HEFT vs SMEFT

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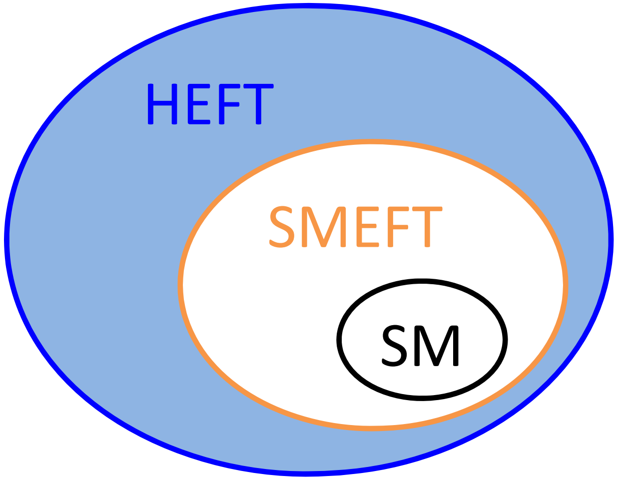
Xiaochuan Lu

University of Oregon

arXiv: 2008.08597,

with Timothy Cohen, Nathaniel Craig, and Dave Sutherland



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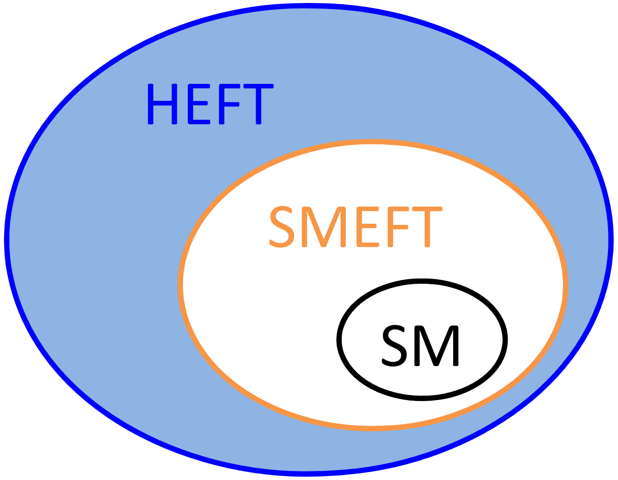

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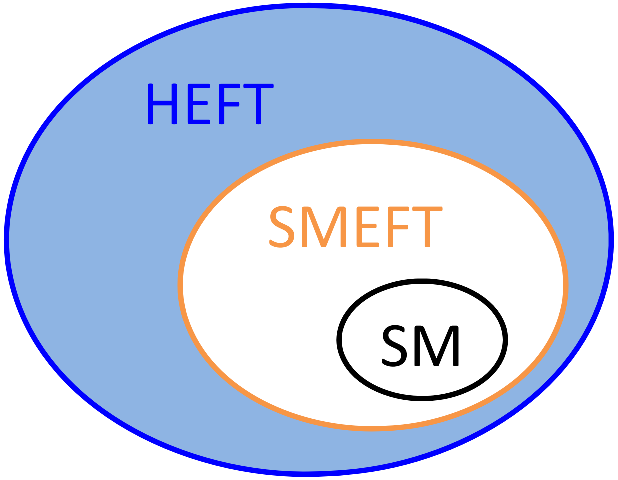

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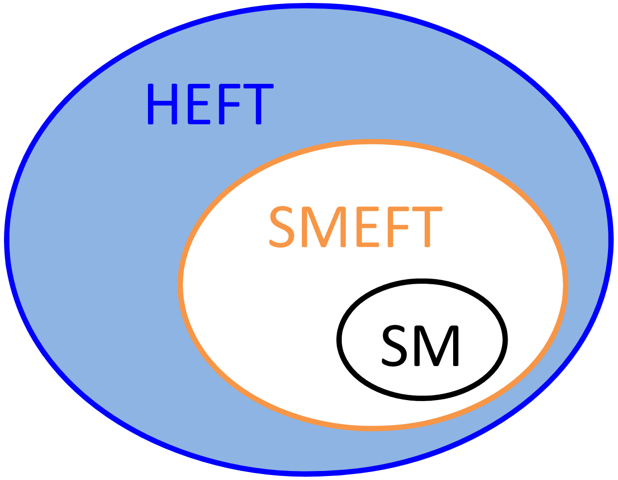

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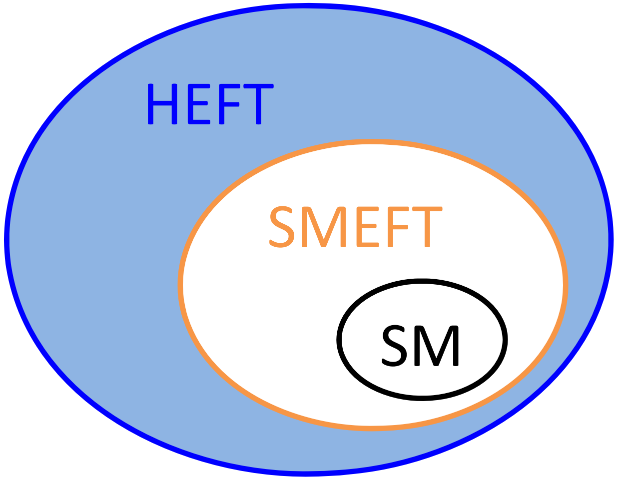

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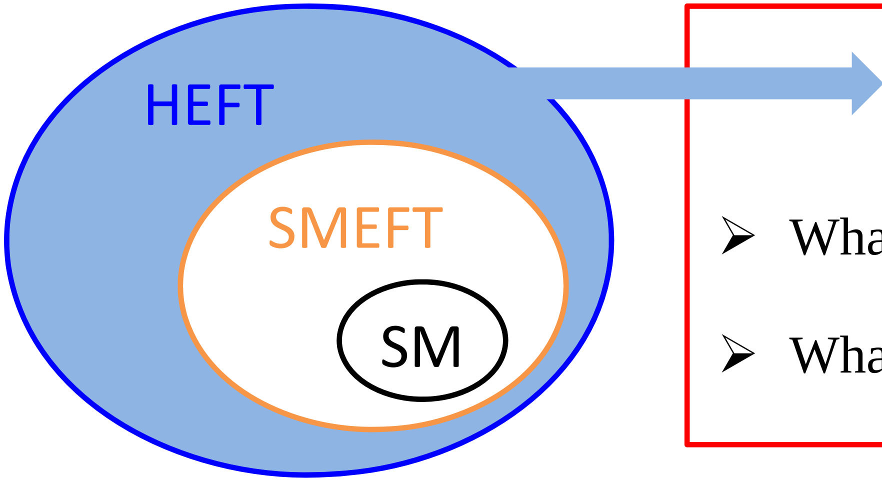

Outline

- What is HEFT?
- What UV physics generate HEFT?

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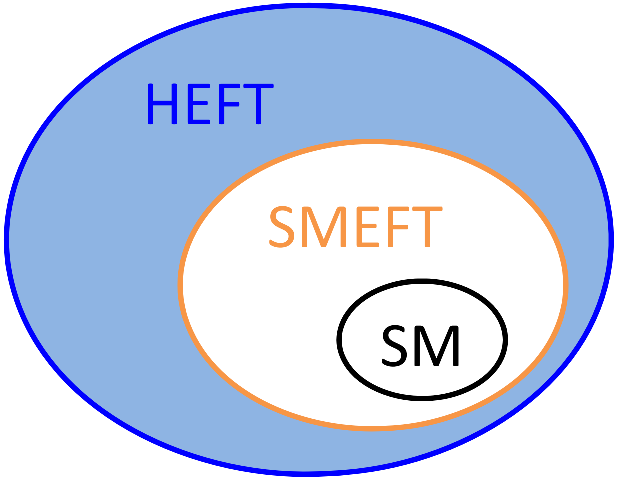

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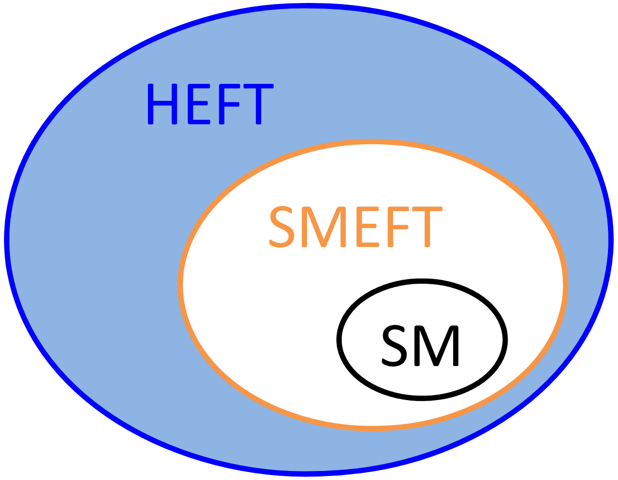
$$\mathcal{L}_{\text{HEFT}}\left(h, U\right)$$

$$\mathcal{L}_{\text{SMEFT}}\left(H\right)$$



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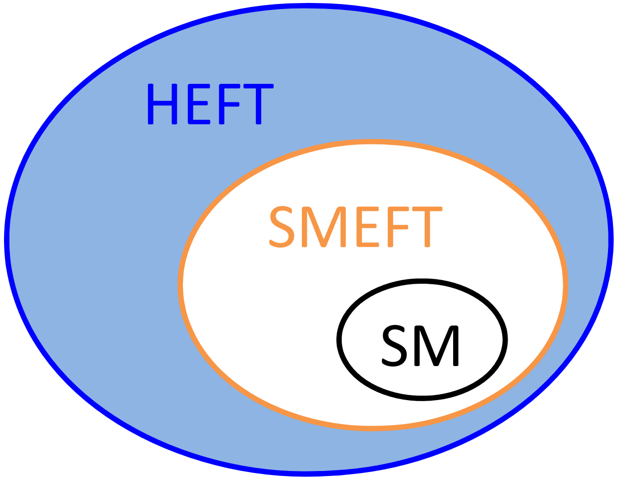
$$\begin{cases} \text{SMEFT:} \ (x, y) \\ \text{HEFT:} \ (r, \theta) \end{cases} \Rightarrow \quad r = \sqrt{x^2 + y^2}$$

Non-analytic



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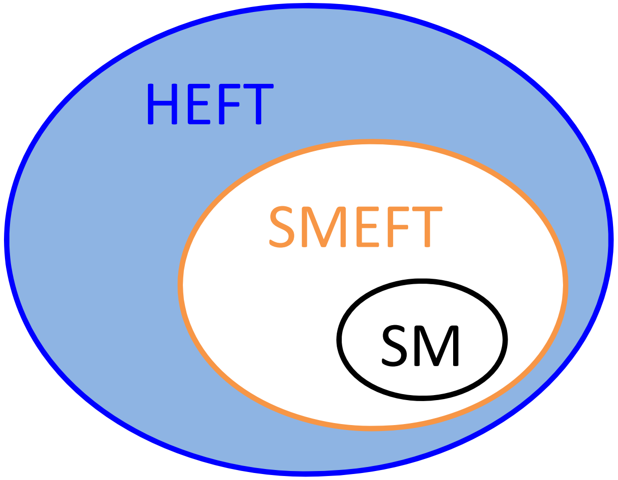

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$$\sqrt{x} = \sqrt{x_0 + (x - x_0)} = \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}(x - x_0) + \cdots$$

All order

Adam Falkowski and Riccardo Rattazzi: (arXiv: 1902.05936)

UV cut-off. Our distinction between analytic and non-analytic lagrangians coincides with the distinction, in use in the Higgs EFT community, between linear (so-called SMEFT) and non-linear (so-called HEFT) effective theory, or equivalently between h being or not being part of a $SU(2)_W$ doublet. We however believe our classification is more adequate and enlightening from a physical point of view.

$$V(H) \supset \sqrt{2H^\dagger H} = v + h$$

$$(v+h)^{2k+1} = \left(\sqrt{2H^\dagger H}\right)^{2k+1}$$

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Rodrigo Alonso, Elizabeth E. Jenkins, Aneesh V. Manohar: (arXiv: 1605.03602)

One can convert the SMEFT Lagrangian to HEFT form using Eq. (2.11) to switch from Cartesian and polar coordinates. One can attempt to convert from HEFT to SMEFT form using

$$\frac{\phi}{(\phi\cdot\phi)^{1/2}} = \boldsymbol{n} \tag{2.30}$$

with $(\phi\cdot\phi)^{1/2}$ some function of h. This substitution gives a Lagrangian $L(\phi)$ that need not be analytic in ϕ. However, if there is an $O(4)$ fixed point, then there is a suitable change of variables such that the resulting Lagrangian is analytic in ϕ.

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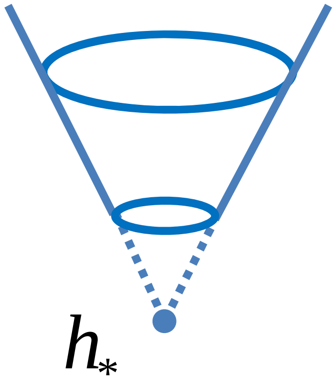

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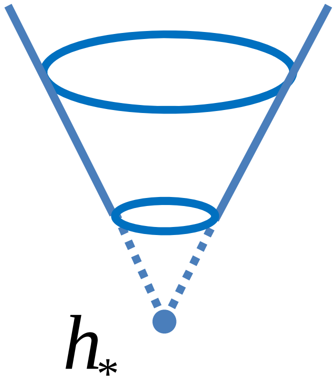

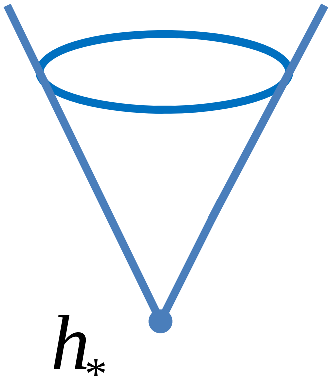

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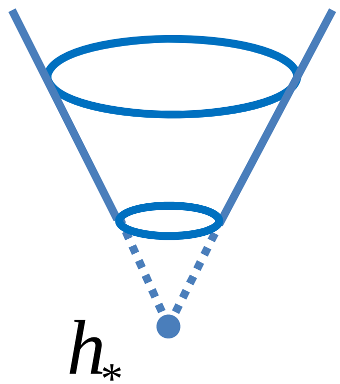

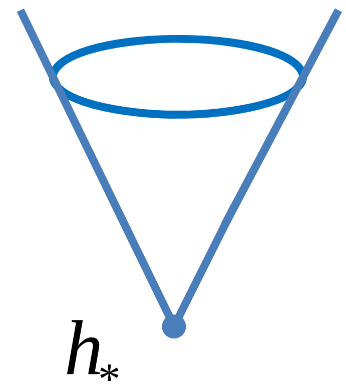

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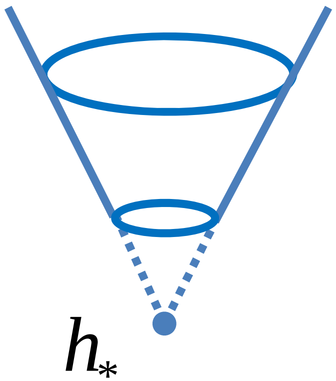
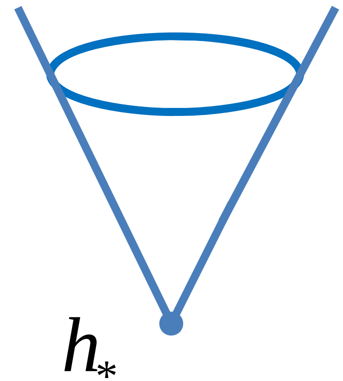
UV theories that will generate HEFT?





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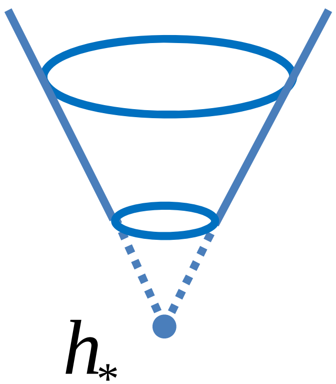

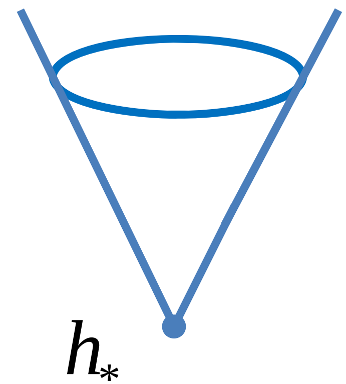

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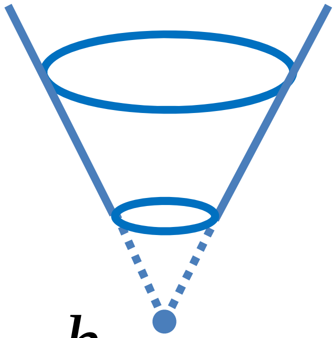
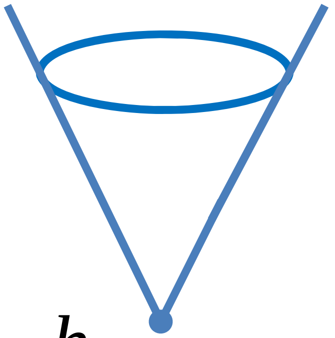


- Mass fully from electroweak symmetry breaking



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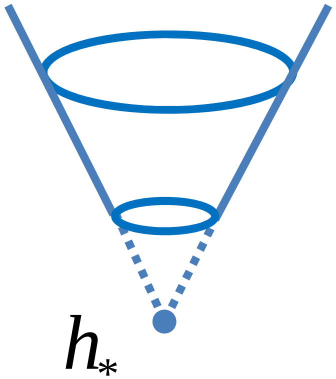

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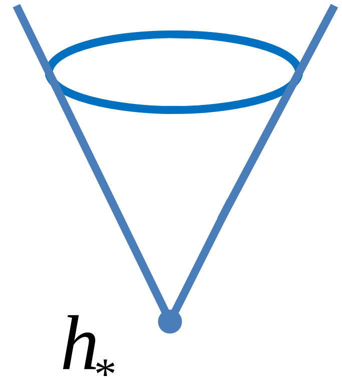

Leading Order Criterion

AJM (arXiv: 1605.03602)

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Matching to all-order in fields

Coleman-Weinberg: $\mathcal{L}_{\text{UV}}[\phi,\Phi] \supset -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U(\phi)\right]\Phi$

$$\mathcal{L}_{\text{EFT}}[\phi] = \frac{i}{2}\ln\det\left(\partial^2 + M^2 + U\right) = \frac{i}{2}\text{Tr}\ln\left(\partial^2 + M^2 + U\right)$$

$$= \frac{i}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4}\text{tr}\left[\ln\left(-p^2 + M^2 + U\right) + \frac{1}{6\left(p^2 - M^2 - U\right)^3}(\partial U)^2\right]$$

$$= \frac{1}{2}\int d^4x \frac{1}{16\pi^2}\text{tr}\left[\frac{1}{2}\left(M^2 + U\right)^2\left(\ln\frac{\mu^2}{M^2+U} + \frac{3}{2}\right) + \frac{1}{12}\frac{1}{M^2+U}(\partial U)^2 + \cdots\right]$$

Matching to all-order in fields

Coleman-Weinberg: $$\mathcal{L}_{\text{UV}}[\phi,\Phi] \supset -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U(\phi)\right]\Phi$$

$$\mathcal{L}_{\text{EFT}}[\phi] = \frac{i}{2}\ln\det\left(\partial^2 + M^2 + U\right) = \frac{i}{2}\mathrm{Tr}\ln\left(\partial^2 + M^2 + U\right)$$

$$= \frac{i}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4}\mathrm{tr}\left[\ln\left(-p^2 + M^2 + U\right) + \frac{1}{6\left(p^2 - M^2 - U\right)^3}(\partial U)^2\right]$$

$$= \frac{1}{2}\int d^4x \frac{1}{16\pi^2}\mathrm{tr}\left[\frac{1}{2}\left(M^2 + U\right)^2\left(\ln\frac{\mu^2}{M^2 + U} + \frac{3}{2}\right) + \frac{1}{12}\frac{1}{M^2 + U}(\partial U)^2 + \cdots\right]$$

If $\left[U, \partial_\mu U\right] \neq 0$: see App. D in 2008.08597

Example: A heavy Singlet

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} - \frac{1}{2} S\left(\partial^2 + M^2 + \kappa |H|^2\right) S \qquad , \qquad m_S^2 = M^2 + \frac{1}{2}\kappa v^2$$

$M^2 > 0$, SMEFT $\qquad$ $M^2 = 0$, HEFT

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$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{16\pi^2}\left[\frac{1}{4}\left(M^2 + \kappa|H|^2\right)^2 \left(\ln\frac{\mu^2}{M^2 + \kappa|H|^2} + \frac{3}{2}\right) + \frac{1}{24}\frac{\kappa^2}{M^2 + \kappa|H|^2}\left(\partial|H|^2\right)^2\right]$$

$$vF(h) = v + h \quad , \quad K^2 \equiv 1 + \frac{1}{96\pi^2}\frac{\kappa^2 |H|^2}{M^2 + \kappa|H|^2}$$

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$$R = \frac{1}{16\pi^2} \left[\frac{1}{2K^4} \frac{\kappa^2 M^2}{\left(M^2 + \kappa |H|^2 \right)^2} + \frac{1}{2K^2} \frac{\kappa^2}{M^2 + \kappa |H|^2} \right]$$

Summary

- A geometric criterion to tell a HEFT Lagrangian from SMEFT
- An understanding of what UV theories would generate HEFT
- A functional method to match to all order in *fields* (not *derivatives* yet)
- A few UV examples show that the leading order criterion works