

The Y states and other vectors in e^+e^- annihilation

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第五届论坛：The Y states and other vectors in e+e- annihilation

主持人：沈肖雁 研究员



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腾讯会议室ID：6733913824

会议时间：2020年7月22日 15:00 – 16:30

报告人：苑长征 研究员（高能所）

摘要：Besides the peaks observed in inclusive hadronic cross section in e+e- annihilation, a few Y structures, Y(4260), Y(4360), Y(4660), are observed in exclusive hadronic cross sections in the charmonium energy region. In this talk, some new experimental results from BESIII experiment will be presented together with relevant results in bottomonium and other energy regions. Theoretical efforts are called for to understand the nature of these vector structures.

报告人简介：

1991年获华东师范大学物理学学士学位，
1997年获中国科学院高能物理研究所博士学位，
1997年至1999年中国高等科学技术中心，博士后；
1999年至2001年法国科研中心核物理与粒子物理研究所直线加速器实验室(LAL, IN2P3-CNRS)，
博士后；
2001年起，中国科学院高能物理研究所，研究员。
现任BESIII合作组发言人。参加北京谱仪（BES、BESIII）和Belle、Belle II实验，带领研究团队发现了带电类粲偶素Zc(3900)、Zc(4020)和矢量类粲偶素Y(4660)等。在国际核心刊物上发表论文多篇。（参见SLACInspire数据库 <http://inspirehep.net/author/profile/Chang.Zheng.Yuan.1>）

顾问委员：(按姓氏拼音排序)

陈莹（中国科学院高能物理研究所），高原宁（北京大学），李海波（中国科学院高能物理研究所），梁作堂（山东大学），刘川（北京大学），吕才典（中国科学院高能物理研究所），马建平（中国科学院理论物理研究所），彭海平（中国科学技术大学），乔从丰（中国科学院大学），沈肖雁（中国科学院高能物理研究所），许怒（中国科学院近代物理所），苑长征（中国科学院高能物理研究所），张肇西（中国科学院理论物理研究所），张宗桦（中国科学院高能物理研究所），赵光达（北京大学），赵强（中国科学院高能物理研究所），赵政国（中国科学技术大学），郑汉青（北京大学），郑阳恒（中国科学院大学），朱世琳（北京大学），邹冰松（中国科学院理论物理研究所）

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致谢与声明

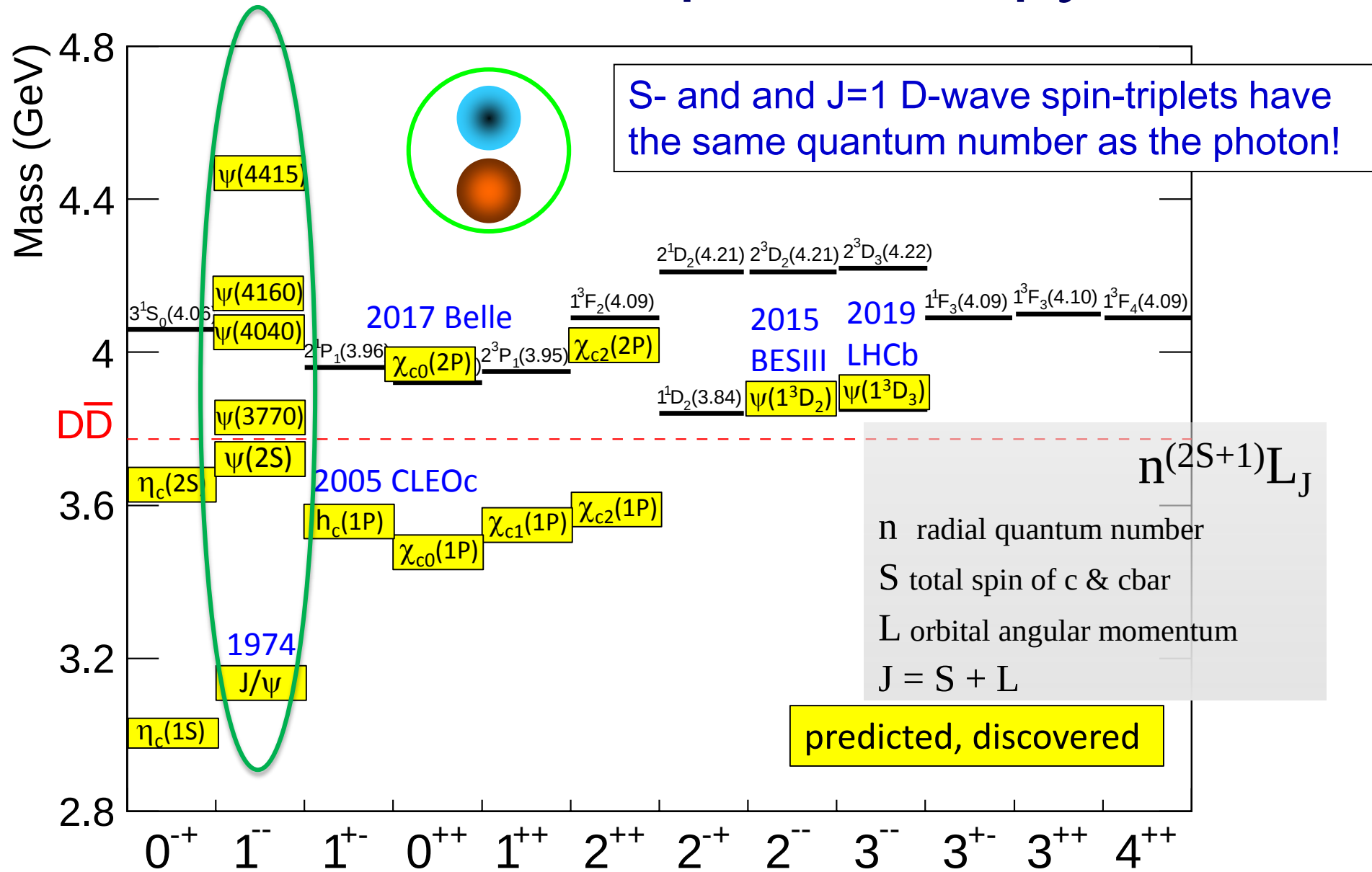
- 感谢吴佳俊等的邀请
- 整理一下数据
- 分享一些思考
- 提出几个问题
- 内容

— 矢量粲偶素/类粲偶素

— 矢量底偶素/类底偶素

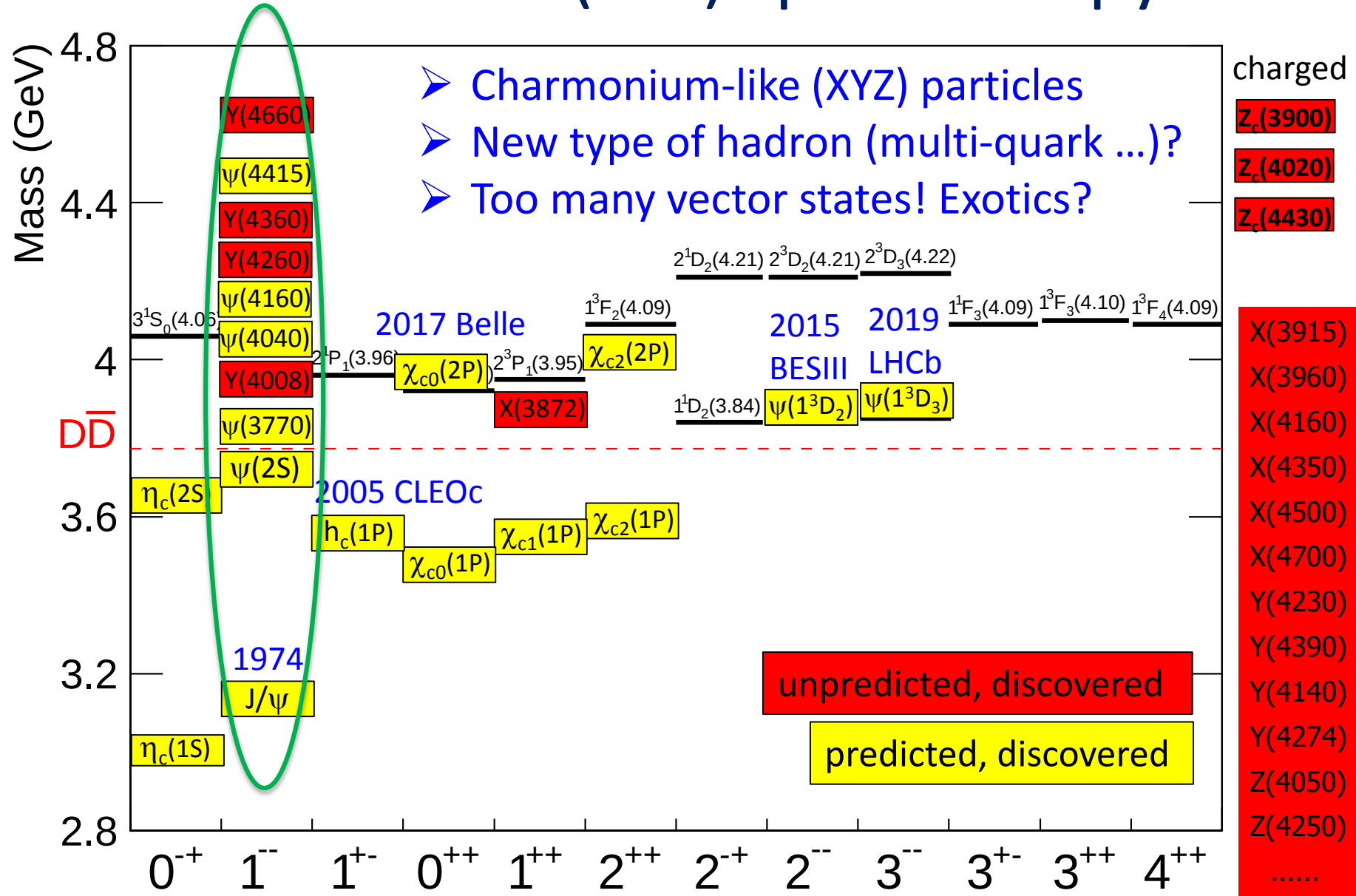
— 矢量轻夸克偶素

Charmonium spectroscopy



Godfrey & Isgur, PRD32, 189 (1985)

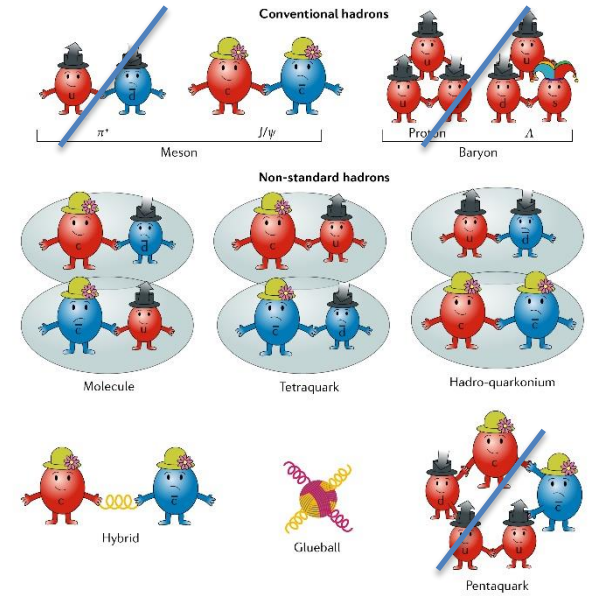
Charmonium(like) spectroscopy



Godfrey & Isgur, PRD32, 189 (1985)

What are these ψ 's & Y 's? [mass > $2m_D$]

- Vector charmonia
 - 1^3D_1 , 3^3S_1 , 2^3D_1 , 4^3S_1 , 3^3D_1 , 5^3S_1 , 4^3D_1 , 6^3S_1 , ...
- Vector charmonium hybrids
- Vector tetraquark states
- Vector hadro-charmonia
- Vector hadronic molecules ($\omega\chi_{cJ}$, $\bar{D}D_1$, $\bar{\Lambda}_c\Lambda_c$,
- Kinematic effects (thresholds, coupled channels, interference, ...)
-

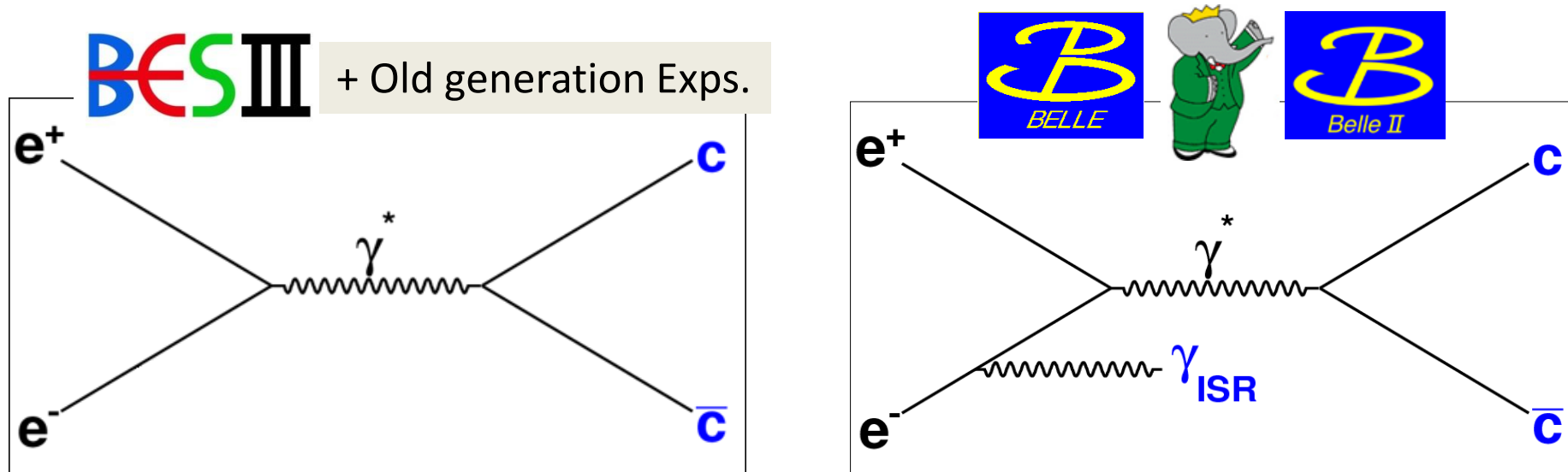


arXiv: 2001.01164

➔ 哪些实验信息是可靠的？

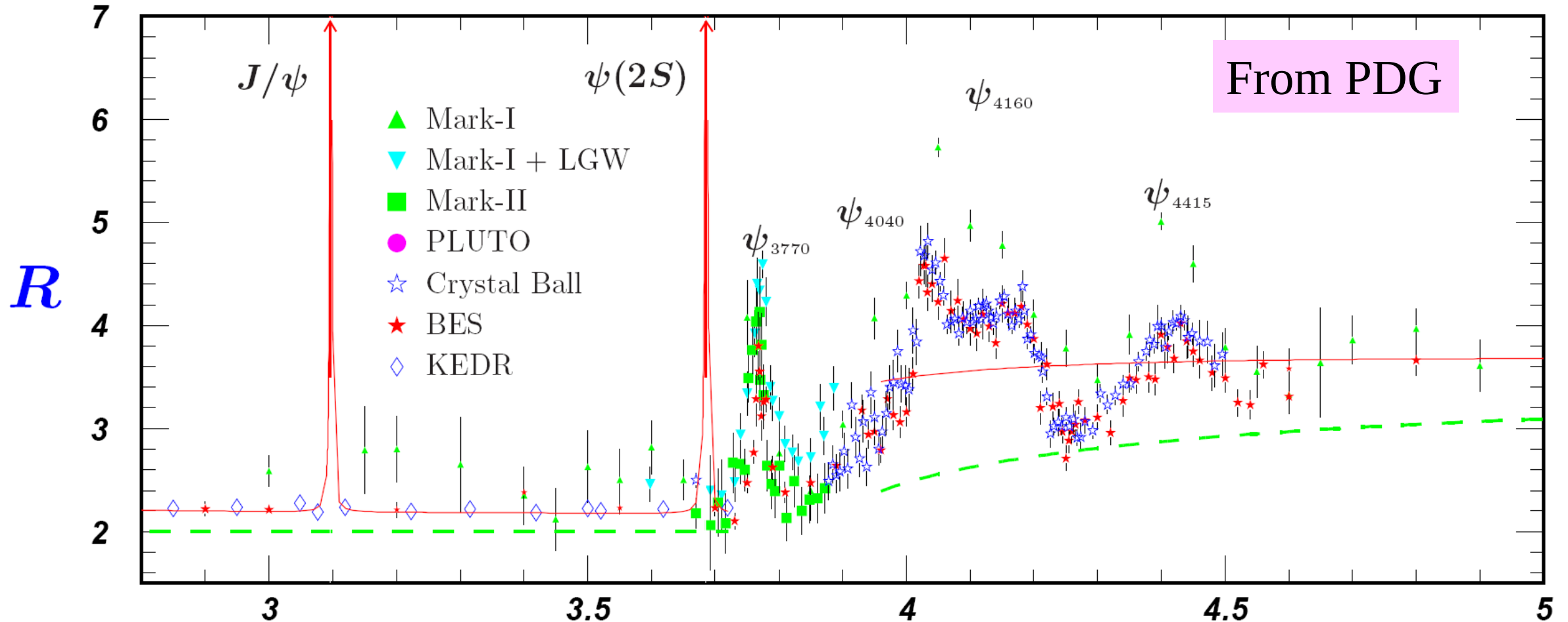
How did we know?

(ψ 's from inclusive, Y 's from exclusive)



ISR = initial state radiation

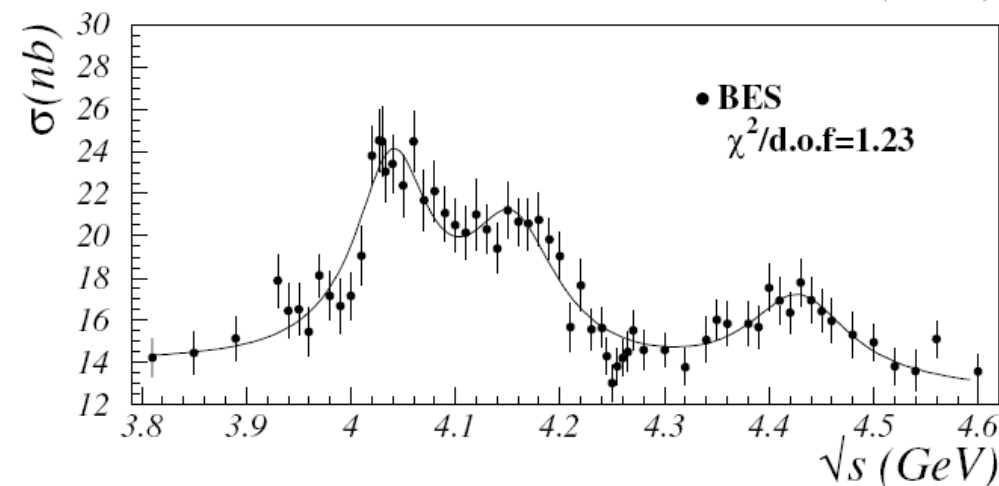
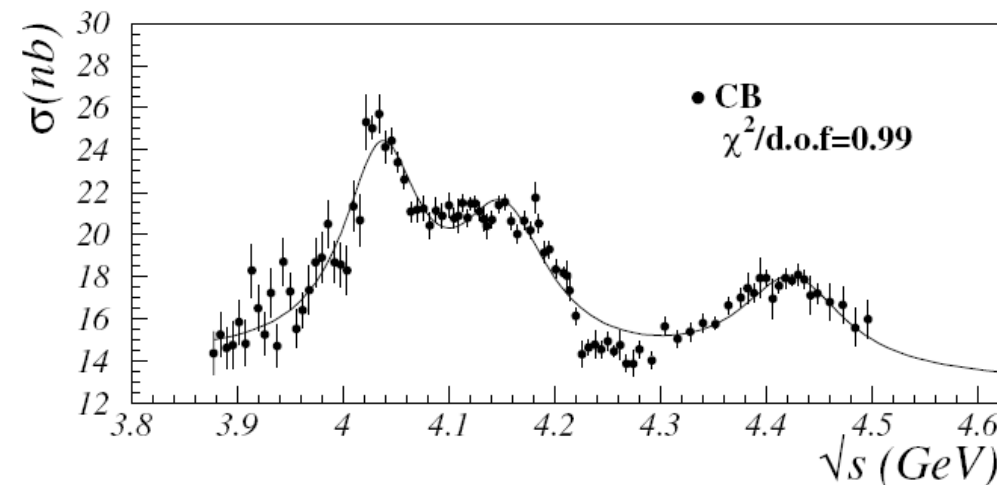
Inclusive cross sections, most precise data from BESII & Crystal Ball



Fit by K. K. Seth

Incoherent sum of continuum+3 BWs

- BES data (M, Γ, Γ_{ee})
 - $\psi(4040)$
 - 4040 ± 1 MeV
 - 89 ± 6 MeV
 - 0.91 ± 0.13 keV
 - $\psi(4160)$
 - 4155 ± 5 MeV
 - 107 ± 16 MeV
 - 0.84 ± 0.13 keV
 - $\psi(4415)$
 - 4429 ± 9 MeV
 - 118 ± 35 MeV
 - 0.64 ± 0.23 keV



$$\sigma_{BW}(\sqrt{s}) = (3\pi/4p^2)\Gamma_{ee}\Gamma_h/[(\sqrt{s} - M)^2 + (\Gamma_{\text{tot}}/2)^2]$$

Fit by BES

$$R_{\text{the}} = R_{\text{con}} + R_{\text{res}}$$

Coherent sum of 4 BWs

$$R_{\text{con}} = C_0 + C_1(W - 2M_{D^\pm}) + C_2(W - 2M_{D^\pm})^2$$

Include $\bar{c}c$, may also interfere with R_{res}

$$R_{\text{res}} = \frac{\sigma_{\text{res}}}{\sigma_{\mu\mu}^0} = \frac{12\pi}{s} [|\mathcal{T}_{\psi'}|^2 + |\mathcal{T}_{\text{res}}|^2]$$

Amplitude of $r \rightarrow f$

$$|\mathcal{T}_{\text{res}}|^2 = \sum_f \left| \sum_r \mathcal{T}_r^f(W) \right|^2$$

$$\mathcal{T}_r^f(W) = \frac{M_r \sqrt{\Gamma_r^{ee} \Gamma_r^f}}{W^2 - M_r^2 + i M_r \Gamma_r} e^{i\delta_r}$$

Mass dependent width

$$\Gamma_r^f(W) = \hat{\Gamma}_r \sum_L \frac{Z_f^{2L+1}}{B_L}$$

→ BR of ψ decays

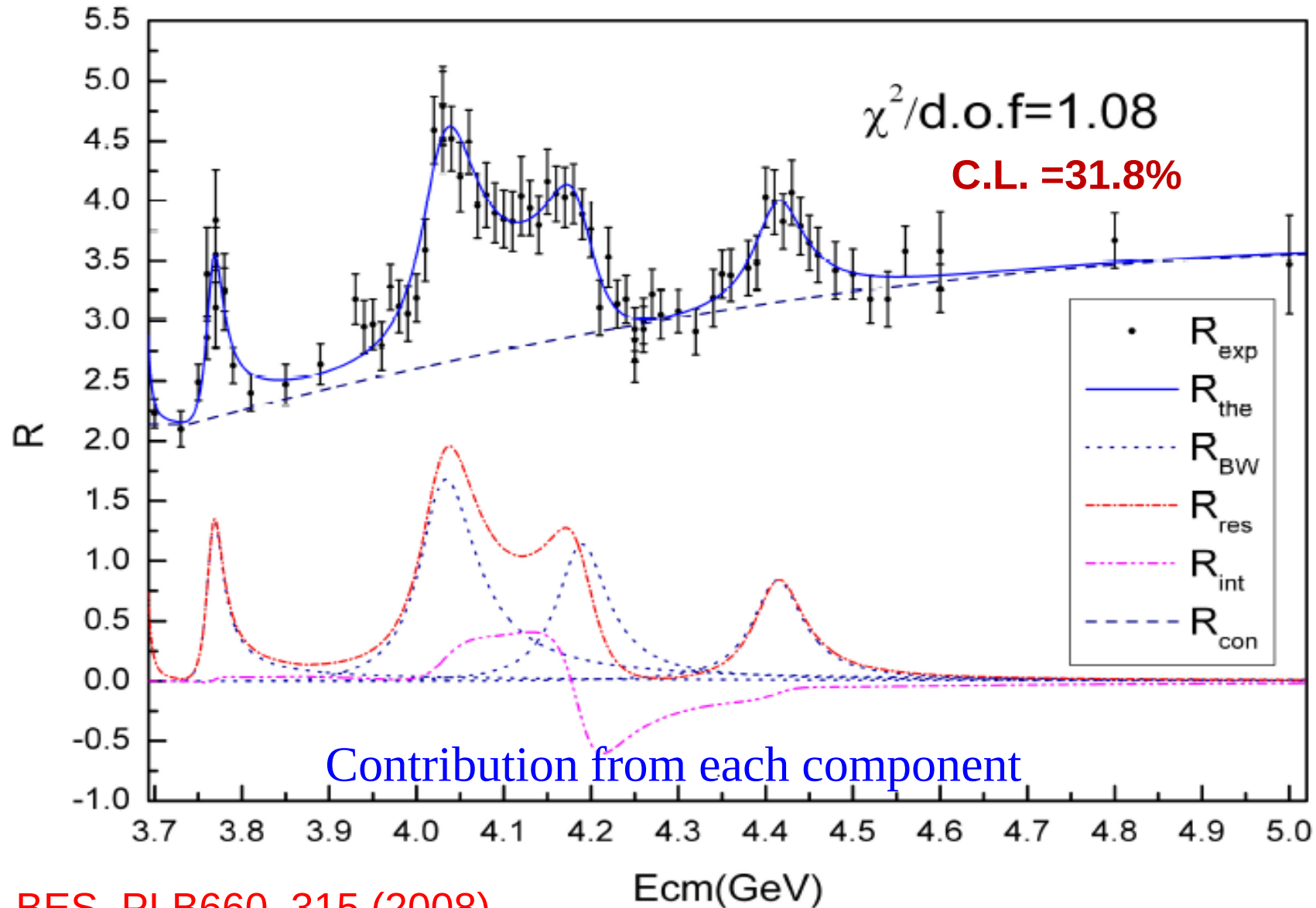
$$\psi(3770) \Rightarrow D\bar{D};$$

$$\psi(4040) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s;$$

$$\psi(4140) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, D_s\bar{D}_s^*;$$

$$\psi(4415) \Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, D_s\bar{D}_s^*, \\ D_s^*\bar{D}_s^*, D\bar{D}_1, D\bar{D}_2^*.$$

Fit by BES



Final results from BES

The resonance parameters of the high mass charmonia in this work together with the values in PDG2004 [11], PDG2006 [12] and Seth's evaluations [13] based on Crystal Ball and BES data. The total width $\Gamma_{\text{tot}} \equiv \Gamma_r(M)$ in Eq. (9)

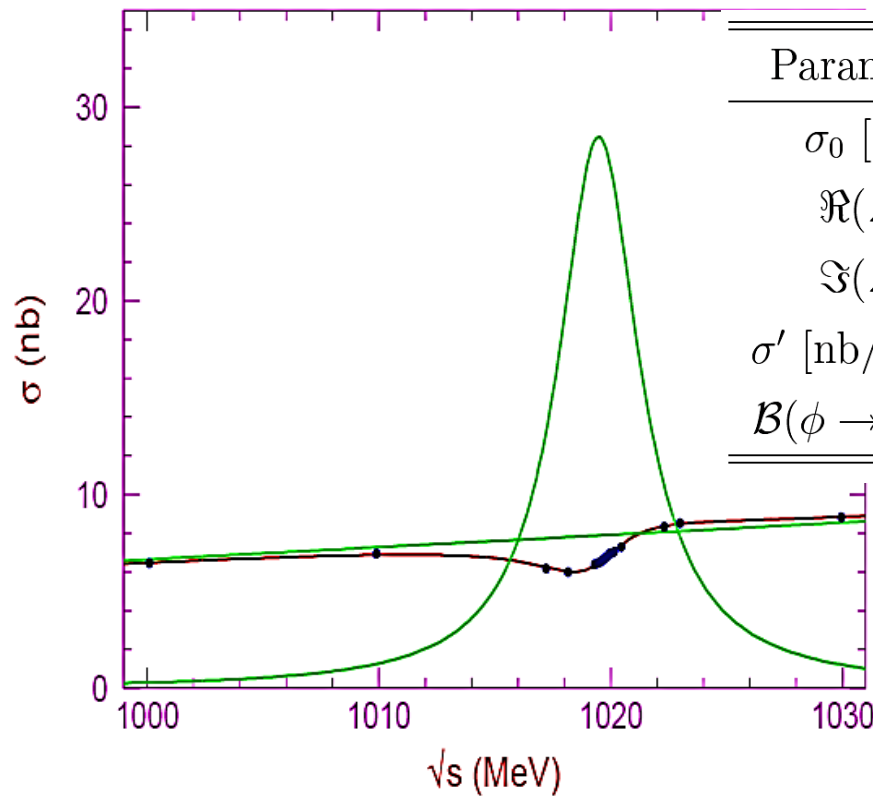
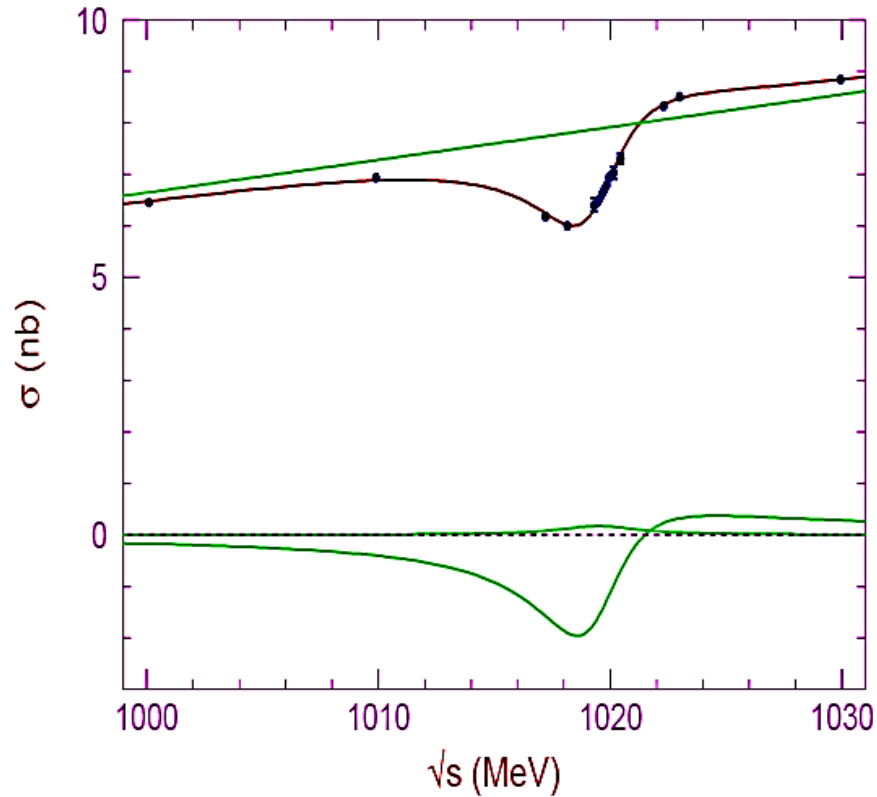
		$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV/ c^2)	PDG2004	3769.9 ± 2.5	4040 ± 10	4159 ± 20	4415 ± 6
	PDG2006	3771.1 ± 2.4	4039 ± 1	4153 ± 3	4421 ± 4
	CB (Seth)	–	4037 ± 2	4151 ± 4	4425 ± 6
	BES (Seth)	–	4040 ± 1	4155 ± 5	4455 ± 6
	BES (this work)	3772.0 ± 1.9	4039.6 ± 4.3	4191.7 ± 6.5	4415.1 ± 7.9
Γ_{tot} (MeV)	PDG2004	23.6 ± 2.7	52 ± 10	78 ± 20	43 ± 15
	PDG2006	23.0 ± 2.7	80 ± 10	103 ± 8	62 ± 20
	CB (Seth)	–	85 ± 10	107 ± 10	119 ± 16
	BES (Seth)	–	89 ± 6	107 ± 16	118 ± 35
	BES (this work)	30.4 ± 8.5	84.5 ± 12.3	71.8 ± 12.3	71.5 ± 19.0
Γ_{ee} (keV)	PDG2004	0.26 ± 0.04	0.75 ± 0.15	0.77 ± 0.23	0.47 ± 0.10
	PDG2006	0.24 ± 0.03	0.86 ± 0.08	0.83 ± 0.07	0.58 ± 0.07
	CB (Seth)	–	0.88 ± 0.11	0.83 ± 0.08	0.72 ± 0.11
	BES (Seth)	–	0.91 ± 0.13	0.84 ± 0.13	0.64 ± 0.23
	BES (this work)	0.22 ± 0.05	0.83 ± 0.20	0.48 ± 0.22	0.35 ± 0.12
δ (degree)	BES (this work)	0	130 ± 46	293 ± 57	234 ± 88

Potential problems

- Peaks = resonances
- Contribution from the Y states was not considered
- Many assumptions
 - Only two-body open-charm decays, multi-body + charmonium neglected
 - Decay width $\propto$ phase space only
 - Same relative phase for different final states
 - No interference between continuum & resonant charm final states
 - BW parametrization for resonances
- Multiple solutions ignored

Multiple solutions

$$\sigma(\sqrt{s}) = \sigma_{\text{nr}}(\sqrt{s}) \cdot \left| 1 - Z \frac{M_\phi \Gamma_\phi}{D_\phi(\sqrt{s})} \right|^2$$



Parameter	Solution I	Solution II
σ_0 [nb]	7.88 ± 0.04	7.88 ± 0.08
$\Re(Z)$	0.106 ± 0.004	0.106 ± 0.006
$\Im(Z)$	-0.103 ± 0.003	-1.90 ± 0.006
σ' [nb/MeV]	0.064 ± 0.002	0.064 ± 0.006
$\mathcal{B}(\phi \rightarrow \omega\pi^0)$	4.61×10^{-5}	7.62×10^{-3}

Figure 2: Fit to the $e^+e^- \rightarrow \omega\pi^0$ cross sections as a function of center-of-mass energy.

CZY, X.H. Mo, P. Wang, arXiv:0911.4791. Same problem exists in fitting R data.
More on multiple solutions: K. Zhu et al., IJMPA26, 4511 (2011)

Multiple solutions in R-fit

Simplified parameterization of the resonance amplitudes

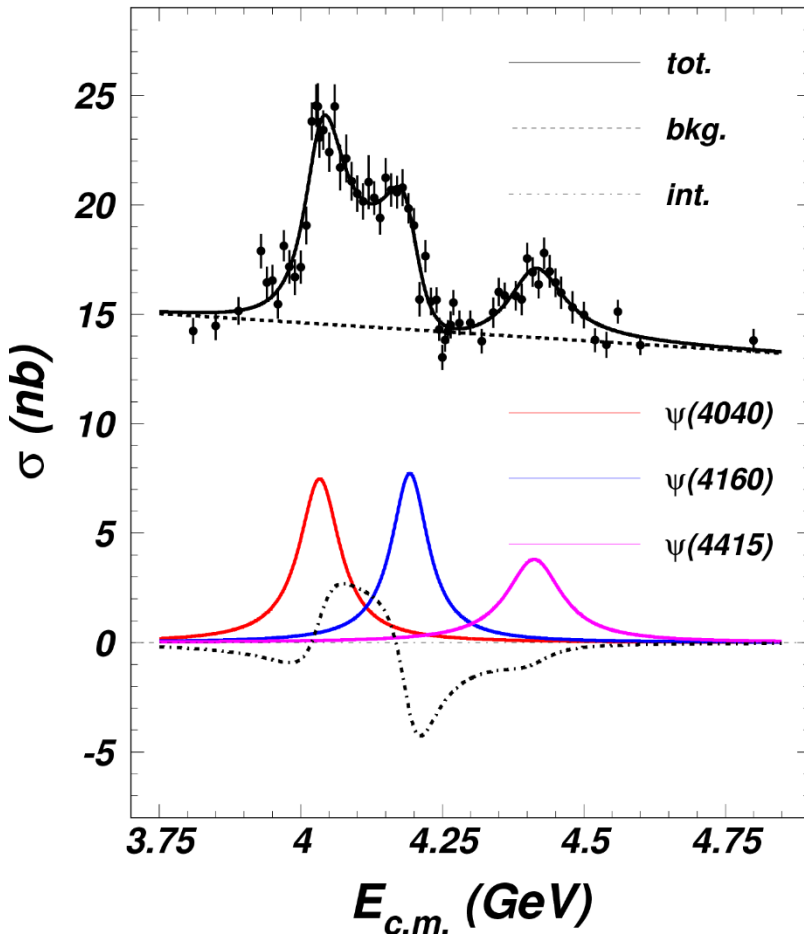
$$\sigma^{the.} = \sigma^{res.} + \sigma^{con.}$$

$$\sigma^{con.} = A + B(\sqrt{s} - 2M_{D^\pm})$$

$$\sigma^{res.}(s) = \left| \sum_{j=1}^3 T_j(s) \right|^2$$

$$T_j(s) = \frac{\sqrt{12\pi\Gamma_j^h\Gamma_j^{ee}} e^{i\phi_j}}{s - M_j^2 + iM_j\Gamma_j^t}$$

Multiple solutions in BES data



Parameter	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV)	4034 ± 6	4193 ± 7	4412 ± 15
Γ_t (MeV)	87 ± 11	79 ± 14	118 ± 32
$\Gamma_{ee}^{(1)}$ (keV)	0.66 ± 0.22	0.42 ± 0.16	0.45 ± 0.13
$\phi^{(1)}$ (radian)	0 (fixed)	2.7 ± 0.8	2.0 ± 0.9
$\Gamma_{ee}^{(2)}$ (keV)	0.72 ± 0.24	0.73 ± 0.18	0.60 ± 0.25
$\phi^{(2)}$ (radian)	0 (fixed)	3.1 ± 0.7	1.4 ± 1.2
$\Gamma_{ee}^{(3)}$ (keV)	1.28 ± 0.45	0.62 ± 0.30	0.59 ± 0.20
$\phi^{(3)}$ (radian)	0 (fixed)	3.7 ± 0.4	3.8 ± 0.8
$\Gamma_{ee}^{(4)}$ (keV)	1.41 ± 0.12	1.10 ± 0.15	0.78 ± 0.17
$\phi^{(4)}$ (radian)	0 (fixed)	4.1 ± 0.1	3.2 ± 0.3

Same for all solutions

*Sol. I
(same as BES)*

Sol. II

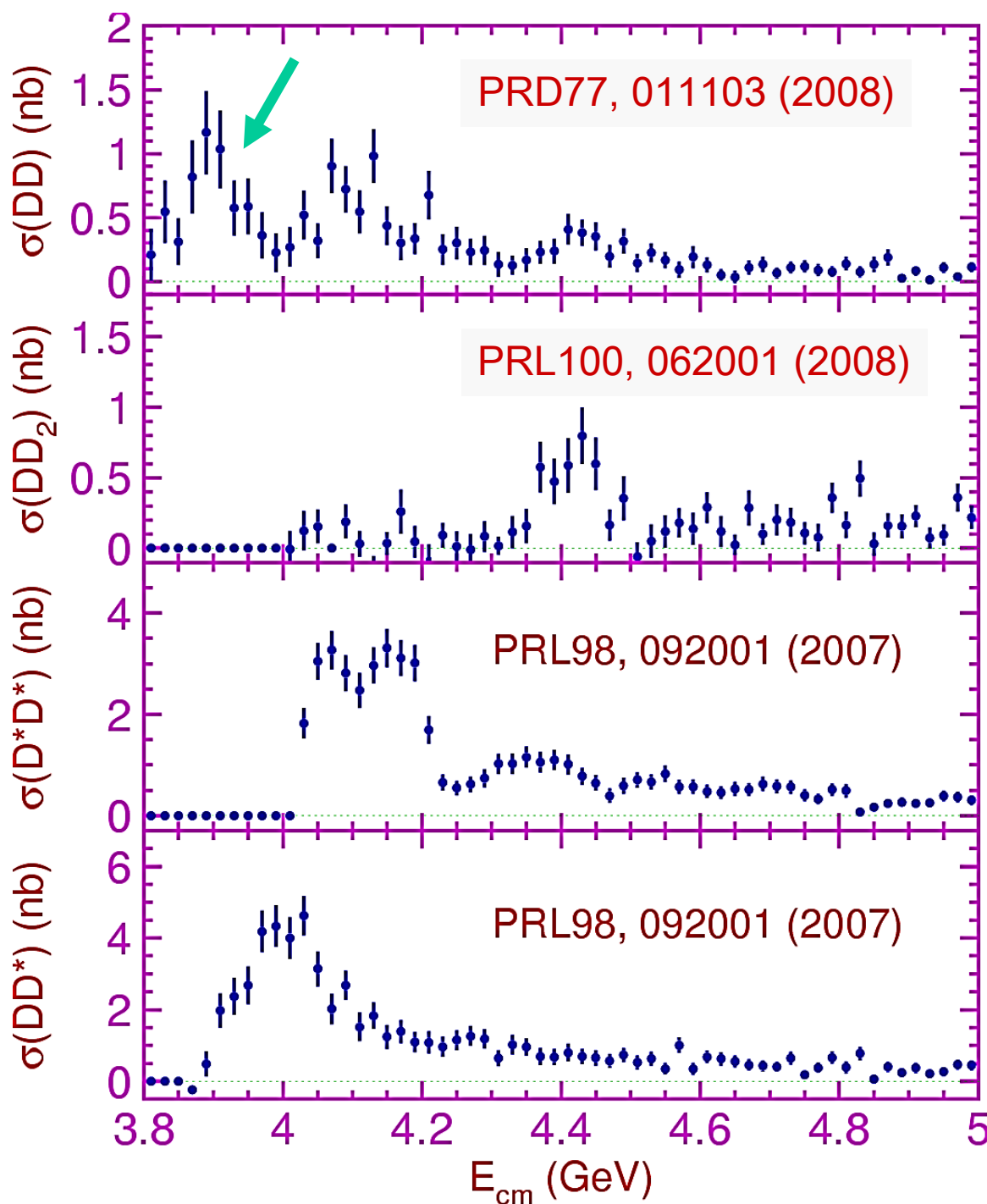
Sol. III

Sol. IV

4 solutions with the same fit quality!

BES official fit is only one of the four possible solutions!

Belle R_D Scan via ISR

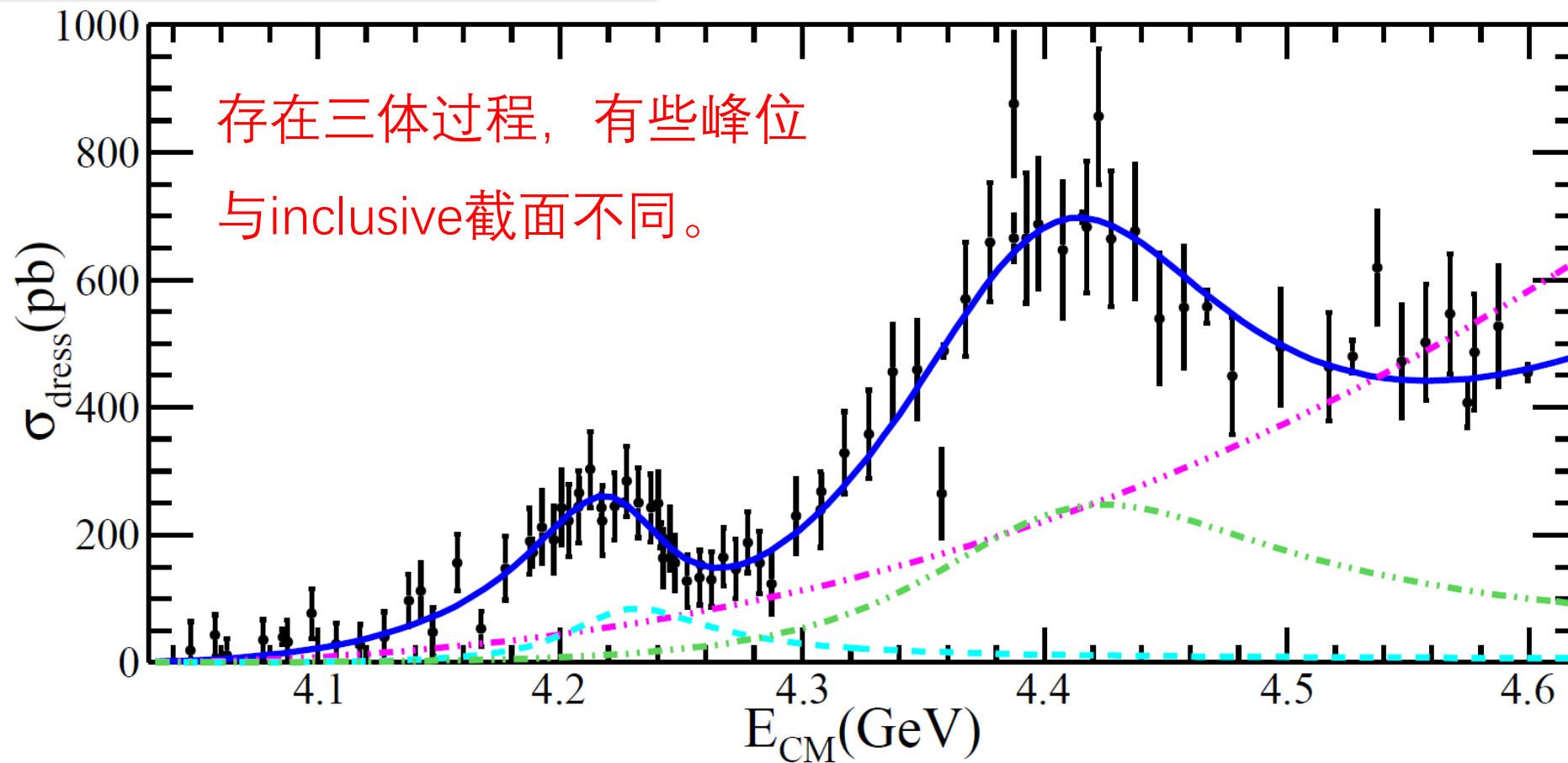


Continuous energy scan. Full mass range in one experiment, errors large due to low efficiency of ISR & D tag.

- 有的峰在单举截面中看不到;
- 两体过程分支比不只跟相空间有关;
- 每个道的干涉都可能不同;
- 每个道都可能可能有非共振振幅。

$$\sigma_{\text{dress}} = \frac{N^{\text{obs}}}{\mathcal{L}(1 + \delta^r) B(D^0 \rightarrow K^- \pi^+) \varepsilon}$$

$$\sigma_{\text{dress}}(m) = |c \cdot \sqrt{P(m)} + e^{i\phi_1} B_1(m) \sqrt{\frac{P(m)}{P(M_1)}} + e^{i\phi_2} B_2(m) \sqrt{\frac{P(m)}{P(M_2)}}|^2$$



Fit with a phase space term (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$\pi^+\pi^-D^+D^-$

存在四体过程，有些峰位
与inclusive截面不同。

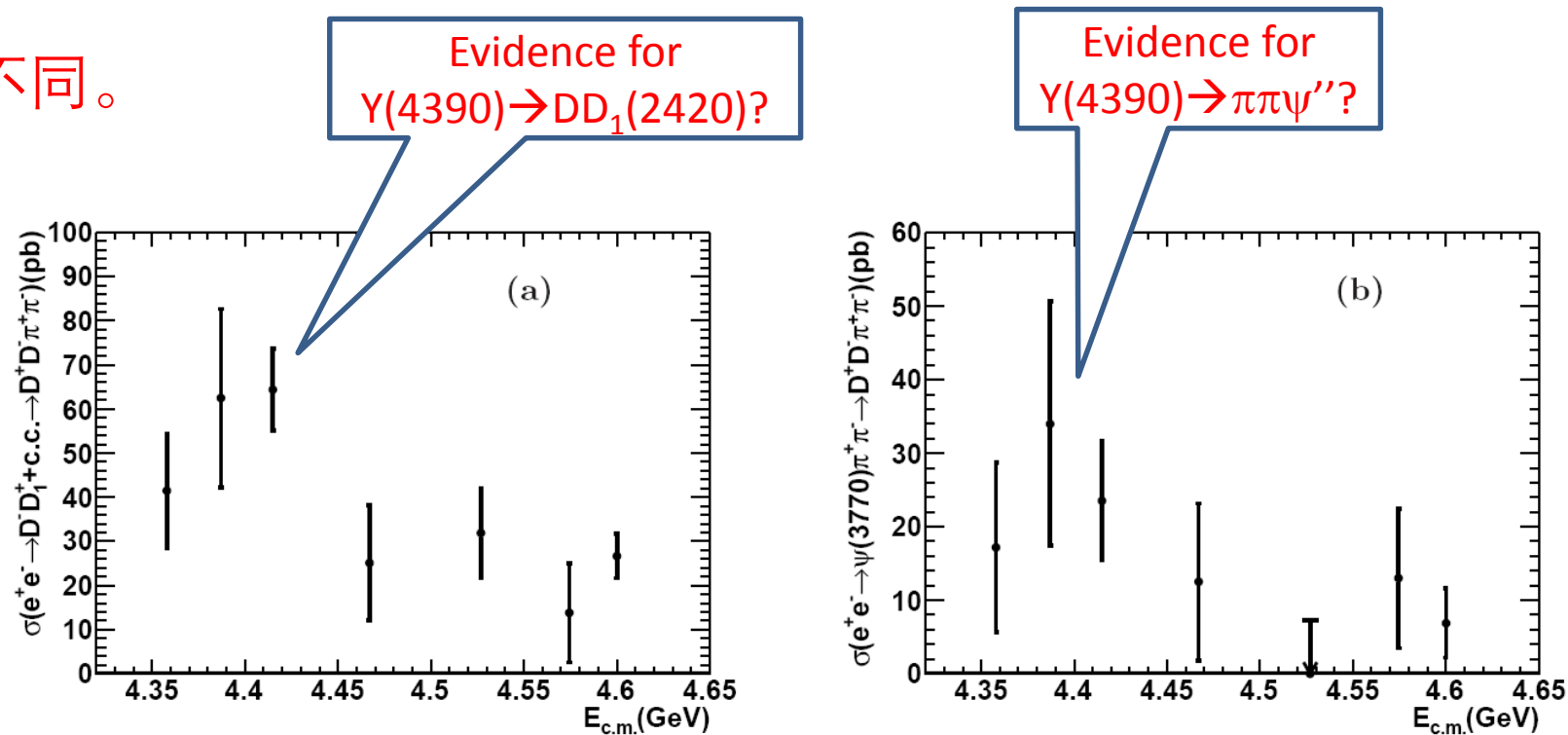
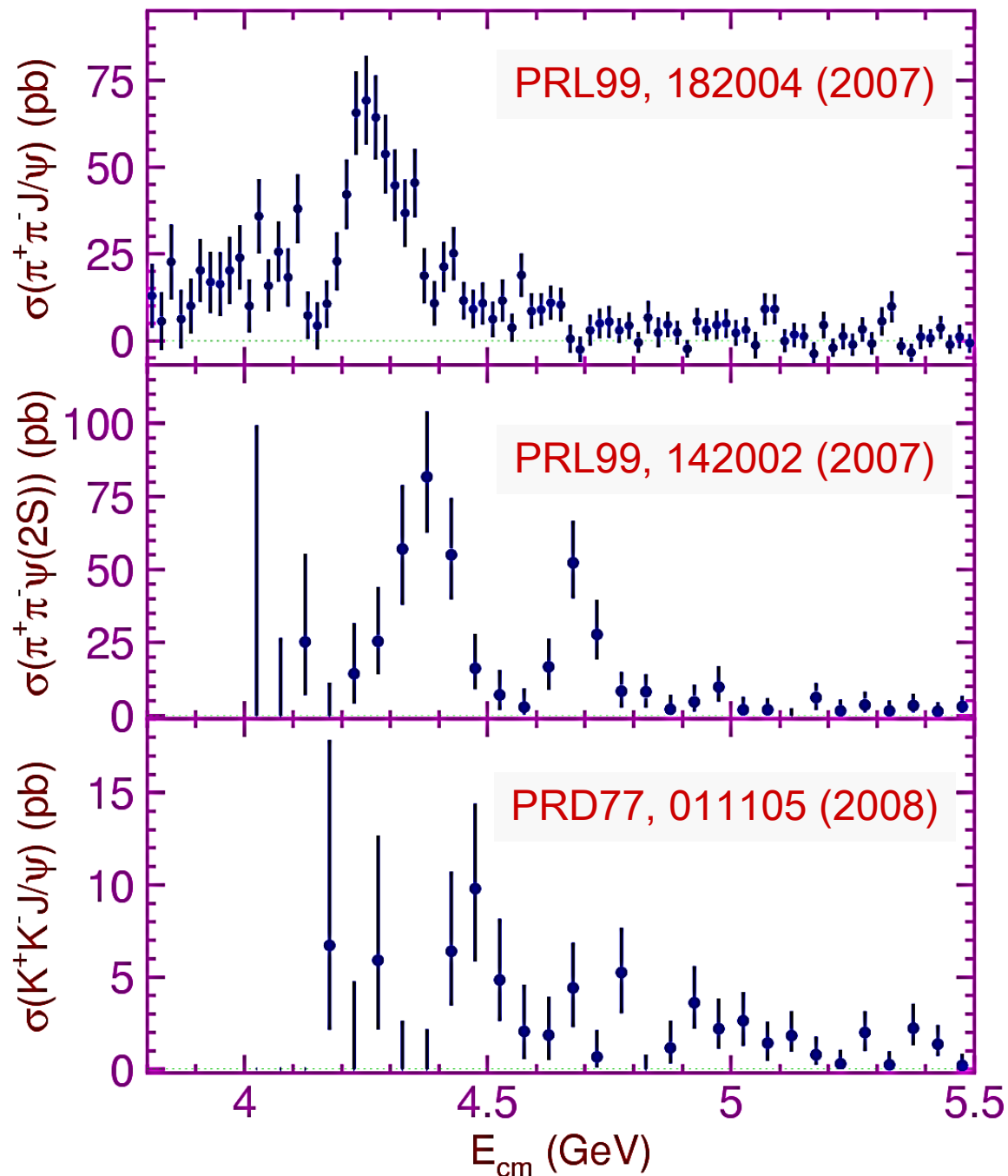


FIG. 4. The measured Born cross sections of the signal processes (a) $e^+e^- \rightarrow D_1(2420)^+ D^- + c.c. \rightarrow D^+ D^- \pi^+ \pi^-$ and (b) $e^+e^- \rightarrow \psi(3770) \pi^+ \pi^- \rightarrow D^+ D^- \pi^+ \pi^-$. The short horizontal line is the upper limit of cross section.

Belle ISR on R_{cc} (charmonium+hadrons)

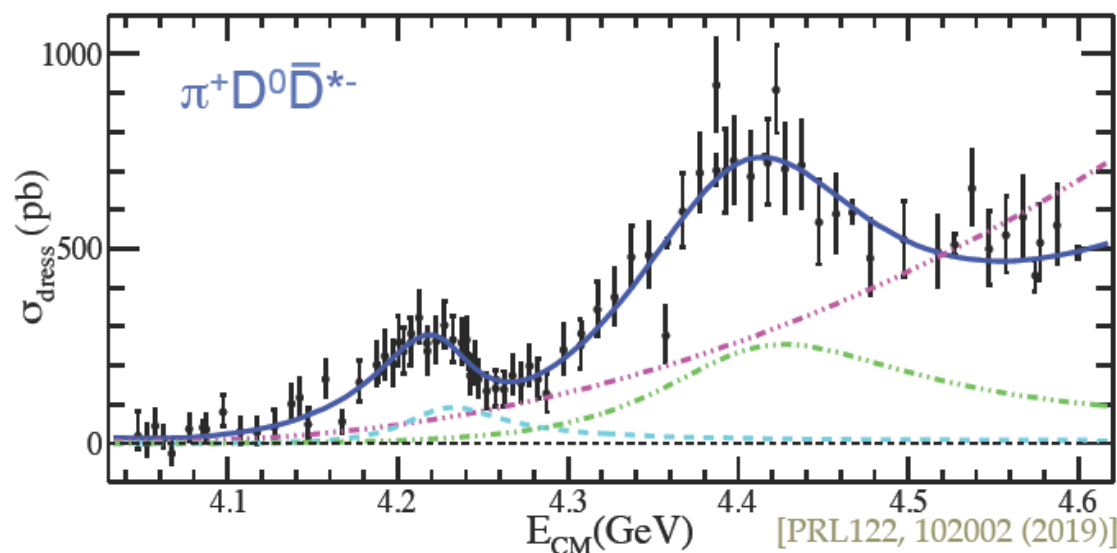
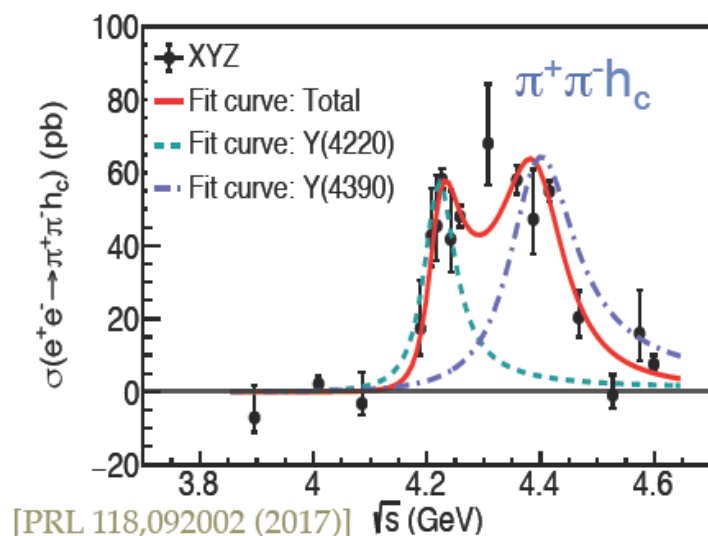
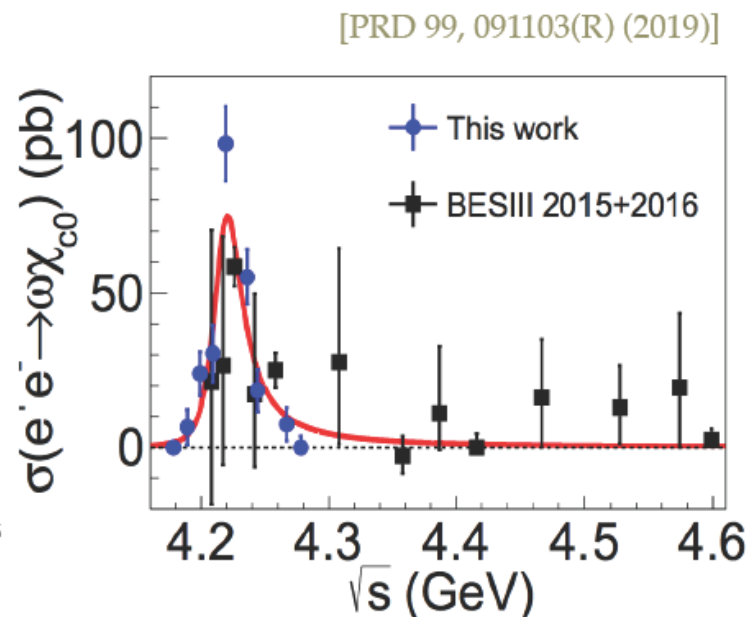
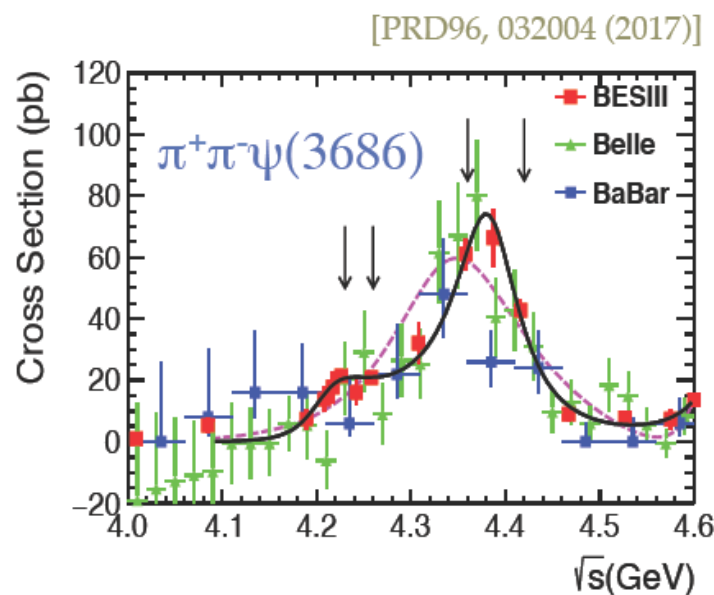
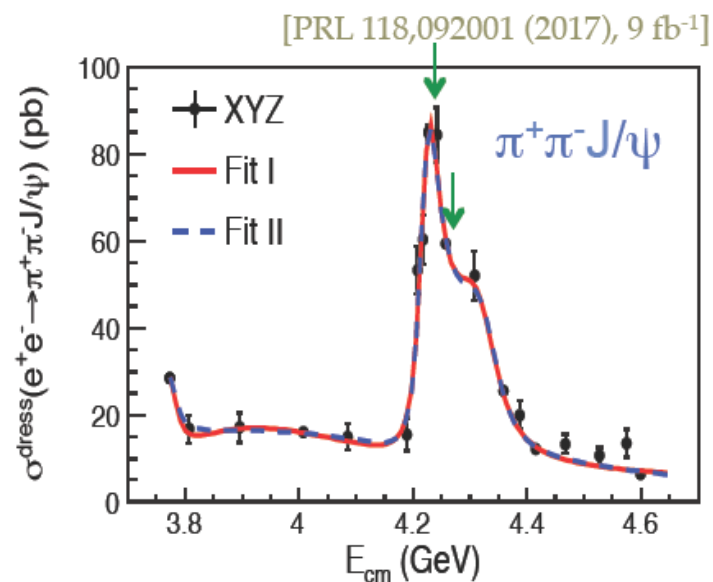
Full reconstruction of the hadronic
system, no ISR photon tag



存在hidden charm末态，其中的结构与inclusive截面以及open charm截面有很大不同。

存在与 ψ 激发态位置不同的 $Y(4008)$ 、 $Y(4260)$ 、 $Y(4360)$ 、 $Y(4660)$ 等。

Y states with high precision data



Combined fit to 4 modes

- $\omega\chi_{c0}, \pi^+\pi^-h_c, \pi^+\pi^-J/\psi, D^0D^{*-}\pi^+ + c.c.$

$$BW(\sqrt{s}) = \frac{\sqrt{12\pi\Gamma_{e^+e^-}\mathcal{B}_f\Gamma}}{s - M^2 + iM\Gamma} \sqrt{\frac{PS_n(\sqrt{s})}{PS_n(M)}}$$

$$\begin{aligned}\sigma_{\omega\chi_{c0}}(\sqrt{s}) &= |BW_1(\sqrt{s})|^2, \\ \sigma_{\pi^+\pi^-h_c}(\sqrt{s}) &= |BW_1(\sqrt{s}) + BW_3(\sqrt{s}) \cdot e^{i\phi_1}|^2, \\ \sigma_{\pi^+\pi^-J/\psi}(\sqrt{s}) &= |BW_0(\sqrt{s}) + BW_1(\sqrt{s}) \cdot e^{i\phi_2} + BW_2(\sqrt{s}) \cdot e^{i\phi_3}|^2, \\ \sigma_{D^0D^{*-}\pi^+ + c.c.}(\sqrt{s}) &= |\sqrt{PS_3(\sqrt{s})} + BW_1(\sqrt{s}) \cdot e^{i\phi_4} + BW_3(\sqrt{s}) \cdot e^{i\phi_5}|^2,\end{aligned}$$

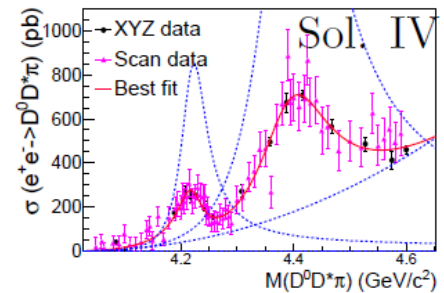
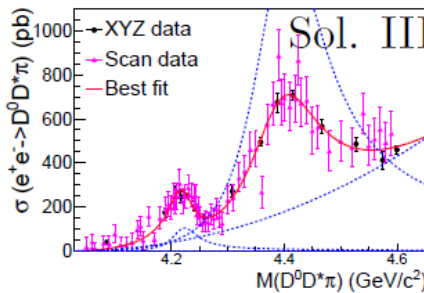
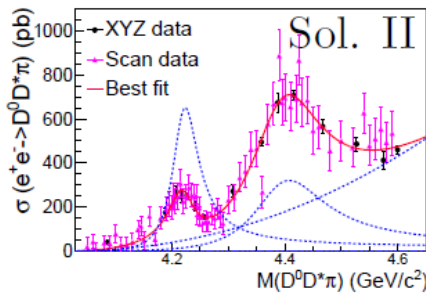
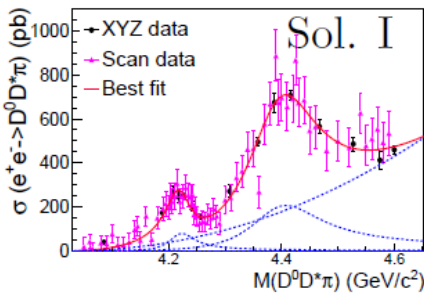
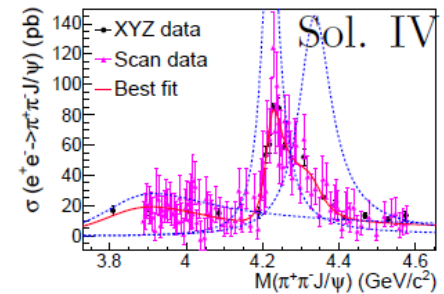
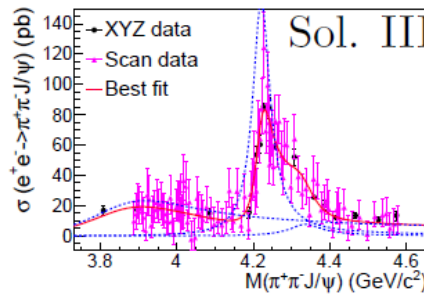
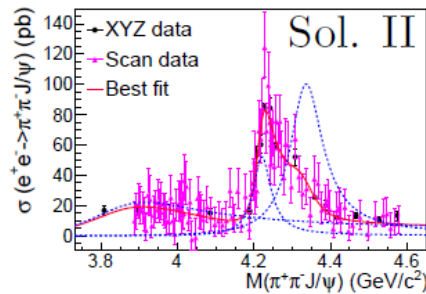
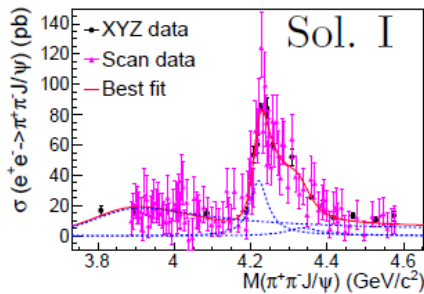
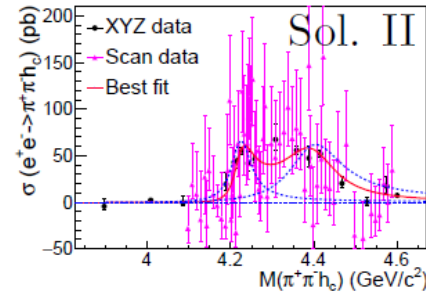
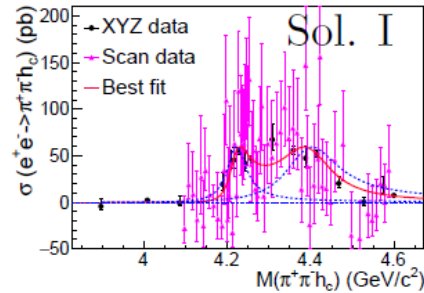
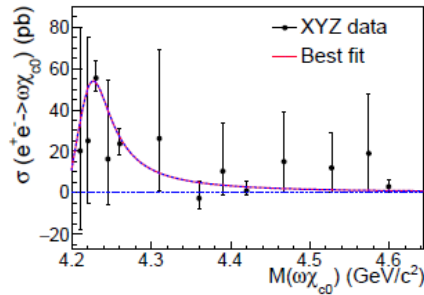
BW_1, BW_2, BW_3, BW_4 represent Y(4008), Y(4220), Y(4320), Y(4390)

Least square fit

$$\chi^2 = \sum_{j=1}^4 \sum_{i=1}^n \frac{(\sigma_{ij}^{data} - \sigma_{ij}^{fit})^2}{\delta_{ij}^2}$$

Fit results

- $M = 4219.6 \pm 3.3(stat) \pm 5.1(sys) \text{ MeV}/c^2$
- $\Gamma = 56.0 \pm 3.6(stat) \pm 6.9(sys) \text{ MeV}$



Fit results

	$Y(4008)$	$Y(4220)$	$Y(4320)$	$Y(4390)$
M	3846.3 ± 45.5	4219.6 ± 3.3	4333.2 ± 19.9	4391.5 ± 6.3
Γ	345.6 ± 58.2	56.0 ± 3.6	104.3 ± 44.9	153.2 ± 11.4

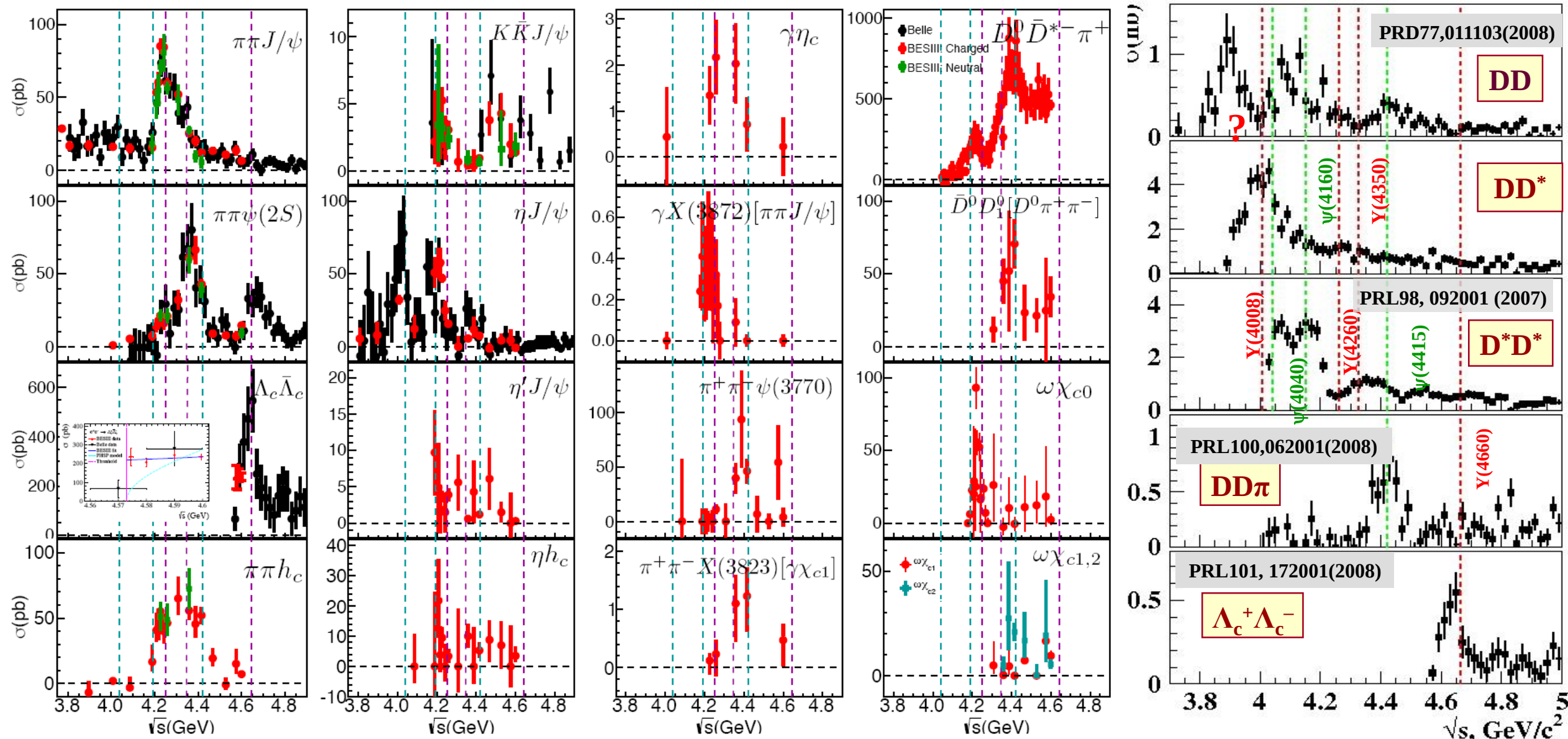
	Solution I	Solution II	Solution III	Solution IV
$(\mathcal{B}_{\omega\chi_{c0}} \times \Gamma_{e^+e^-})_{Y(4220)}$	3.4 ± 0.4			
$(\mathcal{B}_{\pi^+\pi^-h_c} \times \Gamma_{e^+e^-})_{Y(4220)}$	4.0 ± 1.1	4.0 ± 1.1		
$(\mathcal{B}_{\pi^+\pi^-h_c} \times \Gamma_{e^+e^-})_{Y(4390)}$	11.7 ± 2.4	11.7 ± 2.5		
ϕ_1	3.1 ± 0.4	-3.2 ± 0.4		
$(\mathcal{B}_{\pi^+\pi^-J/\psi} \times \Gamma_{e^+e^-})_{Y(4008)}$	5.5 ± 0.3	6.6 ± 0.7	6.9 ± 0.7	8.3 ± 0.7
$(\mathcal{B}_{\pi^+\pi^-J/\psi} \times \Gamma_{e^+e^-})_{Y(4220)}$	2.5 ± 0.2	3.5 ± 0.7	10.5 ± 1.1	15.1 ± 1.3
ϕ_2	0.1 ± 0.1	0.8 ± 0.3	-1.8 ± 0.2	-1.0 ± 0.1
$(\mathcal{B}_{\pi^+\pi^-J/\psi} \times \Gamma_{e^+e^-})_{Y(4320)}$	0.7 ± 0.2	13.3 ± 3.8	1.0 ± 0.5	19.4 ± 3.2
ϕ_3	2.2 ± 0.2	-2.0 ± 0.2	1.4 ± 0.6	-2.7 ± 0.1
$(\mathcal{B}_{D^0D^{*-}\pi^++c.c.} \times \Gamma_{e^+e^-})_{Y(4220)}$	5.3 ± 0.6	43.3 ± 3.2	6.9 ± 0.8	56.7 ± 4.2
ϕ_4	2.2 ± 0.1	-2.2 ± 0.1	-2.7 ± 0.1	-0.8 ± 0.1
$(\mathcal{B}_{D^0D^{*-}\pi^++c.c.} \times \Gamma_{e^+e^-})_{Y(4390)}$	39.7 ± 4.3	61.6 ± 6.6	265.5 ± 16.6	412.0 ± 26.0
ϕ_5	1.9 ± 0.1	1.5 ± 0.2	4.7 ± 0.1	4.2 ± 0.1

What are the Y states?

- At least one well determined Y state [Y(4220)]
- Is it the same as $\psi(4160)$?
- Are the Y(4360) and Y(4660) real resonances different from the $\psi(4415)$ and other ψ states?

Are the structures observed in inclusive and exclusive cross sections produced by the same states?

After we have measured all the e^+e^- annihilation cross sections, what do we do to get the resonant parameters of the vector charmonium(-like) states?



Inclusive fit: coupled channels

EICHTEN, GOTTFRIED, KINOSHITA,
LANE, AND YAN
PRD 21 203 (1980)

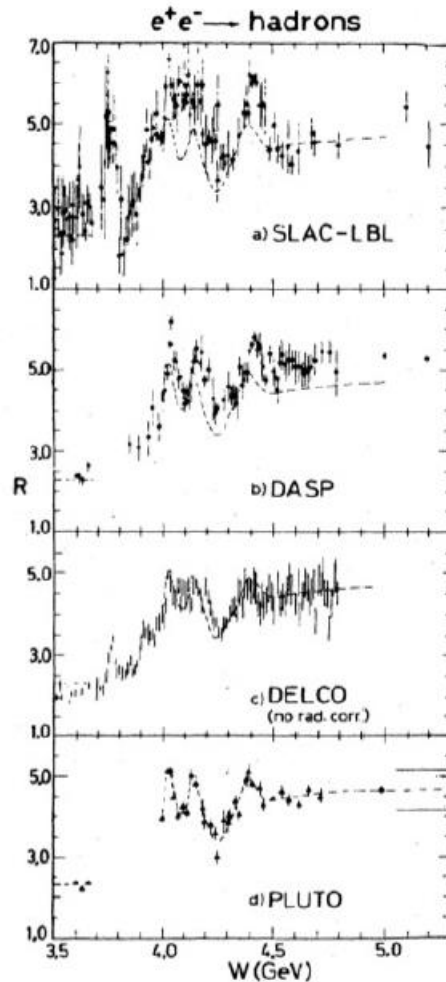


FIG. 15. Results of R (including $e^+e^- \rightarrow \tau^+\tau^-$) from four experiments: (a) SLAC-LBL (Ref. 44), (b) DASP (Ref. 46), (c) DELCO (Ref. 45), (d) PLUTO (Ref. 47). The curves represent a hand-drawn line through the PLUTO data. The band in Fig. 15(d) indicates the systematic errors of the PLUTO measurement. The plots shown were compiled by G. Feldman.

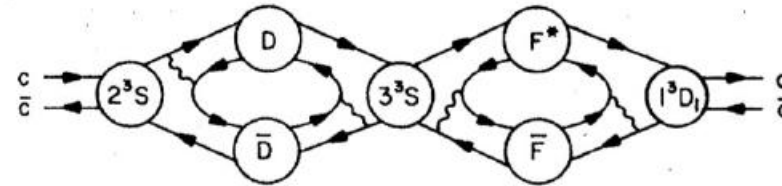


FIG. 8. The propagation of a $c\bar{c}$ pair in the presence of open and closed decay channels as described in the Green's function $\mathcal{G}$.

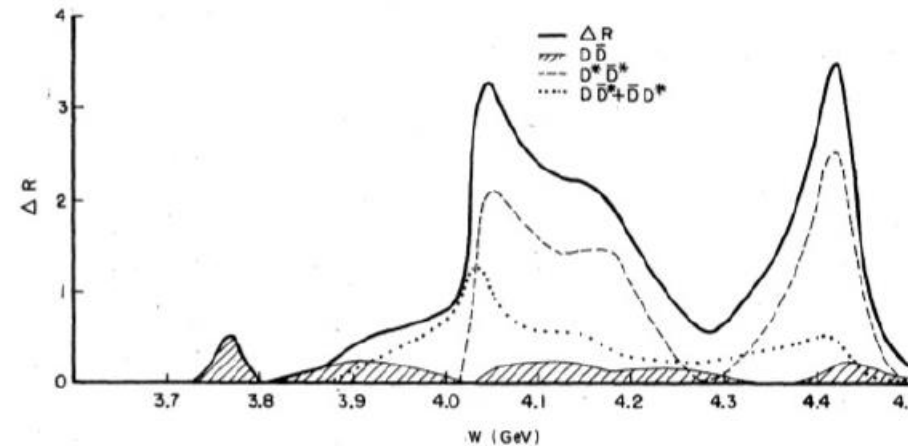


FIG. 13. The charm contribution to R in the region $3.7 < W < 4.5$ GeV as computed in the coupled-channel model. Contributions from $F_1\bar{F}_2$ channels are included but not indicated separately since they are too small; they are shown in Fig. 12.

Recent K-matrix fit by T. V. Uglov

$$S = 1 + 2iA,$$

$$A = K(1 - iK)^{-1},$$

$$AA^\dagger = \frac{1}{2i}(A - A^\dagger).$$

Ensures unitarity

i runs over $D^{(*)}\bar{D}^{(*)}$ channels,
 α runs over ψ 's

$$(P^{-1}(s))_{\alpha\beta} = (M_\alpha^2 - s)\delta_{\alpha\beta} - i \sum_m G_{m\alpha} G_{m\beta}$$

$$K_{ij} = \sum_\alpha G_{i\alpha}(s) \frac{1}{M_\alpha^2 - s} G_{j\alpha}(s),$$

$$\Gamma_{e\alpha} \equiv \Gamma(\psi_\alpha \rightarrow e^+ e^-) = \frac{\alpha g_{e\alpha}^2}{3M_\alpha^3}.$$

Electron width

$$G_{i\alpha}^2(s) = g_{i\alpha}^2 \frac{k_i^{2l_i+1}}{\sqrt{s}} \theta(s - s_i)$$

Coupling constant

$$\Gamma_{i\alpha} \equiv \Gamma(\psi_\alpha \rightarrow [D^{(*)}\bar{D}^{(*)}]_i) = \frac{g_{i\alpha}^2}{M_\alpha^2} [p_i(M_\alpha)]^{2l_i+1}$$

Partial decay width

$$A_{ij} = \sum_{\alpha\beta} G_{i\alpha}(s) P_{\alpha\beta}(s) G_{j\beta}(s)$$

$$\sigma_i(s) = \frac{4\pi\alpha}{s^{5/2}} [p_i(s)]^{2l_i+1} \left| \sum_{\alpha,\beta} g_{e\alpha} P_{\alpha\beta}(s) g_{i\beta} \right|^2$$

Cross-section

27

g is real, so there will be no multiple solutions!

Exclusive channels

Isospin-conjugated modes should be treated independently
It doubles number of channels

$D\bar{D},$	2 channels,
$D\bar{D}^*,$	4 channels,
$D_2\bar{D},$	4 channels,
$[D^*\bar{D}^*]_{S=0}^P,$	2 channels,
$[D^*\bar{D}^*]_{S=2}^P,$	2 channels,
$[D^*\bar{D}^*]_{S=2}^F,$	2 channels.

$D^0 D^- \pi^+$
is dominated by
 $D\bar{D}_2$
corrected to

$$\frac{\mathcal{B}(D_2 \rightarrow D\pi)}{(\mathcal{B}(D_2 \rightarrow D\pi) + \mathcal{B}(D_2 \rightarrow D^*\pi))}$$

ratio

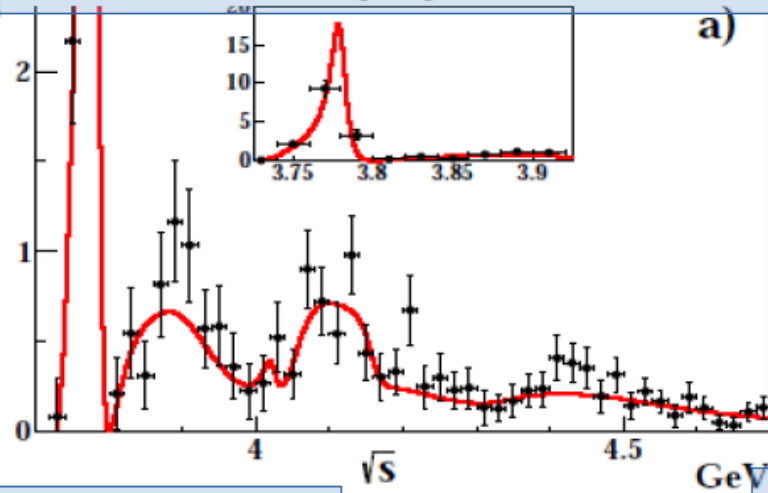
$\psi(2S), \psi(3770), \psi(4040), \psi(4160), \psi(4415)$

16 channels, 5 ψ -states

Results

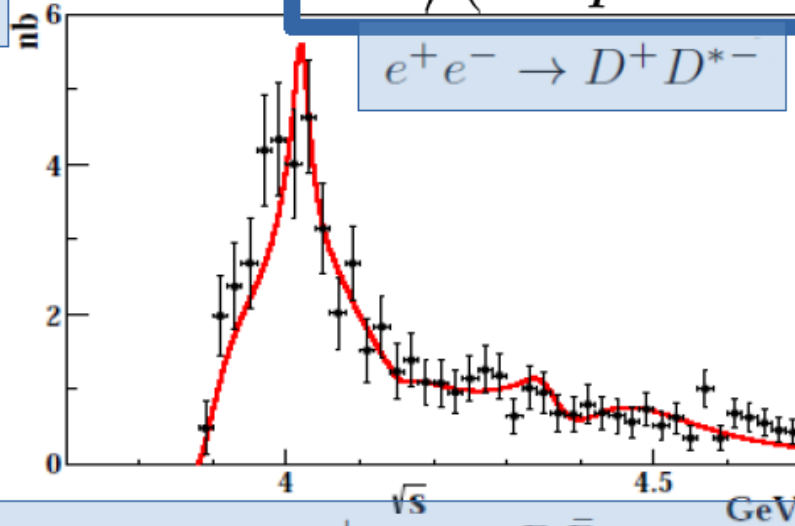
$$e^+e^- \rightarrow D\bar{D}$$

$$([e^+e^- \rightarrow D^+D^-] + [e^+e^- \rightarrow D^0\bar{D}^0])$$

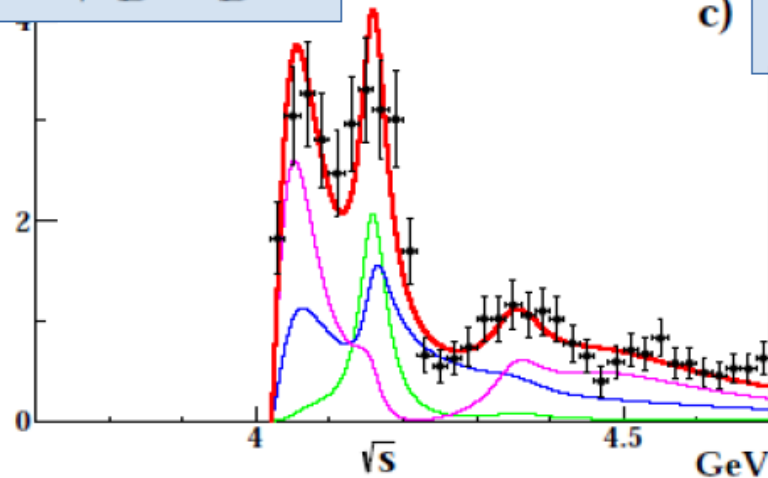


$$\chi^2/n.d.f. = 158/(191point - 33par)$$

$$e^+e^- \rightarrow D^+D^{*-}$$

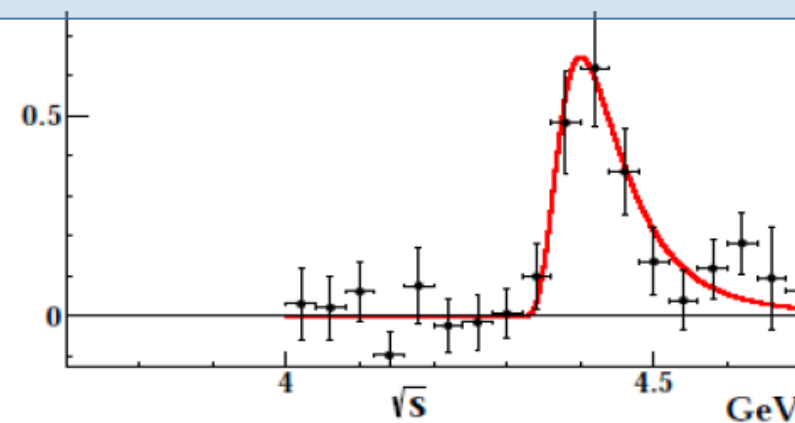


$$e^+e^- \rightarrow D^{*+}D^{*-}$$



$$e^+e^- \rightarrow D\bar{D}\pi$$

$$([e^+e^- \rightarrow D^0D^-\pi^+] + [e^+e^- \rightarrow \bar{D}^0D^+\pi^-])$$



Results (II)

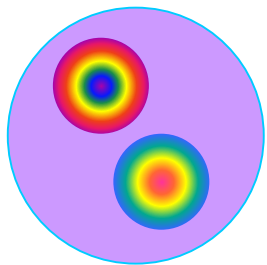
	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5
PDG name	$\psi(2S)$	$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M , MeV	3686* (fixed)	3782 ± 1	4115 ± 14	4170 ± 7	4515 ± 18
Coupling constants $g_{i\alpha}$ $\alpha = 1 \dots 5$, $i = D\bar{D}, D\bar{D}^*, etc$					
$D\bar{D}$	3.0 ± 0.3	-1.8 ± 0.3	-0.1 ± 0.1	0.3 ± 0.1	-0.1 ± 0.1
$D\bar{D}^*$	-4.7 ± 0.5	-3.1 ± 0.3	2.4 ± 0.2	-0.0 ± 0.7	-0.7 ± 0.2
$[D^*\bar{D}^*]_{S=0}^P$	4.8 ± 0.5	6.9 ± 0.9	-0.1 ± 0.2	0.6 ± 0.5	-0.3 ± 0.1
$[D^*\bar{D}^*]_{S=2}^P$	-21.7 ± -2.3	-3.1 ± -0.4	0.5 ± 0.9	-0.3 ± -0.2	1.5 ± -0.3
$[D^*\bar{D}^*]_{S=0}^F$, MeV $^{-2}$	62.2 ± 15.1	-1.6 ± 5.4	-1.0 ± 2.8	8.0 ± 1.4	0.2 ± 0.6
$D_2\bar{D}$, MeV $^{-1}$	-8.2 ± 29.3	25.2 ± 7.7	-23.5 ± 3.3	-1.0 ± 7.4	-1.5 ± 1.4
Partial decay widths $\Gamma_{i\alpha}$, MeV					
e^+e^-	2.354* (fixed)	0.2 ± 0.0	1.6 ± 0.3	0.7 ± 0.4	1.4 ± 0.3
D^+D^-	-	5.6 ± 1.7	0.4 ± 0.8	4.3 ± 2.6	0.5 ± 1.0
$D^0\bar{D}^0$	-	7.5 ± 2.2	0.4 ± 0.8	4.5 ± 2.7	0.5 ± 1.0
D^+D^{*-}	-	-	110.7 ± 23.5	0.0 ± 0.5	32.8 ± 17.4
$[D^*\bar{D}^*]_{S=0}^P$	-	-	0.1 ± 0.2	3.6 ± 6.5	5.9 ± 2.6
$[D^*\bar{D}^*]_{S=2}^P$	-	-	1.2 ± 6.8	0.7 ± 0.3	118.0 ± 729.4
$[D^*\bar{D}^*]_{S=0}^F$	-	-	0.2 ± 1.0	58.6 ± 22.9	2.3 ± 14.2
$D_2^+D^-$	-	-	-	-	11.7 ± 21.1

Still lots of problems in current model

- Model has room to improve (step function, wide resonances, ...)
- Only two-body charmed meson final states considered
- Charmed strange meson production not included
- Charmonium final states not included
- Model constraints used; parameters constrained to PDG values
- Y states not included (nor tested if they are needed)
- Fit quality is bad
- ...

Seems the right direction for understanding the physics

Similar situation in bottomonium energy region

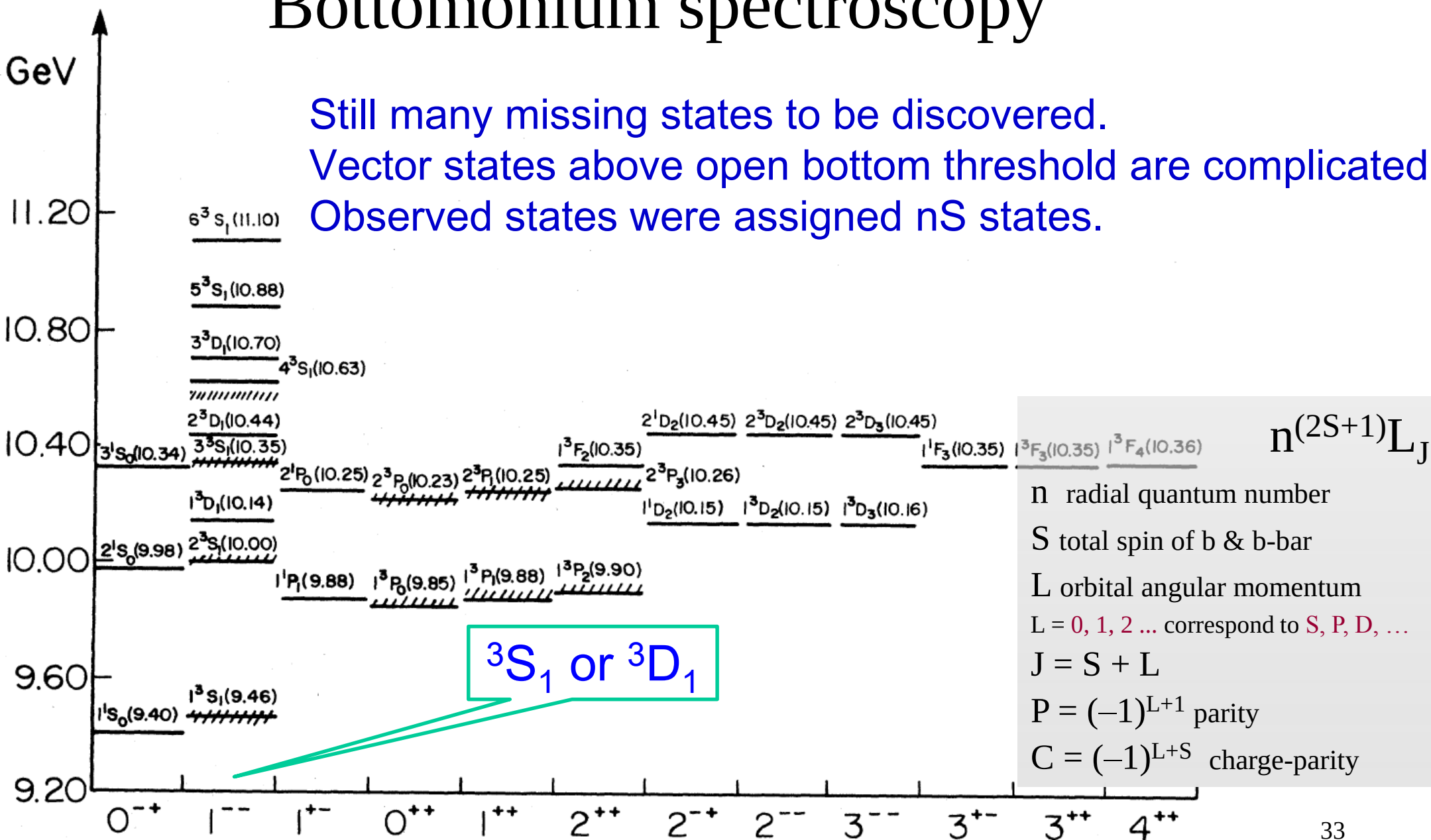


Bottomonium spectroscopy

Still many missing states to be discovered.

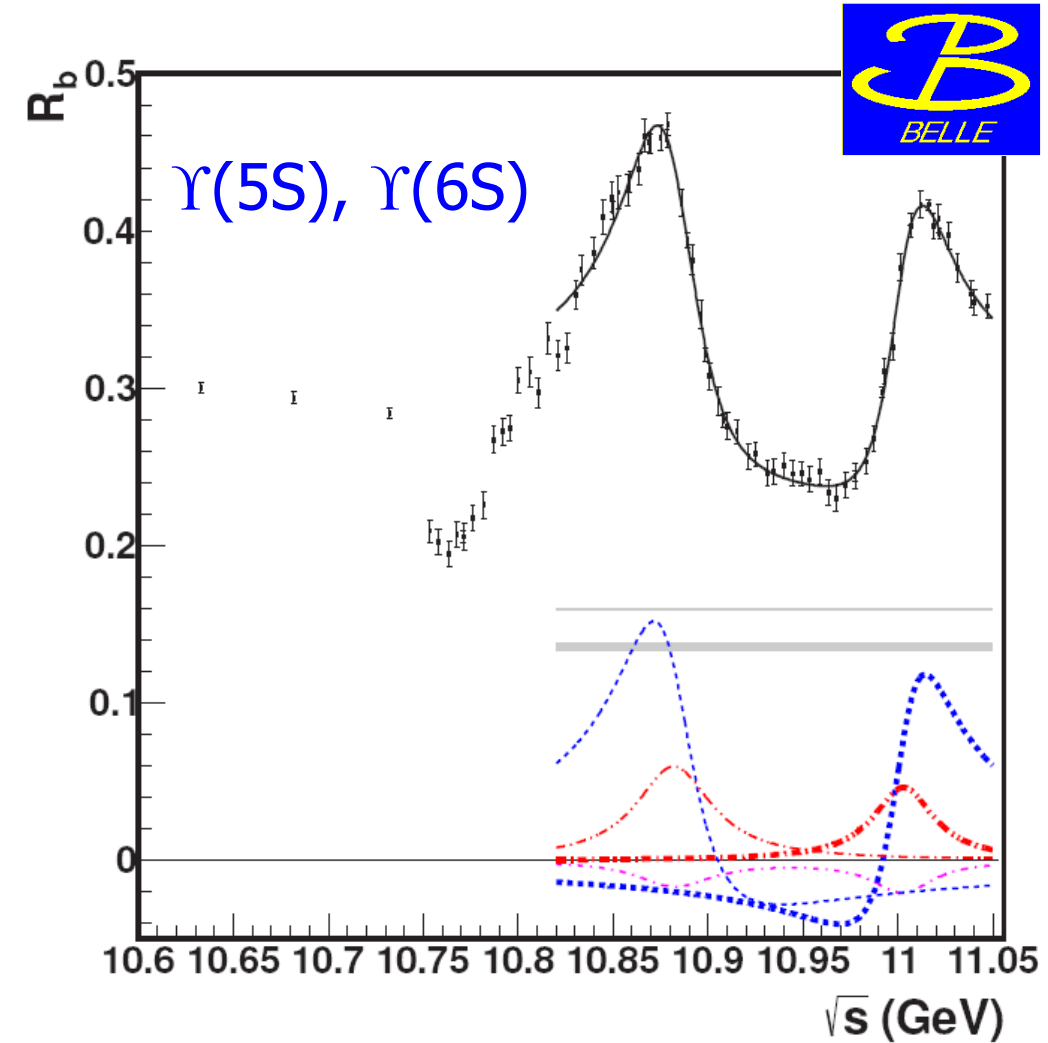
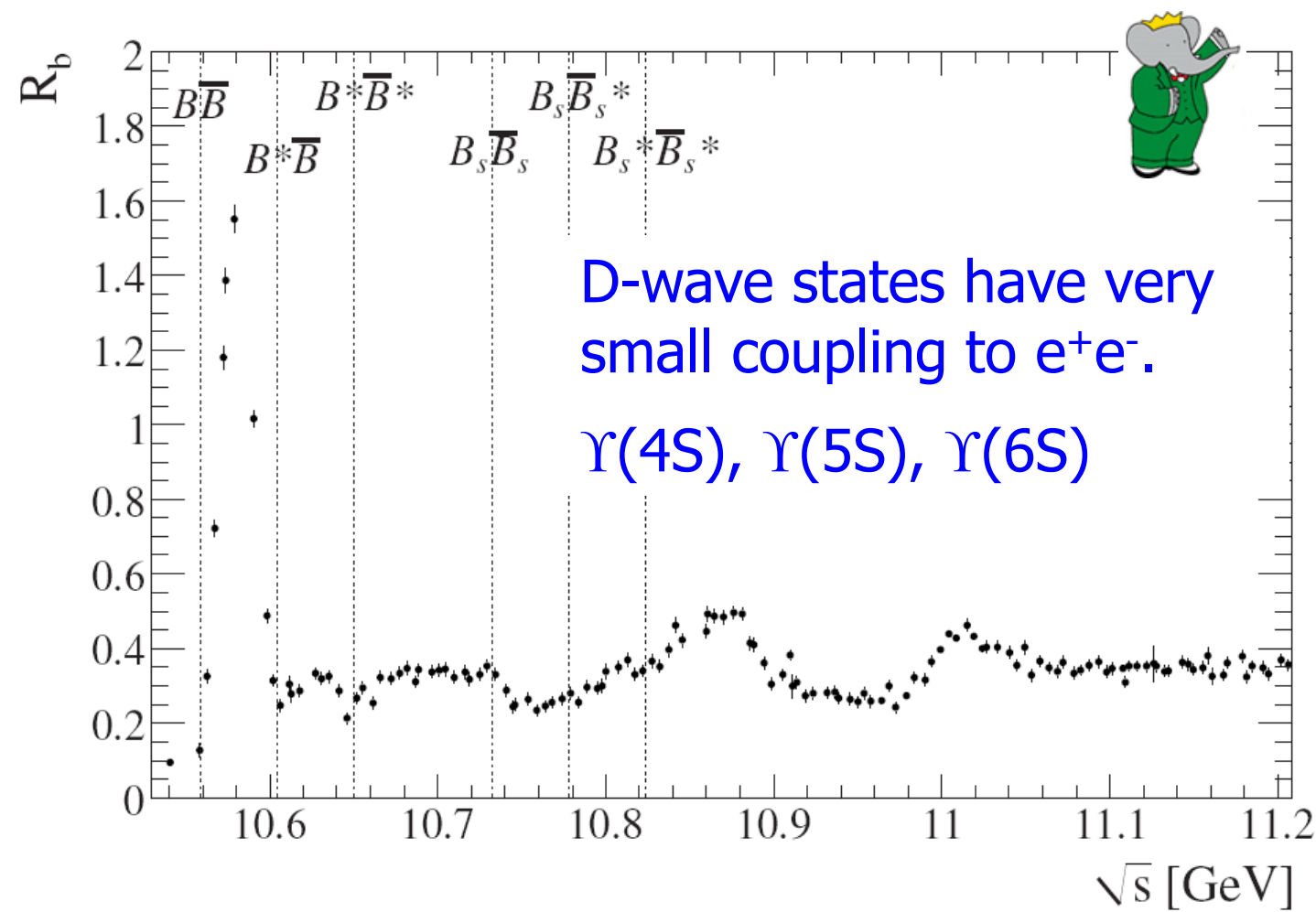
Vector states above open bottom threshold are complicated.

Observed states were assigned nS states.



S. Godfrey, N. Isgur, PRD 32, 189 (1985)

BaBar & Belle measured inclusive cross sections



Υ states are peaks in inclusive cross sections



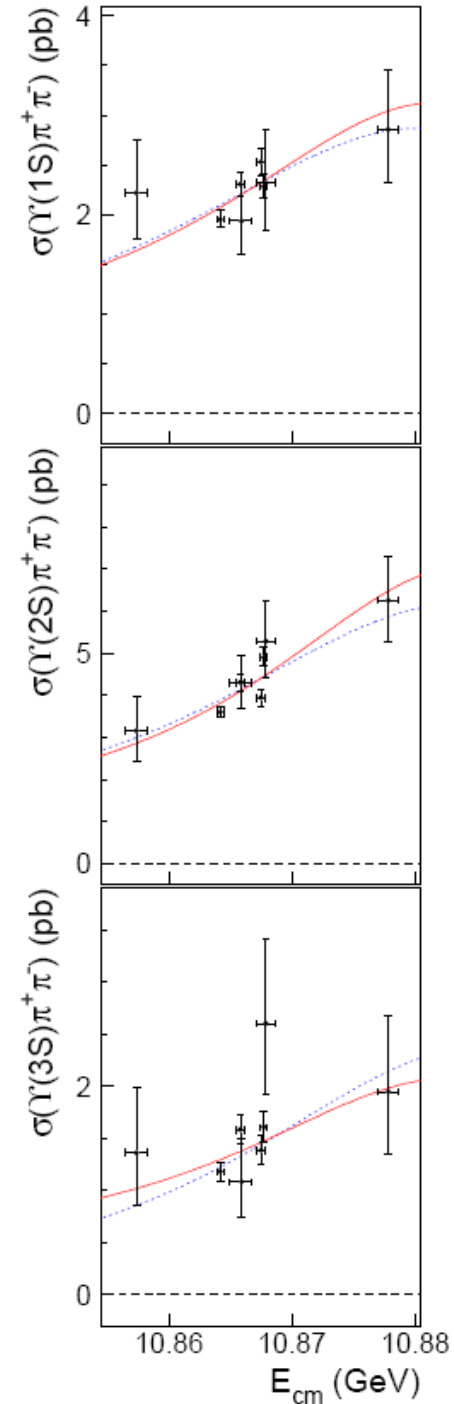
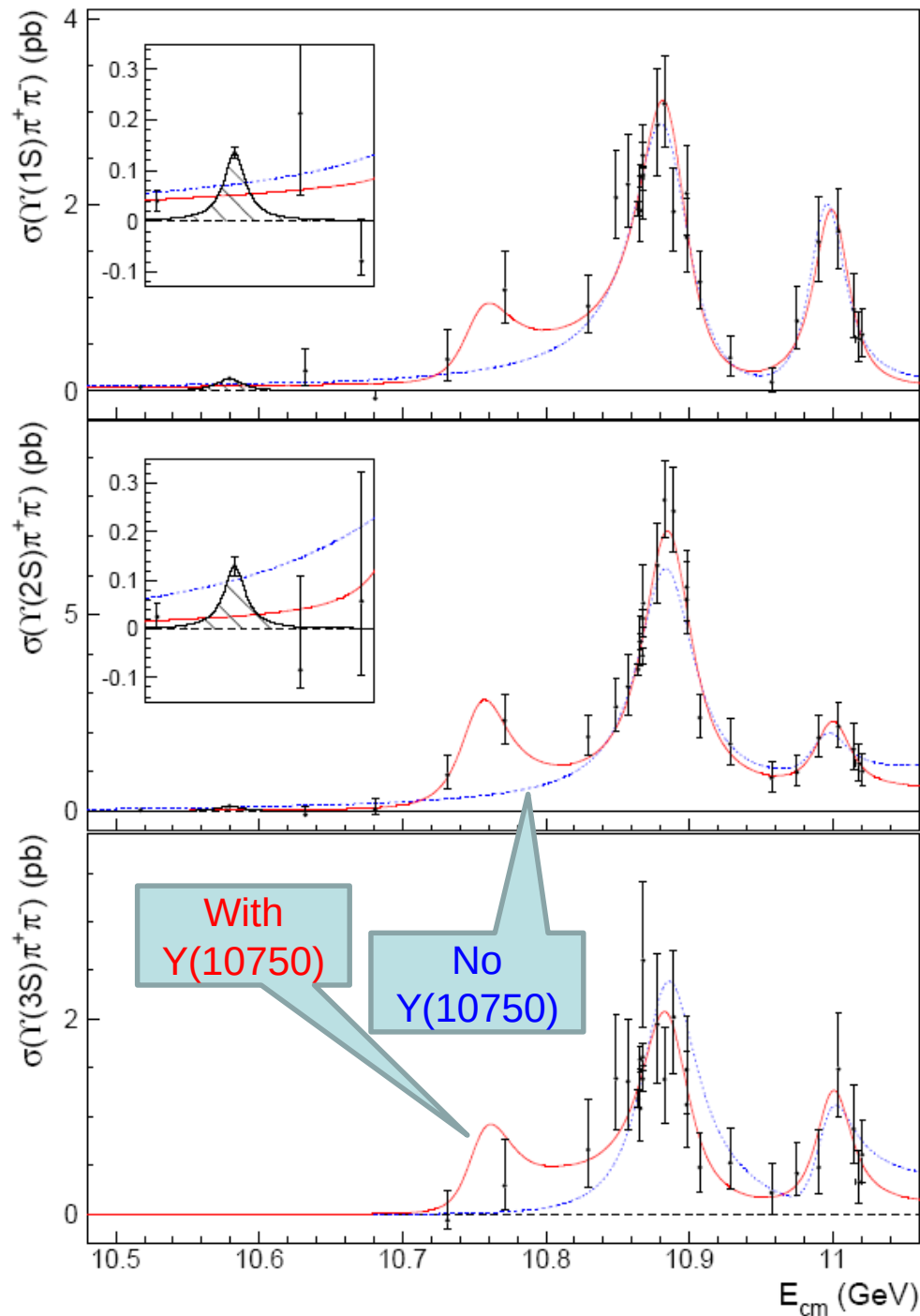
$Y(10750)$ in $e^+e^- \rightarrow \pi^+\pi^-Y(nS)$

Significance of the $Y(10750)$:

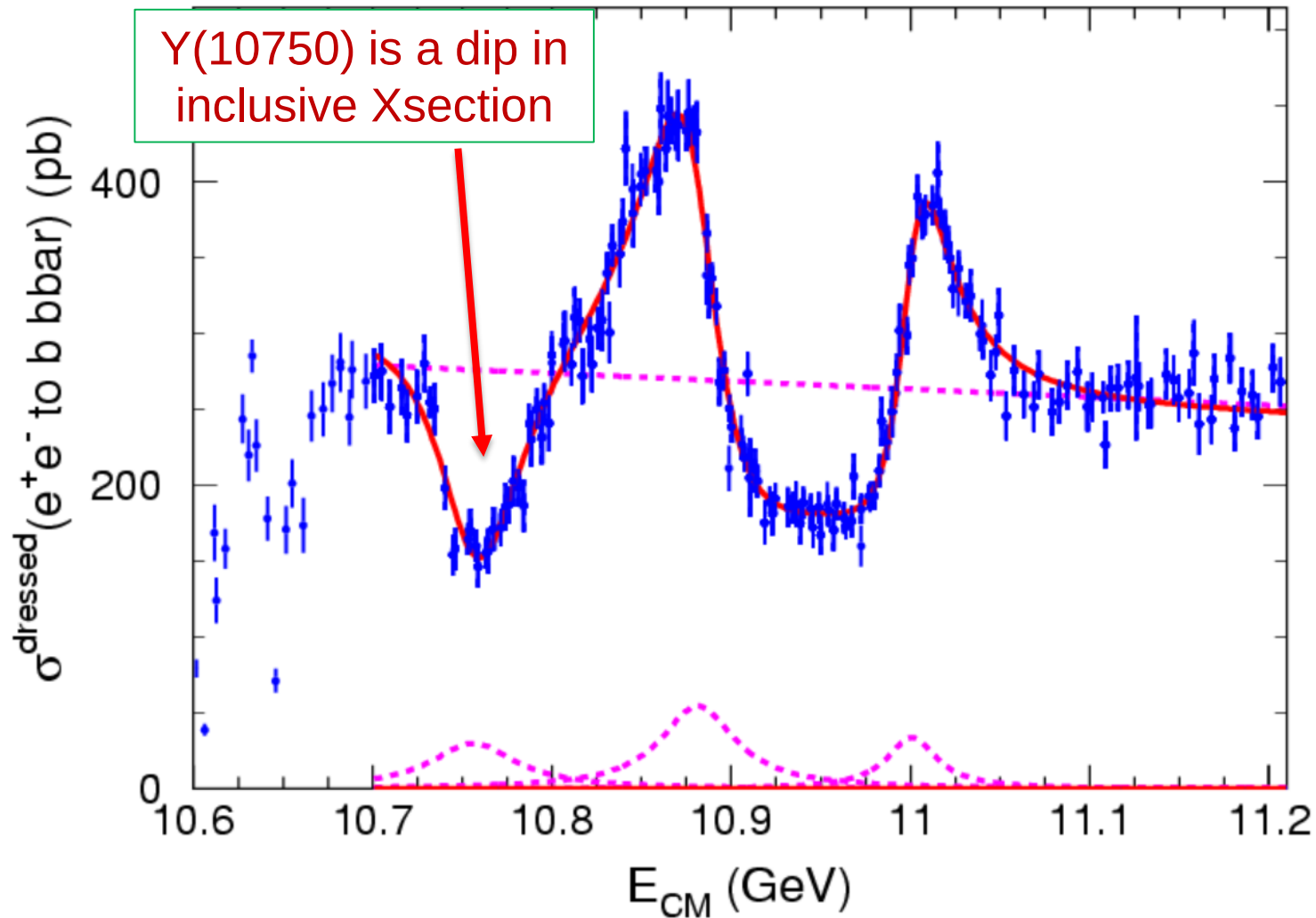
- 5.1σ in $Y(2S)$
- 5.2σ in all modes

With all kinds of systematic effects considered!

arXiv:1905.05521, JHEP



Fit to inclusive cross sections with $Y(10750)$



Coherent sum of a continuum amplitude ($\propto 1/\sqrt{s}$) and 3 BW functions (constant width).

Free parameters:

- Mass M
- width Γ
- leptonic partial width Γ_{ee}
- relative phase ϕ

Fit results (mass & width)

	$\Upsilon(5S)$	$\Upsilon(6S)$	$\Upsilon(10750)$
Mass (MeV)	10882 ± 1	11001 ± 1	10761 ± 2
Width (MeV)	50 ± 2	35 ± 2	48 ± 3

From fit to BaBar & Belle R_b data;
Statistical error only

	$\Upsilon(10860)$	$\Upsilon(11020)$	$\Upsilon(10750)$
M (MeV/c ²)	$10885.3 \pm 1.5^{+2.2}_{-0.9}$	$11000.0^{+4.0}_{-4.5} {}^{+1.0}_{-1.3}$	$10752.7 \pm 5.9^{+0.7}_{-1.1}$
Γ (MeV)	$36.6^{+4.5}_{-3.9} {}^{+0.5}_{-1.1}$	$23.8^{+8.0}_{-6.8} {}^{+0.7}_{-1.8}$	$35.5^{+17.6}_{-11.3} {}^{+3.9}_{-3.3}$

From fit to Belle
 $e^+e^- \rightarrow \pi^+\pi^-\Upsilon(nS)$

PDG2020

	$\Upsilon(5S)$	$\Upsilon(6S)$
Mass (MeV)	$10885.2^{+2.6}_{-1.6}$	11000^{+4}_{-4}
Width (MeV)	37^{+4}_{-4}	24^{+8}_{-6}

The results can be very wrong!

- Model (continuum + BWs)
- Inclusive data only
- No open bottom cross sections

Can we understand the ρ - ω - ϕ data in K-matrix formalism

- only 3 resonances
- limited (6) decay modes, all known well
- Only $\bar{K}K$ threshold important

ρ decays

- $\pi^+\pi^-$ $\sim 100\%$
- $\pi^+\pi^-\pi^0$ $\sim 10^{-4}$
- $\gamma\pi^0$ $\sim 10^{-4}$
- $\gamma\eta$ $\sim 10^{-4}$

ω decays

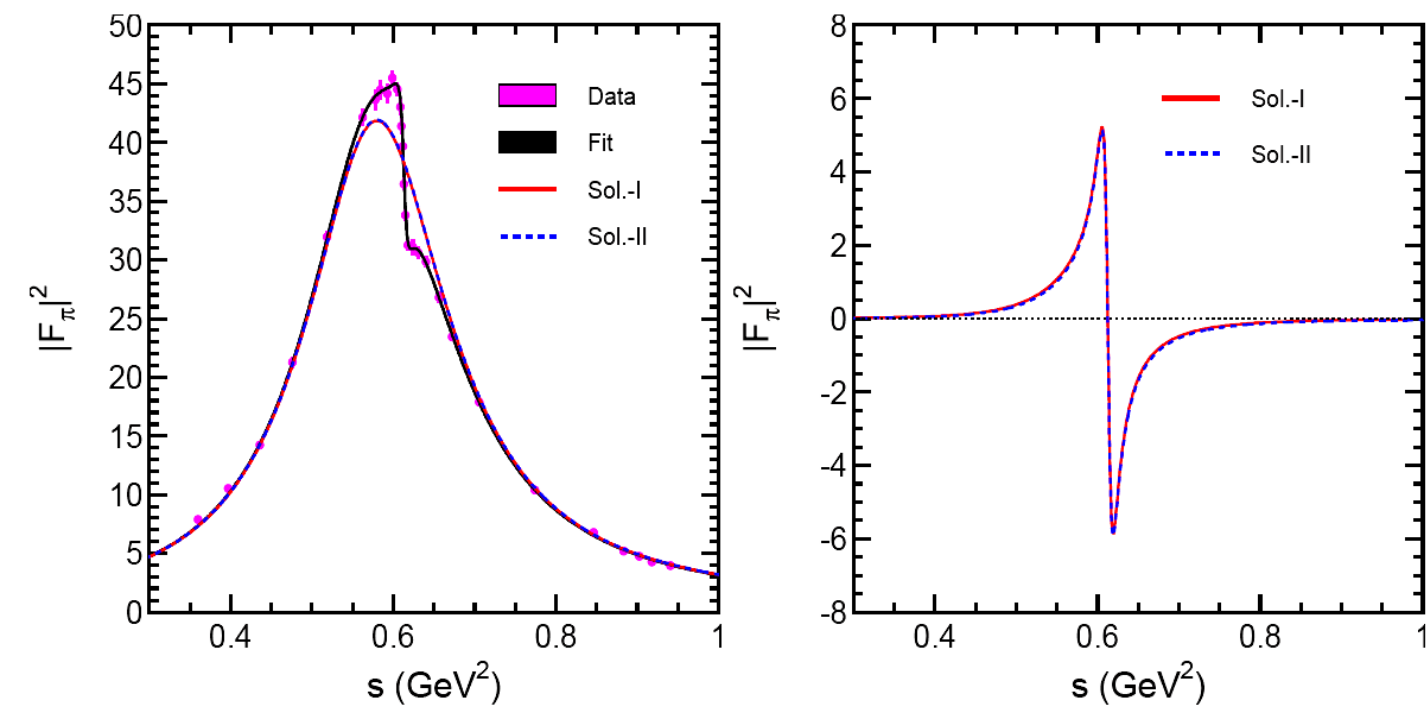
- $\pi^+\pi^-\pi^0$ $\sim 89\%$
- $\gamma\pi^0$ $\sim 8.4\%$
- $\pi^+\pi^-$ $\sim 1.5\%$
- $\gamma\eta$ $\sim 10^{-4}$

ϕ decays

- K^+K^- $\sim 49\%$
- $K_S K_L$ $\sim 34\%$
- $\pi^+\pi^-\pi^0$ $\sim 15\%$
- $\gamma\eta$ $\sim 1.3\%$
- $\gamma\pi^0$ $\sim 10^{-3}$
- $\pi^+\pi^-$ $\sim 10^{-5}$

Caution: These BF's may not be reliable!

Multiple solutions in $e^+e^- \rightarrow \pi^+\pi^-$



$$F_{\pi}^{\text{GS}}(s) = \frac{\text{BW}_{\rho}^{\text{GS}}(s) \left[1 + \delta \frac{s}{m_{\omega}^2} P_{\omega}(s) \right] + \beta \text{BW}_{\rho'}^{\text{GS}}(s)}{1 + \beta}$$

Parameter	Solution I	Solution II	Davier [8]
m_{ρ} [MeV]	775.9 ± 0.5		—
Γ_{ρ} [MeV]	146.0 ± 0.8		—
$ \delta $ [$\times 10^{-3}$]	1.62 ± 0.06	21.97 ± 0.04	—
ϕ_{δ} [$^{\circ}$]	10.1 ± 1.4	86.56 ± 0.17	—
$ \beta $	0.086 ± 0.004		—
$\Delta \mathcal{B}^{\text{mixing}}$ [%]	-0.03 ± 0.01	$+0.04 \pm 0.01$	-0.01 ± 0.01
$\Delta a_{\mu}^{\text{mixing}}$ [10^{-10}]	$+2.5 \pm 0.2$	$+1.6 \pm 0.2$	$+2.80 \pm 0.19$

Figure 3: Fit to the $e^+e^- \rightarrow \pi^+\pi^-$ form factors below $s = 1 \text{ GeV}^2$ measured at CMD2

This was simply overlooked for a few decades.

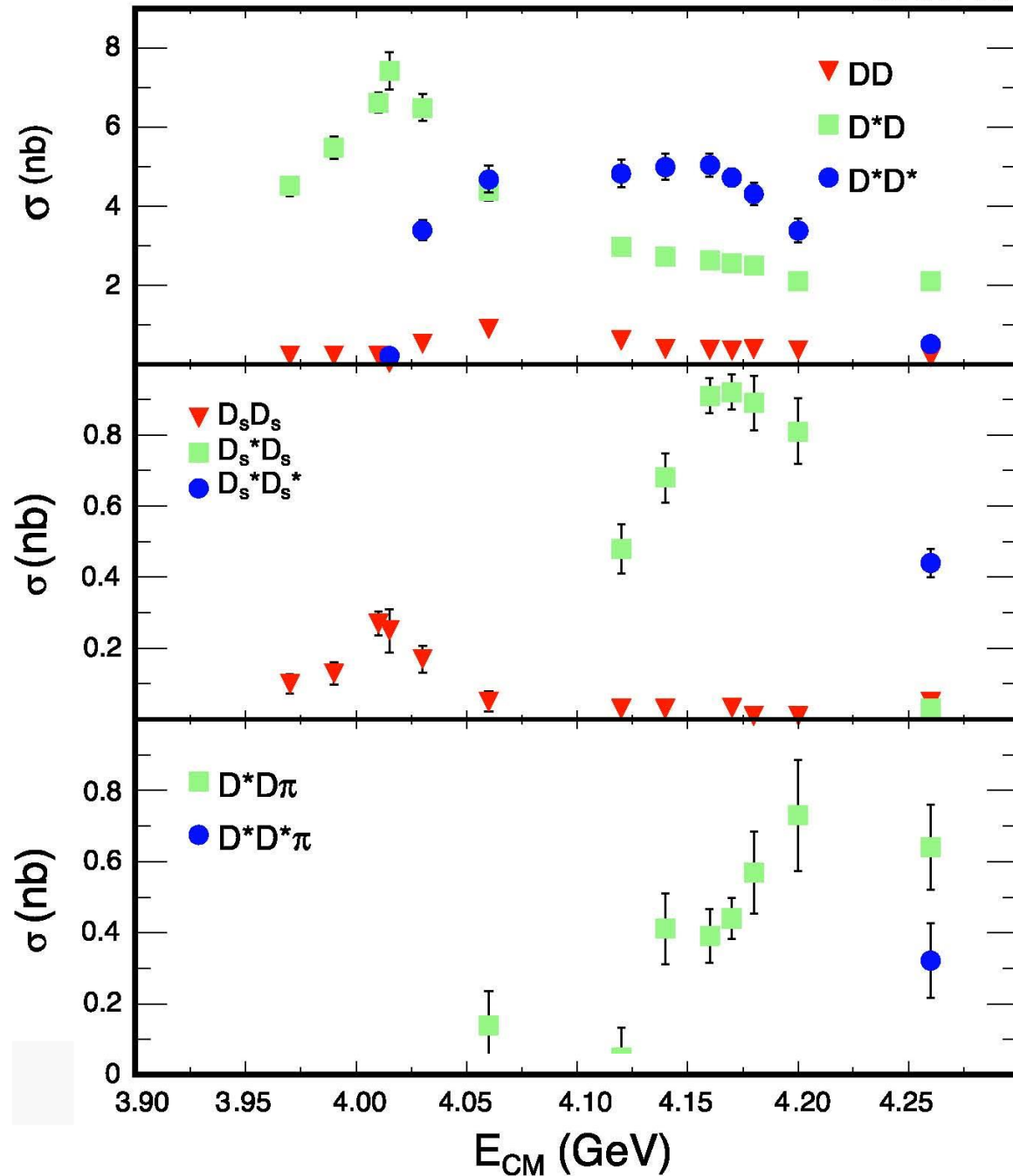
Can K-matrix explain the non-zero phase & pick up one solution for us?

Summary

- We do not understand the ψ 's
- We do not understand the Y 's
- So it is difficult to understand the Y 's
- May we start from understanding ρ - ω - ϕ , to find a way to understand the other vectors?

Thanks a lot!

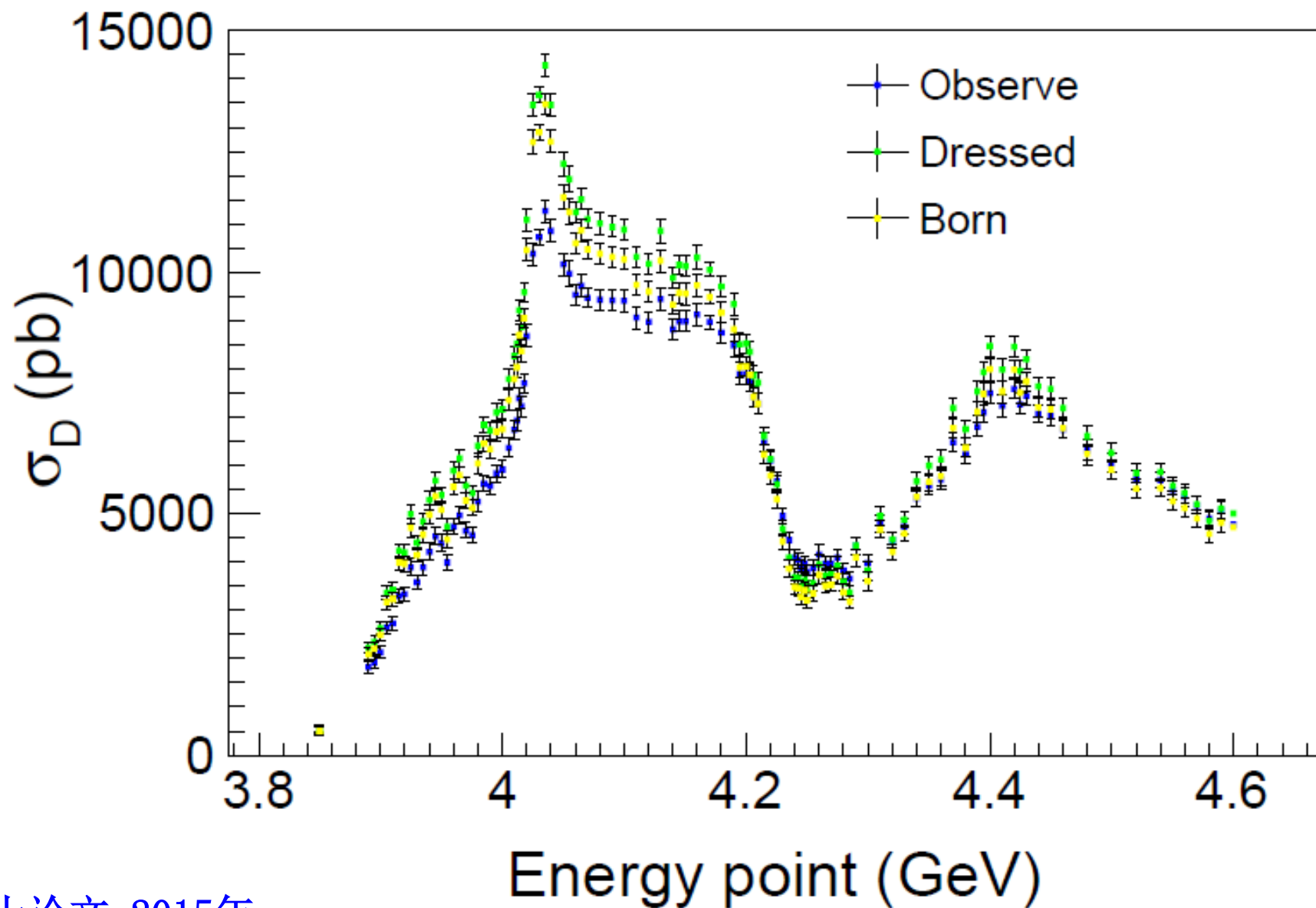
More on R_D & Y states

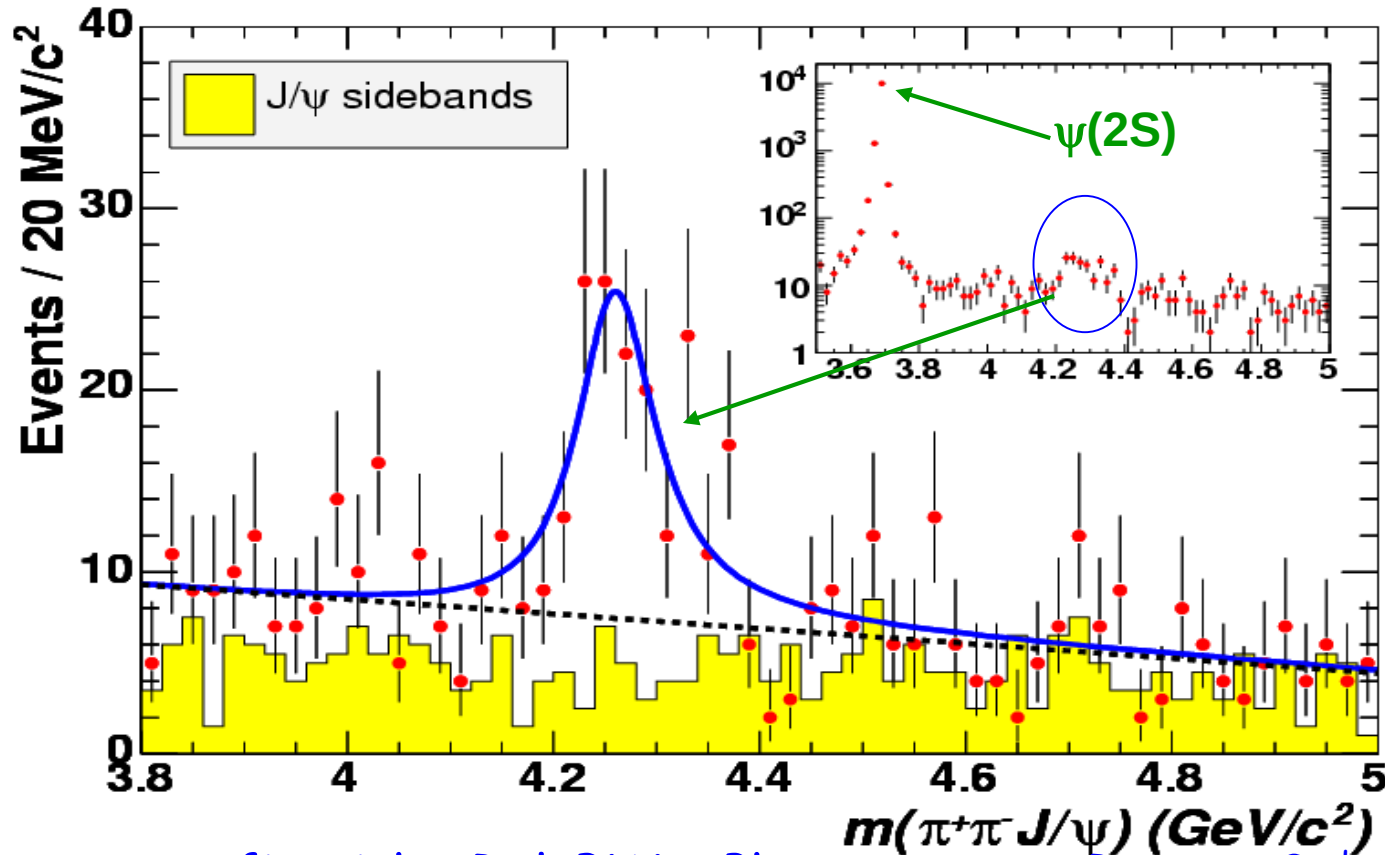


R_D Scan (3.97-4.26 GeV)

CLEOc, PRD80, 072001 (2009)

$E_{\text{c.m.}}$ (MeV)	$\int \mathcal{L} dt$ (pb $^{-1}$)
3970	3.85
3990	3.36
4010	5.63
4015	1.47
4030	3.01
4060	3.29
4120	2.76
4140	4.87
4160	10.16
4170	178.89
4180	5.67
4200	2.81
4260	13.11

Cross Section for $e^+e^- \rightarrow D+X$ 

BaBar discovers the Y(4260) in ISR $\pi^+\pi^-J/\psi$ 

BaBar: 232 fb⁻¹

>8 σ significance structure
called Y(4260)

$M(J/\psi\pi\pi)$ of $\psi(2S)$ with J/ψ
constraint is well described by
Cauchy shape funct.

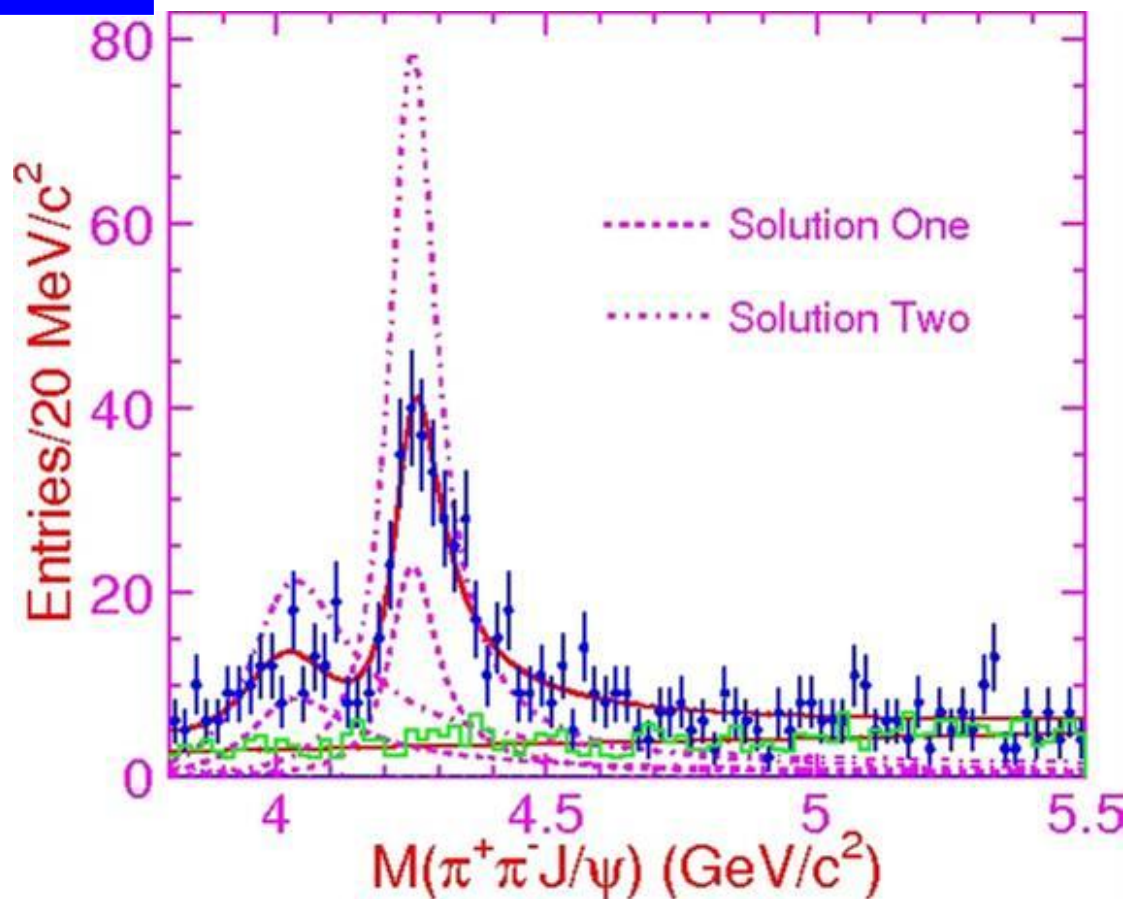
- fit with Rel-BW $\times$ PhaseSpace $\otimes$ Reso + 2nd polynomial (BKGD)
- fit-probability (χ^2) is about 2.6%, $N_{\text{events}} = 125 \pm 23$

$$m = 4259 \pm 8_{-6}^{+2} \text{ MeV}$$

$$\Gamma = 88 \pm 23_{-4}^{+6} \text{ MeV}$$

$$\Gamma(Y \rightarrow e^+e^-) \cdot B(Y \rightarrow \pi^+\pi^-J/\psi) = 5.5 \pm 1.0_{-0.7}^{+0.8} \text{ eV}$$

$e^+e^- \rightarrow \pi^+\pi^-J/\psi$ via ISR



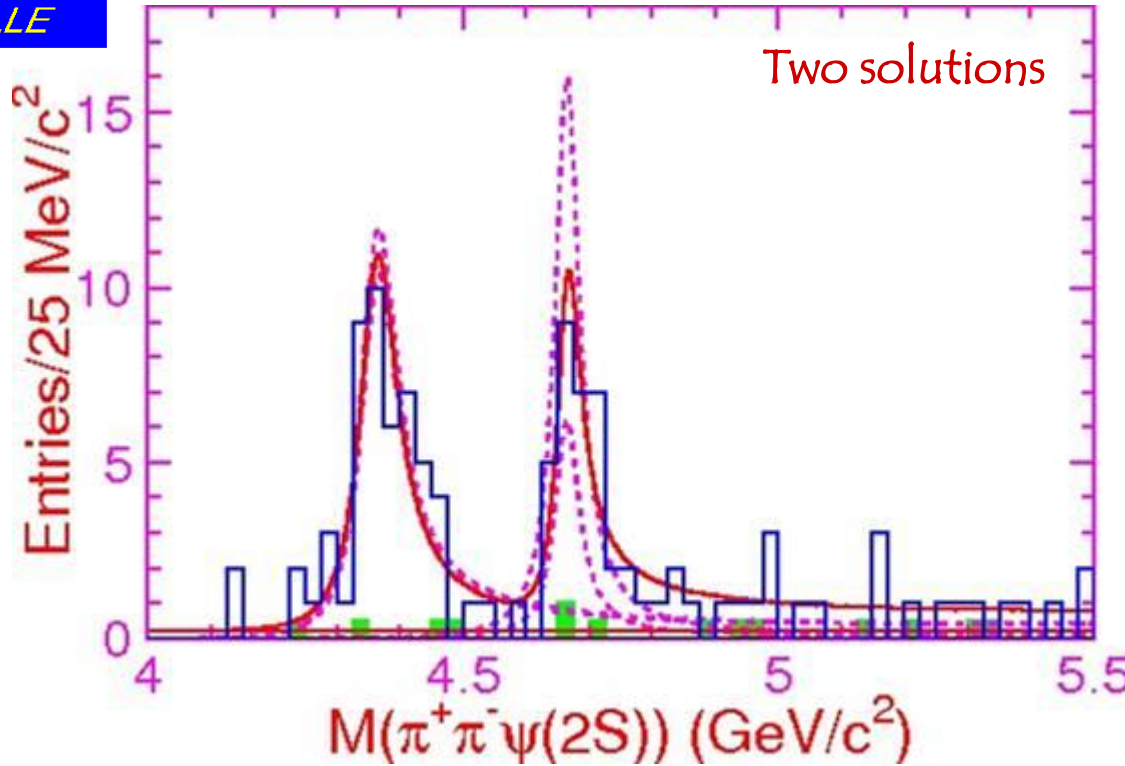
2-BW fit with interference better describes the data:
Y(4260) parameters are different (especially peak
cross section – large uncertainty)

- **Non resonant $J/\psi\pi\pi$?**
- **Re-scattering $ee \rightarrow D^{(*)}D^{(*)} \rightarrow J/\psi\pi\pi$?**
- **Another broad state ?**
 - Check the latter hypothesis and influence of interference of Y(4260) with non-Y contribution:
 - Fit with 2 coherent BWs
 - Two-fold ambiguity in amplitude (constructive-destructive interference) + model uncertainty due to ψ' tail

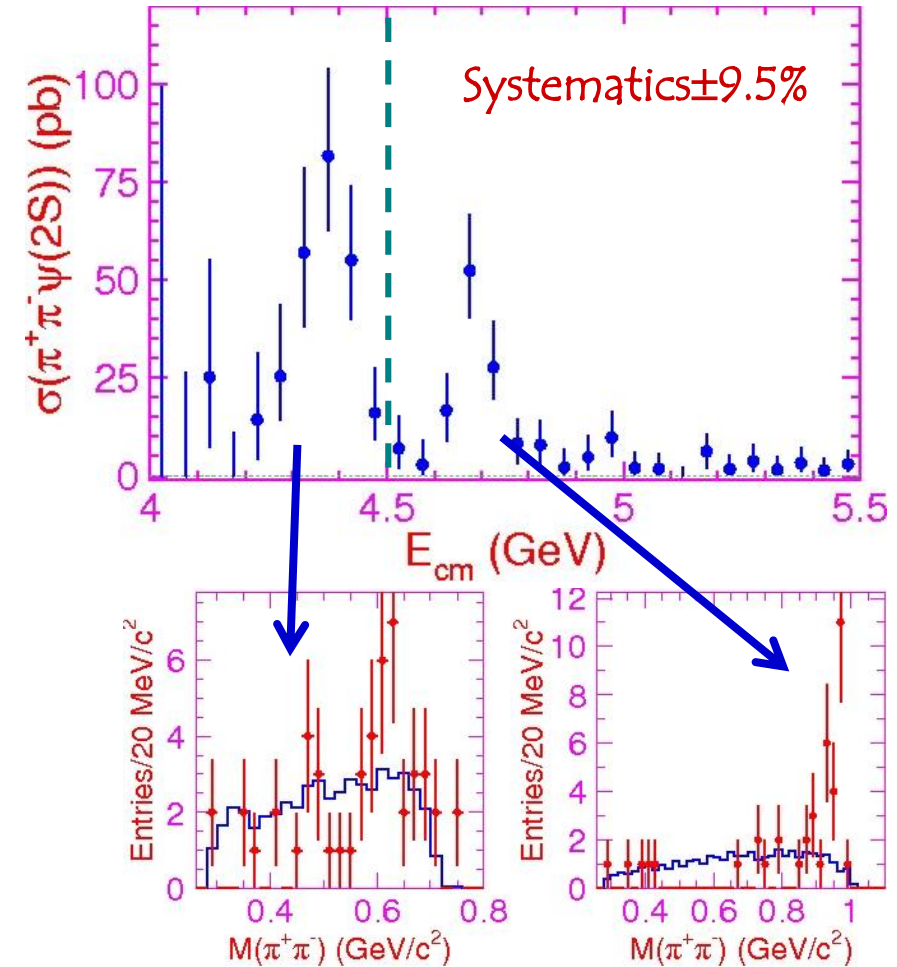
Parameters	Solution I	Solution II
$M(R1)$	$4008 \pm 40^{+114}_{-28}$	
$\Gamma_{\text{tot}}(R1)$	$226 \pm 44 \pm 87$	
$\mathcal{B} \cdot \Gamma_{e^+e^-}(R1)$	$5.0 \pm 1.4^{+6.1}_{-0.9}$	$12.4 \pm 2.4^{+14.8}_{-1.1}$
$M(R2)$	$4247 \pm 12^{+17}_{-32}$	
$\Gamma_{\text{tot}}(R2)$	$108 \pm 19 \pm 10$	
$\mathcal{B} \cdot \Gamma_{e^+e^-}(R2)$	$6.0 \pm 1.2^{+4.7}_{-0.5}$	$20.6 \pm 2.3^{+9.1}_{-1.7}$
ϕ	$12 \pm 29^{+7}_{-98}$	$-111 \pm 7^{+28}_{-31}$



Belle discovers $Y(4360)$ & $Y(4660)$ in $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$



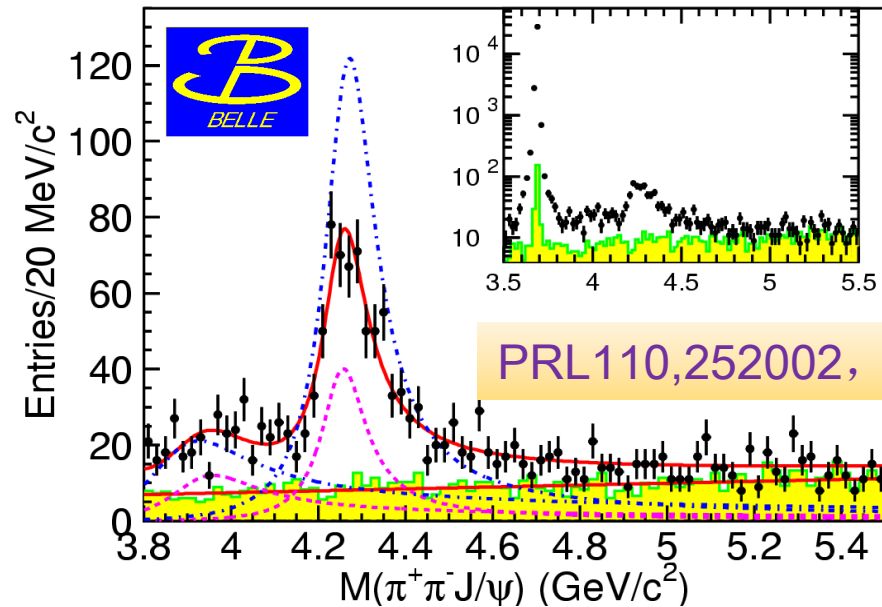
Parameters	Solution one	Solution two
$M(Y(4360))$	$4361 \pm 9 \pm 9$	
$\Gamma_{\text{tot}}(Y(4360))$	$74 \pm 15 \pm 10$	
$\mathcal{B} \cdot \Gamma_{e^+e^-}(Y(4360))$	$10.4 \pm 1.7 \pm 1.5$	$11.8 \pm 1.8 \pm 1.4$
$M(Y(4660))$	$4664 \pm 11 \pm 5$	
$\Gamma_{\text{tot}}(Y(4660))$	$48 \pm 15 \pm 3$	
$\mathcal{B} \cdot \Gamma_{e^+e^-}(Y(4660))$	$3.0 \pm 0.9 \pm 0.3$	$7.6 \pm 1.8 \pm 0.8$
ϕ	$39 \pm 30 \pm 22$	$-79 \pm 17 \pm 20$



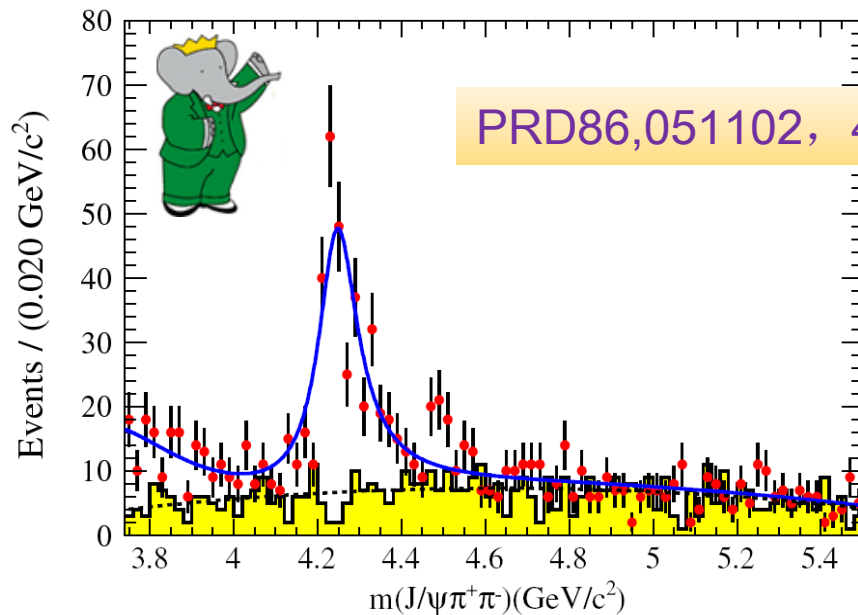
$Y(4360)$ – consistent with BaBar
 $Y(4660)$ – NEW (5.8σ)

The Y states

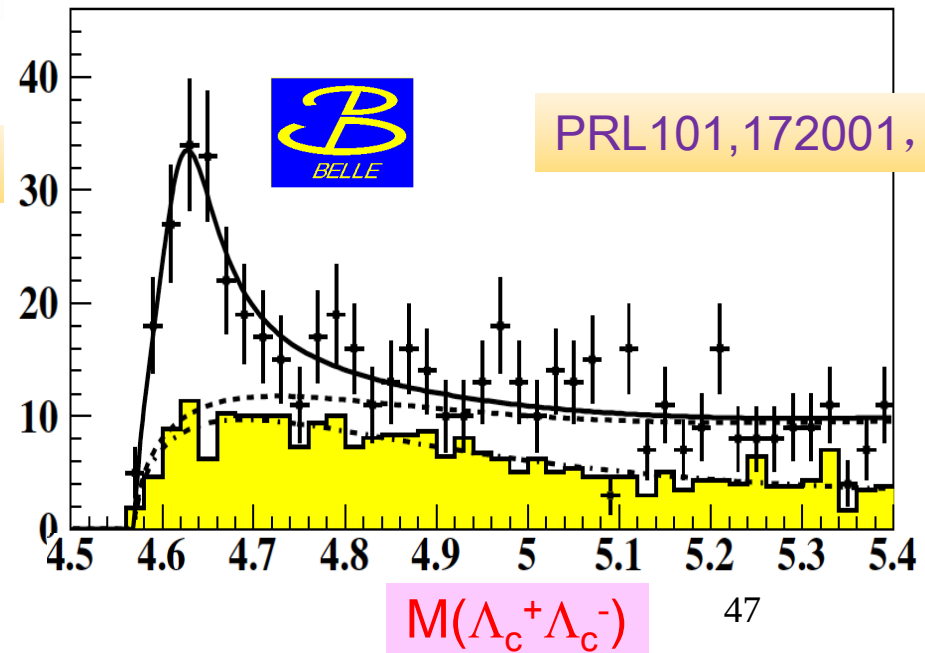
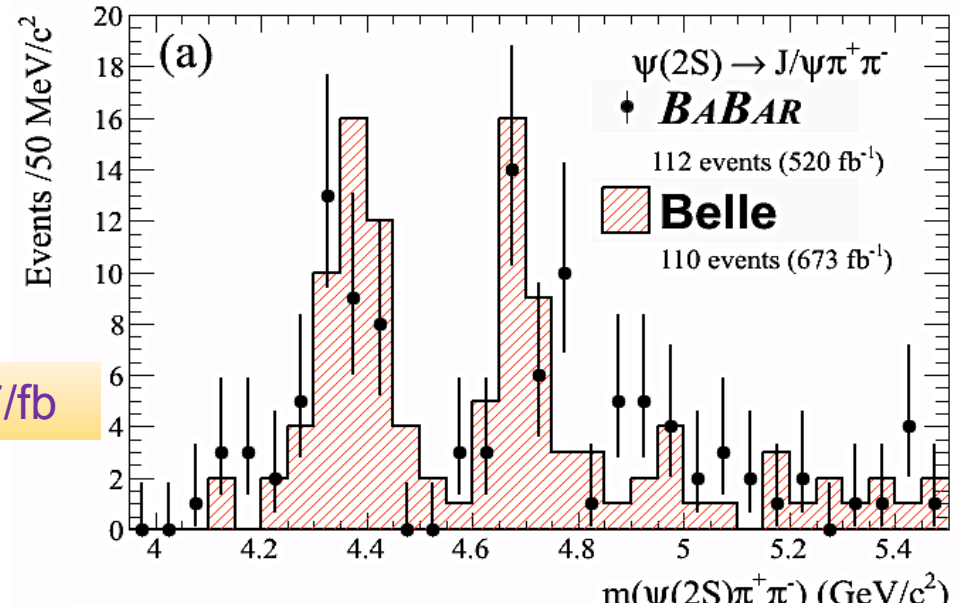
Belle: PRL99,142002, 670/fb
BaBar: PRD89, 111103, 520/fb



PRL110,252002, 967/fb



PRD86,051102, 454/fb

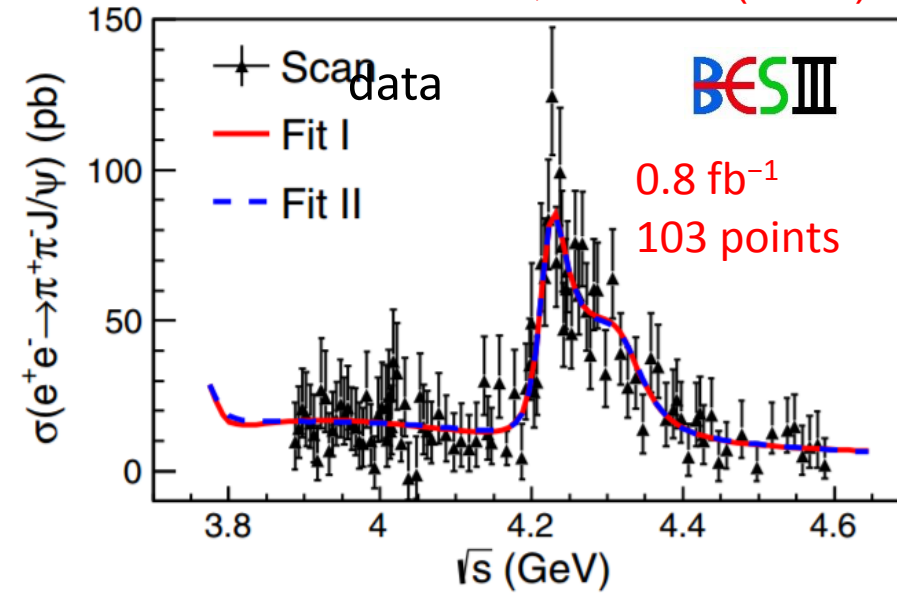
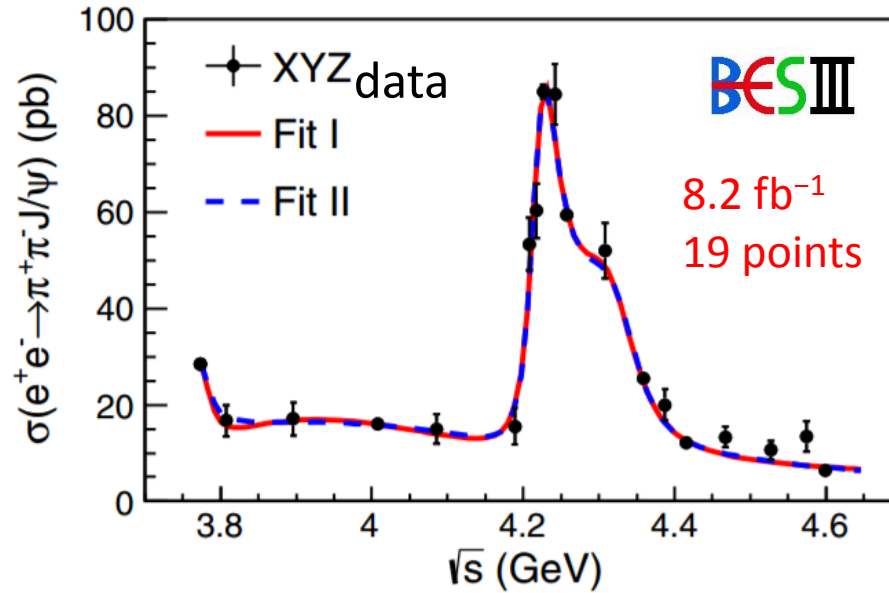


PRL101,172001, 695/fb

$M(\Lambda_c^+\Lambda_c^-)$

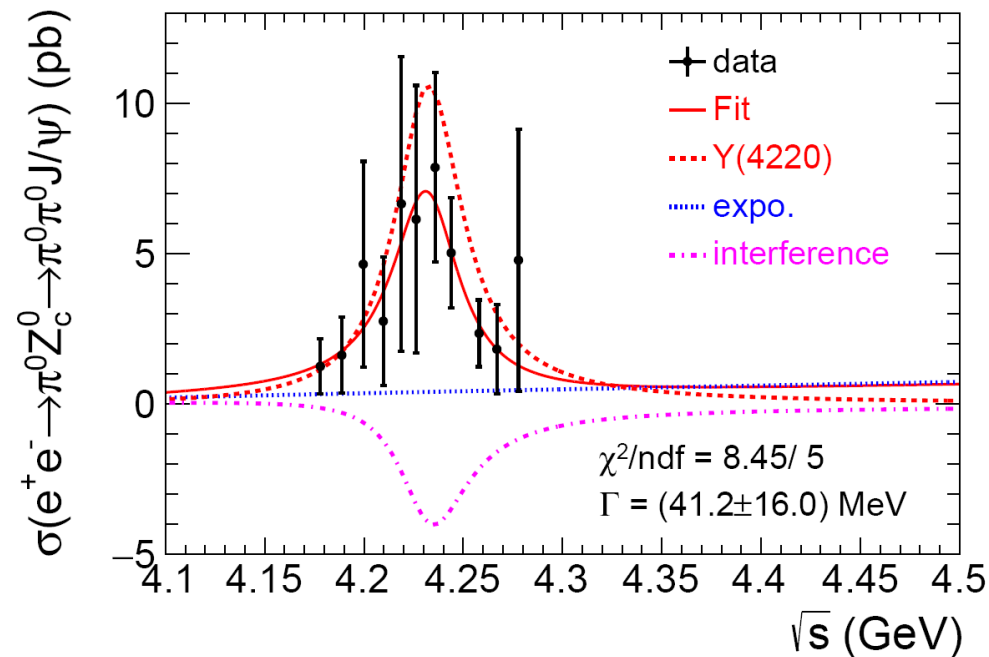
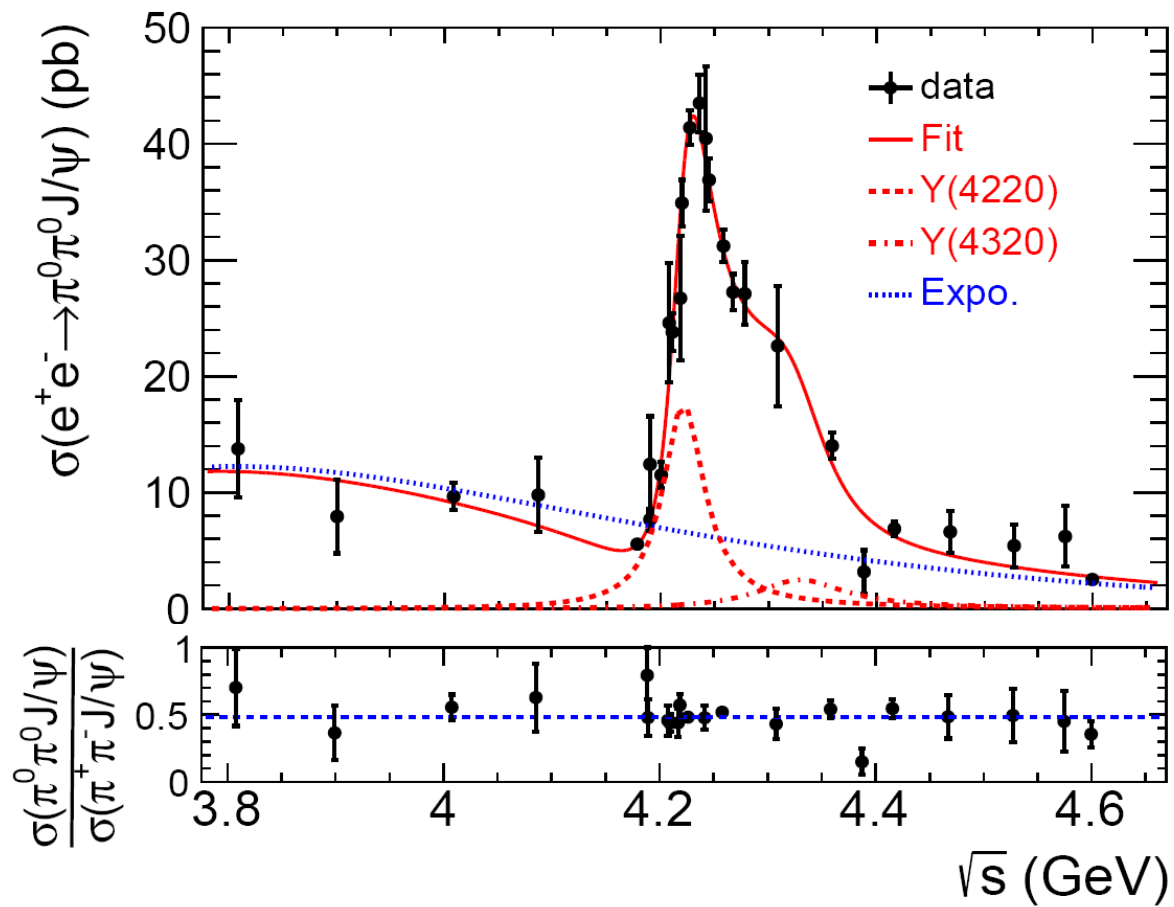
$e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross section

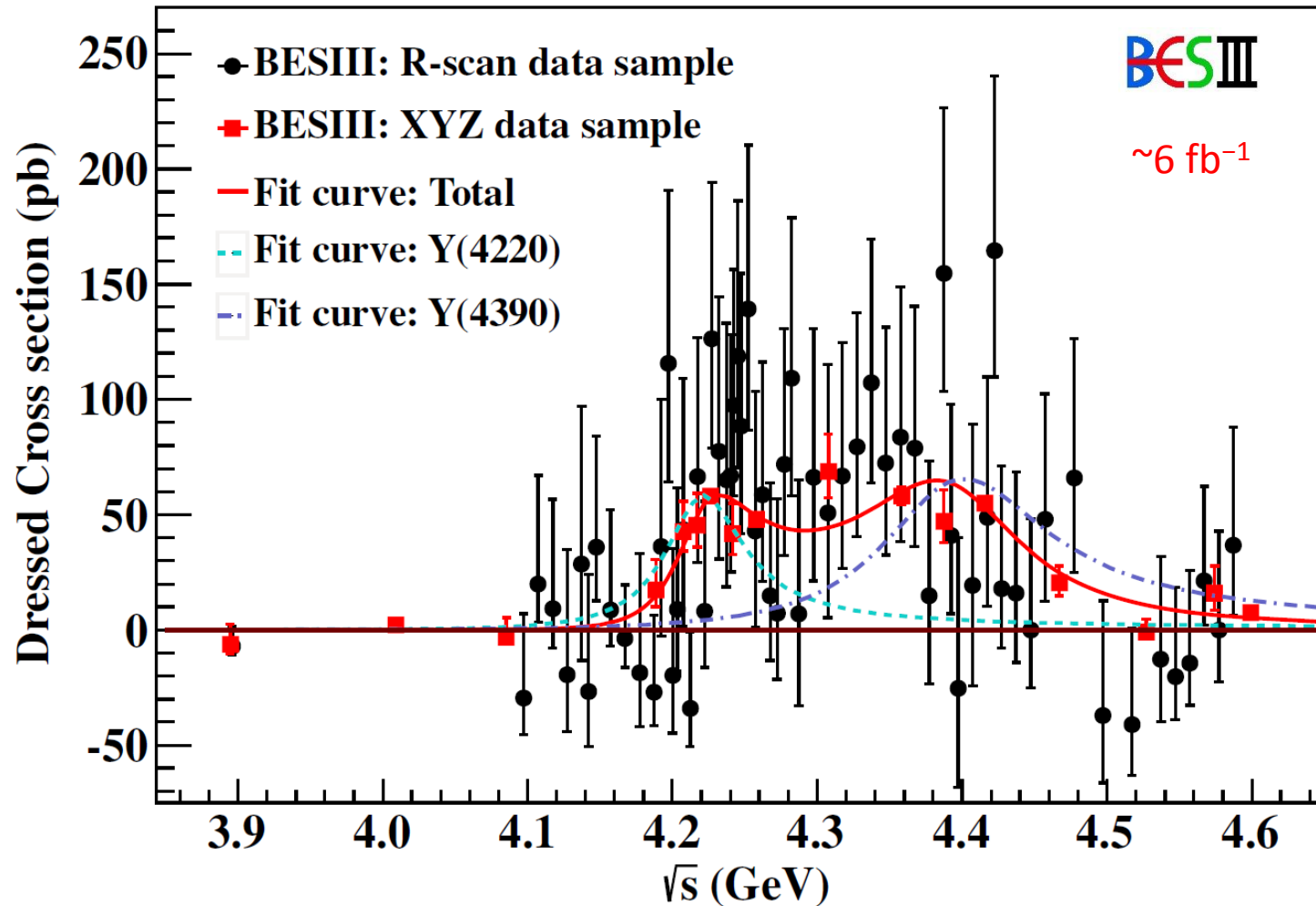
PRL118, 092001 (2017)



- Most precise cross section measurement to date from BESIII
- Fit I = $|BW_1 + BW_2 * e^{i\phi_2} + BW_3 * e^{i\phi_3}|^2$ or Fit II = $|\exp + BW_2 * e^{i\phi_2} + BW_3 * e^{i\phi_3}|^2$ (other fits ruled out)
- $M = 4222.0 \pm 3.1 \pm 1.4$ MeV (**lower**)
- $\Gamma = 44.1 \pm 4.3 \pm 2.0$ MeV (**narrower**)
- A 2nd resonance Y_2 with $M = 4320.0 \pm 10.4 \pm 7.0$ MeV/c²
 $\Gamma = 101.4^{+25.3}_{-19.7} \pm 10.2$ MeV
- Observed for the **first time**, significance > **7.6σ**

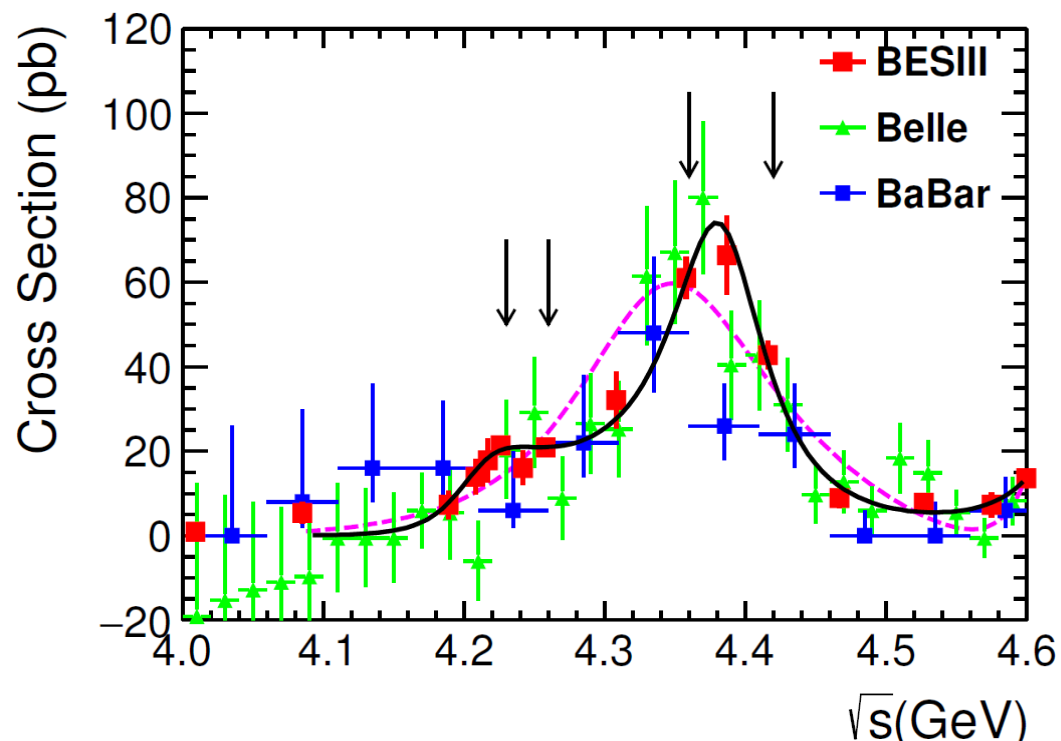
BESIII: 2004.13788





➤ $M_1 = 4218.4^{+5.5}_{-4.5} \pm 0.9 \text{ MeV}/c^2$, $\Gamma_1 = 66.0^{+12.3}_{-8.3} \pm 0.4 \text{ MeV} \rightarrow Y(4220)$

➤ $M_2 = 4391.5^{+6.3}_{-6.8} \pm 1.0 \text{ MeV}/c^2$, $\Gamma_2 = 139.5^{+16.2}_{-20.6} \pm 0.6 \text{ MeV} \rightarrow Y(4390)$

The Ys in $e^+e^- \rightarrow \pi^+\pi^-\psi'$ 

arXiv:1703.08787,
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The Y(4220) is necessary
(significance = 5.8σ)

Fix parameters of the
Y(4660) to Belle results

Parameters	Solution I	Solution II
$M(Y4220)$ (MeV/ c^2)	4209.5 ± 7.4	
$\Gamma(Y(4220))$ (MeV)	80.1 ± 24.6	
$\mathcal{B}\Gamma^{e^+e^-}(Y(4220))$ (eV)	0.8 ± 0.7	0.4 ± 0.3
$M(Y4390)$ (MeV/ c^2)	4383.8 ± 4.2	
$\Gamma(Y(4390))$ (MeV)	84.2 ± 12.5	
$\mathcal{B}\Gamma^{e^+e^-}(Y(4390))$ (eV)	3.6 ± 1.5	2.7 ± 1.0
ϕ_1 (rad)	3.3 ± 1.0	2.8 ± 0.4
ϕ_2 (rad)	0.8 ± 0.9	4.7 ± 0.1