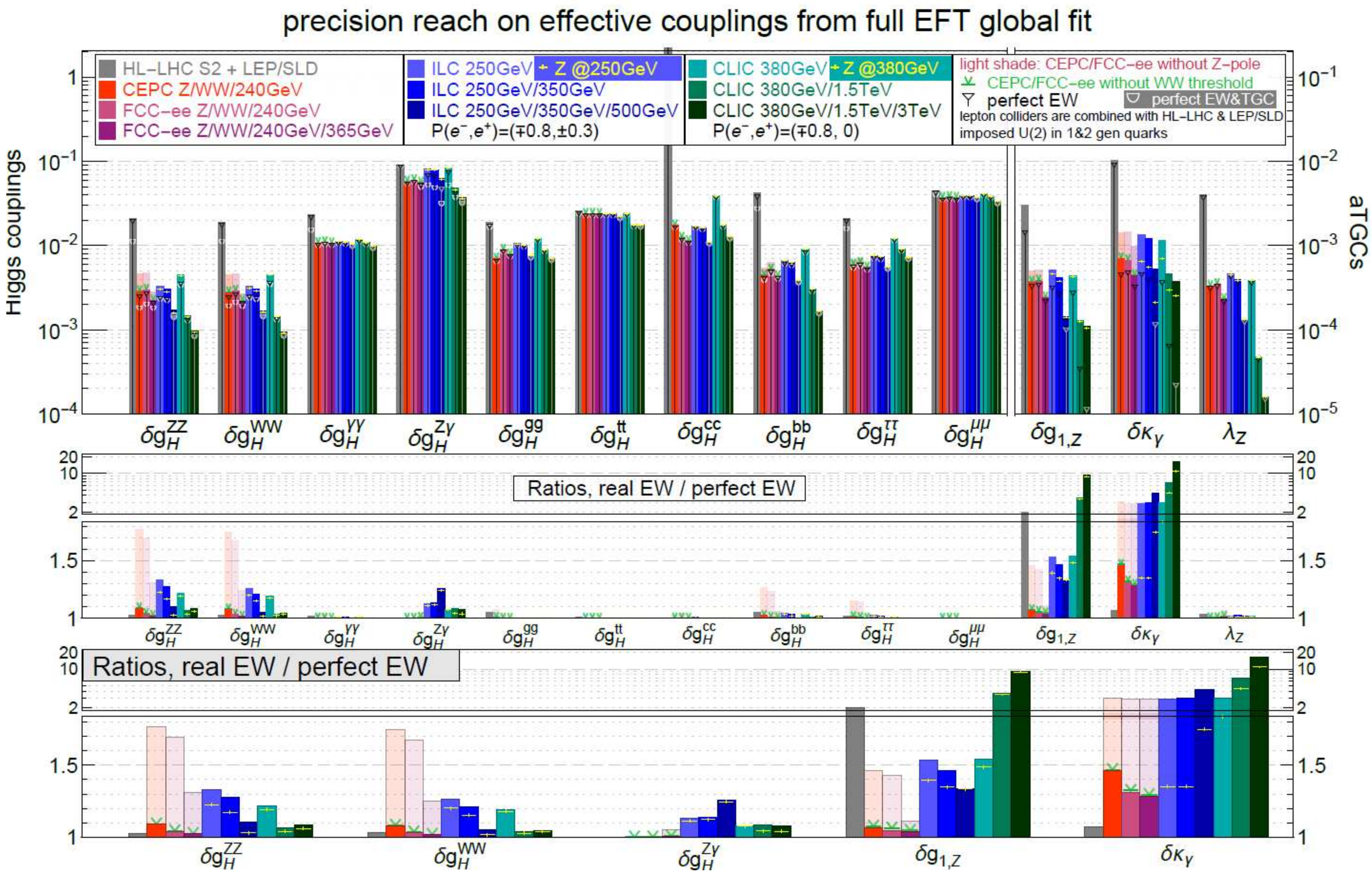
$\longrightarrow$ dominated by s-channel photon, less room for new physics

$\longrightarrow$ QCD still perturbative

naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

New physics interpretation: correlations

- Z-pole run essential for CEPC / FCC-ee



de Blas, Durieux, Grojean, Gu, Paul ’19

Main challenges

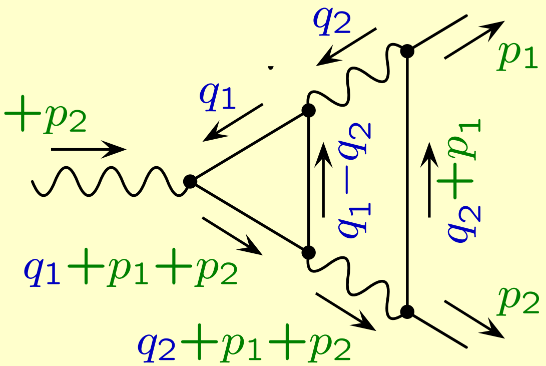
For Higgs and WW physics:

- Full $\mathcal{O}(\alpha^2)$ for $2 \to 2$ processes
- $\mathcal{O}(\alpha_s^4)$ QCD corrections
- Matching to Monte-Carlo tools
- Also need $O(\alpha)$ (or better?) corrections for backgrounds: $e^+e^-b\bar{b}$, $\nu\bar{\nu}b\bar{b}$, etc.
  $\longrightarrow$ Technology exists, but work needed Denner, Dittmaier, Roth, Wieders '05

For Z pole:

- 3-loop EW and mixed EW-QCD corrections for Zff vertices
- Leading 4-loop effects
- Initial-final QED effects / mergering multi-loop and Monte-Carlo

Theory calculations



Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)

Integrals over loop momenta:

$$\int d^4q_1 d^4q_2 \; f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(1000) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods are more general, but computing intensive
- Special numerical techniques can balance precision and evaluation time

Analytic calculations

- Mostly used for diagrams with few mass scales
- Reduce to **master integrals** with integration-by-parts and other identities
  Chetyrkin, Tkachov ’81; Gehrmann, Remiddi ’00; Laporta ’00; ...

Public programs:

Public programs:	`Reduze`	von Manteuffel, Studerus ’12
	`FIRE`	Smirnov ’13,14
	`LiteRed`	Lee ’13
	`KIRA`	Maierhoefer, Usovitsch, Uwer ’17

⟶ Large need for computing time and memory

- Evaluate master integrals with differential equations or Mellin-Barnes rep.
  Kotikov ’91; Remiddi ’97; Smirnov ’00,01; Henn ’13; ...

⟶ Result in terms of Goncharov polylogs / multiple polylogs

⟶ Some problems need iterated elliptic integrals / elliptic multiple polylogs
Broedel, Duhr, Dulat, Trancredi ’17,18
Ablinger er al. ’17

⟶ Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios,
  $e.g.\ M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

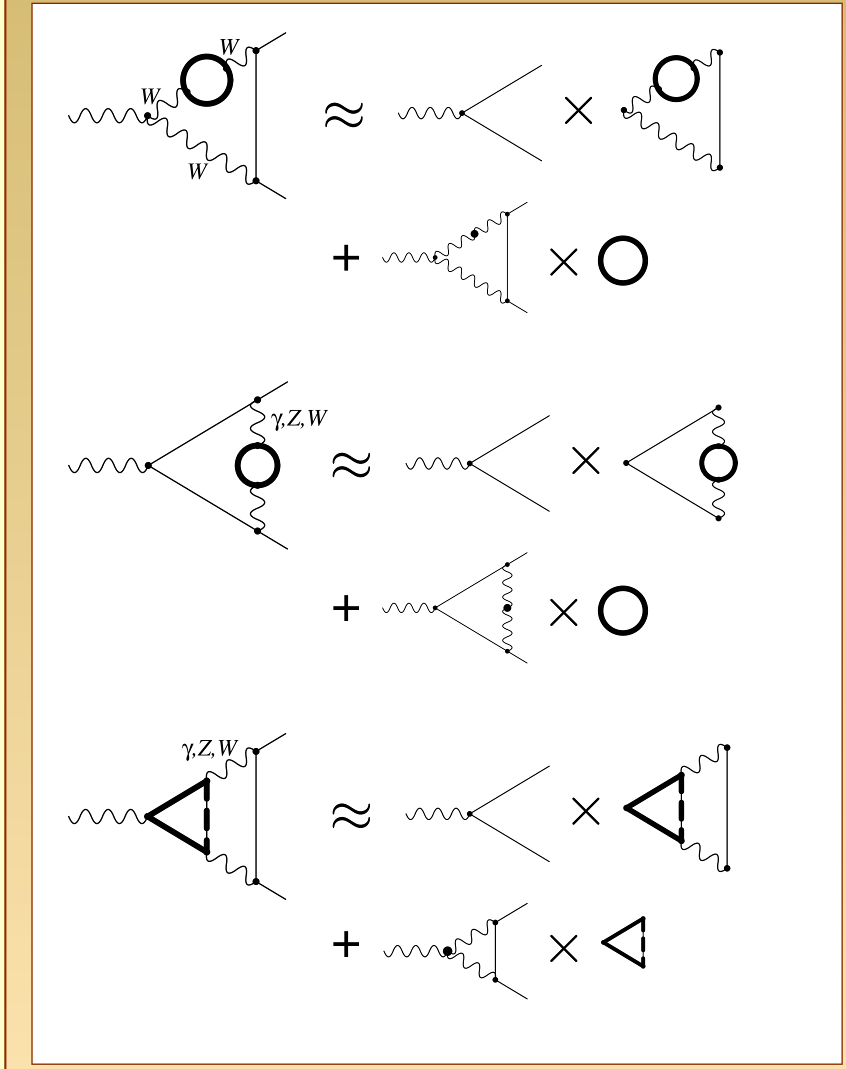
⟶ Used in some 2/3-scale problems

⟶ Public programs:

exp Harlander, Seidensticker, Steinhauser '97
asy Pak, Smirnov '10

⟶ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Numerical integration

Two general approches:

⟶ Automated treatment of UV/IR divergencies

⟶ No restriction on number of loops or legs

- **Sector decomposition:**

  Public programs:
  `SecDec` Carter, Heinrich ’10; Borowka et al. ’12,15,17
  `FIESTA` Smirnov, Tentyukov ’08; Smirnov ’13,15

- **Mellin-Barnes representations:**

  Public programs:
  `MB/MBresolve` Czakon ’06; Smirnov, Smirnov ’09
  `AMBRE/MBnumerics` Gluza, Kajda, Riemann ’07
  Dubovyk, Gluza, Riemann ’15
  Usovitsch, Dubovyk, Riemann ’18

- Diagrams with internal thresholds can cause numerical instabilities
- Specialized techniques (for some type of diagrams) often improve computing time, robustness, precision (but not automated)

Summary

- **Precision measurements** at future e^+e^- colliders require 1–2 orders improvement in SM theory calculations and tools
- Uncertainties from **perturbative** and **non-perturbative** theory and **input parameters** require much work, but can also be mitigated through choice of measurements and analysis
- **Direct determination** of α_s, m_t, $\alpha(M_\mathsf{Z})$ at e^+e^- colliders is important
- Other lower-energy experiments can provide additional input:
  BELLE II, BES, **...**

Backup slides

Theory and parametric uncertainties

	CEPC	perturb. error with 3-loop[†]	Param. error CEPC*	main source
M_W [MeV]	1	1	2.1	m_t, $\Delta\alpha$
Γ_Z [MeV]	0.5	0.15	0.15	m_t, α_s
R_b [10^{-5}]	4.3	5	< 1	
$\sin^2\theta^\ell_\mathrm{eff}$ [10^{-5}]	<1	1.5	2	m_t, $\Delta\alpha$

[†] **Theory scenario:** $\mathcal{O}(\alpha\alpha_\mathrm{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\mathrm{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\mathrm{s})$,
leading 4-loop $\mathcal{O}(\alpha_\mathrm{t}^{4-n}\alpha_\mathrm{s}^n)$,
[N_f^n = at least n closed fermion loops, $\alpha_\mathrm{t} = y_\mathrm{t}^2/(4\pi)$]

Parametric inputs:

***CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_\mathrm{s} = 0.0002$, $\delta M_\mathrm{Z} = 0.5$ MeV,
$\delta(\Delta\alpha) = 5 \times 10^{-5}$

Theory uncertainties in extraction of pseudo-observables

- Subtraction of QED radiation contributions

  → Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A$_{\text{FB}}$**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

  → $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}

  → Improvement needed for ILC/FCC-ee

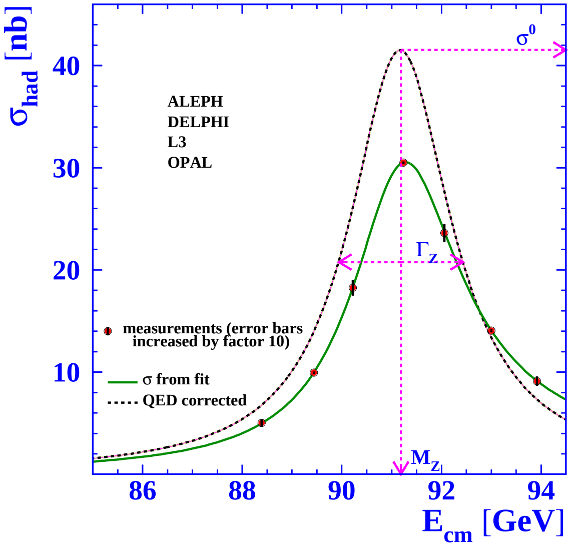
- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

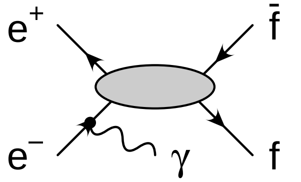
  see, e.g., Bardin, Grünewald, Passarino '99

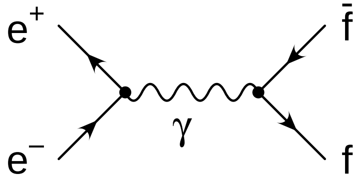
  → $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

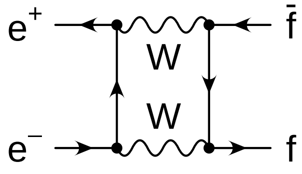
  → Sensitivity to some NP beyond EWPO

LEP EWWG '05









Z-pole asymmetries

Left-right asymmetry: (using polarization e^- beams)

$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e + \Delta A_{\gamma Z} + \Delta A_\gamma$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2} \qquad \sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$

Limited by systematic uncertainty of P_{e^-}

0.5% at SLD, 0.1% possible in future

Karl, List ’17

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4}$ $\Rightarrow$ $\delta \sin^2\theta^{\ell}_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathsf{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_\mathsf{s}$, $\alpha\alpha_\mathsf{s}^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_\mathsf{s}$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_\mathsf{s}$
$\sin^2\theta^\ell_\mathsf{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathsf{s}$

- Theory error estimate is not well defined, ideally $\Delta_\mathsf{th} \ll \Delta_\mathsf{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26\text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30\text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23\text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035\text{ MeV}$$

$$\mathcal{O}(\alpha_{\text{bos}}^2) \sim \mathcal{O}(\alpha_{\text{bos}})^2 \sim 0.1\text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{\text{bos}}^2) \sim \Gamma_Z \alpha^2 \sim 0.1\text{ MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{\text{lq}}}{\pi}\alpha_s^2 \sim 0.29\text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5\text{ MeV}$

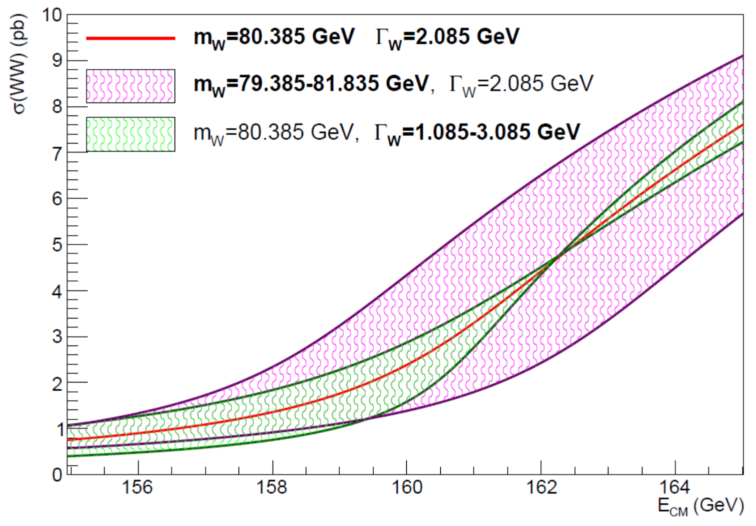
WW threshold

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold
- a) Corrections near threshold enhanced by

  $1/\beta$ and $\ln\beta$

  $$\beta \sim \sqrt{1-4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$

  Denner, Dittmaier, Roth, Wieders '05

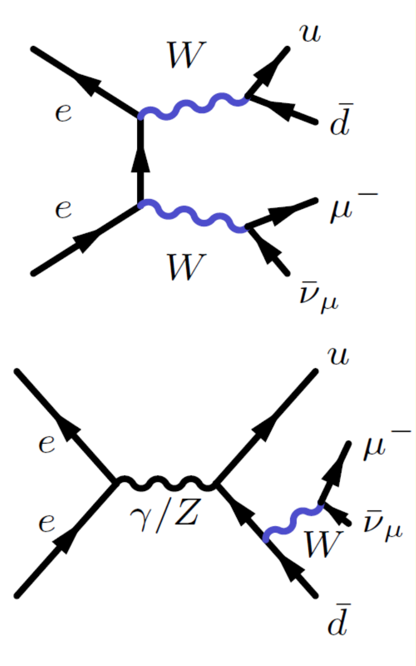
- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$

  Beneke, Falgari, Schwinn, Signer, Zanderighi '07

  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\mathrm{th}} M_W \sim 3$ MeV

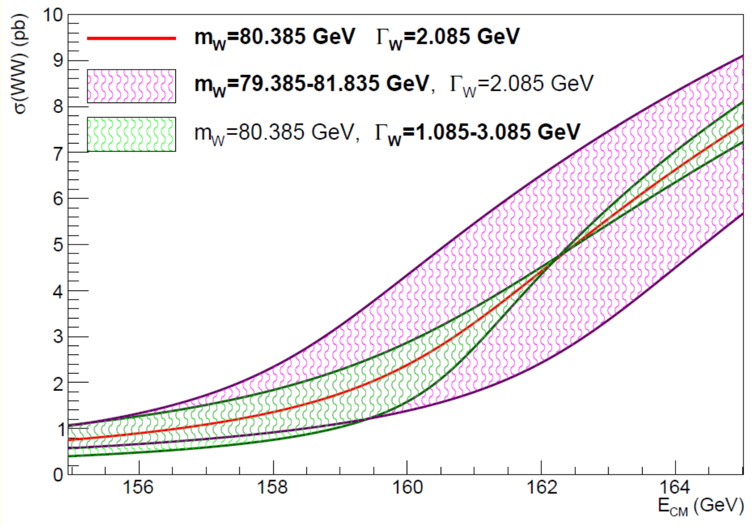
    Actis, Beneke, Falgari, Schwinn '08

  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{\mathrm{th}} M_W \lesssim 0.6$ MeV



WW threshold

- High-precision measurement of M_{W} from $e^+e^- \to W^+W^-$ at threshold



- a) Corrections near threshold enhanced by
  $1/\beta$ and $\ln\beta$

$$\beta \sim \sqrt{1 - 4\frac{M_{\mathrm{W}}^2 - iM_{\mathrm{W}}\Gamma_{\mathrm{W}}}{s}} \sim \sqrt{\Gamma_{\mathrm{W}}/M_{\mathrm{W}}}$$

  b) Non-resonant contributions are important

- Resummation of soft photon radiation

Jadach, Płaczek, Skrzypek ’19