Effective Theory Analysis of Electroweak Data

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Why are we here?

- Standard Model Effective Field Theory (SMEFT) is an important method to deal with the small deviation from SM
  - Help finding New Physics Beyond Standard Model
  - Constraining the parameters of the new model
- Electroweak measurement (especially with on-shell Z and W boson production) is among the most accurately measured observables
- EFT Analysis on Electroweak Data has been performed many times, What's new about our study?
  - We introduce the top coupling through loops
  - A more complete Electroweak dataset is used

SMEFT

- SMEFT can describe new particles' contribution by introducing new degrees of freedom (higher-dimensional operators constructed out of only SM fields)

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{L}_{d5} + \frac{1}{\Lambda^2}\mathcal{L}_{d6} + \cdots$$

- Λ is the energy scale of new interaction (set as 1TeV in our study)

- SM Lagrangian are dimension 4. Assuming lepton number conservation, dimension 6 operators has the NLO contribution to the observables

Our EFT Lagrangian

- We can impose $U(2)_u \otimes U(2)_d \otimes U(2)_q \otimes U(3)_l \otimes U(3)_e$ on Warsaw basis and set the mass of leptons and first two generation quarks as 0

$$
\begin{aligned}
\mathcal{L}_{dim6} = & \frac{c_{t\varphi}}{\Lambda^2}Q^{33}_{t\varphi} + \frac{c_{b\varphi}}{\Lambda^2}Q^{33}_{d\varphi} + \frac{c_W}{\Lambda^2}Q_W + \frac{c_{\varphi D}}{\Lambda^2}Q_{\varphi D} + \Sigma^3_{i=1}\frac{c^{(1)}_{\varphi l}}{\Lambda^2}Q^{ii(1)}_{\varphi l} + \Sigma^3_{i=1}\frac{c^{(3)}_{\varphi l}}{\Lambda^2}Q^{ii(3)}_{\varphi l} + \Sigma^3_{i=1}\frac{c_{\varphi e}}{\Lambda^2}Q^{ii}_{\varphi e} + \Sigma^2_{i=1}\frac{c^{(1)}_{\varphi q}}{\Lambda^2}Q^{ii(1)}_{\varphi q} \\
& + \Sigma^2_{i=1}\frac{c^{(3)}_{\varphi q}}{\Lambda^2}Q^{ii(3)}_{\varphi q} + \frac{c^{(+)}_{\varphi Q}}{\Lambda^2}Q^{33(+)}_{\varphi q} + \frac{c^{(-)}_{\varphi Q}}{\Lambda^2}Q^{33(-)}_{\varphi q} + \Sigma^2_{i=1}\frac{c_{\varphi u}}{\Lambda^2}Q^{ii}_{\varphi u} + \Sigma^2_{i=1}\frac{c_{\varphi d}}{\Lambda^2}Q^{ii}_{\varphi d} + \frac{c_{\varphi b}}{\Lambda^2}Q^{33}_{\varphi d} + \frac{c_{\varphi t}}{\Lambda^2}Q^{33}_{\varphi u} + \frac{c_{\varphi tb}}{\Lambda^2}Q^{33}_{\varphi ud} \\
& + \frac{c_{tW}}{\Lambda^2}Q^{33}_{uW} + \frac{c_{tB}}{\Lambda^2}Q^{33}_{uB} + \frac{c_{bW}}{\Lambda^2}Q^{33}_{dW} + \frac{c_{bB}}{\Lambda^2}Q^{33}_{dB} + \frac{c_{\varphi WB}}{\Lambda^2}Q_{\varphi WB} + \Sigma^3_{i,j=1}\frac{c_{ll}}{\Lambda^2}Q^{iijj}_{ll} + \Sigma^3_{i,j=1}\frac{c'_{ll}}{\Lambda^2}Q^{ijji}_{ll} \\
& + \Sigma^3_{i=1}\Sigma^2_{j=1}\frac{c^{(1)}_{lq}}{\Lambda^2}Q^{iijj(1)}_{lq} + \Sigma^3_{i=1}\Sigma^2_{j=2}\frac{c^{(3)}_{lq}}{\Lambda^2}Q^{iijj(3)}_{lq} + \Sigma^3_{i=1}\frac{c^{(+)}_{lQ}}{\Lambda^2}Q^{33(+)}_{lq} + \Sigma^3_{i=1}\frac{c^{(-)}_{lQ}}{\Lambda^2}Q^{33(-)}_{lq} + \Sigma^3_{i=1}\Sigma^3_{j=1}\frac{c_{ee}}{\Lambda^2}Q^{iijj}_{ee} \\
& + \Sigma^3_{i=1}\Sigma^2_{j=1}\frac{c_{eu}}{\Lambda^2}Q^{iijj}_{eu} + \Sigma^3_{i=1}\Sigma^2_{j=1}\frac{c_{ed}}{\Lambda^2}Q^{iijj}_{ed} + \Sigma^3_{i=1}\frac{c_{et}}{\Lambda^2}Q^{ii33}_{eu} + \Sigma^3_{i=1}\frac{c_{eb}}{\Lambda^2}Q^{ii33}_{ed} + \Sigma^3_{i,j=1}\frac{c_{le}}{\Lambda^2}Q^{iijj}_{le} + \Sigma^3_{i=1}\Sigma^2_{j=1}\frac{c_{lu}}{\Lambda^2}Q^{iijj}_{lu} \\
& + \Sigma^3_{i=1}\Sigma^2_{j=1}\frac{c_{ld}}{\Lambda^2}Q^{iijj}_{ld} + \Sigma^3_{i=1}\frac{c_{lt}}{\Lambda^2}Q^{ii33}_{lu} + \Sigma^3_{i=1}\frac{c_{lb}}{\Lambda^2}Q^{ii33}_{ld} + \Sigma^2_{j=1}\Sigma^3_{i=1}\frac{c_{qe}}{\Lambda^2}Q^{jjii}_{qe} + \Sigma^3_{i=1}\frac{c_{eQ}}{\Lambda^2}Q^{ii33}_{eq}
\end{aligned}
$$

- **Totally, there are 39 dimension-6 operators** (O_i)

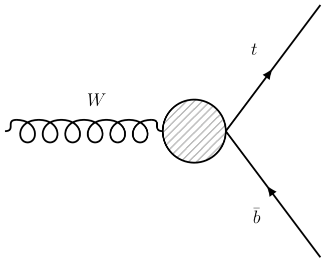
$$\mathcal{L}_{dim6} = \sum_i \frac{c_i}{\Lambda^2} O_i = \sum_i \theta_i O_i$$

Where c_i is called Wilson coefficients

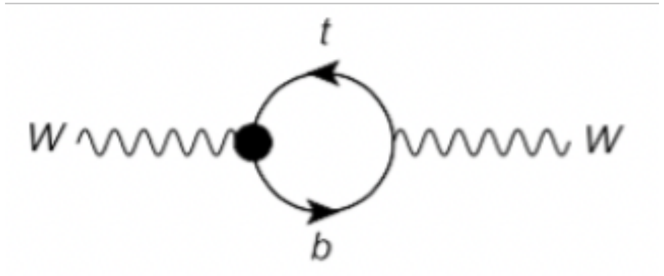
EFT Operators

- How EFT operators contribute to the electroweak process?
  e.g. $O_{tW} = O_{uW}^{33} = (\bar{q}_3 \sigma^{\mu\nu} u_3)\varphi W_{\mu\nu}^{I}$:

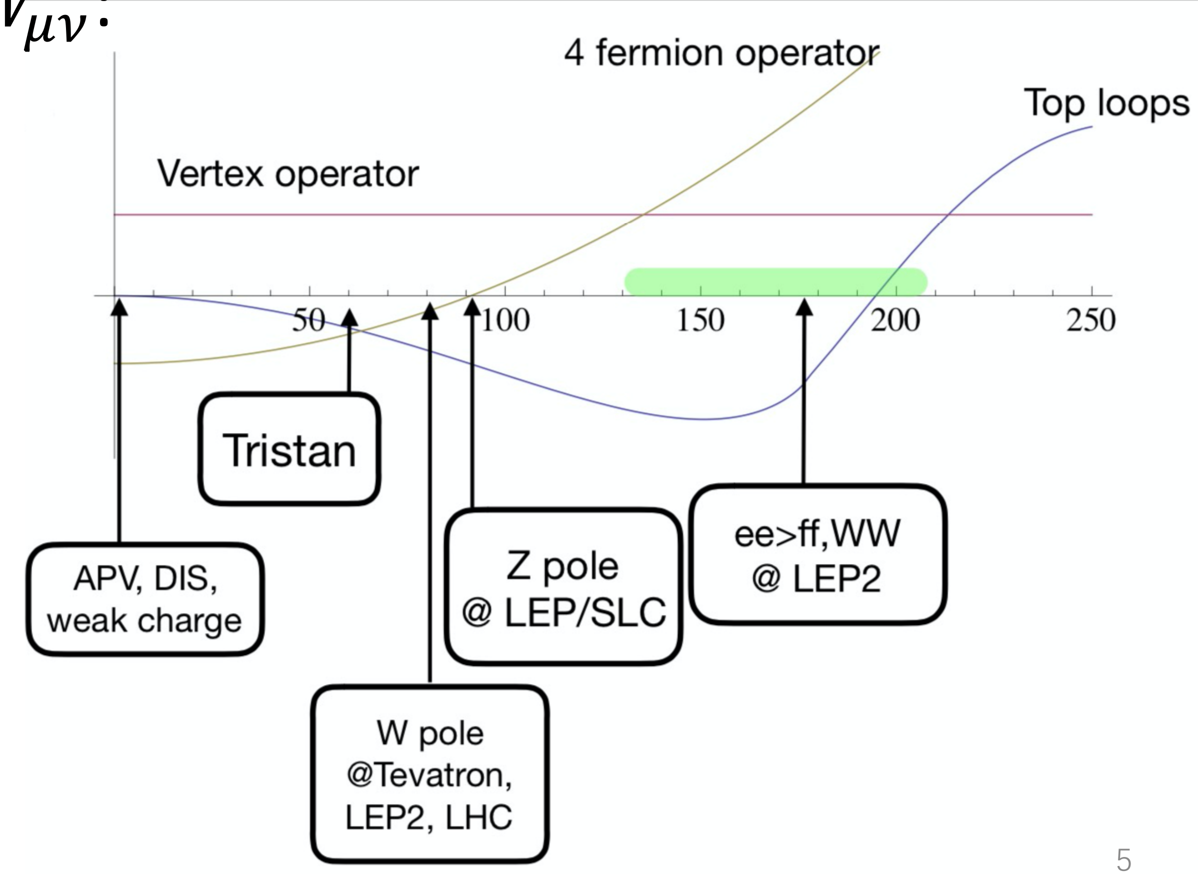
1. Tree-level contribution



2. Loop-level contribution



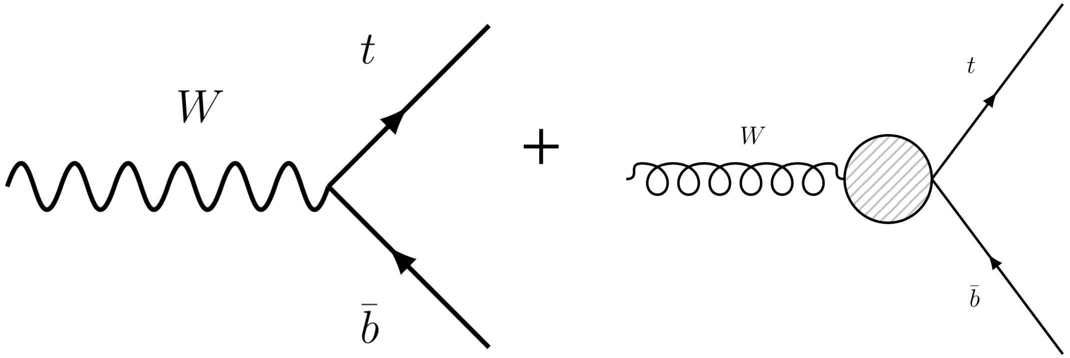
(only consider W boson in the example)



EFT Operators

- O_{tW} on $\Gamma_{W \to t\bar{b}}$

$$\mathcal{M} = \mathcal{M}_{SM} + \mathcal{M}_{dim6}$$



$$\Gamma_{W \to t\bar{b}} \propto |\mathcal{M}_{SM} + \mathcal{M}_{dim6}|^2$$

$$\propto \mathcal{M}_{SM}^2 + 2\mathcal{M}_{SM} \cdot \mathcal{M}_{dim6} + \mathcal{M}_{dim6}^2$$

Γ_{SM} | LO dim6 contribution | High-order contribution

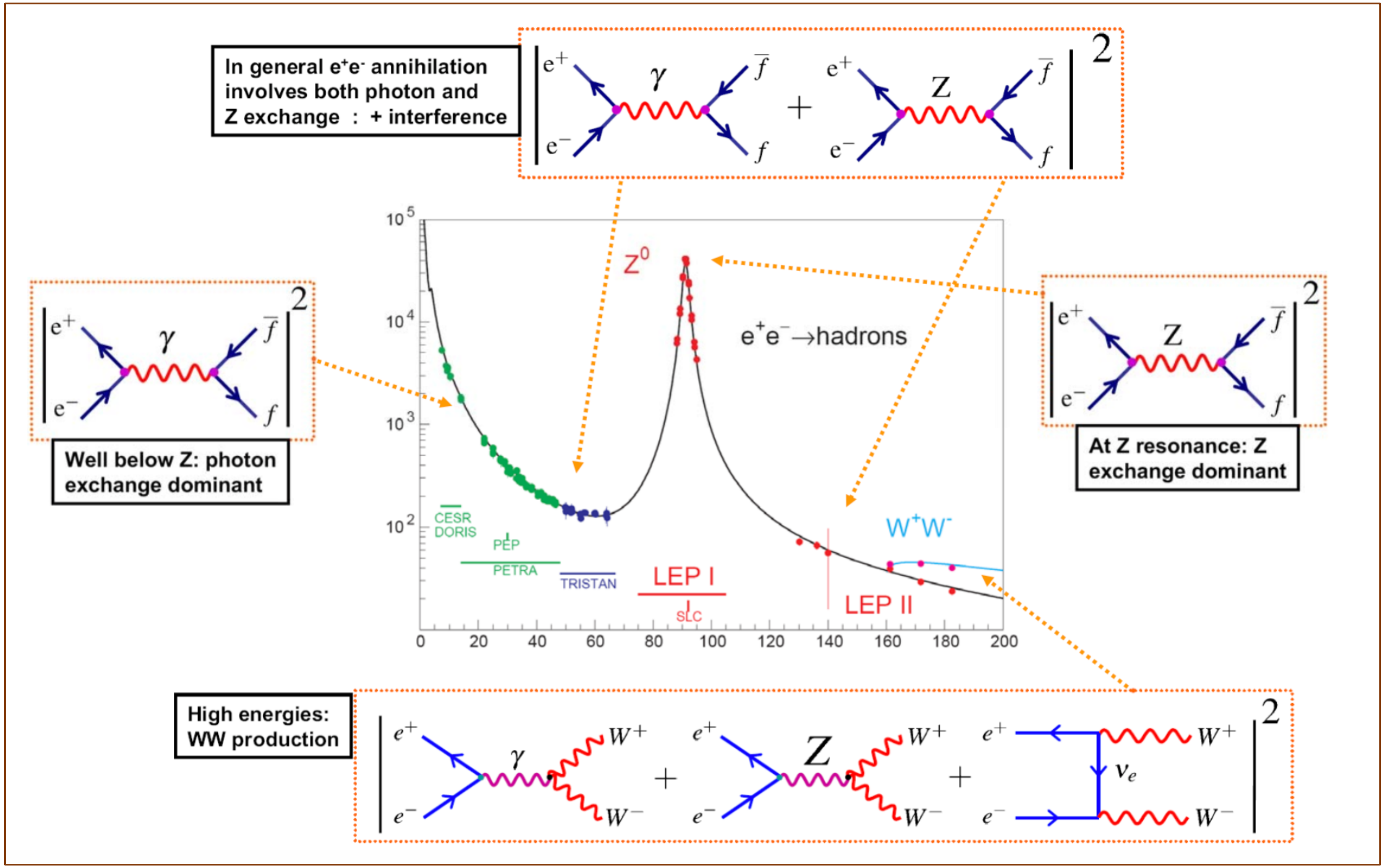
The contribution from different operators can be combined linearly

Process

LEP Experiment:

1. $e^+ e^- \rightarrow f\,\bar{f}$

2. $e^+ e^- \rightarrow W^+ W^-$



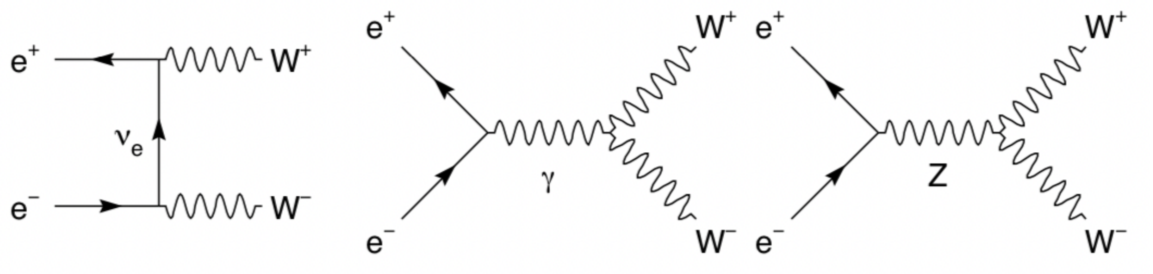
Low Energy:

1. neutrino scattering

2. electron-deuteron scattering

W pair production

- Main feynman diagram:



CC03 diagram

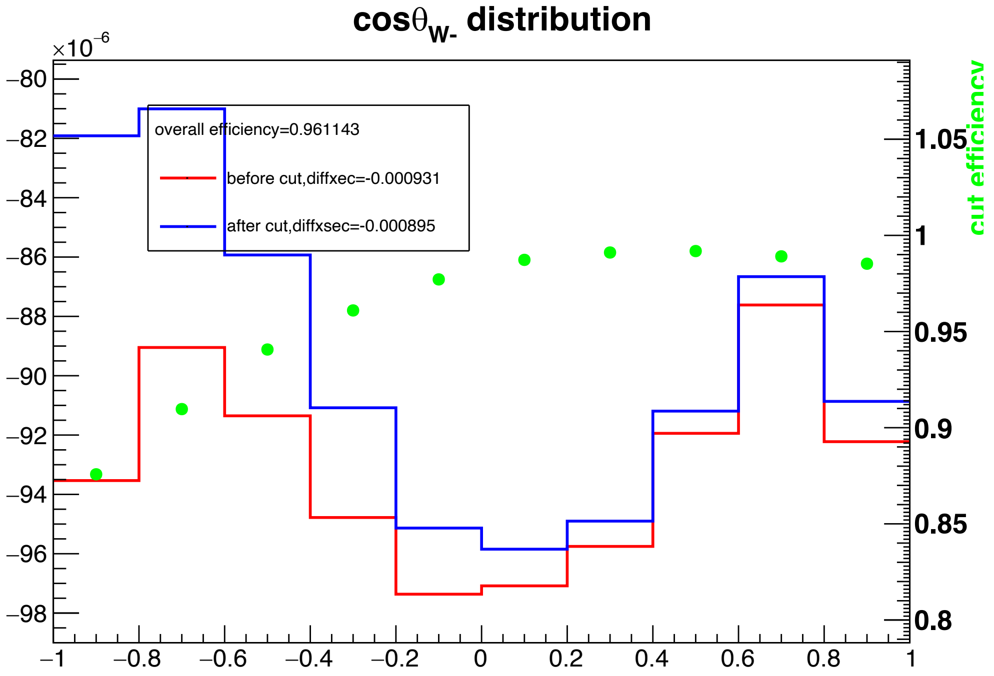
- On-shell or Off-shell?
  - In experiment, W can not be probed directly → it could be off-shell.
  - W- angular distribution is one of the observables → W should be the final state, need to be on-shell.
  - If we simulate $e^+ e^- \rightarrow W^+ W^- \rightarrow 4f$ with W off-shell, which are subset diagrams of process $e^+ e^- \rightarrow 4f$, it will lead to the break of gauge invariance.
  - As a result, we will simulate $e^+ e^- \rightarrow W^+ W^- \rightarrow l\ v\ q\ \bar{q}$ with W on-shell!

W pair production: W- angular distribution

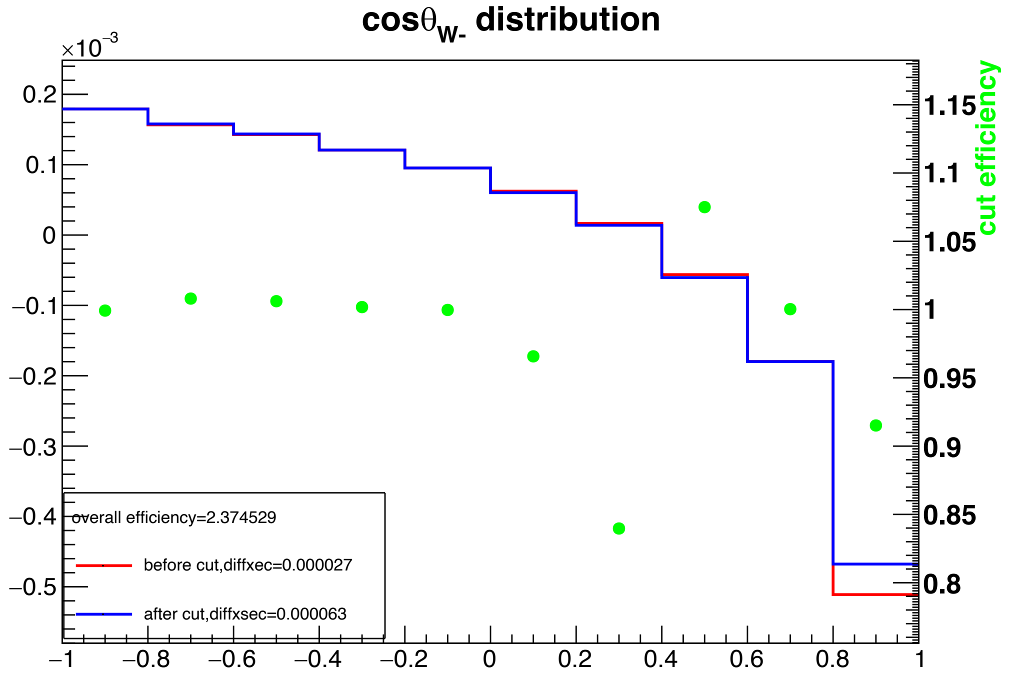
- Angular Distribution: $d\sigma_{WW}/dcos\theta_{W^-}$,where θ_{W^-} is the polar angle between W- and beam. (divided into 10 bins)
- Decay Channel:$e^+ e^- \to W^- W^+ \to l\ v\ q\ \bar{q}\ (l = e, \mu)$ BR~30%
- Angular cut:the angle between the lepton and beam >20 degree
  - Due to the large background in the small angle region at LEP, an angular cut is applied. Additionally, it greatly reduce the difference between $4f$ process and CC03 diagrams.
  - In the experiment, the overall cut efficiency is around 92%, which corresponds to our SM simulation.
  - The cut efficiency in every bin is different.
  - The cut efficiency in every dim6 operators' contribution is different.

W- angular distribution: Angular cut

- cut efficiency of some New Physics operators' contribution:



cpe



cwww

ecm=189GeV

χ^2 Fit

- Introduce χ^2 to measure the goodness of fitting

$$\chi^2 = \left(\vec{y} - A\vec{\theta}\right)^T V^{-1}(\vec{y} - A\vec{\theta})$$

$\vec{y}$ is the difference between the experimental value and SM prediction of observables, A_{ij} is the i-th operator’s contribution to j-th observable , *V* is the covariance matrix of observables

- Set $\nabla\chi^2 = 0$, we can get the Least Square estimator θ and its covariance matrix U:

$$\hat{\vec{\theta}} = (A^T V^{-1} A)^{-1} A^T V^{-1} \vec{y} = B\vec{y}$$

$$U = BVB^T = (A^T V^{-1} A)^{-1}$$

χ^2 Fit: Result

- Marginalized Bound (1σ):
  - Floating all the operator together
  - Altogether, there are 436 observables and 34 operators with non-zero contribution
  - At the best fit point, $\chi^2 = 430.212$
  - At the zero point (which stands for no new physics), $\chi^2 - \chi^2_{min} = 54.7962$, that means it can be included within 2.47σ region.

```
cHQM:49.8513   +-22.6275
cHQP:0.731309  +-0.499049
cpt :10.5563   +-7.42657
cHtb:-464.448  +-321.925
ctW :-3.15425  +-11.6353
cbW :-9.14372  +-5.51773
ctB :85.9912   +-42.1346
cpDC:-3.30731  +-2.36376
cpl3:1.32176   +-0.476584
cpWB:0.984566  +-0.95908
cllp:0.0702804 +-0.122517
cll :0.0190819 +-0.126997
cpl :0.59523   +-0.631228
cle :0.0515366 +-0.0380569
cpe :1.15044   +-1.26211
cpq :-0.163339 +-0.248193
cpq3:1.31961   +-0.484772
cpu :-0.662563 +-0.858626
cpd :0.342007  +-0.86693
clq1:8.08432   +-2.67817
clq3:3.20932   +-1.09643
clu :-2.37828  +-0.779431
cld :-14.3654  +-4.84658
ceq :10.0286   +-3.33585
ceu :-7.7074   +-2.52024
ced :-13.0978  +-4.32221
cee :0.0467404 +-0.0575042
cpb :0.598353  +-0.608004
clQP:-2.52211  +-1.22993
clb :-4.73505  +-2.95697
ceqt:-2.3986   +-1.40424
ceb :6.24316   +-3.06574
cWWW:-3.36584  +-1.21611
cbB :-7.29405  +-3.49446
```

χ^2 Fit: Result

- Individual Bound (1σ):
  - Floating one operator at one time
  - Most of the operators contain the zero point within the 1σ region

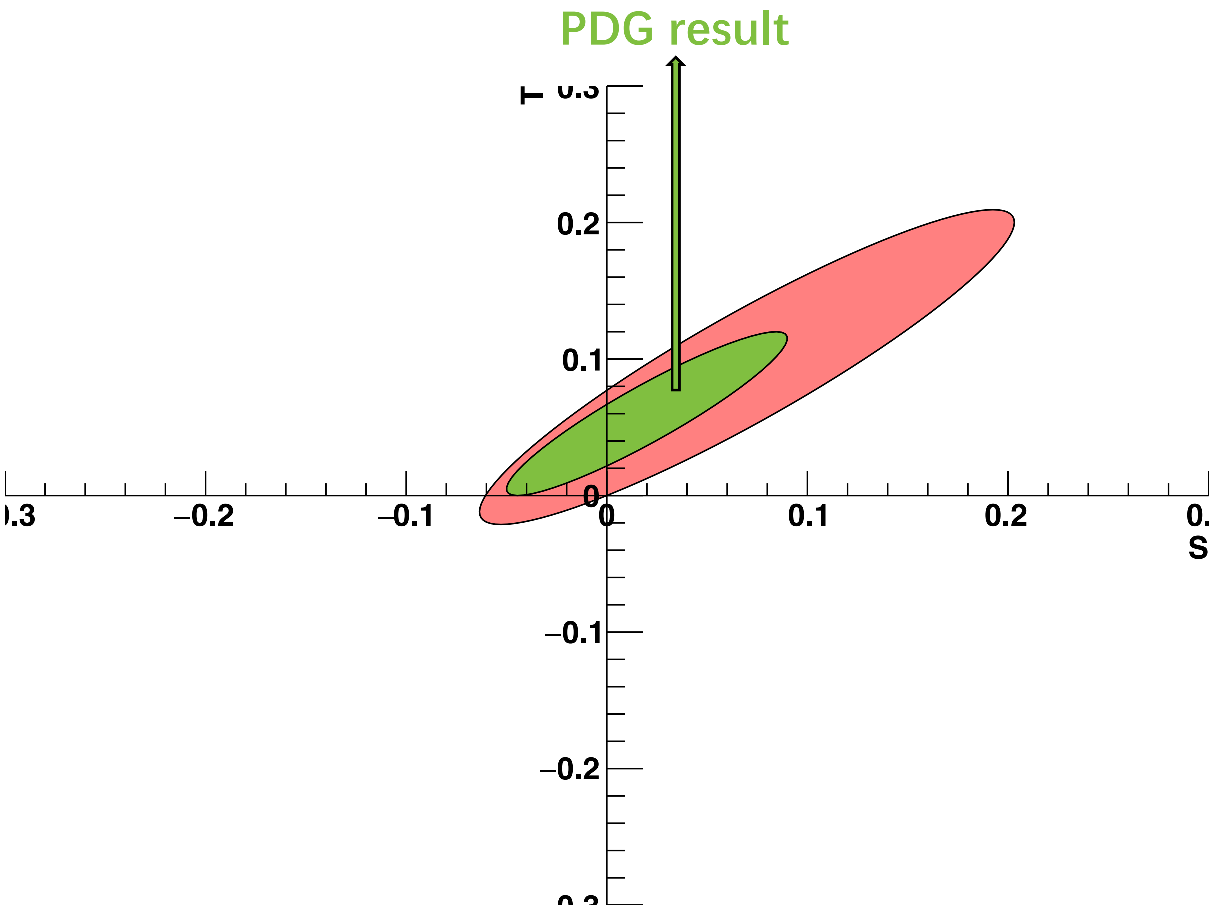
```
cHQM:-0.23668  +-0.194806
cHQP:0.0156382 +-0.0196963
cpt :0.194587  +-0.170321
cHtb:-11.2626  +-8.88384
ctW :-0.154758 +-0.474507
cbW :0.117362  +-0.129101
ctB :0.234797  +-0.330889
cpDC:-0.00929742+-0.00735525
cpl3:-0.0035438+-0.00346937
cpWB:-0.00220751+-0.00257292
cllp:0.00575134+-0.00556312
cll :0.0131002 +-0.0154819
cpl :0.00194129+-0.00421061
cle :0.0095601 +-0.0181492
cpe :-0.00206655+-0.00539786
cpq :-0.0222767+-0.0294185
cpq3:0.00421476+-0.00754999
cpu :0.0135304 +-0.0334877
cpd :-0.0581777+-0.0416187
clq1:0.013926  +-0.0322003
clq3:0.0351024 +-0.0137771
clu :-0.0722251+-0.054454
cld :0.0856226 +-0.0481595
ceq :-0.0591577+-0.0307013
ceu :-0.0780053+-0.033423
ced :0.0066563 +-0.0414798
cee :0.0070912 +-0.0165346
cpb :-0.164183 +-0.106388
clQP:-0.0264519+-0.0372579
clb :0.0800751 +-0.205033
ceqt:-0.00673712+-0.102396
ceb :-0.0536198+-0.0916119
cWWW:-0.148192 +-0.568469
cbB :-0.110743 +-0.114168
```

χ^2 Fit: Result

- Two operators fit (cHQP & cpWB):
  - They are proportional to the Peskin–Takeuchi parameters (T & S), which are important in electroweak fit.
  - Fitting Result (1σ):

    S:0.0698+-0.1333

    T:0.0941+-0.1153
  - Compared with PDG result, we share a similar shape, they have better constraint due to the Higgs data.
  - Compared with other similar study with Electroweak Data, we have a bit tighter result due to more complete data(ee>ll differential xsec)



Thank you!