Light pseudoscalar meson parton distribution function with Dyson-Schwinger Equations

based on: arXiv: 1905.05208, 1912.07529, 2006.14075

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https://indico.ihep.ac.cn/event/11793/

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EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS

FONDAZIONE BRUNO KESSLER
FUTURE BUILT ON KNOWLEDGE

Outline

- **Introduction**
- Dyson-Schwinger Equations
- Pion parton distribution functions
- Kaon parton distribution functions
- Summary and outlook

The pion

- Phenomenology
  - Mediate the interaction between nucleons, Yukawa interaction.
  - Lightest hadron: $\pi^0 \sim 135 MeV$.
  - Pseudoscalar meson: $J^P = 0^-$.
  - Nambu-Goldstone boson of spontaneously broken chiral symmetry.
    - Chiral limit $m_u = 0$, massless pion $m_\pi = 0$.
- The dynamic mechanism driven these phenomenology is QCD.

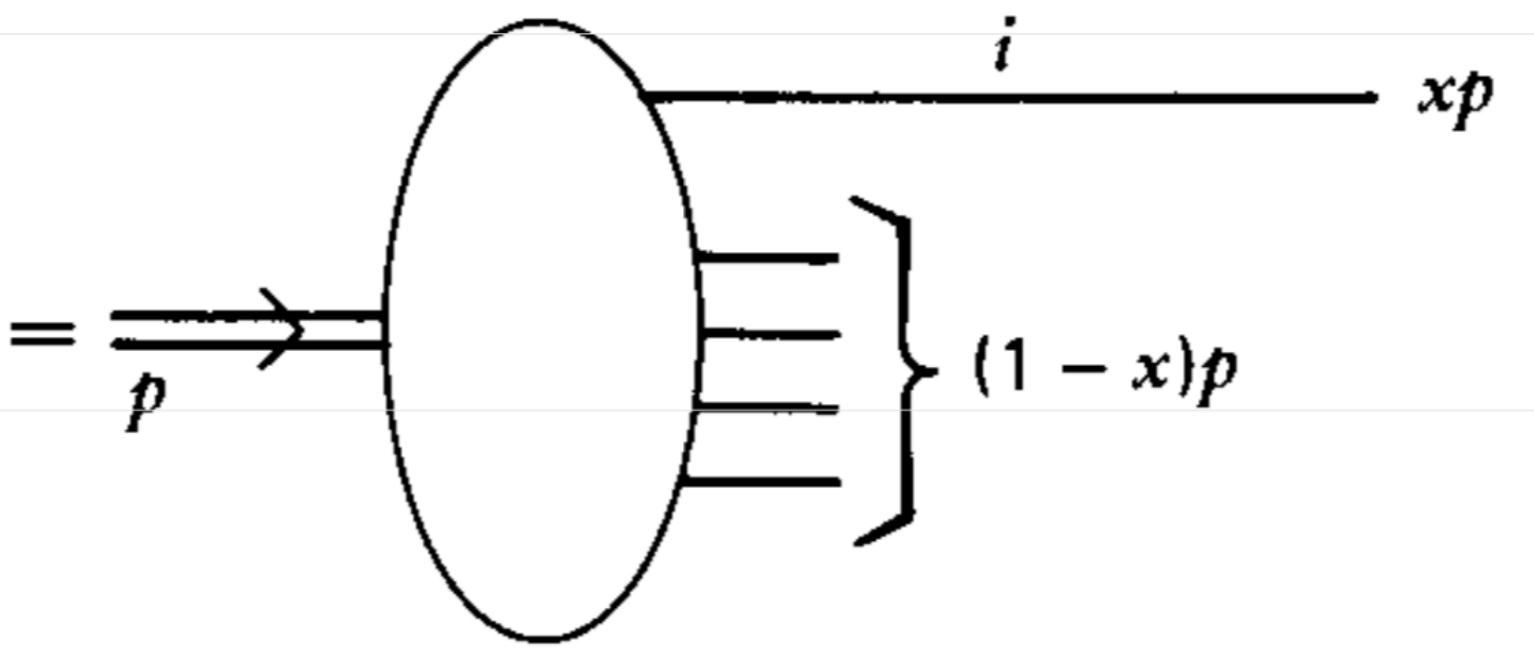
The pion

- Parton model:
  - Valence quarks : 1 up quark + 1 down antiquark
  - Sea quarks
  - Gluons

- Infinite many body dynamic system of quarks and gluons.

- Parton distribution function
  - Probability that parton i carries a fraction x of the pion's momentum p.

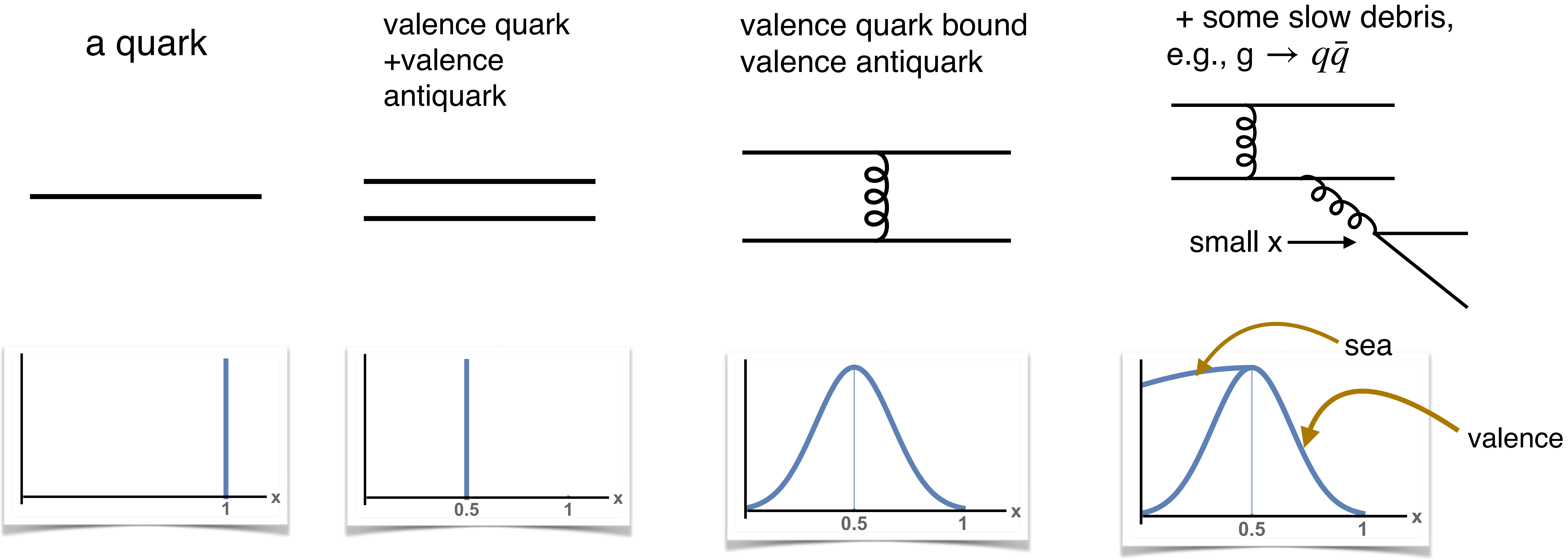
$$f_i(x) = \frac{dP_i}{dx} =$$



The pion parton distribution function (PDF)

If pion is

then $f(x)$ is

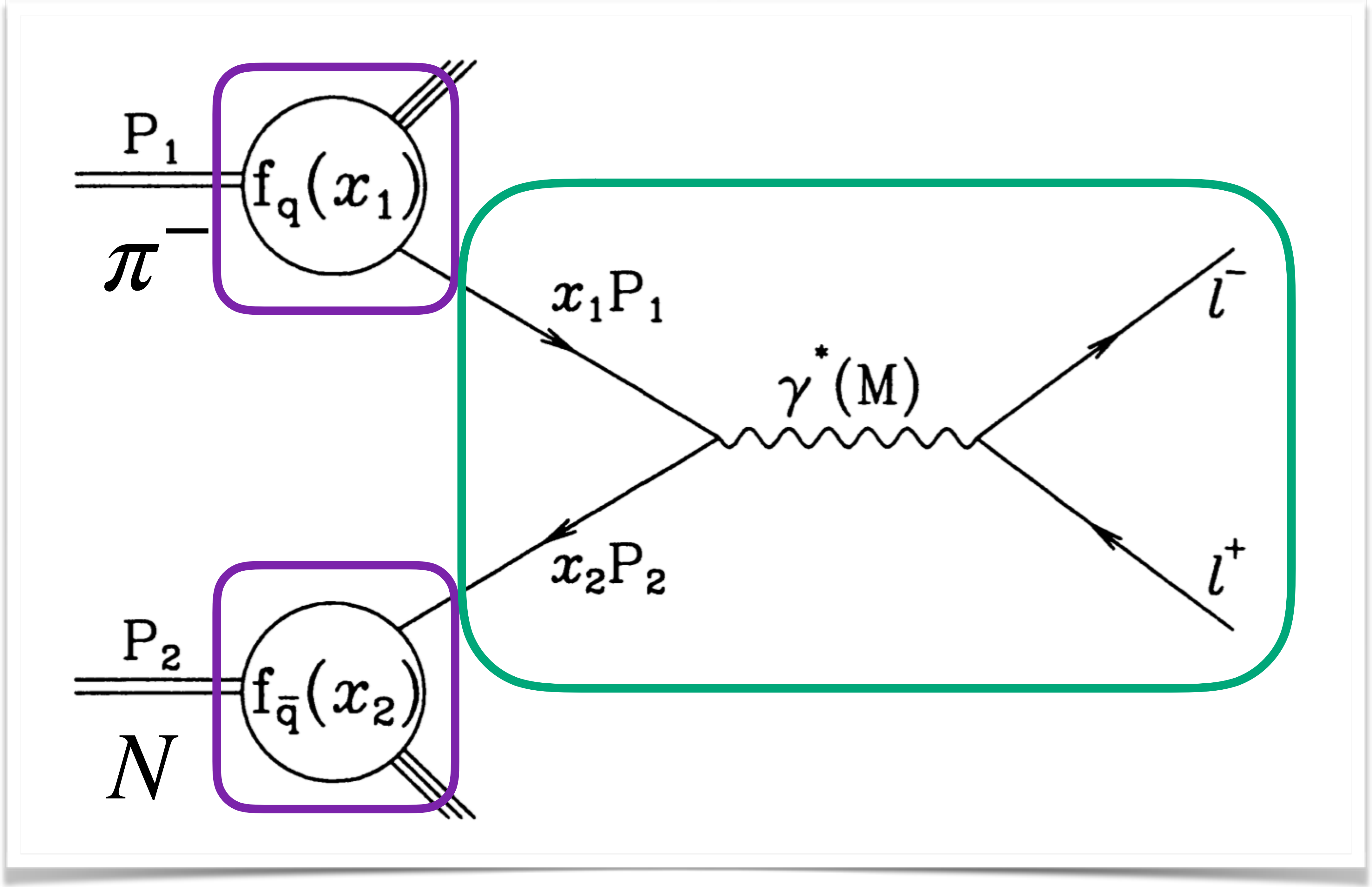


Halzen and Martin (1984)

PDFs depend on the detailed dynamics of the pion, not *a priori* known and obtained from experiment.

Experiments: Valence distribution, $q\bar{q} \to \gamma^\star \to l^+l^-$

- π-induced Drell-Yan $\pi^- N \to \mu^+\mu^- X$



✱ cross section

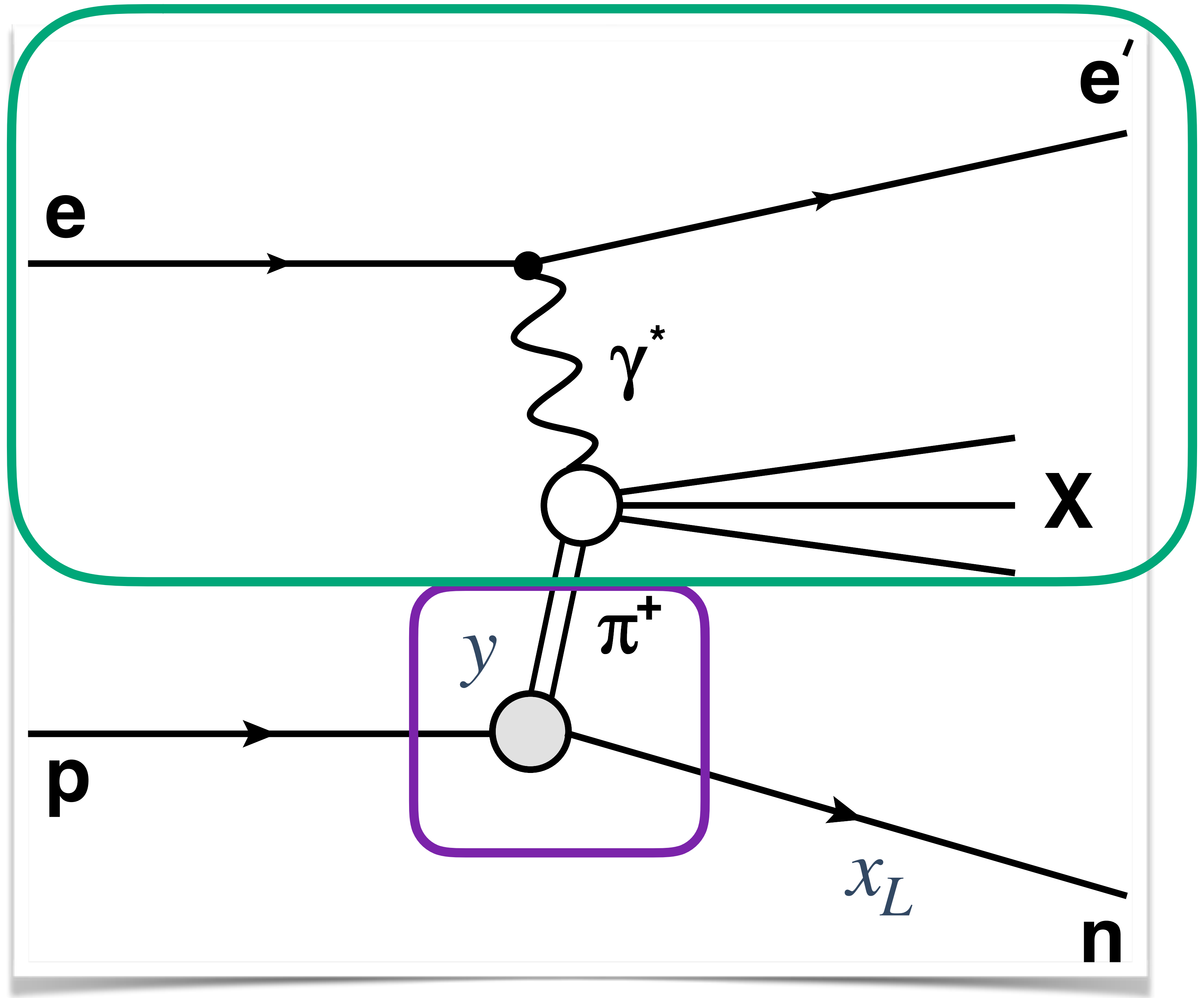
$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^N(x_N, \mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_N, Q/\mu)$$

- $f_i^\pi(x_\pi, \mu)$: pion PDF.

☑ CERN: NA3 (1983), NA10 (1985), Omega (1980) ☑ FNAL: E615 (1989)

Experiments: Valence distribution, $q\bar{q} \to \gamma^\star \to l^+ l^-$

- Leading Neutron DIS $ep \to e'nX$



☑ HERA: ZEUS (2002), H1 (2010)

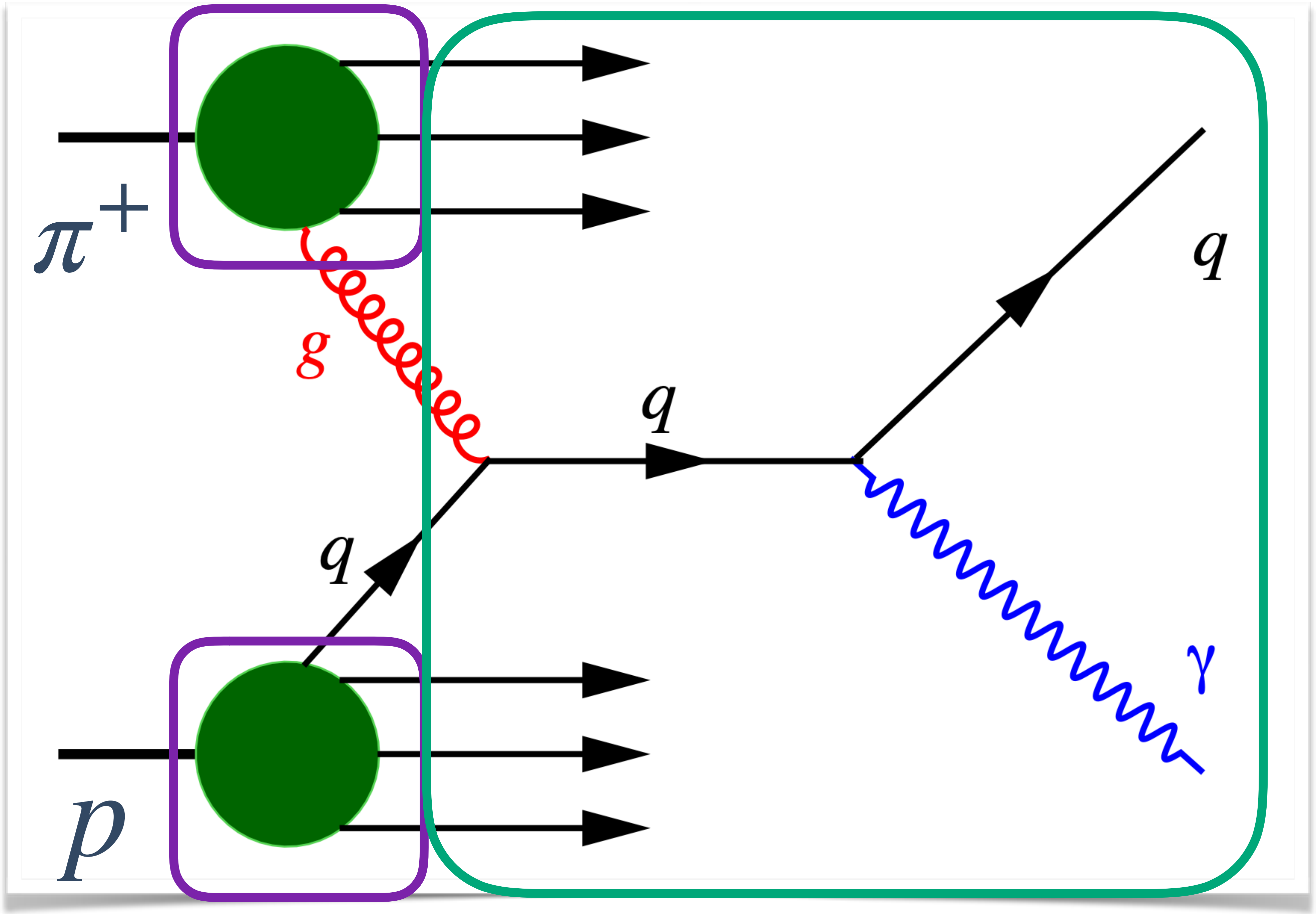
Sullivan process, diffractive process

$$\frac{d\sigma}{dx\,dQ^2\,dy} \sim f_{p\to\pi^+ n}(y) \otimes \sum_q \int_{x/y}^{1} \frac{d\xi}{\xi} C(\xi)\, q\left(\frac{x/y}{\xi}, \mu\right)$$

- $q\left(\frac{x/y}{\xi}, \mu\right)$: pion PDF enter indirectly.

Experiments: Gluon distribution, $gq \to \gamma q, \bar{q}q \to \gamma g$

- Prompt photon production, $\pi^+ p \to \gamma X$



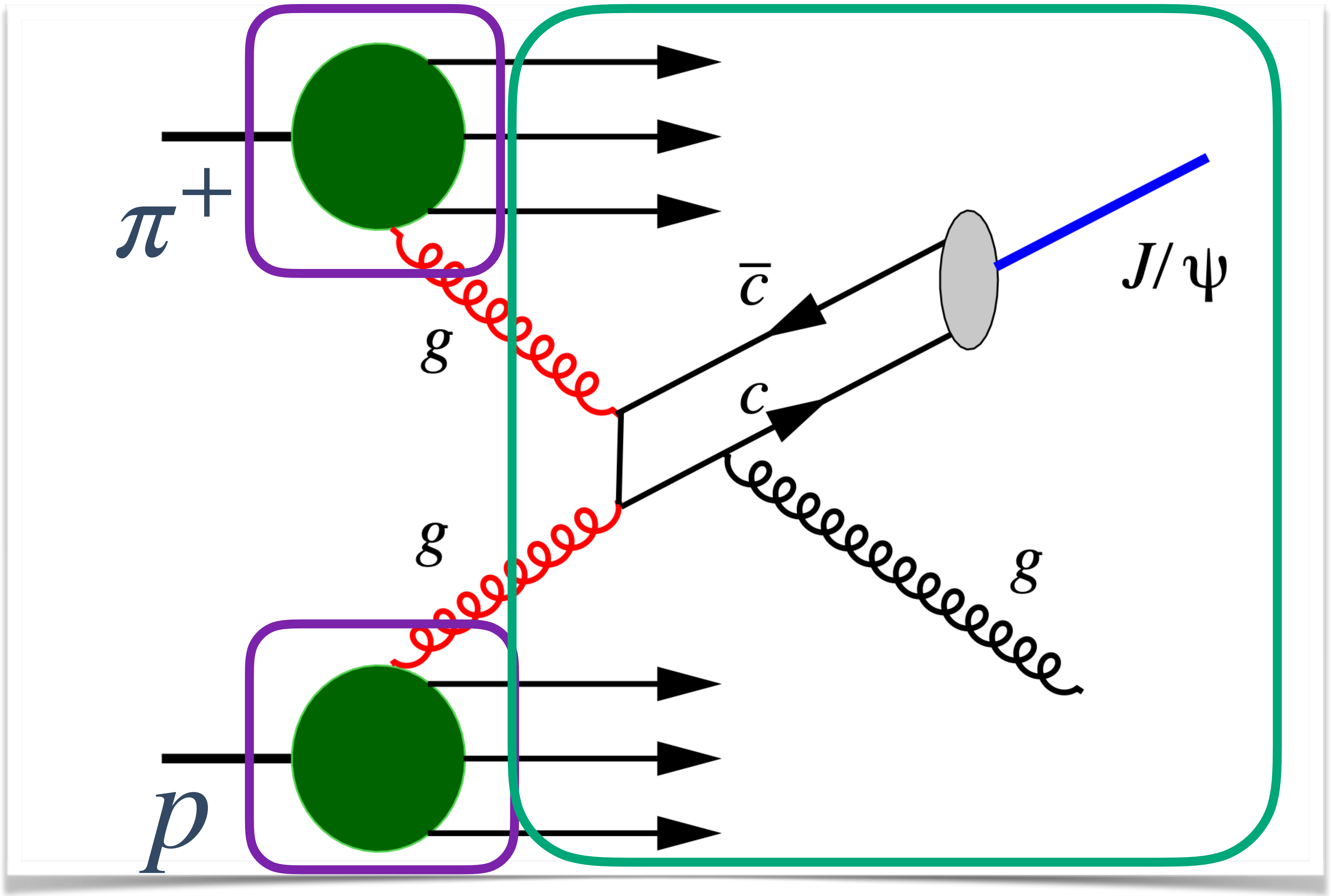
$$d\sigma_{\pi,p} = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i^{\pi}(x_i, \mu) \otimes f_j^{p}(x_j, \mu) \otimes d\sigma_{ij\to\gamma X}(x_i, x_j, \mu)$$

- $f_i^{\pi}(x_i, \mu)$: pion gluon PDF, $i = g$.

- CERN NA24 (1987), WA70 (1988)

Experiments: Gluon and Sea distribution, $gg, \bar{q}q \to c\bar{c} \to J/\psi$

- J/ψ production, $\pi^+ p \to J/\psi\ X$



$$d\sigma_{\pi,p} = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i^{\pi}(x_i, \mu) \otimes f_j^{p}(x_j, \mu) \otimes d\sigma_{ij \to gJ/\psi}(x_i, x_j, \mu)$$

- $f_i^{\pi}(x_i, \mu)$: pion gluon PDF, $i = g$.

- CERN: NA3 (1983)

- Sea distribution: momentum sum rule (MSR)

Experiments:

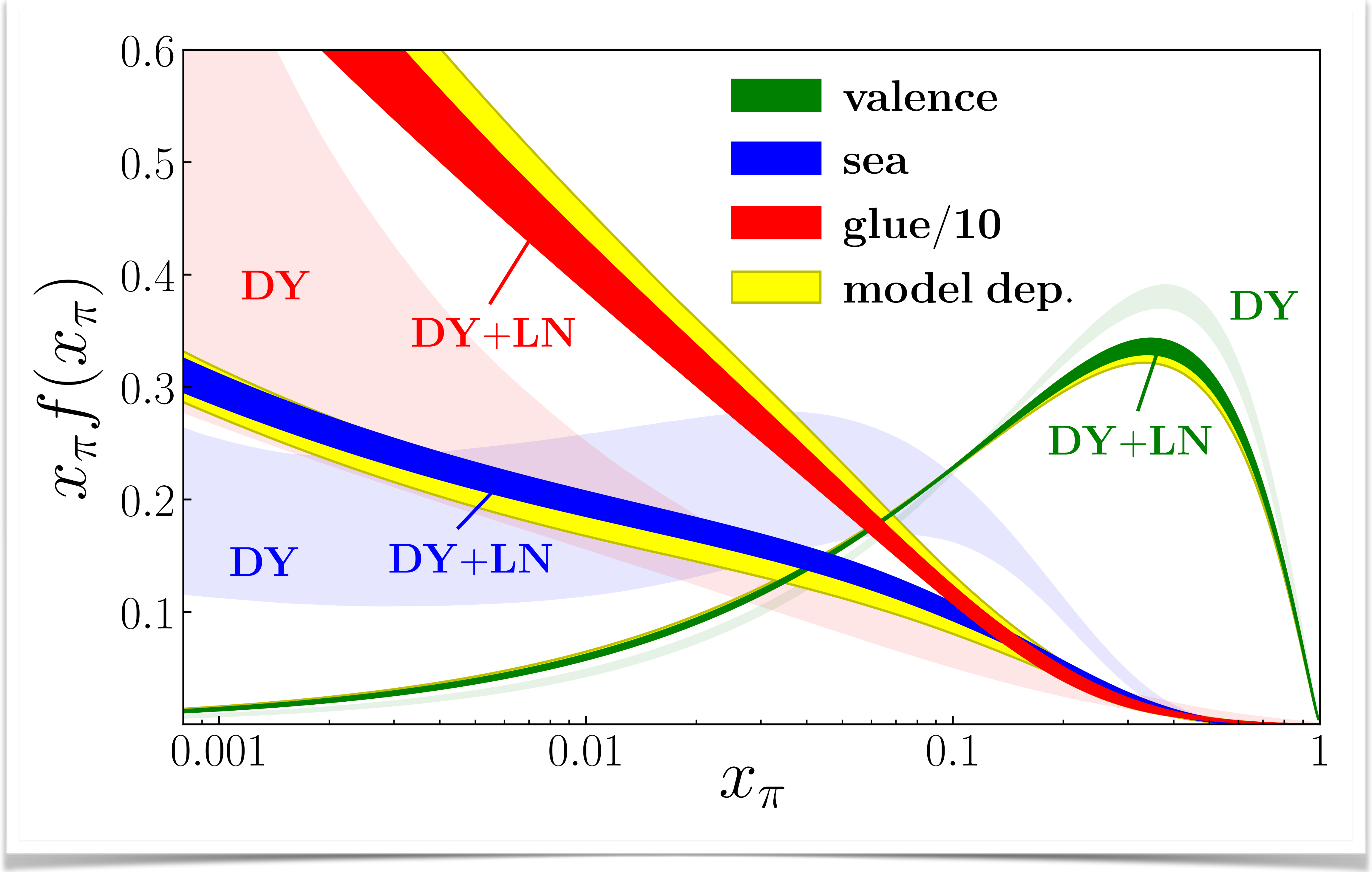
Group	Year	Set	Q_0^2 (GeV^2)	Fac. Sch.	Model	Data	N_f	$\Lambda_{\overline{MS}}^{N_f=4}$ (MeV)
ABFKW	1989	NLO	2.00	$\overline{MS}$	$v^\pi = \gamma X$, DY	WA70,NA24	4	229
					$s^\pi =$ DY	NA3		
					$g^\pi = \gamma X$, MSR	WA70,NA24		
SMRS	1992	10%	4.00	$\overline{MS}$	$v^\pi =$ DY	NA10, E615	4	190
		15%			$s^\pi =$ DY	NA3		
		20%			$g^\pi = \gamma X$, MSR	WA70		
GRV	1992	LO	0.25	LO	$v^\pi =$ ABFKW	WA70,NA24	6	200
		NLO	0.30	$\overline{MS}$	$s^\pi = 0$			
					$g^\pi =$ MSR			
GRSc	1999	LO	0.26	LO	$v^\pi =$ DY, MSR	NA10, E615	3	204
		NLO	0.40	$\overline{MS}$	$s^\pi = (v^\pi/v^p)\, s^p$	H1, ZEUS		299
					$g^\pi = (v^\pi/v^p)\, g^p$	H1, ZEUS		

Michael Klasen, J.Phys.G 28 (2002) 1091-1102

- Valence distribution is mainly determined by DY.
- Gluon distribution is constrained by γX and MSR.
- Leading Neutron DIS can also make contribution.

Leading Neutron (LN) DIS

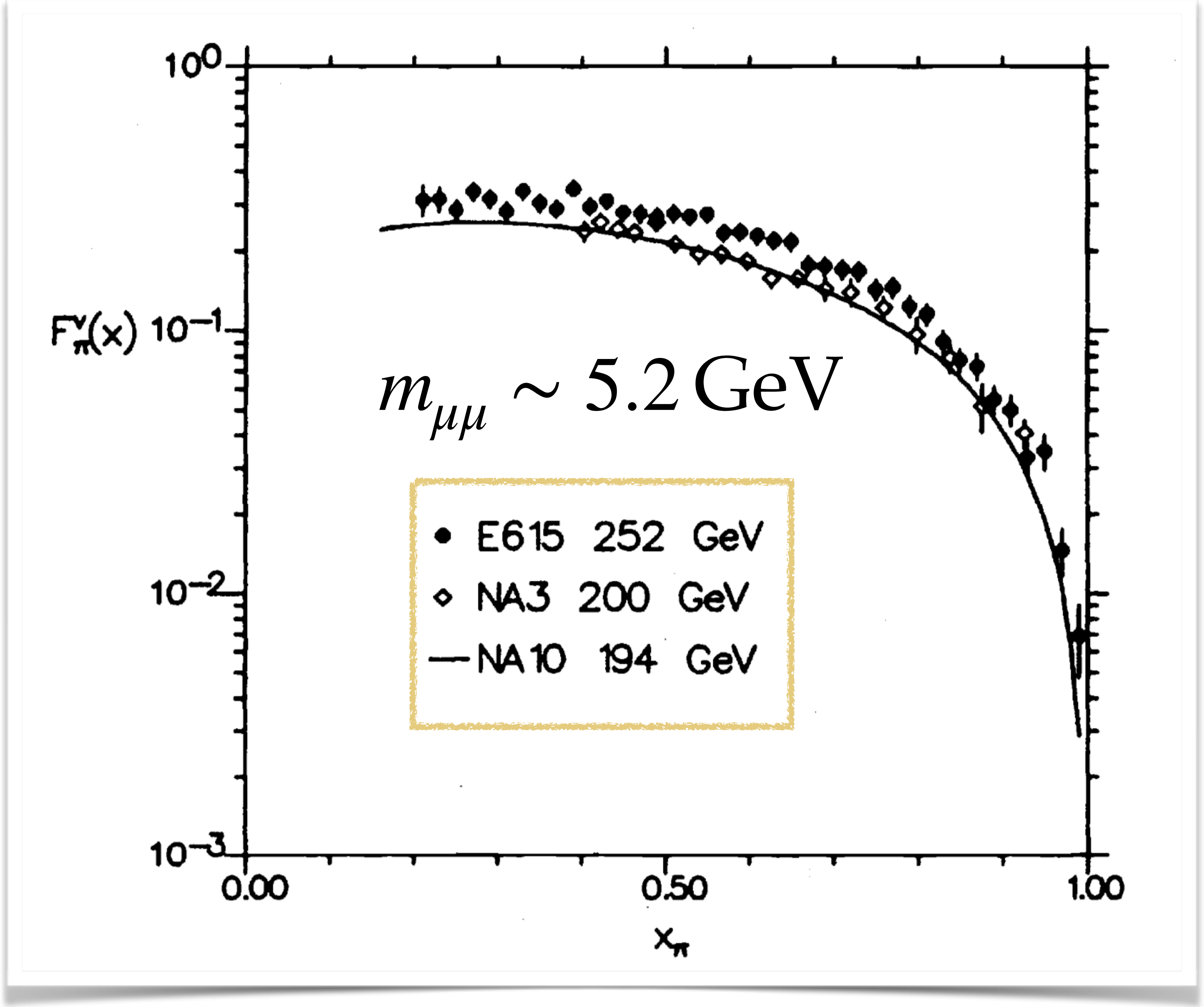
- Global QCD Analysis: JAM Collaboration



Barry, Sato, Melnitchouk, Ji, Phys. Rev. Lett. 121 (2018) no.15, 152001

- DY + LN yields significantly reduced uncertainties on pion sea and gluon distributions at low x.
- Uncertainties on sea and gluon distributions will have impact on valence distribution as a result of momentum sum rule.

Large x behavior of valence PDF



J.S.Conway et al.. Phys. Rev. D 39 (1989) 92-122

$$F_\pi^v(x) = A^v[x^\alpha(1-x)^\beta + \gamma\frac{2x^2}{9m_{\mu\mu}^2}]$$

CERN NA3 (1983):
$\alpha = 0.45 \pm 0.03, \beta = 1.17 \pm 0.02.$

CERN NA10 (1985):
$\alpha = 0.40 \pm 0.02, \beta = 1.17 \pm 0.03.$

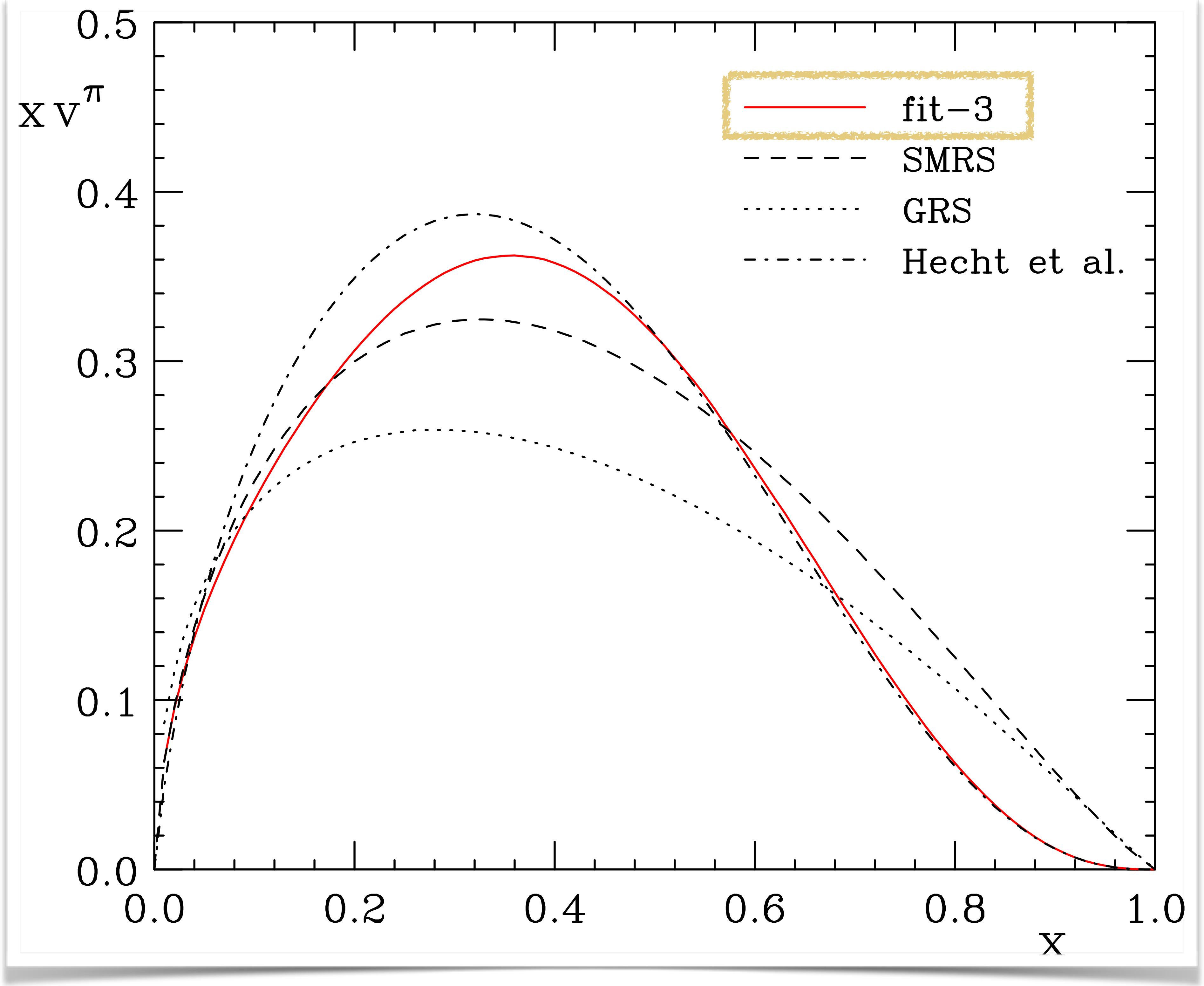
FNAL E615 (1989):
$\alpha = 0.60 \pm 0.03, \beta = 1.26 \pm 0.04.$

$\gamma\frac{2x^2}{9m_{\mu\mu}^2}$: higher twist.

Perturbative QCD: hadronic $Q^2, x \to 1.$	Finite scale calculation or measurement
$\beta = 2$	$\beta = 2 + \delta, (\delta \gtrsim 0)$

E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. 42, 940 (1979)

Soft-Gluon Resummation Reanalysis of E615 Data



Soft-Gluon Resummation and the Valence Parton Distribution Function of the Pion

Matthias Aicher, Andreas Schafer and Werner Vogelsang, PRL 105, 252003 (2010)

E615 (2010): experiment scale
$\beta = 2.03 \pm 0.06$,
Evolve to $Q = 4$ GeV
$\beta = 2.34$.

Hecht et al.. Phys. Rev. C 63, 025213 (2001) DSE in 2001

Non-perturbative nature of pion

- Bound state defined by valence quark and antiquark interacting with strong interaction.
- Nambu-Goldstone boson of spontaneously broken chiral symmetry.
  - Chiral limit $m_u = 0$, massless pion $m_\pi = 0$.
- It is not point-like, internal structure is more complex than is usually imagined.
  - Large x behavior of valence quark PDF.
- Nonperturbative approach.

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- **Dyson-Schwinger Equations**
- Pion parton distribution functions
- Kaon parton distribution functions
- Summary and outlook

Dyson-Schwinger Equations

- Principle of least action
  - Freeman Dyson   Phys. Rev. 75, 1736 (1949)
  - Julian Schwinger   PNAS 37 (7) 452-455 (1951)

$$0 = \int [D\phi] \frac{\delta}{\delta \phi_i} exp[-S + j_i \phi_i]$$

- Equations of motions corresponding to the Green's function.

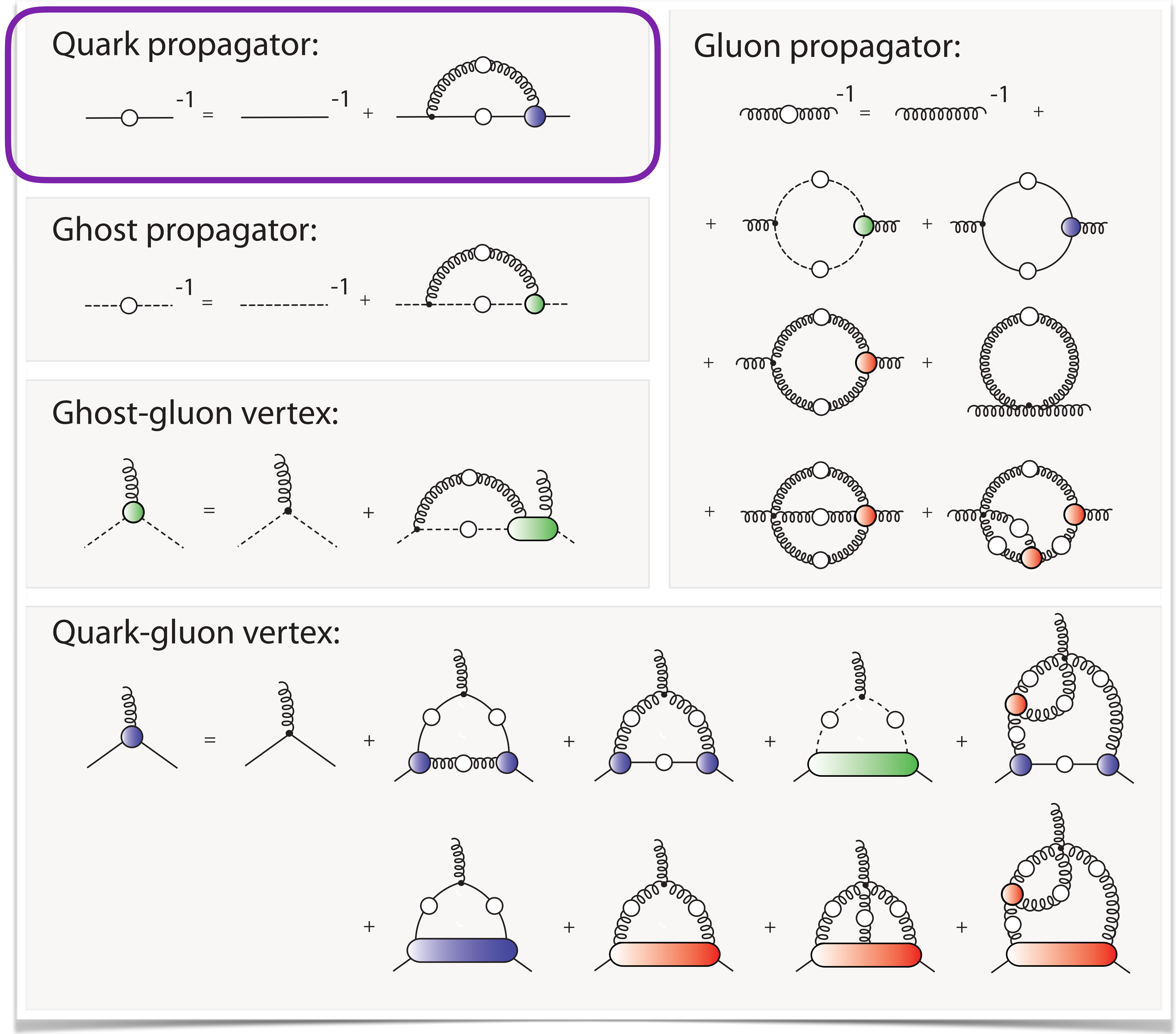
Dyson-Schwinger Equations in QCD

- One-particle Green functions

- Green functions of different orders coupled to each other.
  - Truncation

- Some equations are extremely complicated.
  - Modeling

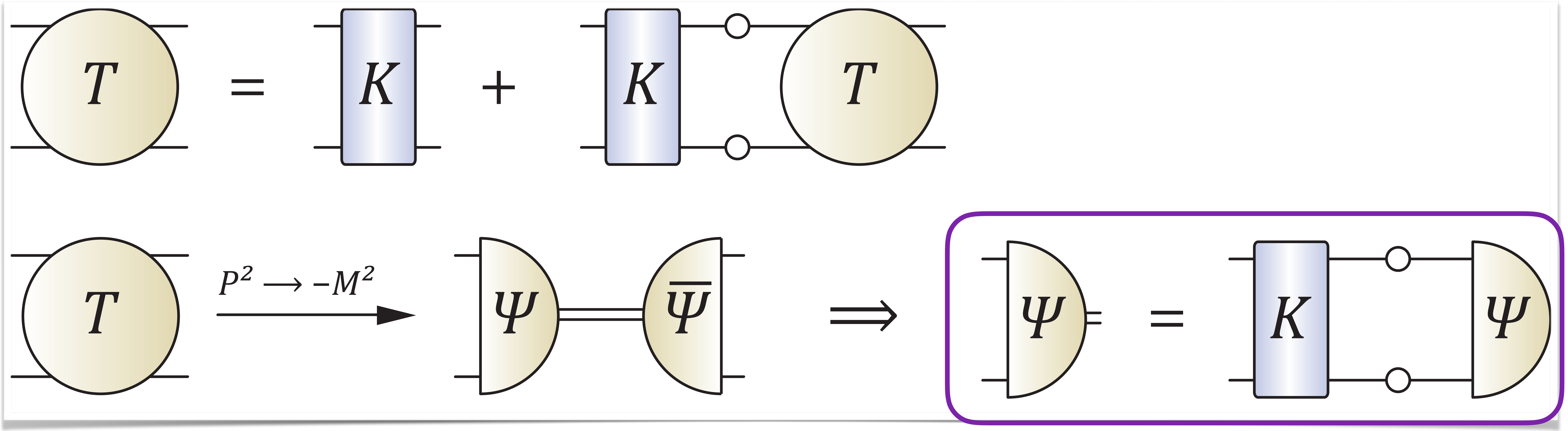
- Nonpurtabative approach



G. Eichmann, arXiv:0909.0703

Bethe-Salpeter Equations

- Two-particle Dyson-Schwinger equation

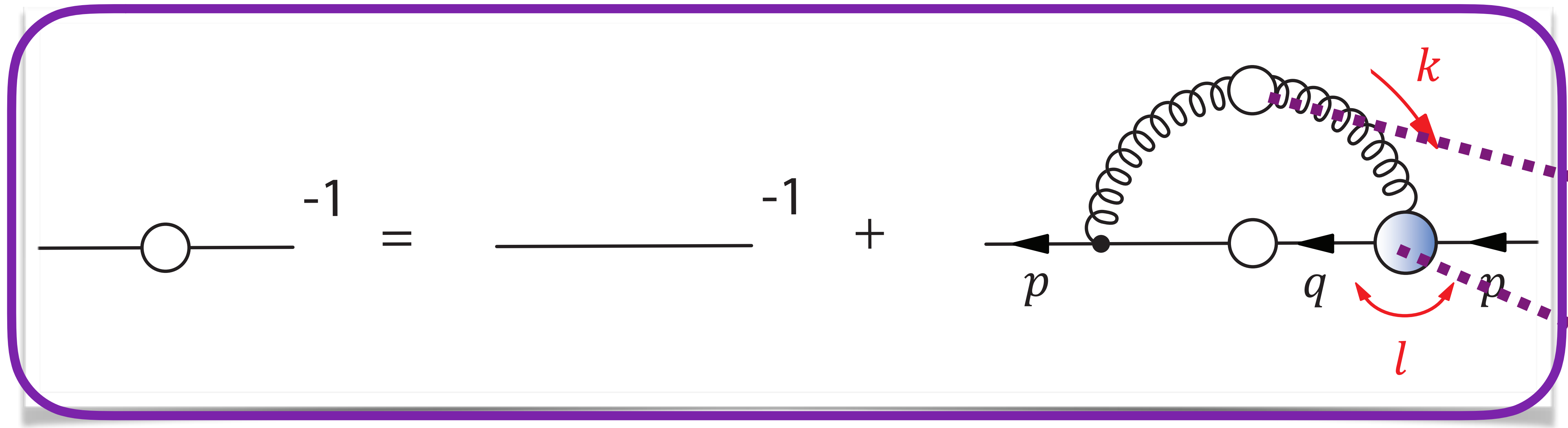


G. Eichmann, arXiv:0909.0703

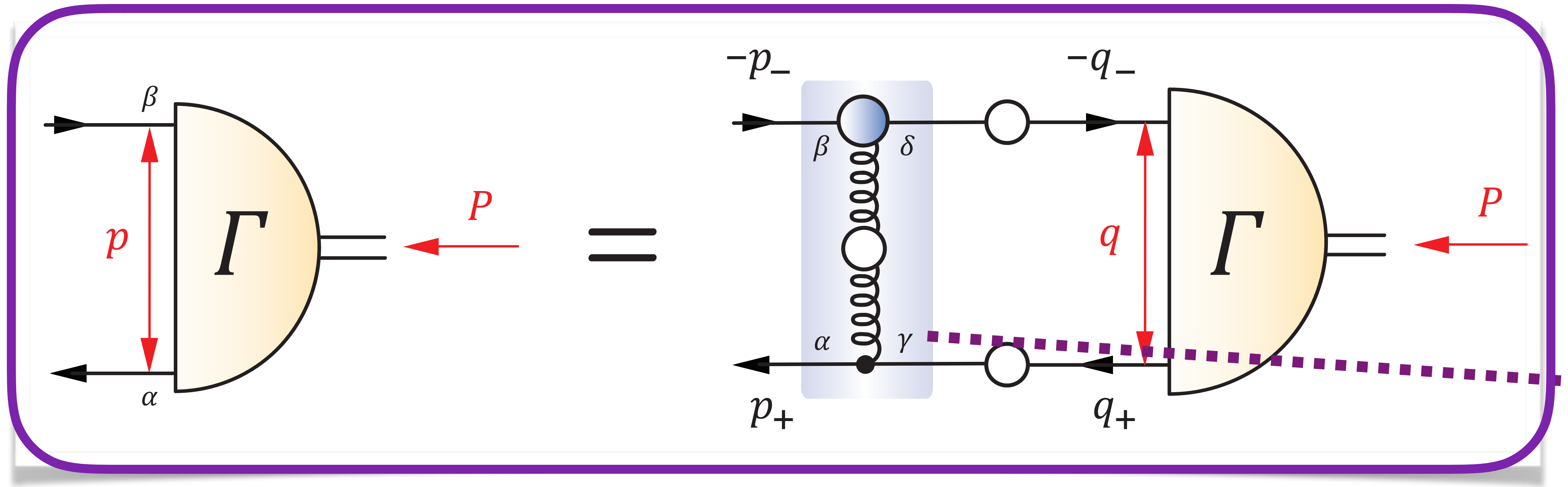
Bottom-up approach

- Green functions of different orders coupled to each other.
- Some equations are extremely complicated.

- Truncation
- Modeling



- Quark propagator
  - gluon propagaor
  - quark gluon vertex



- Bethe-Salpeter amplitude
  - quark-antiquark scattering kernel

G. Eichmann, arXiv:0909.0703

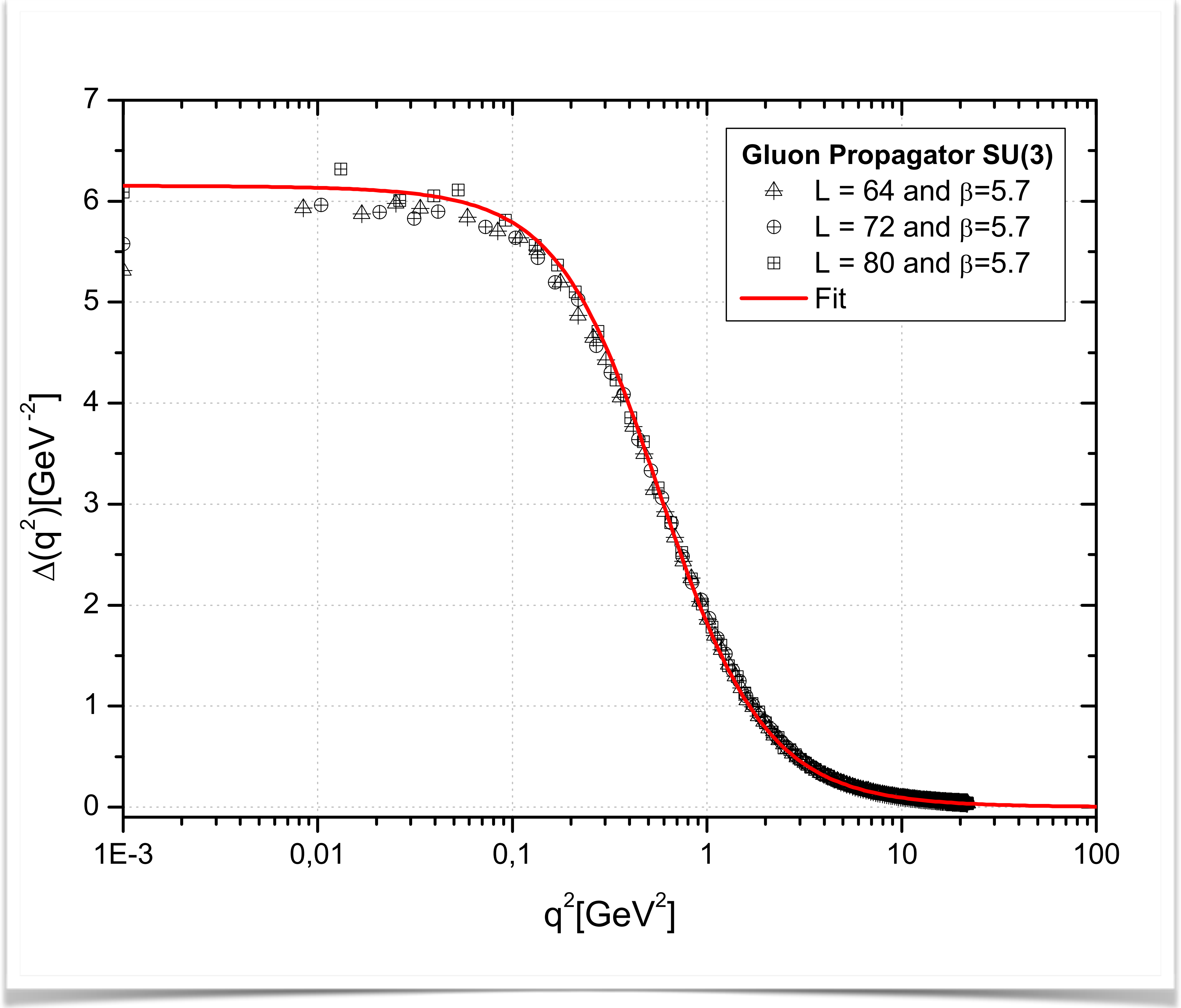
Bottom-up approach

* **Gluon propagaor** $g^2 D_{\mu\nu}(k) = \mathfrak{g}(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$
  - Landau gauge
* Qin-Chang-Liu-Roberts-Wilson (QCLRW)

$$\mathfrak{g}(k^2) = \mathfrak{g}_{IR}(k^2) + \mathfrak{g}_{UV}(k^2)\,, \quad \mathfrak{g}_{IR}(k^2) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2}$$

- $\mathfrak{g}_{UV}(k^2)$ is from one-loop perturbative calculation.
- Gluon propagator is the freezing at a finite (non-vanishing) value.

S. Qin, L. Chang, Y-x. Liu, C.D. Roberts, and D.J. Wilson, PRC 84, 042202R (2011)



A. C. Aguilar, D. Binosi, and J.Papavassiliou, JHEP 07 (2010) 002

Bottom-up approach

- Vector Ward-Green-Takahashi identity
- Axial-Vector Ward-Green-Takahashi identity

$$\begin{cases} iP_\mu \Gamma_\mu(k;P) = S^{-1}(k_+) - S^{-1}(k_-) \\ P_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 + i\gamma_5 S^{-1}(k_-) - 2im\Gamma_5(k;P) \end{cases}$$

- Quark gluon vertex: γ_μ
- Quark-antiquark scattering kernel $\mathcal{K}(k,q,P)_{\alpha\alpha';\beta'\beta}$

$$\begin{cases} \int_{dq} \mathcal{K}(k,q,P)_{\alpha\alpha';\beta'\beta}[S(q_+) - S(q_-)]_{\alpha'\beta'} = -\int_{dq} g^2 \mathcal{D}_{\mu\nu}(k-q)\frac{\lambda^a}{2}\gamma_\mu [S(q_+) - S(q_-)]\frac{\lambda^a}{2}\gamma_\nu \,. \\ \int_{dq} \mathcal{K}(k,q,P)_{\alpha\alpha';\beta'\beta}[S(q_+)\gamma_5 + \gamma_5 S(q_-)]_{\alpha'\beta'} = -\int_{dq} g^2 \mathcal{D}_{\mu\nu}(k-q)\frac{\lambda^a}{2}\gamma_\mu [S(q_+)\gamma_5 + \gamma_5 S(q_-)]\frac{\lambda^a}{2}\gamma_\nu \,. \end{cases}$$

- Rainbow Ladder approximation: $[\mathcal{K}(k,q,P)]^{\mathrm{RL}}_{\alpha\alpha';\beta'\beta} = -g^2 \mathcal{D}_{\mu\nu}(k-q)\left[\frac{\lambda^a}{2}\gamma_\mu\right]_{\alpha\alpha'} \otimes \left[\frac{\lambda^a}{2}\gamma_\nu\right]_{\beta'\beta}$

The pion with Dyson-Schwinger Equations

- Axial-Vector Ward-Green-Takahashi identity in chiral limit $P_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 + i\gamma_5 S^{-1}(k_-)$
- Quark propagator $S^{-1}(k) = i\gamma \cdot k A(k^2) + B(k^2)$ $\lim_{P^2 \to 0} P_\mu \Gamma_{5\mu}(k,0) = i\gamma_5 B(k^2) \neq 0$
- Axial vector vertex $\Gamma_{5\mu}(k,P) \xrightarrow{P^2=-M_\pi^2} \frac{f_\pi P_\mu}{P^2+M^2} \Gamma_\pi(k;P)$ $\lim_{P^2 \to 0} P_\mu \Gamma_{5\mu}(k,0) = i\gamma_5 f_\pi E_\pi(k;P=0)$

- Goldberger Treiman relation: $f_\pi E_\pi(k;P=0) = B(k^2)$

  - Dynamical chiral symmetry breaking (DCSB) ⇔ Nambu-Goldstone theorem
    - Pion exists if, and only if, mass is dynamically generated.
    - Algebraically explain why pion is massless in the chiral limit.
    - Two body problem solved, almost completely, once solution of one body problem is known.

P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B 420 (1998) 267-273

The pion with Dyson-Schwinger Equations

- Axial-Vector Ward-Green-Takahashi identity

$$P_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-) - 2im\Gamma_5(k;P)$$

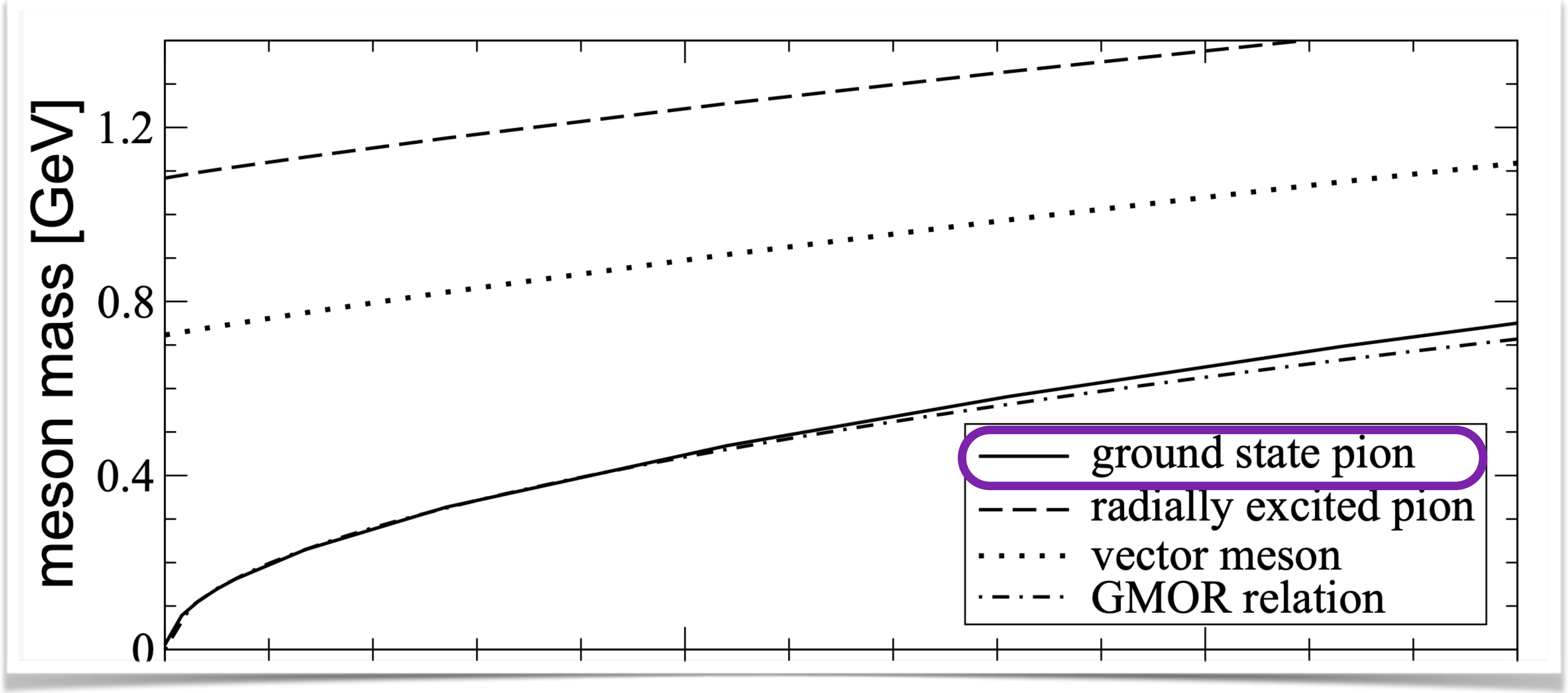
- Axial vector vertex $\Gamma_{5\mu}(k,P) \xrightarrow{P^2=-M_\pi^2} \dfrac{f_\pi P_\mu}{P^2+M^2}\Gamma_\pi(k;P)$

- Pseudoscalar vertex $\Gamma_5(k,P) \xrightarrow{P^2=-M_\pi^2} \dfrac{-ir_\pi}{P^2+M_\pi^2}\Gamma_\pi(k;P)$

- Gell-Mann Oakes Renner (GMOR) relation:

$$f_\pi^2 M_\pi^2 = 2m_u \langle \bar{q}q \rangle^0$$

- Ground state pion: $M_\pi^2 \propto m_u$



P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B 420 (1998) 267-273

P. Maris and P. C. Tandy, Nucl.Phys.B Proc.Suppl. 161 (2006) 136-152

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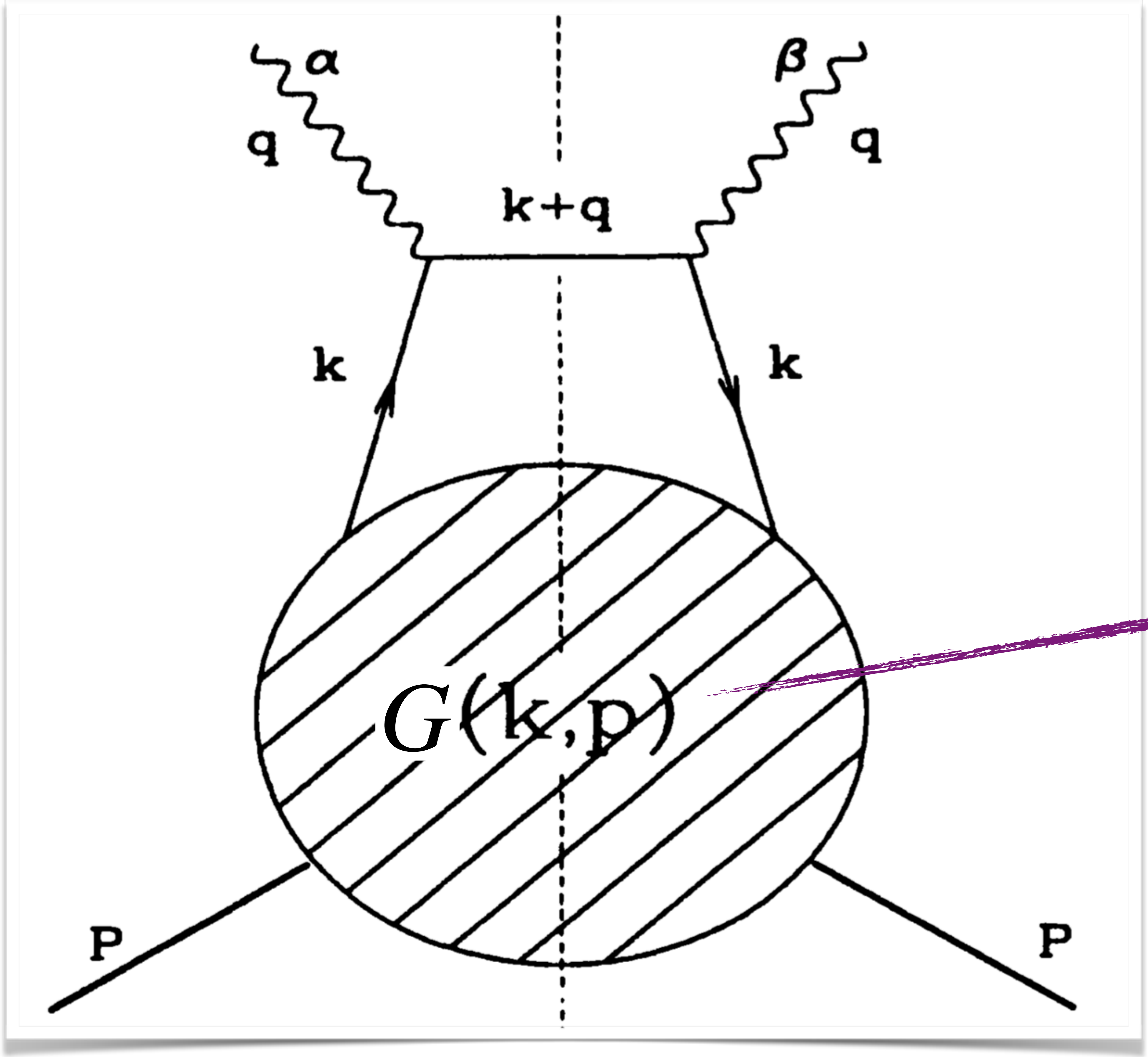
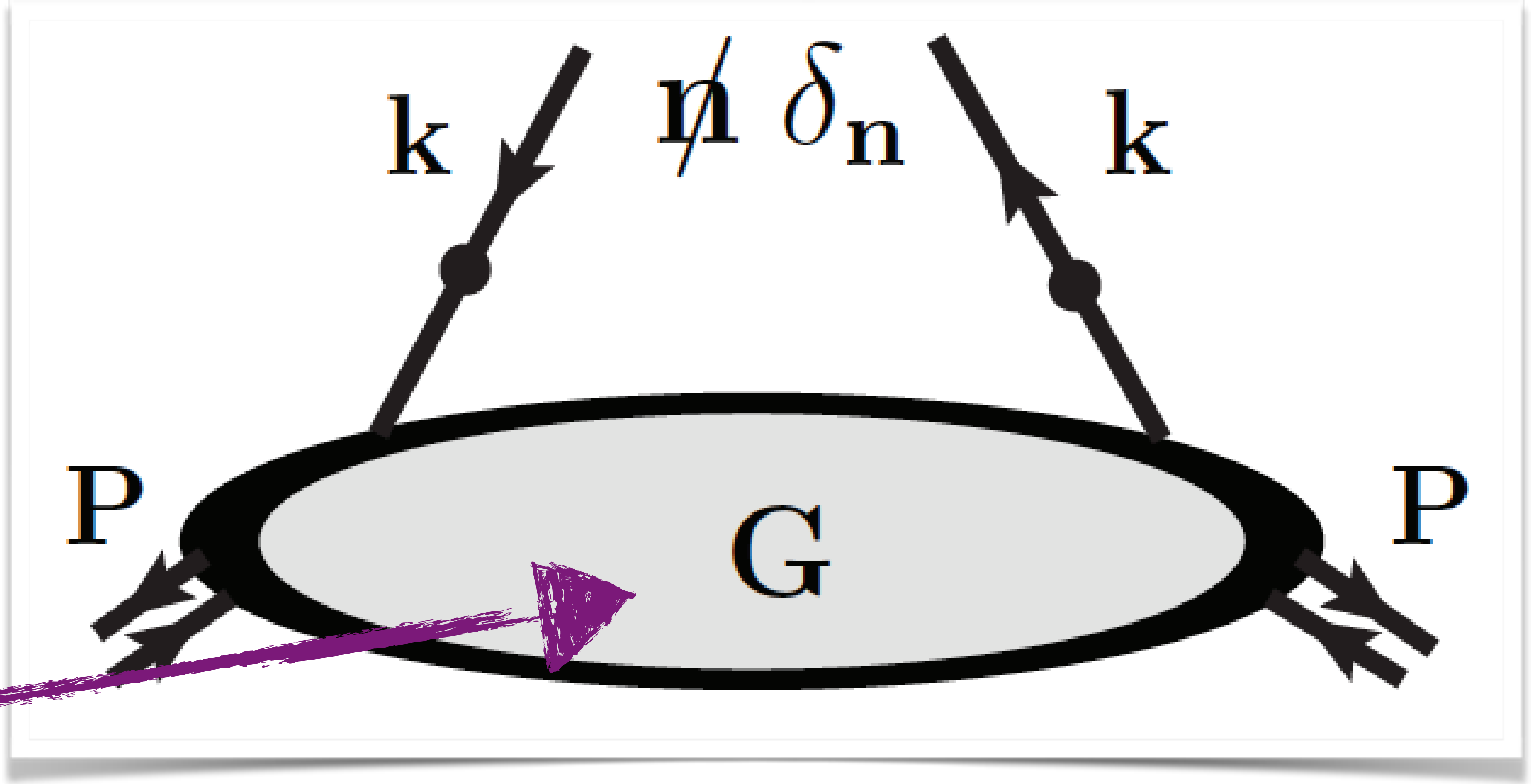
Pion valence quark PDF: general definition

"Handbag diagram"



R. K. Ellis, W. J. Stirling, B. R. Webber, QCD and Collider Physics (2003)

Twist-2 valence quark PDF



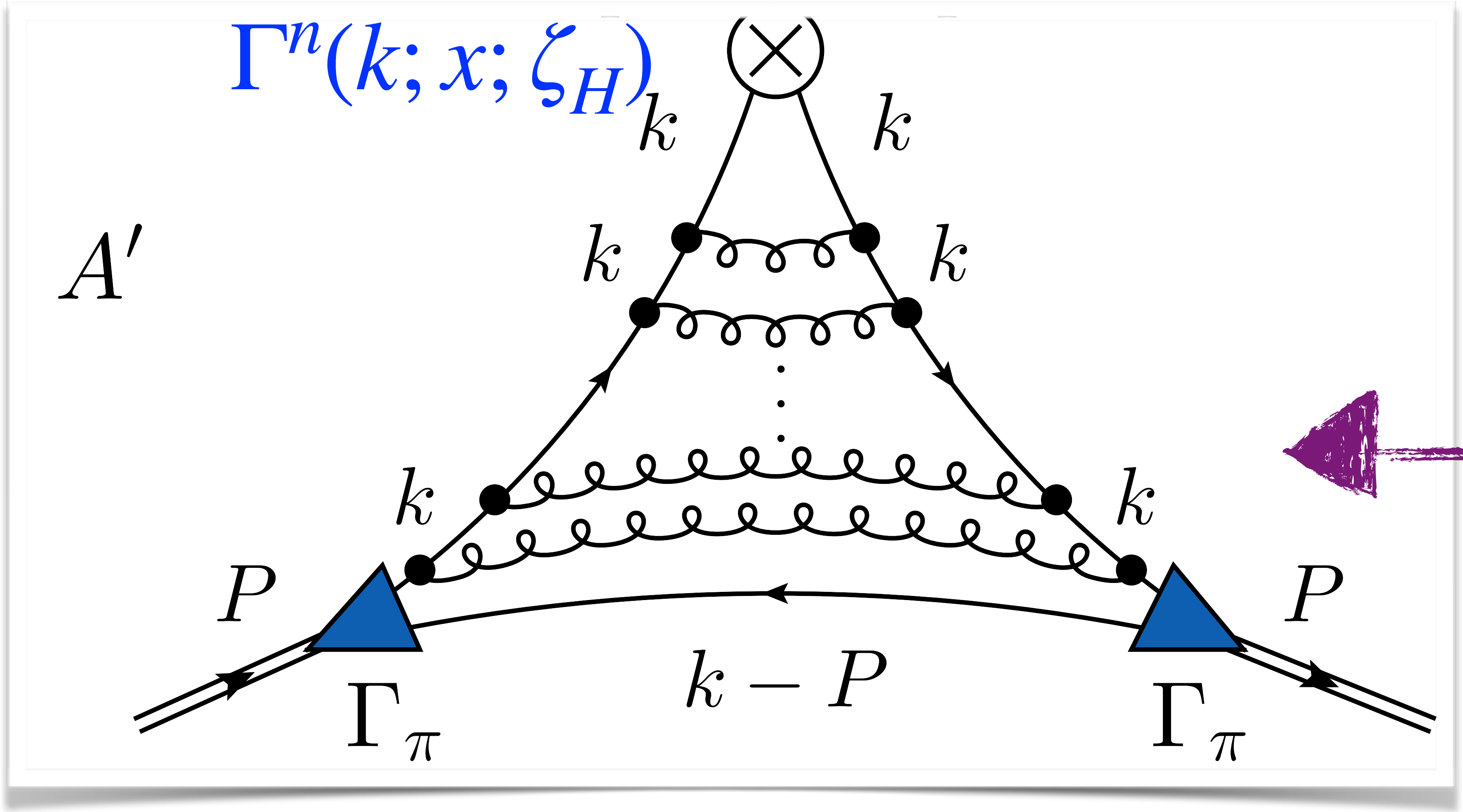
$$q(x) = \int dk \delta(n \cdot k - xn \cdot P) Tr[i\gamma \cdot n G(k, P)]$$

- Scattering amplitude $G(k, P)$

T. Nguyen et al..Phys. Rev. C 83 (2011) 062201

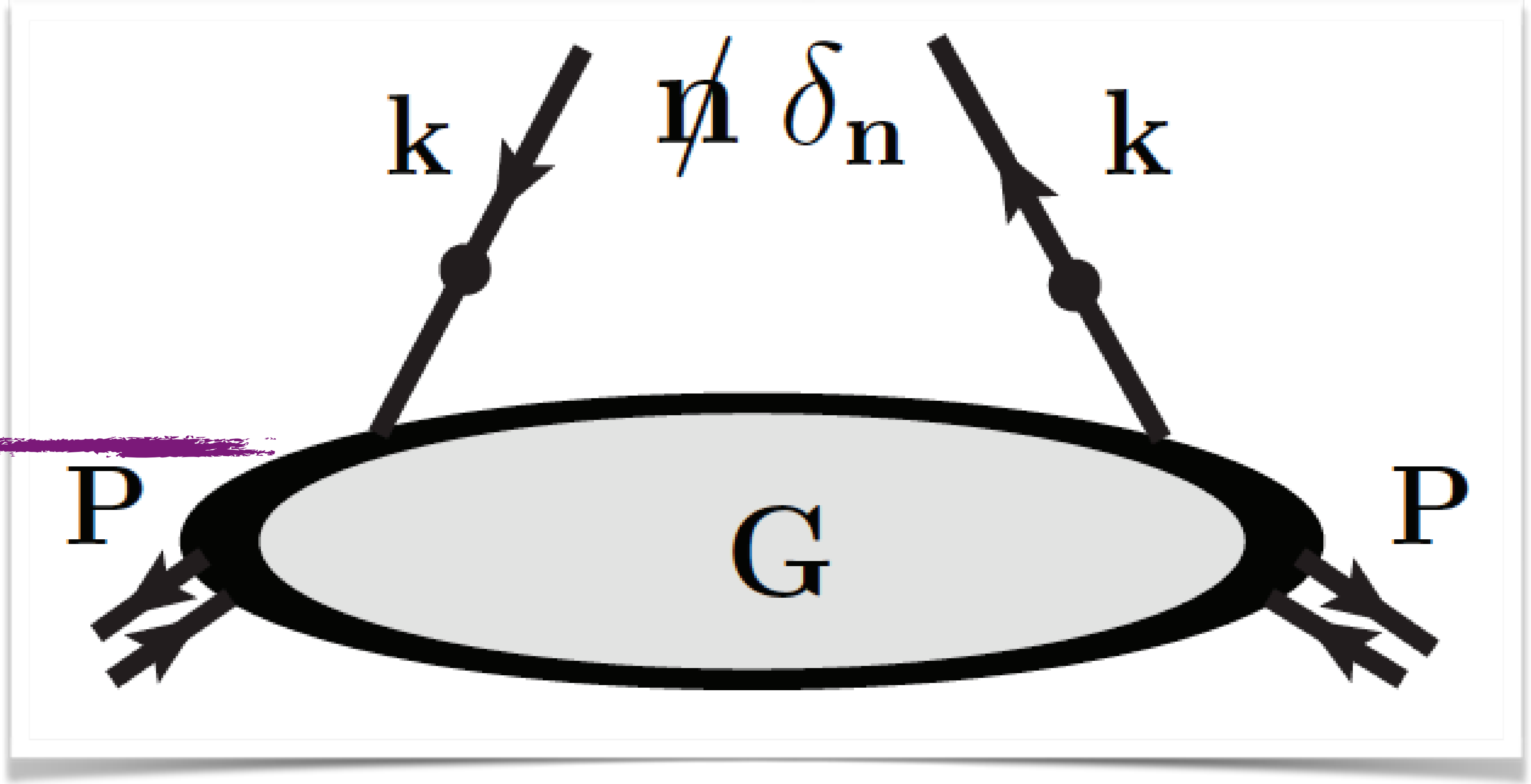
Pion valence quark PDF: rainbow-ladder approximation

Rainbow Ladder approximation



$$
\begin{aligned}
&i\gamma \cdot n G(k, P) \\
&= i\Gamma_\pi(k_\eta, -P)\, S(k_\eta) i\Gamma^n(k;x;\zeta_H)\, S(k_\eta) \\
&\quad \times i\Gamma_\pi(k_{\bar\eta}, P)\, S(k_{\bar\eta})
\end{aligned}
$$

Twist-2 valence quark PDF

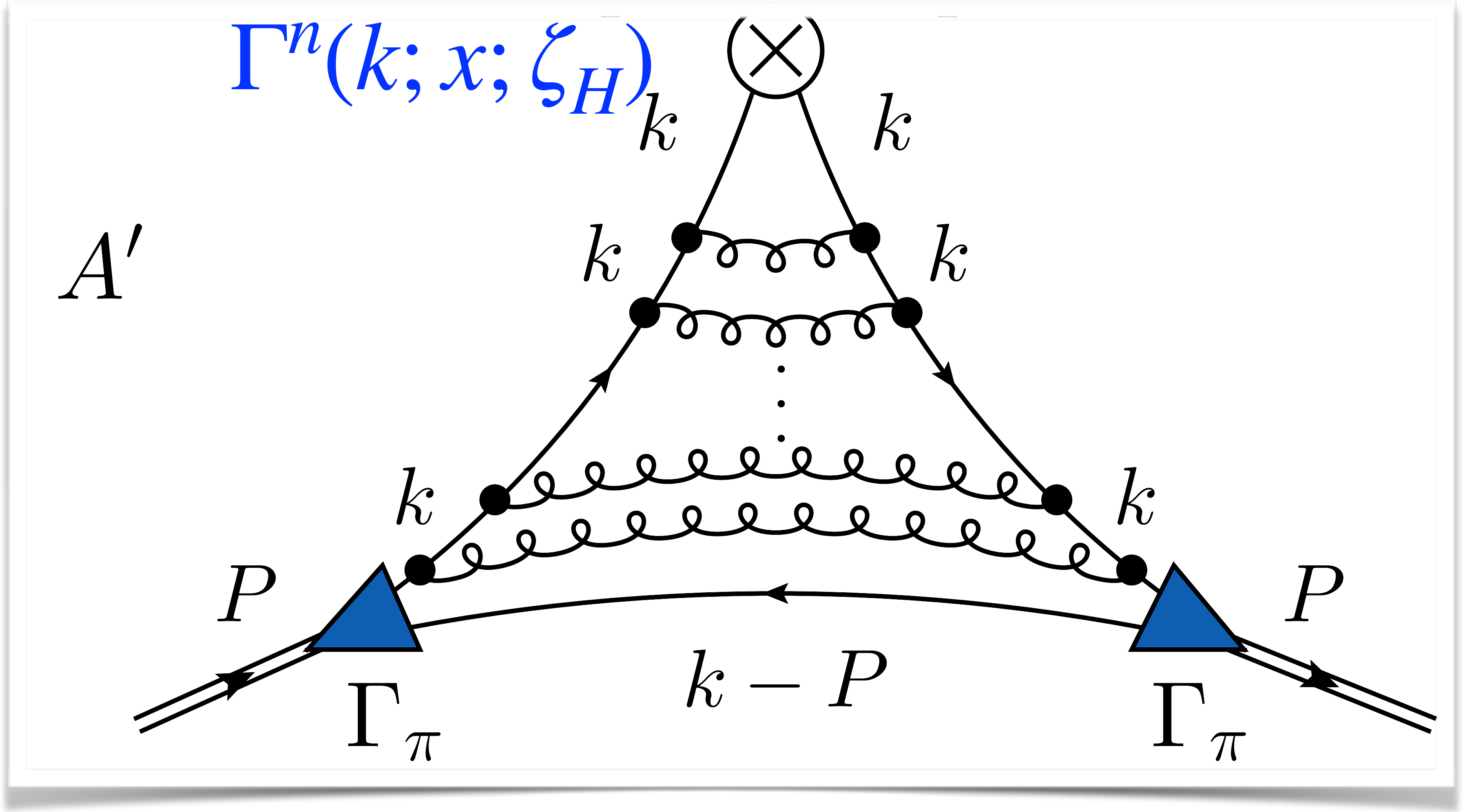


$$q(x) = \int dk \delta(n \cdot k - x n \cdot P) Tr[i\gamma \cdot n G(k, P)]$$

- Scattering amplitude $G(k, P)$

Pion valence quark PDF: use ward identity

Rainbow Ladder approximation



$$i\gamma \cdot n G(k,P)$$
$$= i\Gamma_\pi(k_\eta, -P)\, S(k_\eta) i\Gamma^n(k;x;\zeta_H)\, S(k_\eta)$$
$$\times\, i\Gamma_\pi(k_{\bar\eta}, P)\, S(k_{\bar\eta})$$

- $\Gamma^n(k;x;\zeta_H)$: a generalization of the dressed-quark-photon vertex, satisfies the usual inhomogeneous BSE with the inhomogeneous term is $n \cdot \gamma$.
- Vector Ward-Takahashi Identity

$$iP_\mu \Gamma_\mu(k;P) = S^{-1}(k + P/2) - S^{-1}(k - P/2)$$

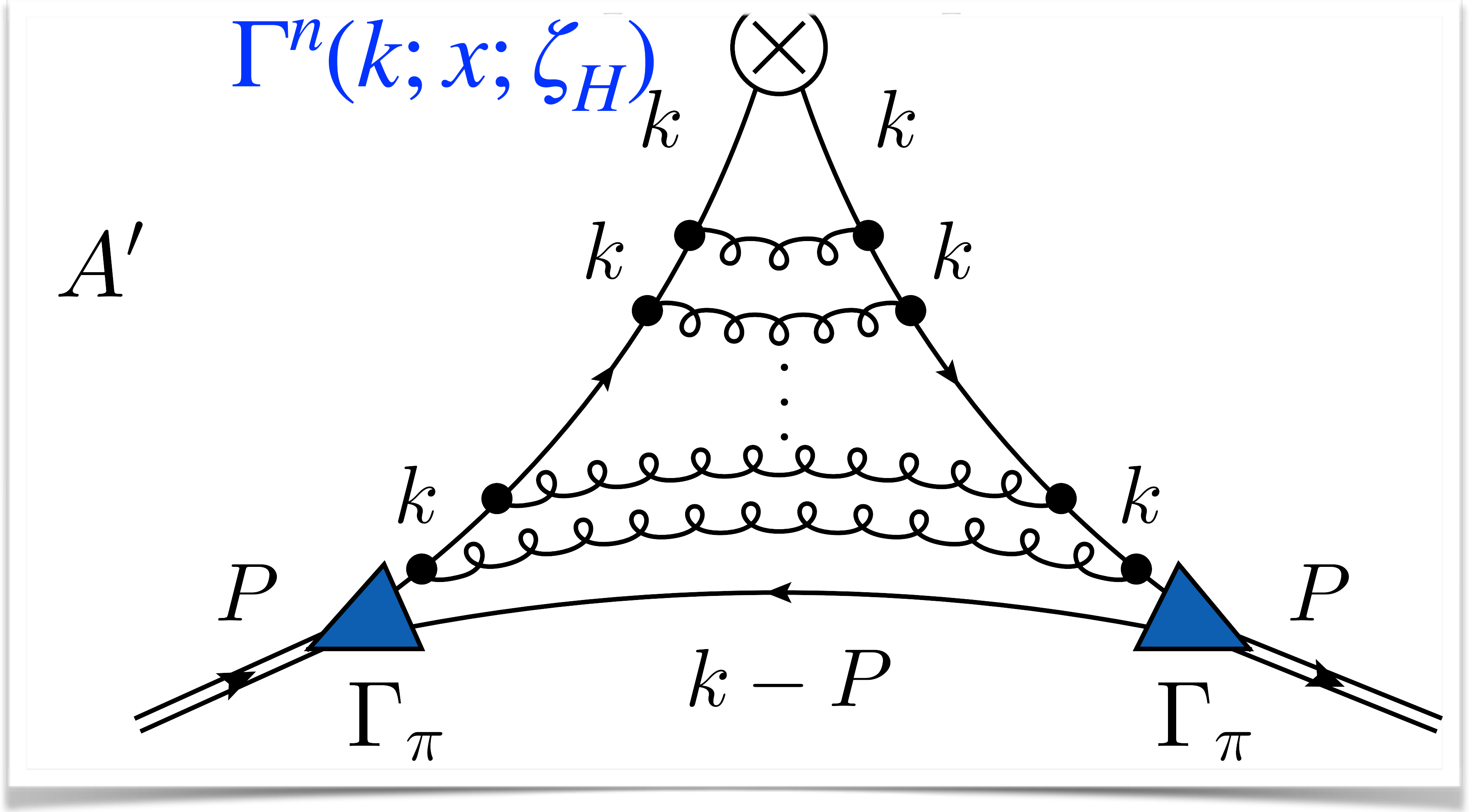
- Differential Ward identity $P = 0$

$$i\Gamma^n(k;x;\zeta_H) = n \cdot \partial_{k_\eta} S^{-1}(k_\eta)$$

T. Nguyen et al..Phys. Rev. C 83 (2011) 062201

Pion valence quark PDF: symmetry-rescue term

Rainbow Ladder approximation



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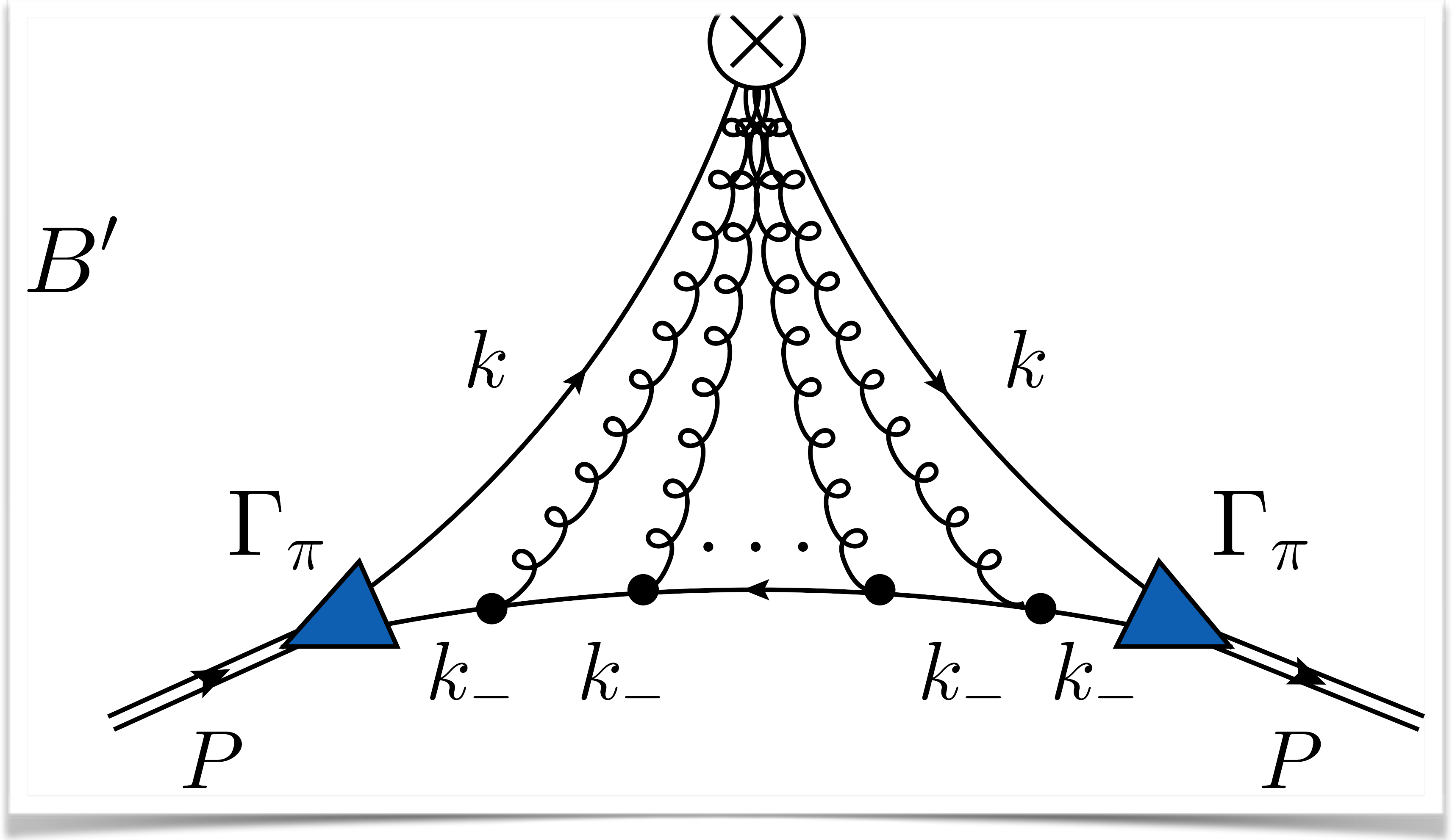


$$i\gamma\cdot n G(k,P) = i\Gamma_\pi(k_\eta,-P)\,S(k_\eta) i\Gamma^n(k;x;\zeta_H)\,S(k_\eta) \times i\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

$$i\gamma\cdot n G(k,P) = \Gamma_\pi^n(k_\eta,-P;\zeta_H) S(k_\eta)\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

L. Chang, C. Mezrag, H. Moutarde, C. D. Roberts, J. Rodríguez-Quintero and P. Tandy, Phys. Lett. B 737 (2014) 23-29

Pion valence quark PDF: symmetry-rescue term

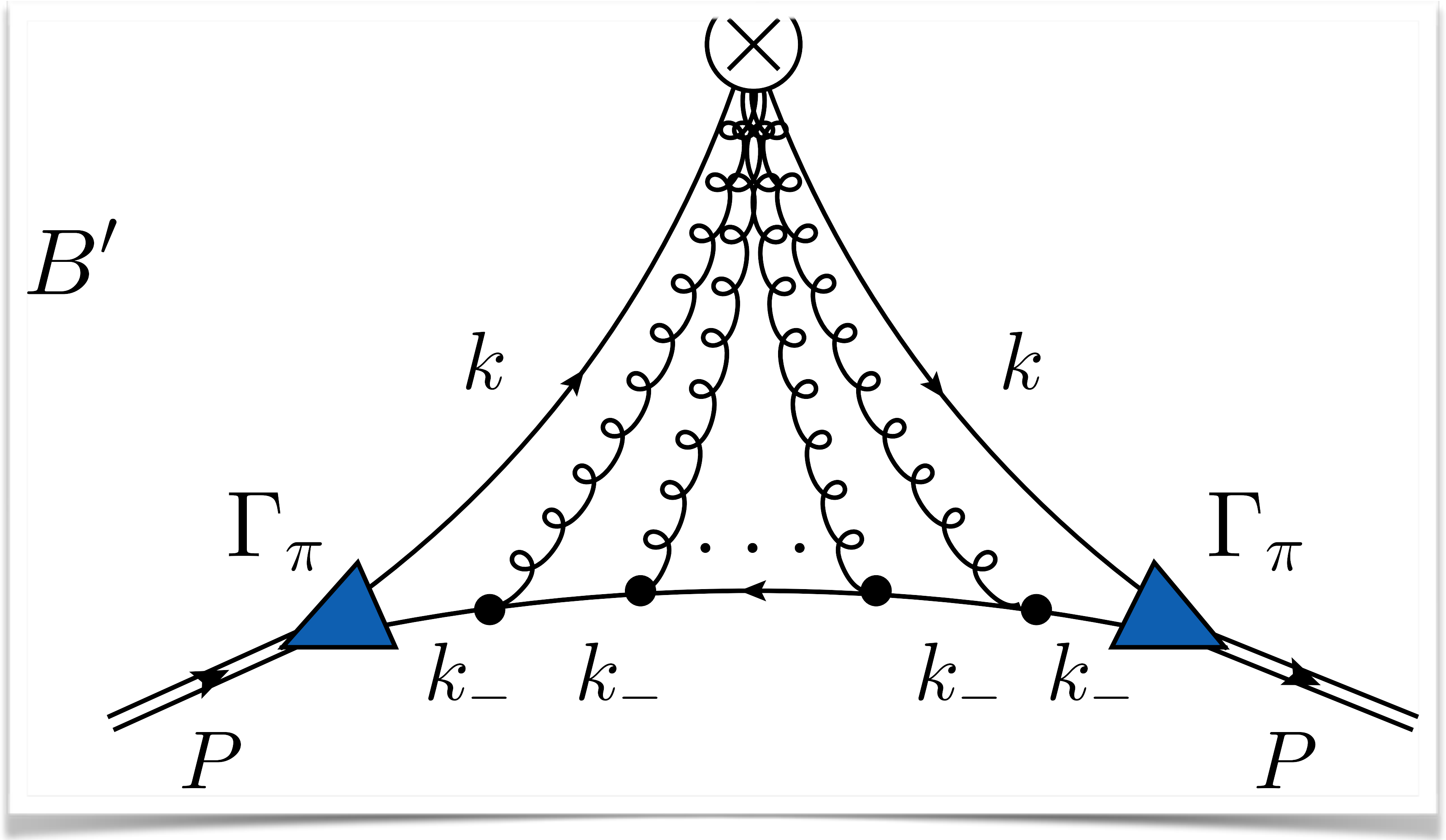
- "Pierced" pion Bethe-Salpeter amplitude

$$\Gamma^n_\pi(k_\eta, -P; \zeta_H) = n \cdot \partial_{k_\eta} \Gamma_\pi(k_\eta, -P; \zeta_H)$$

- Photon acts on Bethe-Salpeter amplitude

- Key point to keep symmetry in pion valence quark PDF $q^\pi(x, \zeta_H) = q^\pi(1-x, \zeta_H)$

A. Faessler, Th. Gutsche, M. A. Ivanov, V. E. Lyubovitskij, and P. Wang, Phys.Rev.D 68 (2003) 014011



$$i\gamma \cdot n G(k, P)$$
$$= \Gamma^n_\pi(k_\eta, -P; \zeta_H) S(k_\eta) \Gamma_\pi(k_{\bar\eta}, P)\, S(k_{\bar\eta})$$

Pion valence quark PDF

Twist-2 valence quark PDF

$$q(x) = \int dk\delta(n \cdot k - xn \cdot P)Tr[i\gamma \cdot nG(k, P)]$$

$$\begin{aligned} & i\gamma \cdot nG(k, P) \\ = & \, i\Gamma_\pi(k_\eta, -P)\, S(k_\eta) i\Gamma^n(k; x; \zeta_H)\, S(k_\eta) \\ & \times i\Gamma_\pi(k_{\bar{\eta}}, P)\, S(k_{\bar{\eta}}) \end{aligned}$$

$+$

$$\begin{aligned} & i\gamma \cdot nG(k, P) \\ = & \, \Gamma^n_\pi(k_\eta, -P; \zeta_H) S(k_\eta) \Gamma_\pi(k_{\bar{\eta}}, P)\, S(k_{\bar{\eta}}) \end{aligned}$$

- Pion PDF $\quad q^\pi(x; \zeta_H) = = N_c \mathrm{tr} \int_{dk} \delta^x_n(k_\eta)\; n \cdot \partial_{k_\eta} \left[\Gamma_\pi(k_\eta, -P; \zeta_H) S(k_\eta) \right] \Gamma_\pi(k_{\bar{\eta}}, P)\, S(k_{\bar{\eta}})$

- Pion is purely a bound-state of a dressed-quark and dressed-antiquark at the hadronic scale ζ_H, therefore $\langle x^1_\pi \rangle_{\zeta_H} = 1/2$, $\Gamma^n_\pi(k_\eta, -P; \zeta_H)$ is necessary.

Pion valence quark PDF: computing inputs

- $q^{\pi}(x;\zeta_H)$ is completely determined once an interaction kernel is specified.

- Bottom up approach:
  - Qin-Chang-Liu-Roberts-Wilson (QCLRW) gluon propagator.
  - Rainbow ladder approximation

- Renormalisation scheme
  - Z_2 & Z_4 defined in chiral limit and invariant for any current quark mass.

- Hadronic scale ζ_H
  - A scale where dressed quasiparticles are the correct degrees-of-freedom.
  - Meson's Poincare covariant wave function must evolve with ζ.

Pion valence quark PDF: hadronic resolving scale ζ_H

- In practice
  - ζ_H is typically used as a parameter.
  - Develops a PDF model that is supposed to be valid at an unspecified scale, which is subsequently identified as ζ_H .
  - Then a target PDF is identified, one that has typically been extracted through a phenomenological analysis of selected experimental data at experiment energy scale ζ_E.
  - The practitioner finally chooses a value of ζ_H so that, after DGLAP evolution $\zeta_H \to \zeta_E$, the model PDF reproduces some property or properties of the target distribution.

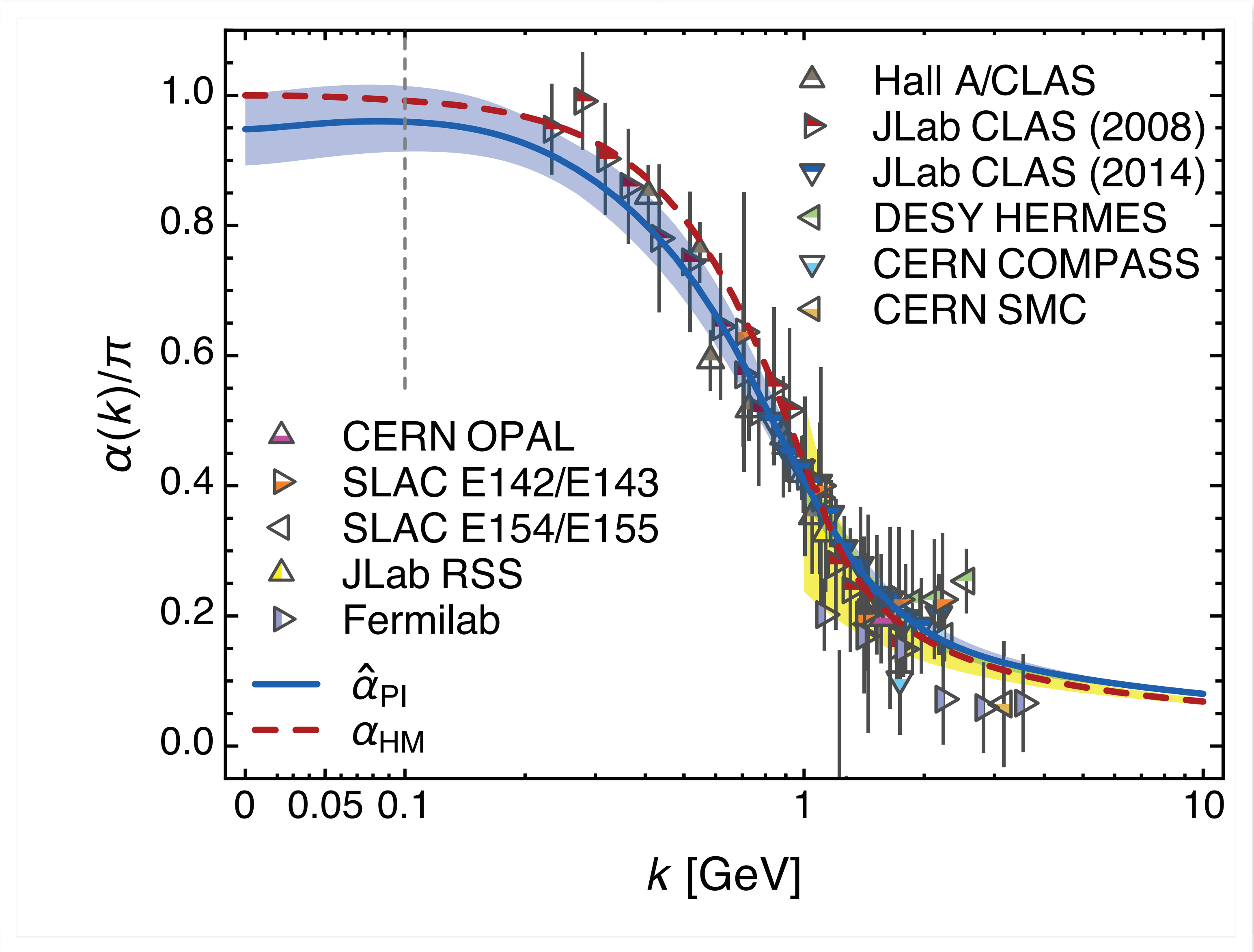
Pion valence quark PDF: hadronic resolving scale ζ_H

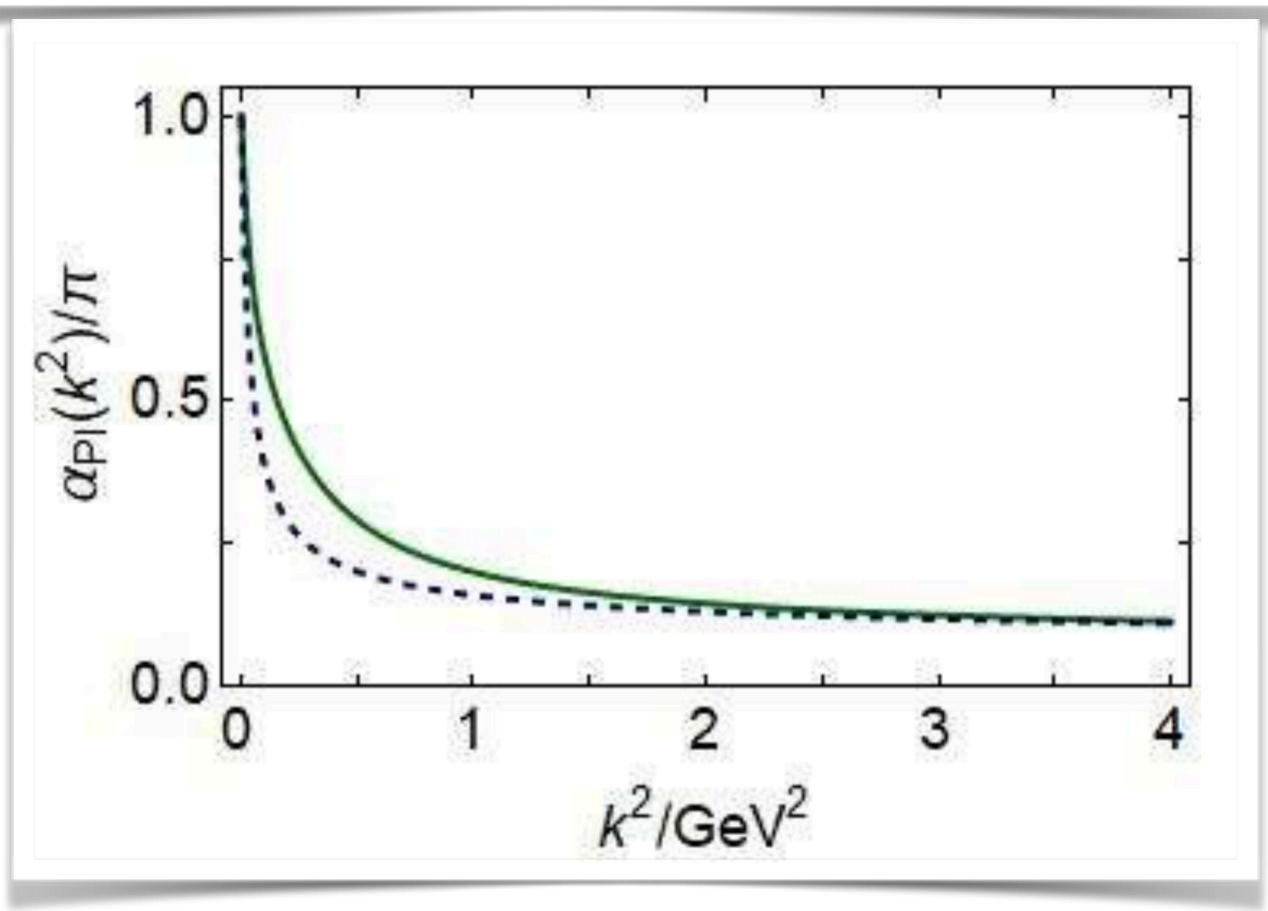
Daniele Binosi et al.. Phys. Rev. D 96, 054026 (2017)

- Process-independent effective charge $\alpha_{\rm PI}(k^2)$
  - Saturates in the infrared: $\alpha_{\rm PI}(0)/\pi \approx 1.$
  - Emergence of a nonzero gluon mass-scale,

$$\alpha_{PI}(k) = \frac{4\pi}{\beta_0 \ln[(m_\alpha{}^2 + k^2)/\Lambda^2_{\rm QCD}]}$$

- $m_\alpha = 0.30\, GeV \gtrsim \Lambda_{\rm QCD}$ is an nonperturbative scale ensures modes with $k^2 \lesssim m_\alpha^2$ are screened from interactions.
- Hadronic resolving scale $\zeta_H = m_\alpha$
- m_α therefore serves to define the natural boundary between soft and hard physics





Solid Green = original Dashed Blue = simplified expression

Pion valence quark PDF: Mellin moments

- $q^\pi(x;\zeta_H)$ is reconstructed from Mellin moments

$$\langle x^m\rangle^\pi_{\zeta_H} = \int_0^1 dx\, x^m q^\pi(x;\zeta_H) = \frac{N_c}{n\cdot P} tr\int_{dk}\left[\frac{n\cdot k_\eta}{n\cdot P}\right]^m \Gamma_\pi(k_{\bar\eta},P)\, S(k_{\bar\eta})\ n\cdot\partial_{k_\eta}\left[\Gamma_\pi(k_\eta,-P)S(k_\eta)\right]$$

- Odd moments are not independent
- Schlessinger point method (SPM)

$$\langle x\rangle^\pi_{\zeta_H} = \frac{1}{2}\langle x^0\rangle^\pi_{\zeta_H} = \frac{1}{2},$$

$$\langle x^3\rangle^\pi_{\zeta_H} = -\frac{1}{4}\langle x^0\rangle^\pi_{\zeta_H} + \frac{3}{2}\langle x^2\rangle^\pi_{\zeta_H},$$

$$\langle x^5\rangle^\pi_{\zeta_H} = \frac{1}{2}\langle x^0\rangle^\pi_{\zeta_H} - \frac{5}{2}\langle x^2\rangle^\pi_{\zeta_H} + \frac{5}{2}\langle x^4\rangle^\pi_{\zeta_H},$$

$$\langle x^7\rangle^\pi_{\zeta_H} = -\frac{17}{8}\langle x^0\rangle^\pi_{\zeta_H} + \frac{21}{2}\langle x^2\rangle^\pi_{\zeta_H} - \frac{35}{4}\langle x^4\rangle^\pi_{\zeta_H} + \frac{7}{2}\langle x^6\rangle^\pi_{\zeta_H}.$$

- Direct can compute m=0~5
- SPM Extrapolate m=6~10

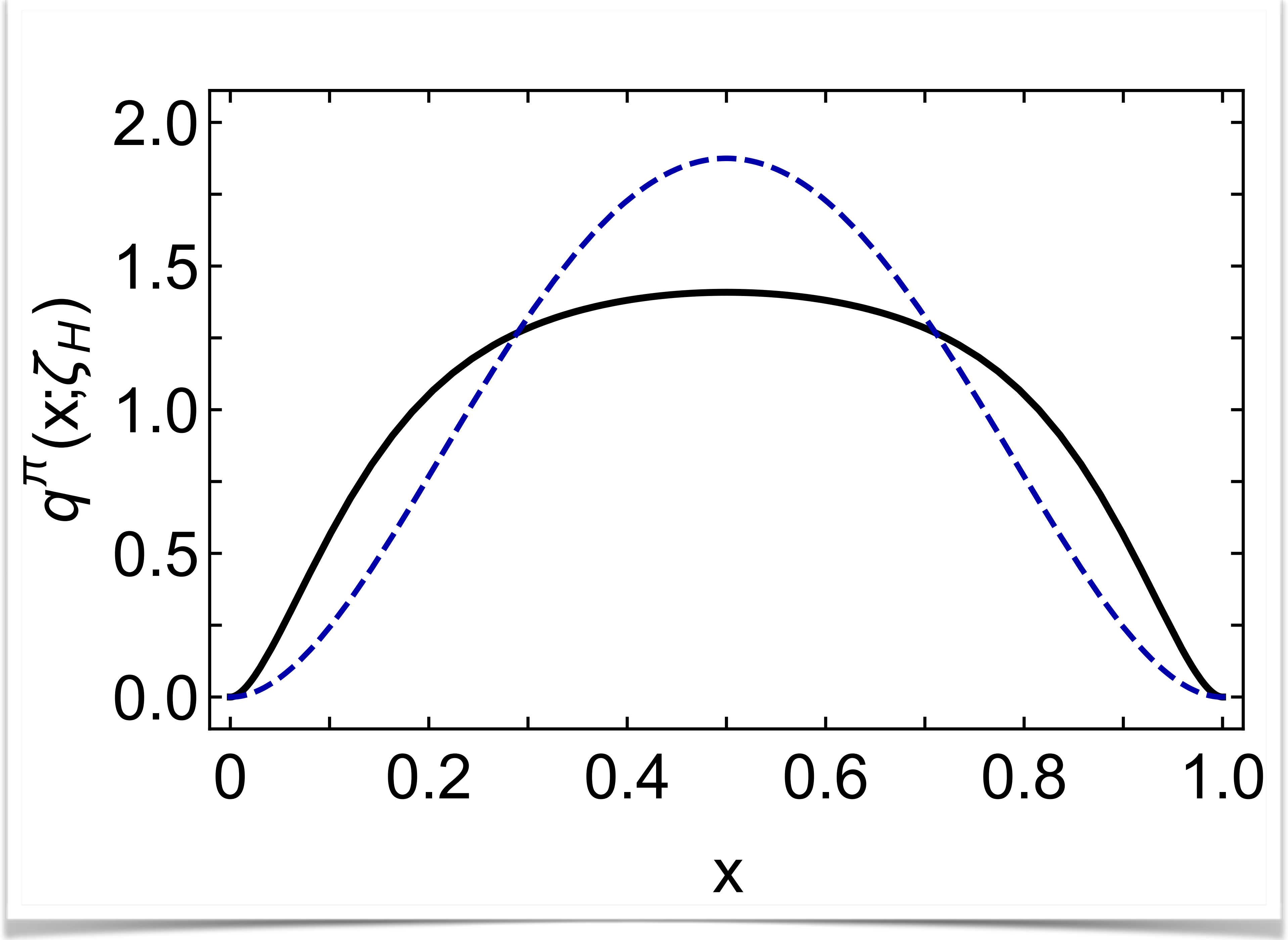
L. Schlessinger and C. Schwartz, Phys. Rev. Lett. 16, 1173 (1966)

L. Schlessinger, Phys. Rev. 167, 1411 (1968)

Pion valence quark PDF at ζ_H

- $q^\pi(x;\zeta_H)$ is reconstructed from Mellin moments

$$q^\pi(x;\zeta_H) = 213.32\, x^2(1-x)^2[1 - 2.9342\sqrt{x(1-x)} + 2.2911\, x(1-x)]$$



Solid Black = $q^\pi(x;\zeta_H)$

Dashed Blue = scale free distribution

$$q_{\rm sf}(x) \approx 30\, x^2(1-x)^2$$

- Broad function
- Dynamical chiral symmetry breaking
- PDA, form factors

M. Ding, K. Raya, D. Binosi, L. Chang, C.D Roberts, S. M. Schmidt, Phys.Rev.D 101 (2020) 5, 054014

Evolution of pion PDFs

- Existing Lattice QCD calculations of low-order moments and phenomenological fits to pion parton distributions are typically quoted at $\zeta_2 = 2$ GeV.

$$\zeta_H = m_\alpha \longrightarrow \zeta_2 = 2 \text{ GeV}$$

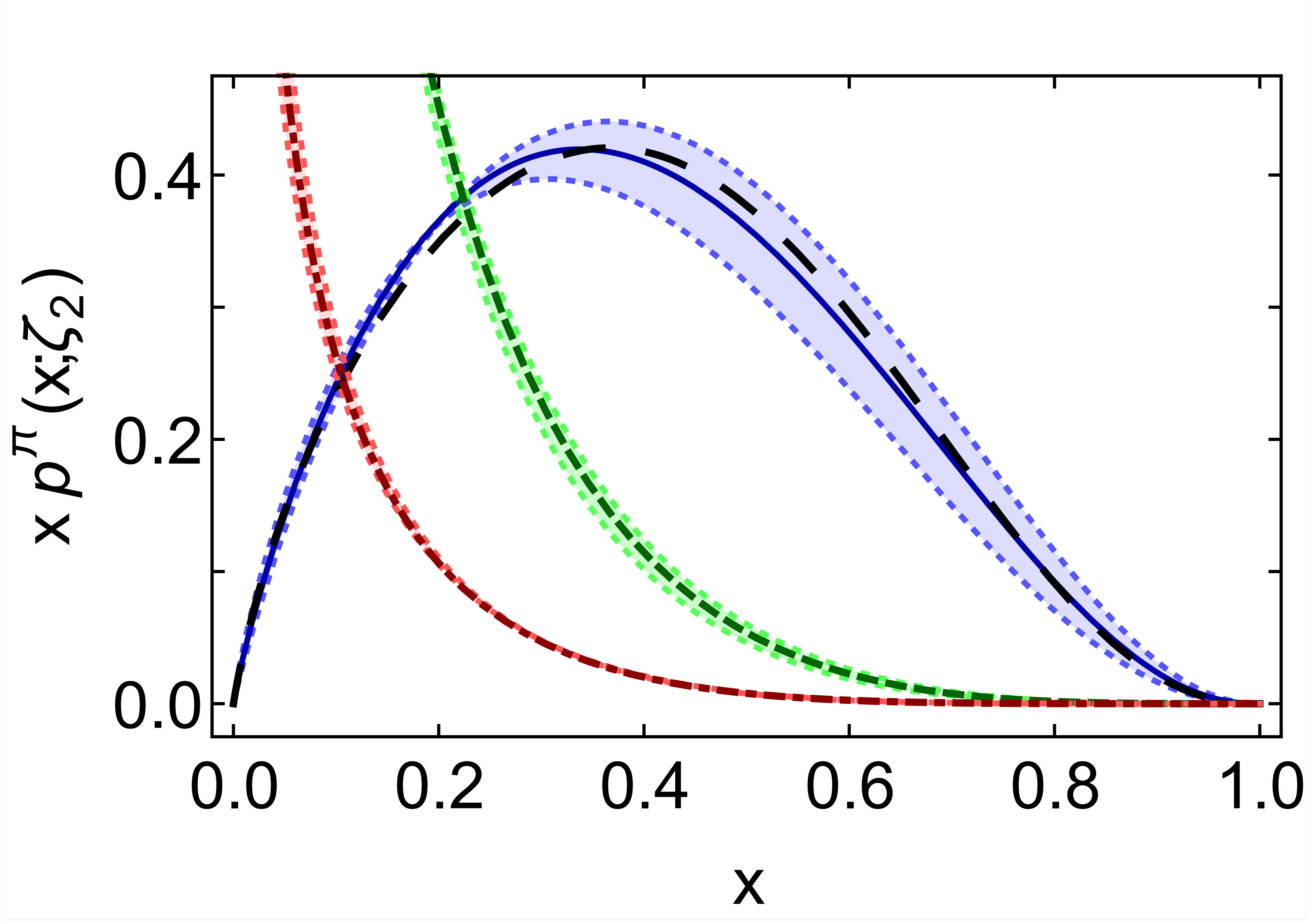
- Experiment takes the average scale $\zeta_5 = 5.2$ GeV.

$$\zeta_H = m_\alpha \longrightarrow \zeta_5 = 5.2 \text{ GeV}$$

K. Wijesooriya, P.E. Reimer, R.J. Holt, Phys.Rev.C 72 (2005) 065203

- Process-independent running coupling $\alpha_{\rm PI}(\zeta_H)/(2\pi) = 0.20\,,\,[\alpha_{\rm PI}(\zeta_H)/(2\pi)]^2 = 0.04.$
  - Leading order DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equation should serve as a good approximation.
- Results report with $\zeta \to (1 \pm 0.1)\zeta$.

Pion valence quark PDF at $\zeta_2 = 2$ GeV



- Solid (blue) curve embedded in shaded band, $q^\pi(x;\zeta_2)$.

$$q^\pi(x) = n_{q^\pi}\, x^\alpha (1-x)^\beta [1 + \rho\, x^{\alpha/4}(1-x)^{\beta/4} + \gamma\, x^{\alpha/2}(1-x)^{\beta/2}]$$

	n_{q^π}	α	β	ρ	γ
ζ_2	9.83	−0.080	2.29	−1.27	0.511
	8.31	−0.127	2.37	−1.19	0.469
	7.01	−0.162	2.47	−1.12	0.453

$$\beta(\zeta_2) = 2.38(9)$$

- Long-dashed (black), ζ_2 result from DSE in 2001.

Hecht et al.. Phys. Rev. C 63, 025213 (2001)

Pion valence quark PDF at $\zeta_2 = 2$ GeV: first moment $\langle x^1 \rangle_u^\pi$

* Low-order moments in comparison with Lattice QCD simulations.

- Both continuum and Lattice QCD results agree

$$\langle 2x \rangle_q^\pi = 0.48(3)$$

- Roughly one-half of the light front momentum fraction is carried by the valence quarks.

ζ_2	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
Ref. [33]	0.24(2)	0.09(3)	0.053(15)
Ref. [34]	0.27(1)	0.13(1)	0.074(10)
Ref. [35]	0.21(1)	0.16(3)	
average	0.24(2)	0.13(4)	0.064(18)
Herein	0.24(2)	0.098(10)	0.049(07)

W. Detmold et al.. Phys. Rev. D 68, 034025 (2003) M. Oehm et al.. Phys. Rev. D 99, 014508 (2019)

D. Brömmel et al. (QCDSF-UKQCD Collaboration), PoS LAT2007, 140 (2007)

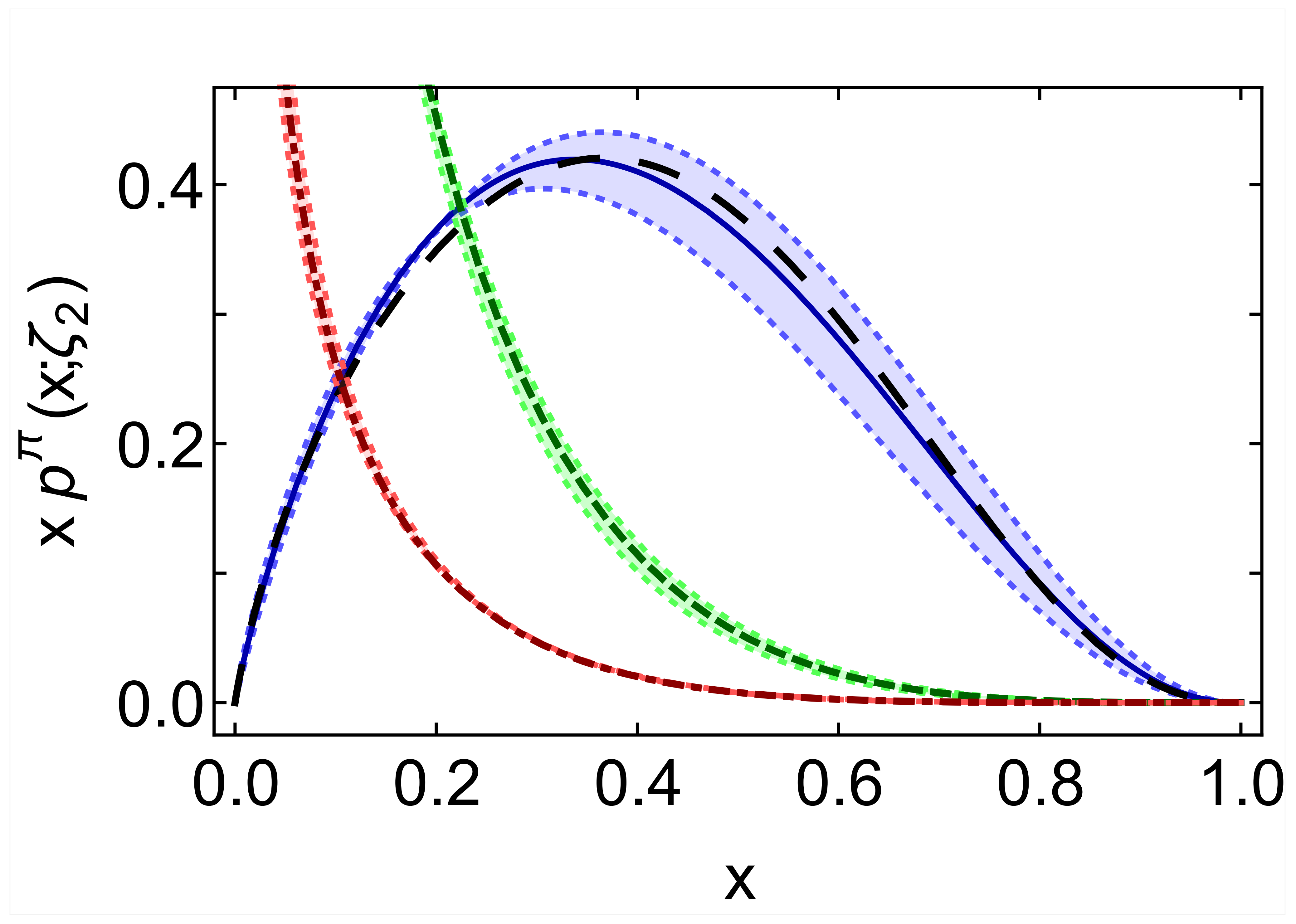
- Global QCD Analysis: phenomenological analysis π-nucleus Drell-Yan and leading neutron DIS data.

$$\langle 2x \rangle_q^\pi = 0.48(1)\,,\zeta = 2.24 \text{ GeV}$$

P. C. Barry et al..,JAM Collaboration, Phys. Rev. Lett. 121, 152001 (2018)

Pion gluon and sea quark PDF at $\zeta_2 = 2$ GeV

- Pion is purely a bound-state of a dressed-quark and dressed-antiquark at the hadronic scale ζ_H, sea and glue distributions are zero at ζ_H.



- Dashed (green), $xg^\pi(x;\zeta_2)$, gluon.
- Dot-dashed (red), $xS^\pi(x;\zeta_2)$, sea-quark.

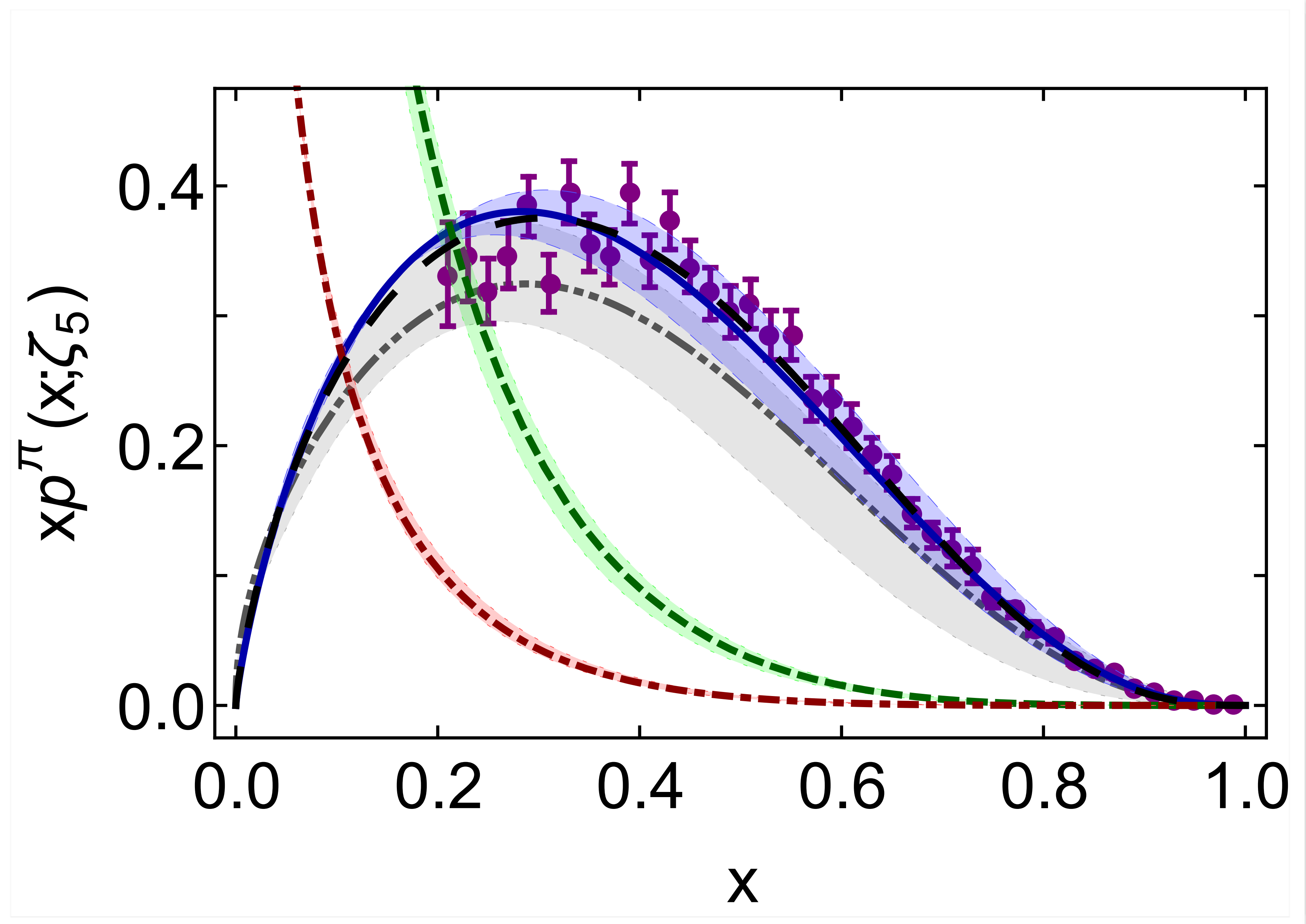
$$xp^\pi(x;\zeta) = A\,x^\alpha\,(1-x)^\beta$$

	p	$\mathcal{A}$	α	β
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13
	S	0.13 ∓ 0.01	-0.53 ∓ 0.05	4.51 ± 0.03

- First Moment $\langle x\rangle_g^\pi = 0.41(2)\,,\ \langle x\rangle_{\rm sea}^\pi = 0.11(2)\,.$
  - Agree with $\langle x\rangle_g^\pi = 0.35(2)\,,\ \langle x\rangle_{\rm sea}^\pi = 0.17(1)\,.$

P. C. Barry et al..(JAM Collaboration), Phys. Rev. Lett. 121, 152001 (2018)

Pion PDFs at $\zeta_5 = 5.2$ GeV



	n_{q^π}	α	β	ρ	γ
	7.81	−0.153	2.54	−1.20	0.505
ζ_5	7.28	−0.169	2.66	−1.21	0.531
	6.48	−0.188	2.78	−1.19	0.555

$\beta(\zeta_5) = 2.66(12)$

- Agree with Lattice QCD $\beta_{\rm lQCD}(\zeta_5) = 2.45(58)$

✱ Solid (blue), $q^{\pi}(x;\zeta_5)$.

$$q^{\pi}(x) = n_{q^\pi}\, x^{\alpha}(1-x)^{\beta}[1 + \rho\, x^{\alpha/4}(1-x)^{\beta/4} + \gamma\, x^{\alpha/2}(1-x)^{\beta/2}]$$

- Long-dashed (black), DSE in 2001 Hecht et al.. Phys. Rev. C 63, 025213 (2001)
- Dot-dot-dashed (grey), Lattice QCD Raza Sabbir Sufian et al.. Phys. Rev. D 99, 074507 (2019)
- Experimental extraction (purple) J.S.Conway et al.. Phys. Rev. D 39 (1989) 92-122
- Rescaled analysis Matthias Aicher et al.. PRL 105, 252003 (2010)

- Low-order moments in comparison with Lattice QCD

ζ_5	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
Ref. [31]	0.17(1)	0.060(9)	0.028(7)
Herein	0.21(2)	0.076(9)	0.036(5)

- Dashed (green), gluon; Dot-dashed (red), sea-quark.
- First Moment $\langle x\rangle_g^\pi = 0.45(1)\,,\ \langle x\rangle_{\rm sea}^\pi = 0.14(2)\,.$

Outline

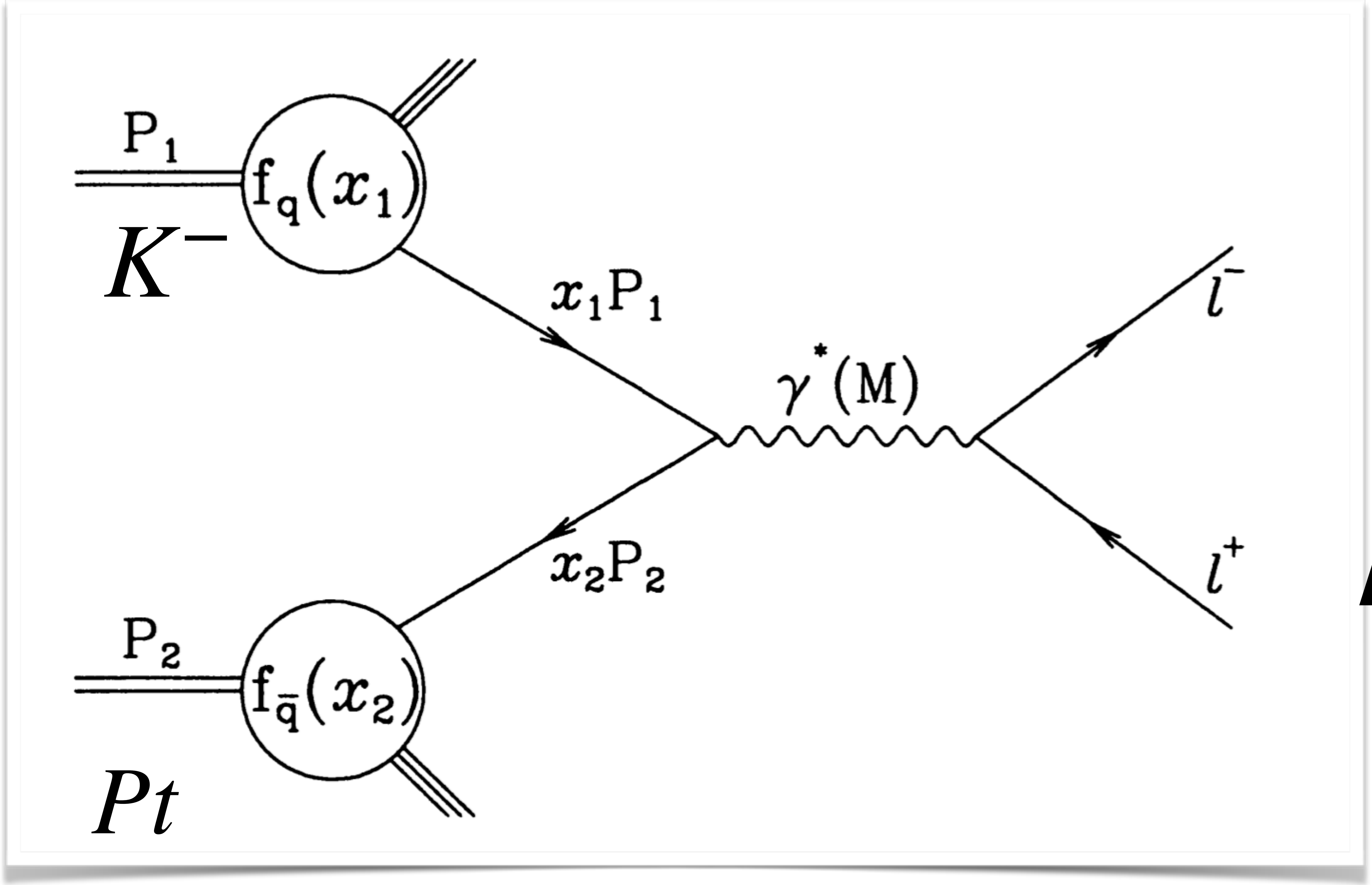
- Introduction
- Dyson-Schwinger Equations
- Pion parton distribution functions
- **Kaon parton distribution functions**
- Summary and outlook

The kaon

- Phenomenology
  - Mass: $K^0 \sim 498 MeV$.
  - Pseudoscalar meson: $J^P = 0^-$.
  - Two sources of mass:
    - Explicit – generated by couplings to Higgs-boson, especially strange quark.
    - Emergent – dynamical consequence of strong interactions.

Experiments: valence quark PDF ratio $u_K(x)/u_\pi(x)$

- K-induced Drell-Yan $K^- Pt \to \mu^+\mu^- X$ / π-induced Drell-Yan $\pi^- Pt \to \mu^+\mu^- X$



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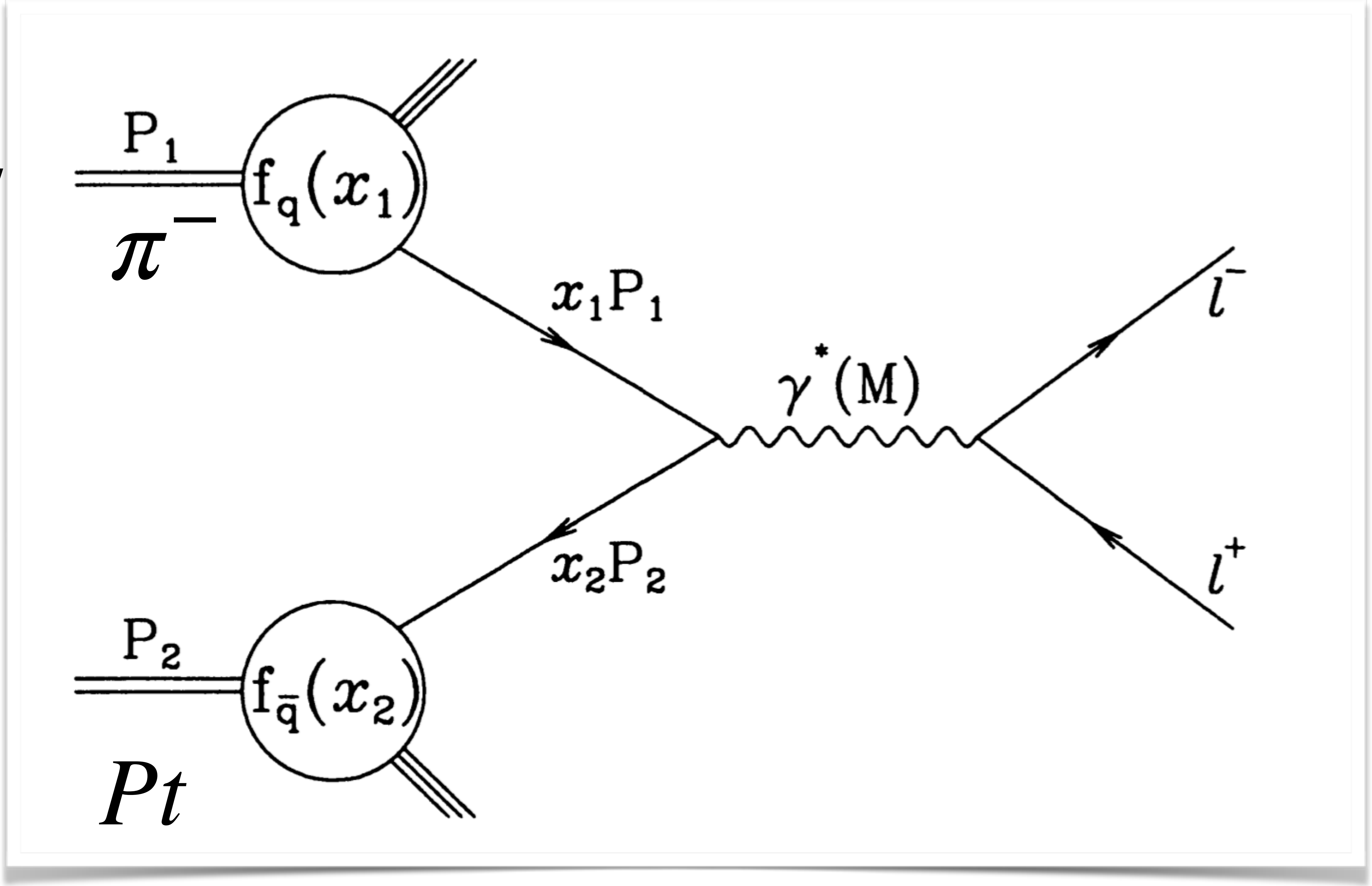
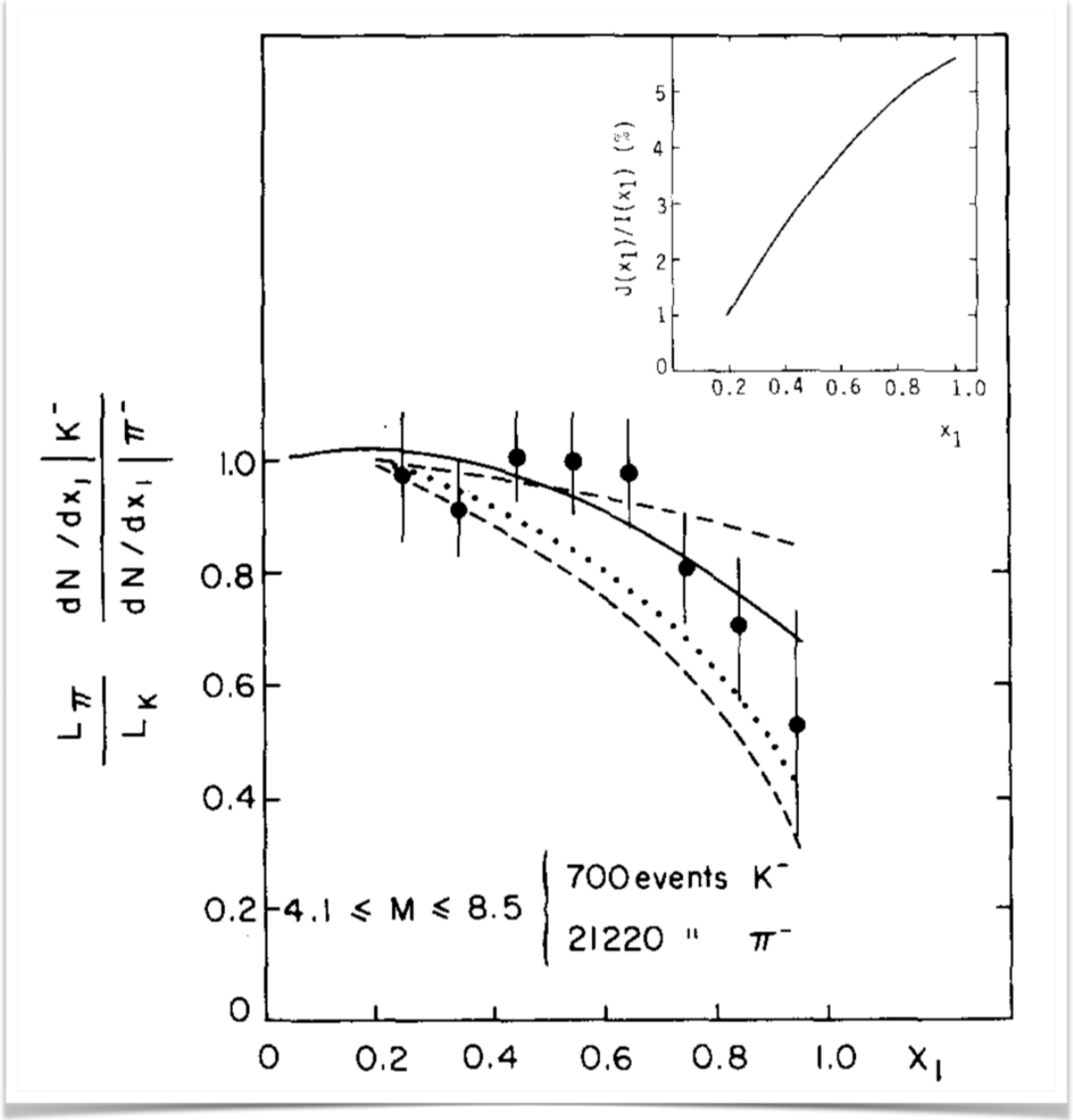


- ☑ CERN: NA3 (1980)

Experiments: valence quark PDF ratio $u_K(x)/u_\pi(x)$

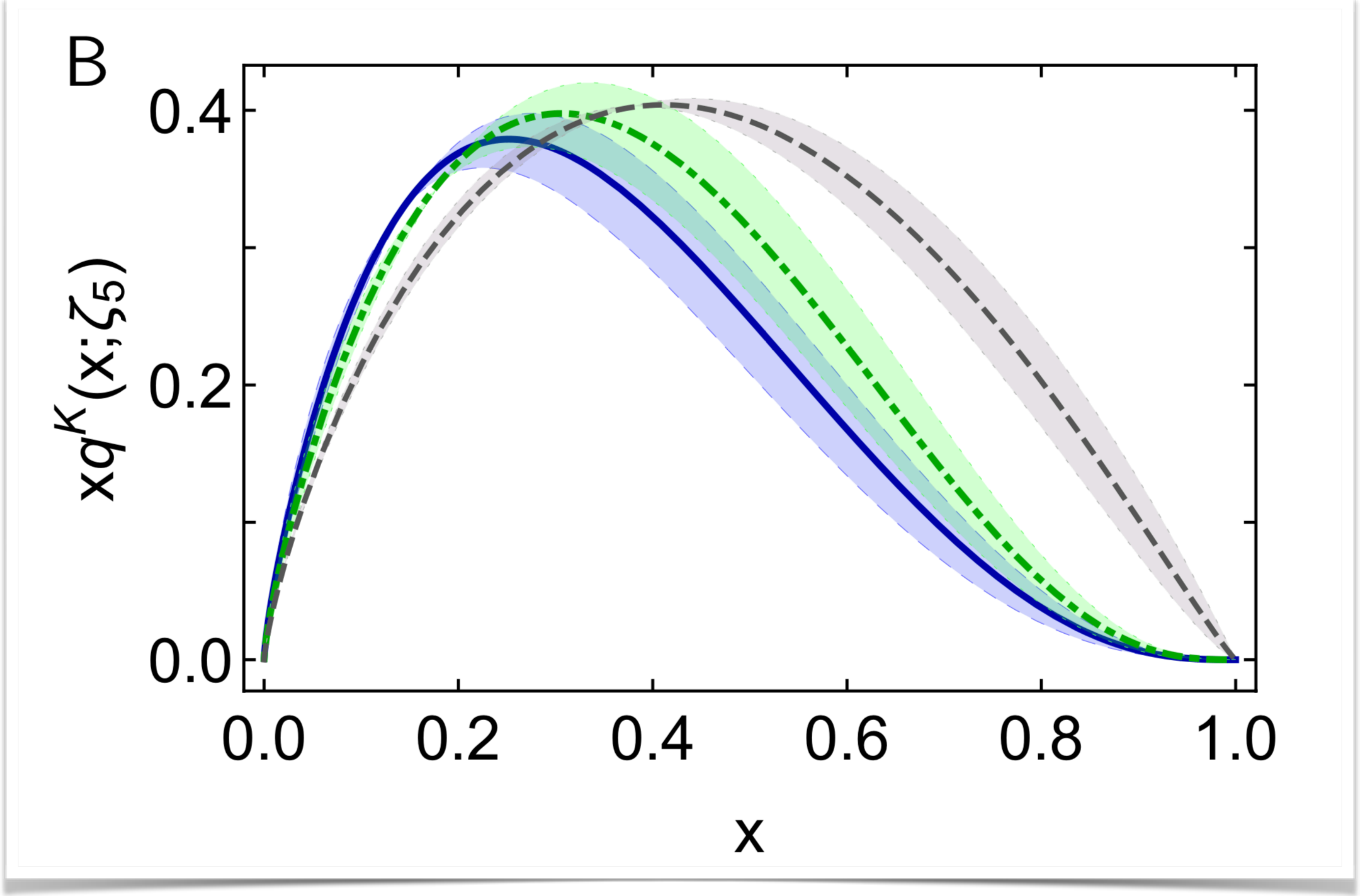
$$R = \frac{u_K(x)}{u_\pi(x)} \approx \frac{L_\pi}{L_K}\frac{dN/dx_1|_K}{dN/dX_1|_\pi}$$

- Striking behavior found in contrast between $u_K(x,\zeta)$ & $u_\pi(x,\zeta)$ at large *x*.
- Significant disparity between these distributions would point to big difference between fractions of π and K momentum carried by other bound state participants, particularly gluons.



J. Badier et al., Phys. Lett. B 93, 354 (1980).

Kaon valence quark PDFs at $\zeta_5 = 5.2$ GeV



- Solid (blue), $u^K(x;\zeta_5)$
- Dot- dashed (green), $s^K(x;\zeta_5)$

$$q^K(x) = n_{q^\pi} x^\alpha (1-x)^\beta [1 + \rho\, x^{\alpha/4}(1-x)^{\beta/4} + \gamma\, x^{\alpha/2}(1-x)^{\beta/2}]$$

- Effective large-x exponent $\beta_{eff}(\zeta)$ by plotting $q^K(x,\zeta)$ against $\ln(1-x)$ on $x \in [0.9, 1.0]$, $\beta_{eff}(\zeta_5) = 2.73(7)$.
- Dashed (grey), Lattice QCD

H.-W. Lin, J.-W. Chen, Z. Fan, J.-H. Zhang and R. Zhang, (arXiv:2003.14128 [hep-lat])

- Low-order moments in comparison with Lattice QCD

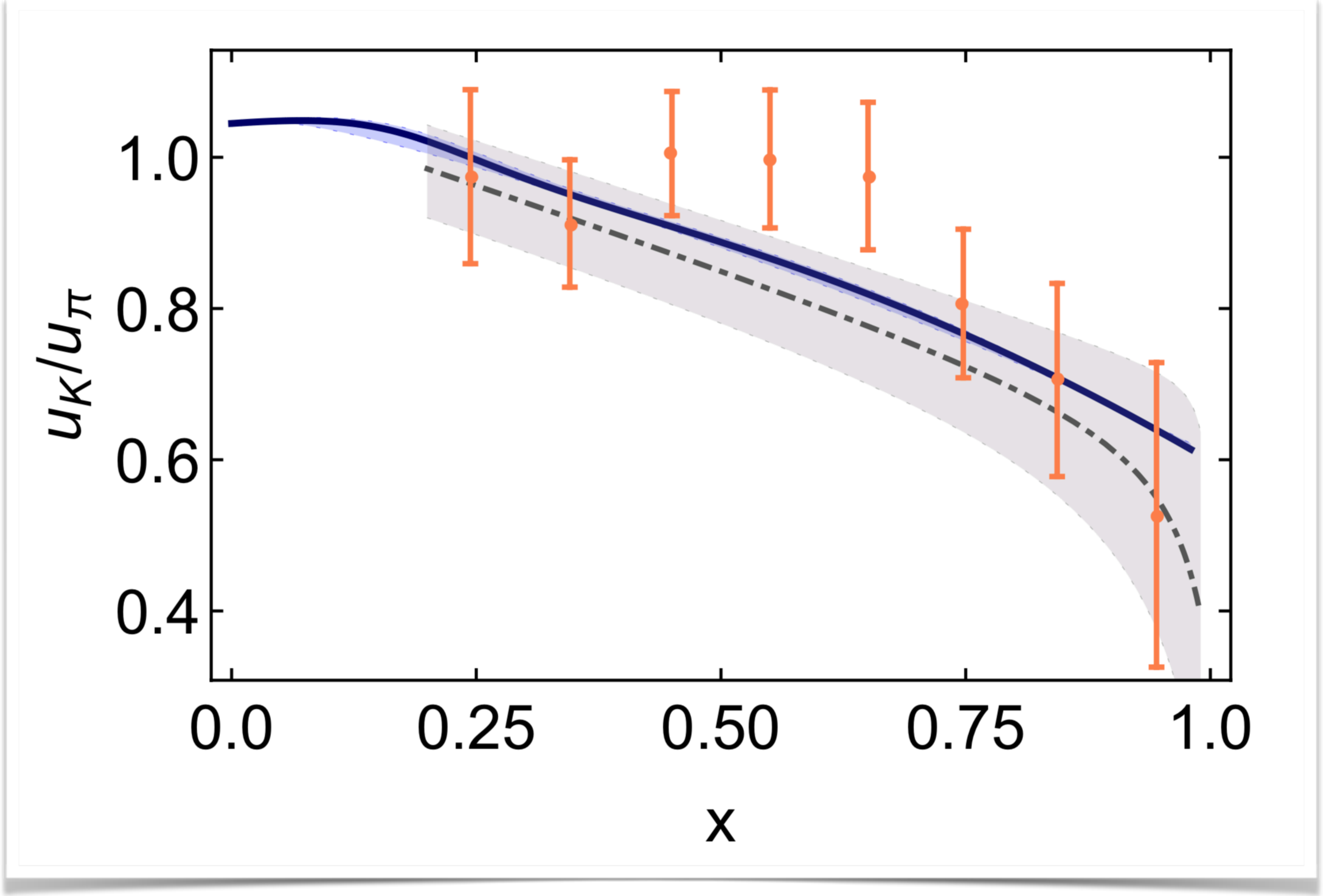
$q \backslash \zeta_5$	$\langle x q^K \rangle$	$\langle x^2 q^K \rangle$	$\langle x^3 q^K \rangle$
u	0.19(2)	0.067(09)	0.030(5)
$\bar{s}$	0.22(2)	0.081(11)	0.038(7)
u	0.193(8)	0.080(7)	0.042(6)
$\bar{s}$	0.267(8)	0.123(7)	0.070(6)

Lattice

- Lattice QCD vs. DSE, lattice QCD PDFs = much harder.
- Lattice QCD PDFs $\ldots(1-x)^\beta$, $\beta = 1.13(16)$

Z.Cui et al. (arXiv:2006.14075 [hep-ph])

Ratio $u_K(x, \zeta_5)/u_\pi(x, \zeta_5)$

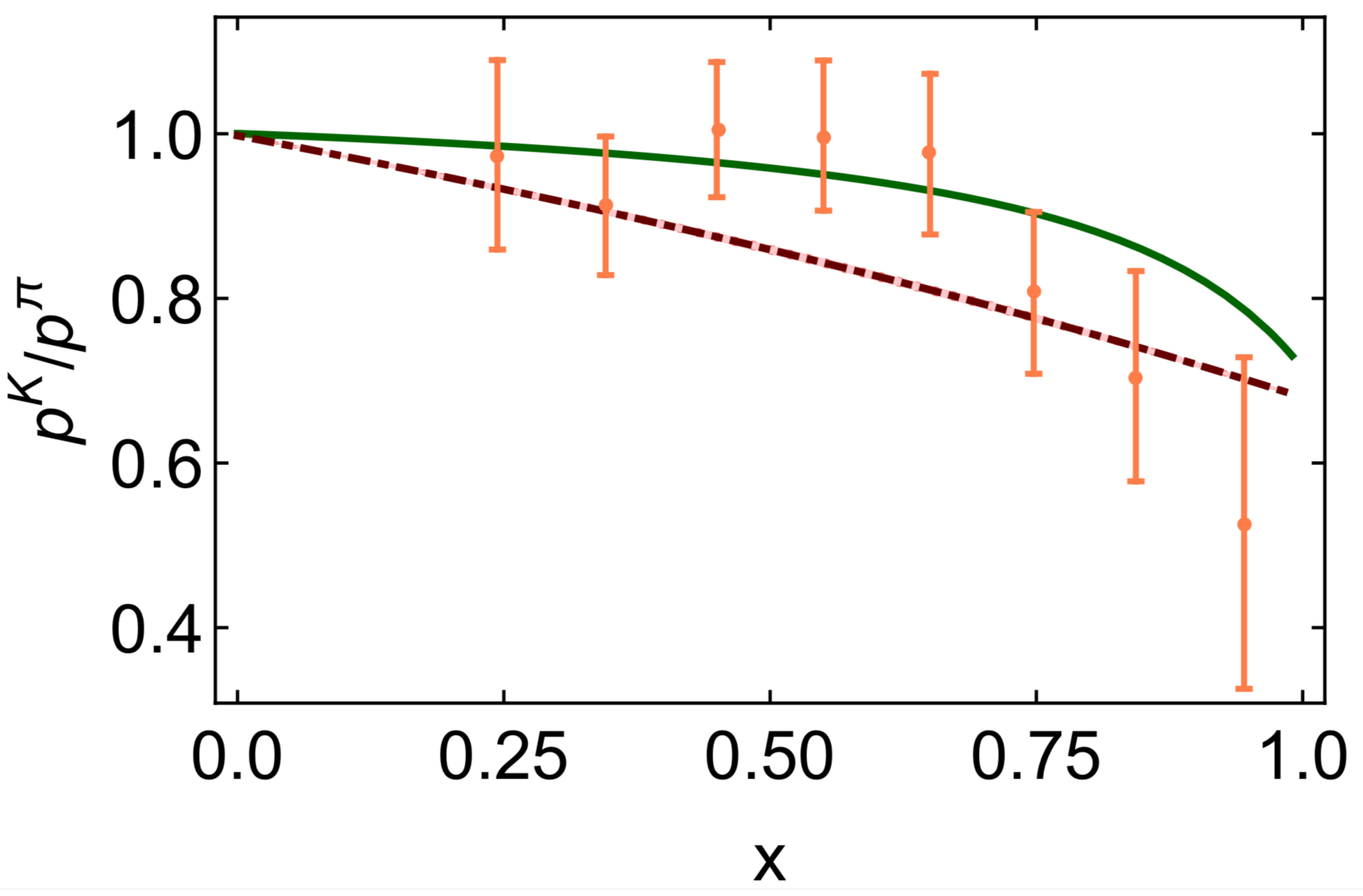


- Solid (blue), $u^K(x;\zeta_5)/u^\pi(x;\zeta_5)$.
- Dot-dashed (grey), Lattice QCD — H.-W. Lin, J.-W. Chen, Z. Fan, J.-H. Zhang and R. Zhang, (arXiv:2003.14128 [hep-lat])
- Experimental extraction (orange) — J. Badier et al., Phys. Lett. B 93, 354 (1980).

- Relative difference between the central Lattice QCD result and DSE prediction is ≈ 5% … despite fact that individual lQCD PDFs are very different from continuum results.

- $u^K(x;\zeta_5)/u^\pi(x;\zeta_5)$ is very forgiving of even large differences between the individual PDFs used to produce the ratio, results for $u^K(x;\zeta_5)$ and $u^\pi(x;\zeta_5)$ separately have greater discriminating power.

Kaon gluon and sea quark PDFs at $\zeta_5 = 5.2$ GeV



- Curious: each of the gluon and sea quark PDF ratios is point-wise similar to the experimentally exacted value of $u^K(x;\zeta_5)/u^\pi(x;\zeta_5)$.

* Solid (green), $g^K(x;\zeta_5)/g^\pi(x,\zeta_5)$.

* Dot-dashed (red), $S^K(x;\zeta_5)/S^\pi(x,\zeta_5)$.

* Experimental extraction (orange) $u^K(x;\zeta_5)/u^\pi(x;\zeta_5)$

J. Badier et al., Phys. Lett. B 93, 354 (1980).

- Kaon's gluon and sea distributions differ from those of the pion only on the valence region $x > 0.2$.
- Mass-dependent splitting functions act primarily to modify valence PDF of the heavier quark.
- Valence PDFs are negligible at low-x, where gluon and sea distributions are large, and vice versa.
- Hence the biggest impact of a change in the valence PDFs must lie at large-x.

- First Moment $\langle x \rangle_g^K = 0.44(2)$, $\langle x \rangle_{sea}^K = 0.14(2)$.

Future Facilities & experiments on pion & kaon PDFs

- ## COMPASS++/AMBER:
  - The Compass++/Amber (proto-) collaboration proposes to establish a “New QCD facility at the M2 beam line of the CERN SPS” and perform in phase-1, i.e. starting in the year 2022, three experiments that will use either muons or hadrons delivered by the existing M2 beam line:
    - (1) Proton charge radius measurement using muon-proton elastic scattering
    - (2) Drell-Yan and J/Psi production experiments using the conventional M2 hadron beam
    - (3) Measurement of proton-induced antiproton production cross sections for dark matter searches.

Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER), arXiv:1808.00848 [hep-ex]

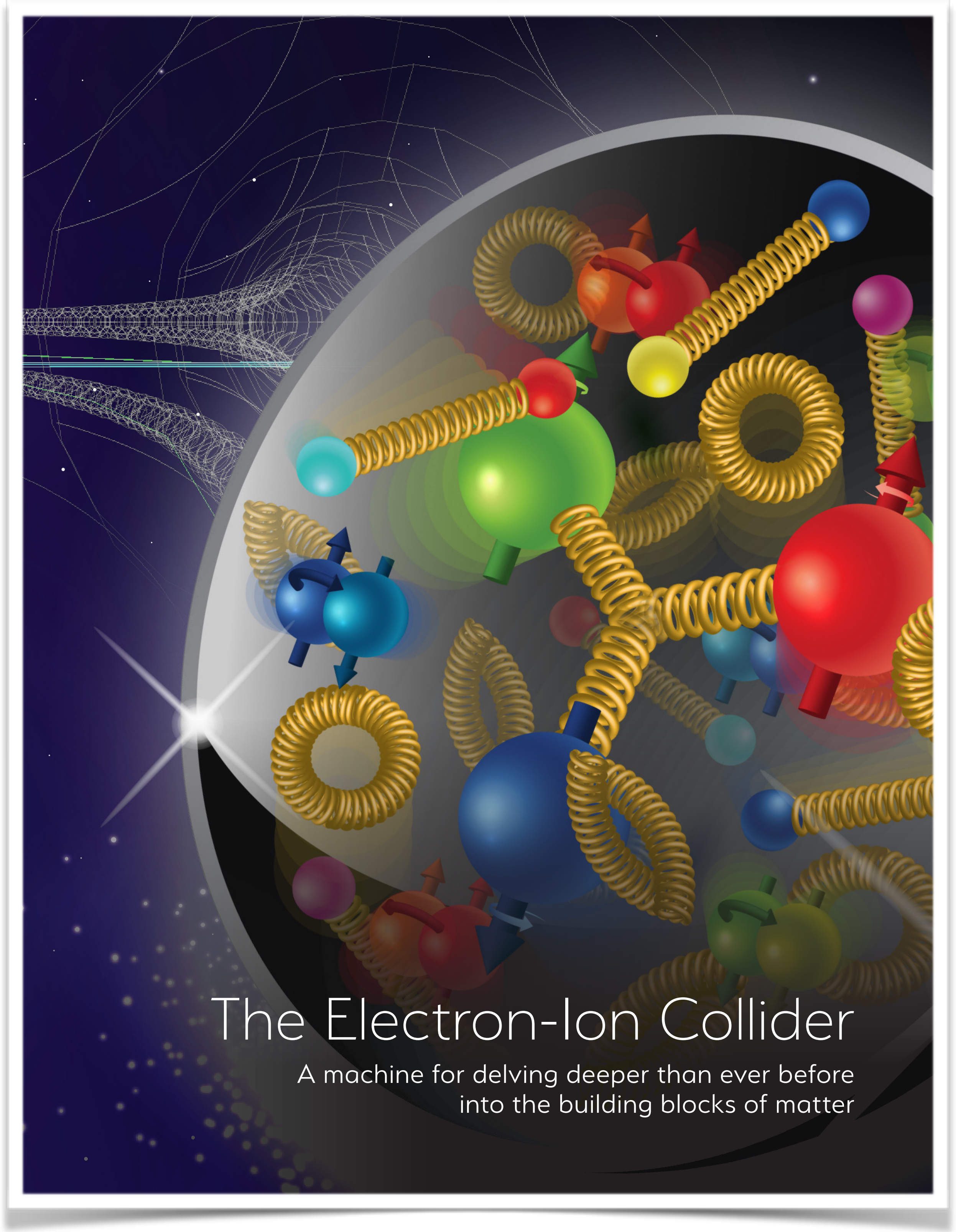
Program	Physics Goals	Beam Energy [GeV]	Beam Intensity $[s^{-1}]$	Trigger Rate [kHz]	Beam Type	Target	Earliest start time, duration	Hardware additions
muon-proton elastic scattering	Precision proton-radius measurement	100	$4 \cdot 10^6$	100	$\mu^\pm$	high-pressure H2	2022 1 year	active TPC, SciFi trigger, silicon veto,
Hard exclusive reactions	GPD E	160	$2 \cdot 10^7$	10	$\mu^\pm$	$NH_3^\uparrow$	2022 2 years	recoil silicon, modified polarised target magnet
Input for Dark Matter Search	$\bar{p}$ production cross section	20-280	$5 \cdot 10^5$	25	p	LH2, LHe	2022 1 month	liquid helium target
$\bar{p}$-induced spectroscopy	Heavy quark exotics	12, 20	$5 \cdot 10^7$	25	$\bar{p}$	LH2	2022 2 years	target spectrometer: tracking, calorimetry
Drell-Yan	Pion PDFs	190	$7 \cdot 10^7$	25	$\pi^\pm$	C/W	2022 1-2 years	
Drell-Yan (RF)	Kaon PDFs & Nucleon TMDs	$\sim$100	10^8	25-50	$K^\pm, \bar{p}$	$NH_3^\uparrow$, C/W	2026 2-3 years	”active absorber”, vertex detector
Primakoff (RF)	Kaon polarisa-bility & pion life time	$\sim$100	$5 \cdot 10^6$	> 10	K^-	Ni	non-exclusive 2026 1 year	
Prompt Photons (RF)	Meson gluon PDFs	≥ 100	$5 \cdot 10^6$	10-100	$K^\pm$ $\pi^\pm$	LH2, Ni	non-exclusive 2026 1-2 years	hodoscope
K-induced Spectroscopy (RF)	High-precision strange-meson spectrum	50-100	$5 \cdot 10^6$	25	K^-	LH2	2026 1 year	recoil TOF, forward PID
Vector mesons (RF)	Spin Density Matrix Elements	50-100	$5 \cdot 10^6$	10-100	$K^\pm, \pi^\pm$	from H to Pb	2026 1 year	

Table 2: Requirements for future programmes at the M2 beam line after 2021. Muon beams are in blue, conventional hadron beams in green, and RF-separated hadron beams in red.

Future Facilities & experiments on pion & kaon PDFs

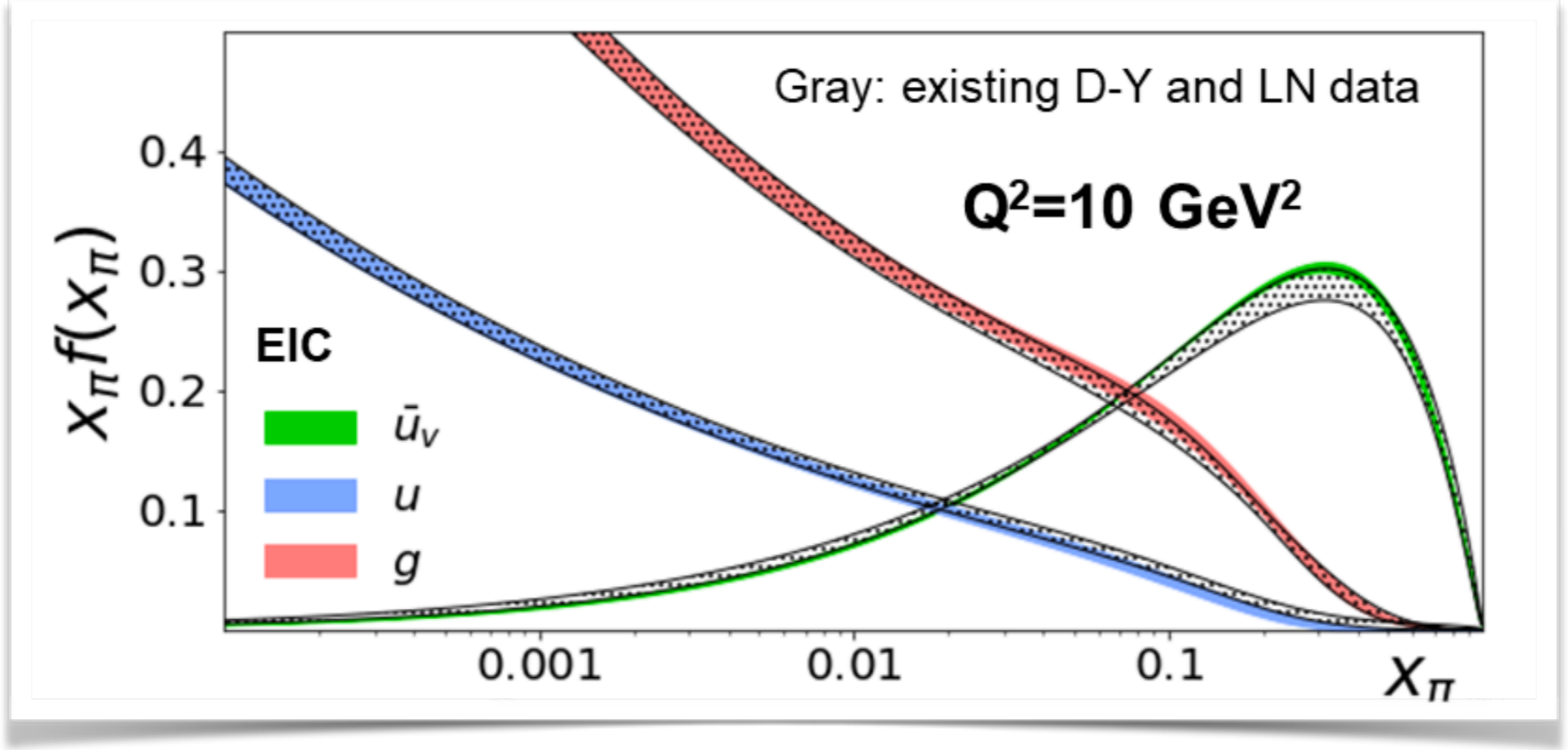
- The Electron Ion Collider (EIC):

  A machine for delving deeper than ever before into the building blocks of matter



- Scientific goals:
  - Precision 3D imaging of protons and nuclei
  - Search for saturation: color glass condensate
  - Solve the proton spin puzzle

Electron Ion Collider: The Next QCD Frontier: Understanding the glue that binds us all, EIC white paper, Eur.Phys.J.A 52 (2016) 9, 268



Sullivan process

A.C. Aguilar et al., Eur. Phys. J. A55 (2019) no.10, 190

- Pion and Kaon Structure Functions

Future Facilities & experiments on pion & kaon PDFs

- Electron Ion Collider in China (EicC): 中国极化电子离子对撞机

研究核子结构和强相互作用的“具有超高分辨率的立体电子显微镜”

- 重要物理
  - 核子的一维纵向结构
  - 核子结构三维成像
  - 核介质效应
  - 强子和奇特强子态
  - 其他重要探索研究
    - π介子的非极化结构函数

中国极化电子离子对撞机计划[J]. 核技术, 2020, 43(2): 20001-020001.

Outline

- Introduction
- Dyson-Schwinger Equations
- Pion parton distribution functions
- Kaon parton distribution functions
- **Summary and outlook**

Summary and outlook

Summary

- Using a Dyson-Schwinger Equations approach, presented a symmetry-preserving calculation of the pion & kaon PDFs.

- Pion: valence quark PDF point-wise behavior agrees with prediction from Lattice QCD as well as experimental extraction; gluon and sea distributions, even though first moments are similar to those from global analysis, point-wise behavior are generally different.

- Kaon: ratio $u_K(x, \zeta_5)/u_\pi(x, \zeta_5)$ is consistent with a single, recent Lattice QCD study as well as experimental extraction; gluon and sea distributions, no results from Lattice QCD available.

* New-era experiments (COMPASS++/AMBER, EIC, and EicC) are capable of discriminating between the results from theoretical studies.

Outlook

- ρ meson PDFs.
- Nucleon PDFs.

Thank you