Lattice calculation of the hadronic light-by-light contribution to the muon magnetic moment

Thomas Blum (UConn / RBRC)

Norman Christ (Columbia)

靳路昶 Luchang Jin (UConn / RBRC)

Masashi Hayakawa (Nagoya)

Taku Izubuchi (BNL / RBRC)

Chulwoo Jung (BNL)

Christoph Lehner (Regensburg / BNL)

Aug 21, 2020

强子物理 在线论坛 (Online via Tencent Meeting)

https://meeting.tencent.com/p/6733913824

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

Masashi Hayakawa (Nagoya)

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Outline

- **Introduction**
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

Muon $g-2$: experiments

$$\vec{\mu} = -g \frac{e}{2m} \vec{s}$$

$$a = \frac{g-2}{2}$$

Authors	Lab	Muon Anomaly	
Garwin et al. '60	CERN	0.001 13(14)	
Charpak et al. '61	CERN	0.001 145(22)	
Charpak et al. '62	CERN	0.001 162(5)	
Farley et al. '66	CERN	0.001 165(3)	
Bailey et al. '68	CERN	0.001 166 16(31)	
Bailey et al. '79	CERN	0.001 165 923 0(84)	
Brown et al. '00	BNL	0.001 165 919 1(59)	(μ^+)
Brown et al. '01	BNL	0.001 165 920 2(14)(6)	(μ^+)
Bennett et al. '02	BNL	0.001 165 920 4(7)(5)	(μ^+)
Bennett et al. '04	BNL	0.001 165 921 4(8)(3)	(μ^-)

World Average dominated by BNL

$$a_\mu = (11659208.9 \pm 6.3) \times 10^{-10}$$

In comparison, for electron

$$a_e = (11596521.8073 \pm 0.0028) \times 10^{-10}$$

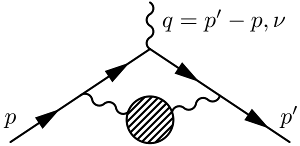
Muon $g-2$: Fermilab E989, J-PARC E34

SM (Model HLbL)	11659182.2 ± 3.8
BNL E821 Exp	11659208.9 ± 6.3
Diff (Exp − SM)	26.7 ± 7.4

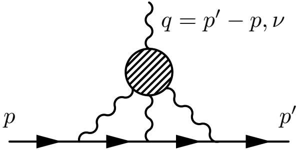
3.6σ deviations

New Physics?

	a_μ	$\times$	10^{10}	
QED incl. 5-loops	11658471.9	$\pm$	0.0	Aoyama, et al, 2012
Weak incl. 2-loops	15.4	$\pm$	0.1	Gnendiger et al, 2013
HVP	693.1	$\pm$	4.0	WP2020
HVP NLO&NNLO	-8.6	$\pm$	0.1	KNT2020
HLbL	9.0	$\pm$	1.7	WP2020
HLbL NLO	0.2	$\pm$	0.1	Colangelo, et al 2014
Standard Model	11659181.0	$\pm$	4.3	WP2020
Experiment	11659208.9	$\pm$	6.3	E821, The $g-2$ Collab. 2006
Difference (Exp-SM)	27.9	$\pm$	7.6	

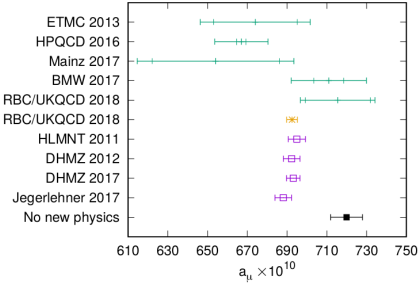


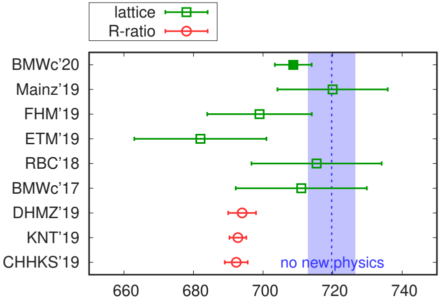
HVP: Hadronic Vacuum Polarization



HLbL: Hadronic Light by Light

HVP: Lattice results

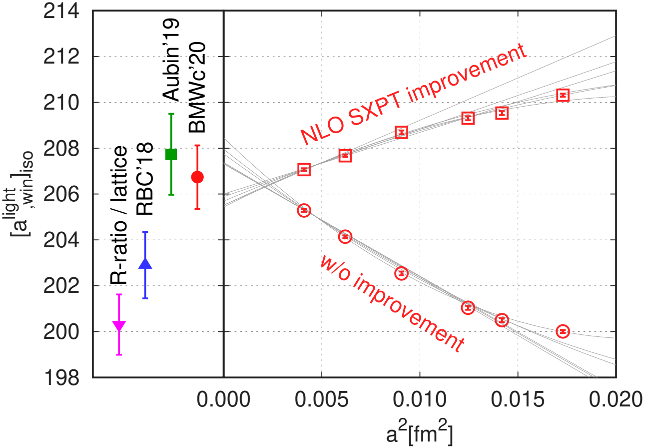




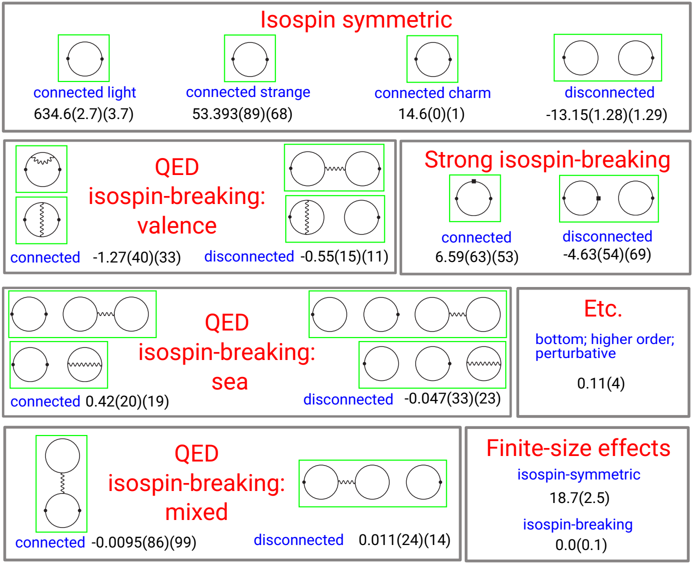
C. Lehner et al. 2018
RBC-UKQCD
(PRL 121, 022003)

Sz. Borsanyi et al. 2020
BMW
(2002.12347)

- Accuracy of lattice has catched up.
- BMW 2.4σ tension with R-ratio.
- More results from different collaborations will appear.



- Light quark connected diagram contribution in a window (from 0.4 fm to 1 fm).



$$10^{10} \times a_\mu^{\text{LO-HVP}} = 708.7(2.8)_{\text{stat}}(4.5)_{\text{sys}}[5.3]_{\text{tot}}$$

HLbL: Analytical approach WP2020

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	}
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

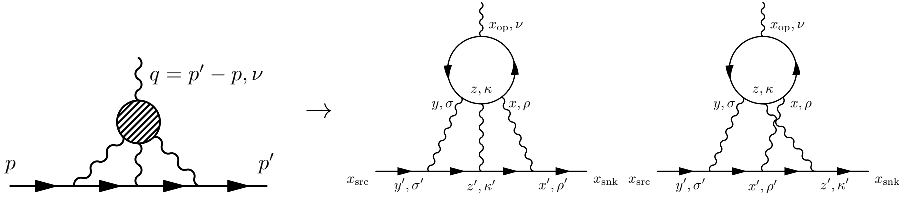
Table 15: Comparison of two frequently used compilations for HLbL in units of 10^{-11} from 2009 and a recent update with our estimate. Legend: PdRV = Prades, de Rafael, Vainshtein ("Glasgow consensus"); N/JN = Nyffeler / Jegerlehner, Nyffeler; J = Jegerlehner.

- Values in the table is in unit of 10^{-11}.
- We will use the unit in 10^{-10} in the rest of the talk.
- The total HLbL contribution is on the order of 10×10^{-10}.

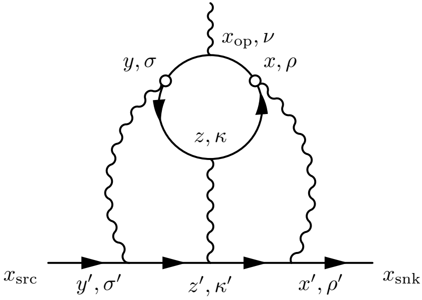
Outline

- Introduction
- **Connected diagrams: exact photon propagator, the moment method**
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

T. Blum et al. 2016 (PRD 93, 014503)



- Gluons and sea quark loops (not directly connected to photons) are included automatically to all orders!
- There are additional four different permutations of photons not shown.
- The photons can be connected to different quark loops. These are referred to as the disconnected diagrams. They will be discussed later.
- First results are obtained by T. Blum et al. 2015 (PRL 114, 012001).



- Two point sources at x, y: randomly sample x and y.
- Importance sampling: focus on small $|x - y|$.
- Complete sampling for $|x - y| \leq 5a$ upto discrete symmetry.

$$\frac{a_\mu}{m_\mu} \bar{u}_{s'}(\vec{0}) \frac{\Sigma}{2} u_s(\vec{0}) = \sum_{r=x-y} \sum_{z} \sum_{x_{\rm op}} \frac{1}{2} (\vec{x}_{\rm op} - \vec{x}_{\rm ref}) \times \bar{u}_{s'}(\vec{0}) i \vec{\mathcal{F}}^C(\vec{0}; x, y, z, x_{\rm op}) u_s(\vec{0})$$

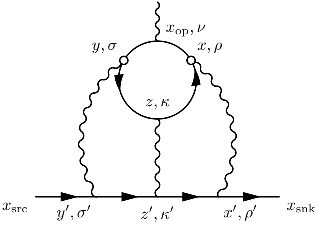
$$\vec{\mu} = \sum_{\vec{x}_{\rm op}} \frac{1}{2} (\vec{x}_{\rm op} - \vec{x}_{\rm ref}) \times \vec{J}(\vec{x}_{\rm op})$$

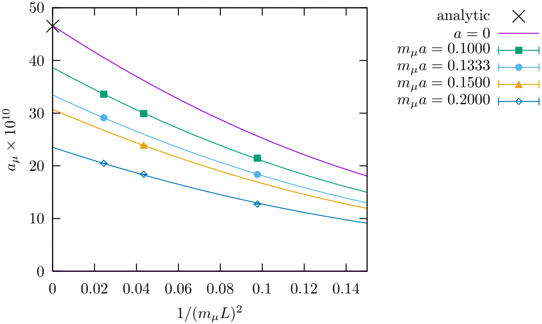
Reorder summation (will discuss later).

- Muon is plane wave, $x_{\rm ref} = (x + y)/2$.
- Sum over time component for $x_{\rm op}$.
- Only sum over $r = x - y$.

Muon leptonic LbL

- We test our setup by computing **muon leptonic light by light** contribution to muon $g-2$.





$$F_2(a,L) = F_2\left(1 - \frac{c_1}{(m_\mu L)^2} + \frac{c_1'}{(m_\mu L)^4}\right)(1 - c_2 a^2 + c_2' a^4) \;\; \rightarrow \;\; F_2 = 46.6(2)\times 10^{-10} \qquad (19)$$

- **Pure QED computation**. Muon leptonic light by light contribution to muon $g-2$. Phys.Rev. D93 (2016) 1, 014503. arXiv:1510.07100.
- Analytic results: $0.371\times(\alpha/\pi)^3 = 46.5\times 10^{-10}$.
- $\mathcal{O}(1/L^2)$ finite volume effect, because the photons are emitted from a conserved loop.

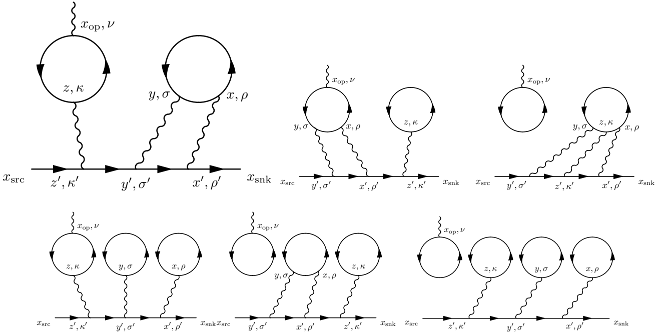
Outline

- Introduction
- Connected diagrams: exact photon propagator, the moment method
- **Disconnected diagrams**
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

T. Blum et al. 2017 (PRL 118, 022005)

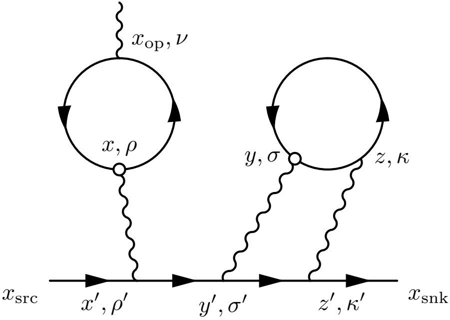
HLbL: disconnected diagrams

- One diagram (the biggest diagram below) do not vanish even in the $\mathrm{SU}(3)$ limit.
- We extend the method and computed this leading disconnected diagram as well.



- Permutations of the three internal photons are not shown.
- **Gluons exchange between and within the quark loops are not drawn.**
- We need to make sure that the loops are connected by gluons by "vacuum" subtraction. So the diagrams are 1-particle irreducible.

HLbL: disconnected formula



- Point x is used as the reference point for the moment method.
- We can use two point source photons at x and y, which are chosen randomly. The points $x_{\rm op}$ and z are summed over exactly on lattice.
- Only point source quark propagators are needed. We compute M point source propagators and all M^2 combinations of them are used to perform the stochastic sum over $r = x - y$.

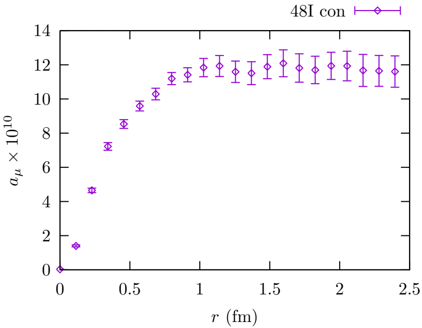
Outline

- Introduction
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- **Results @** $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

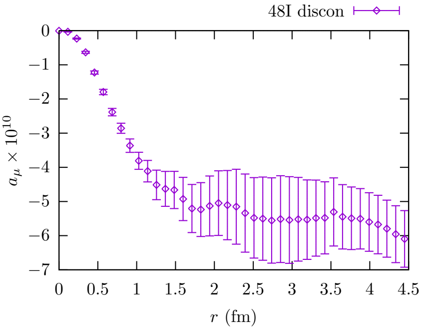
T. Blum et al. 2017 (PRL 118, 022005)

QED$_L$: 48I Results

$$\frac{a_\mu}{m_\mu}\bar{u}_{s'}(\vec{0})\frac{\Sigma}{2}u_s(\vec{0}) = \sum_{r=x-y}\sum_{z}\sum_{x_{\text{op}}}\frac{1}{2}(\vec{x}_{\text{op}}-\vec{x}_{\text{ref}})\times\bar{u}_{s'}(\vec{0})i\vec{\mathcal{F}}^C(\vec{0};x,y,z,x_{\text{op}})u_s(\vec{0})$$



Connected diagrams



Disconnected diagrams

Partial sum is plotted above. Full sum is the right most data point.

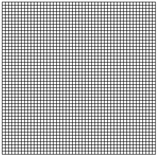
$a_\mu = 5.35(1.35)_{\text{stat}} \times 10^{-10}$ @ $L = 5.5$fm, $1/a = 1.73$GeV, $m_\pi = 139$MeV.

Outline

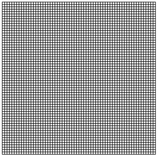
- Introduction
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- **Continuum and infinite volume limit**
- Hybrid continuum limit
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

T. Blum et al. 2020 (PRL 124, 132002)

HLbL: RBC-UKQCD lattices

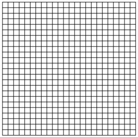


48I: $48^3 \times 96$, $5.5\,\mathrm{fm}$ box

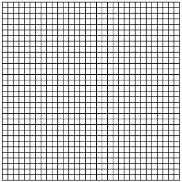


64I: $64^3 \times 128$, $5.5\,\mathrm{fm}$ box

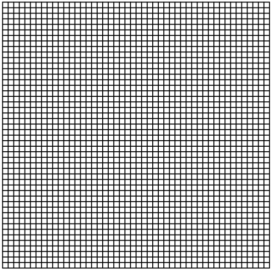
Phys. Rev. D 93, 074505 (2016)



24D: $24^3 \times 64$, $4.8\,\mathrm{fm}$ box



32D: $32^3 \times 64$, $6.4\,\mathrm{fm}$ box

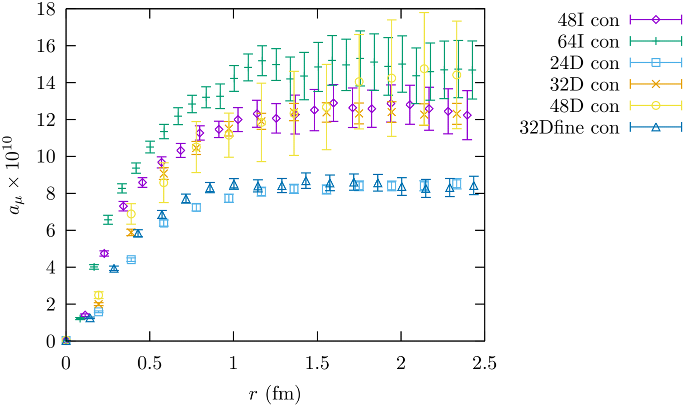


48D: $48^3 \times 64$, $9.6\,\mathrm{fm}$ box

32Dfine: $32^3 \times 64$, $4.8\,\mathrm{fm}$ box

QED_L: Connected diagrams results

$$\frac{a_\mu}{m_\mu}\bar{u}_{s'}(\vec{0})\frac{\Sigma}{2}u_s(\vec{0}) = \sum_{r=x-y}\sum_{z}\sum_{x_{\text{op}}}\frac{1}{2}(\vec{x}_{\text{op}}-\vec{x}_{\text{ref}})\times\bar{u}_{s'}(\vec{0})i\vec{\mathcal{F}}^C(\vec{0};x,y,z,x_{\text{op}})u_s(\vec{0})$$

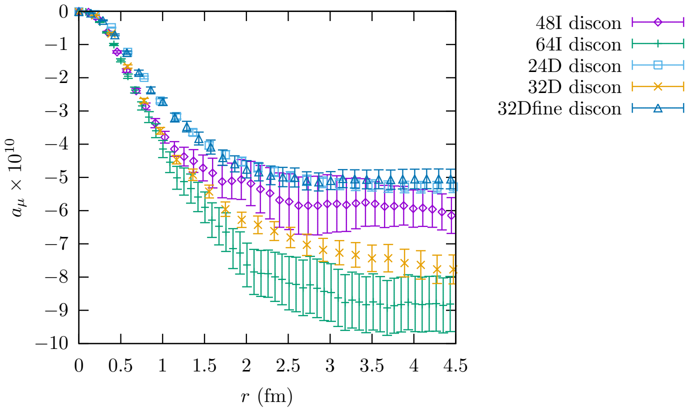


Partial sum is plotted above. Full sum is the right most data point.

T. Blum et al 2020. (PRL 124, 132002)

QED$_L$: Disconnected diagrams results

$$\frac{a_\mu}{m_\mu}\bar{u}_{s'}(\vec{0})\frac{\Sigma}{2}u_s(\vec{0}) = \sum_{r=x-y}\sum_{z}\sum_{x_{\text{op}}}\frac{1}{2}(\vec{x}_{\text{op}}-\vec{x}_{\text{ref}})\times\bar{u}_{s'}(\vec{0})i\vec{\mathcal{F}}^C(\vec{0};x,y,z,x_{\text{op}})u_s(\vec{0})$$



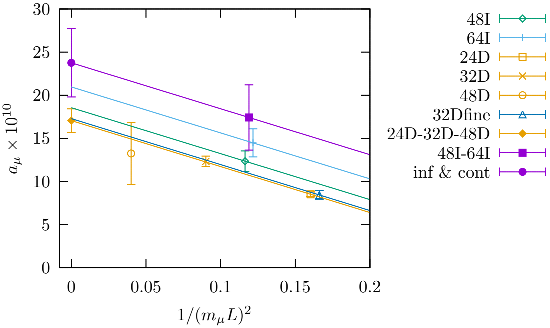
Partial sum is plotted above. Full sum is the right most data point.

T. Blum et al 2020. (PRL 124, 132002)

Inf vol & continuum for connected

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

I-DSDR and Iwasaki ensembles have different $\mathcal{O}(a^2)$ coefficients.

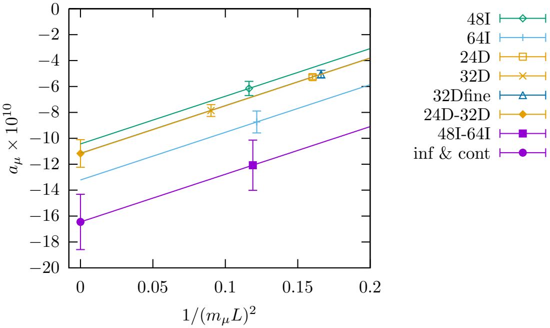


$$a_\mu = 23.76(3.96)_{\mathrm{stat}} \times 10^{-10}$$

Inf vol & continuum for disconnected

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

I-DSDR and Iwasaki ensembles have different $\mathcal{O}(a^2)$ coefficients.

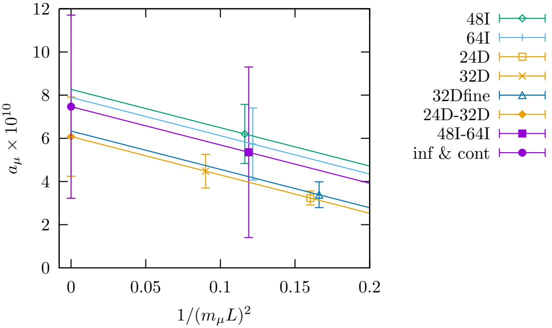


$$a_\mu = -16.45(2.13)_{\mathrm{stat}} \times 10^{-10}$$

Inf vol & continuum for total

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

I-DSDR and Iwasaki ensembles have different $\mathcal{O}(a^2)$ coefficients.



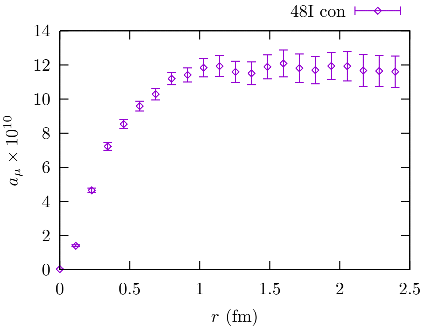
$$a_\mu = 7.47(4.24)_{\mathrm{stat}} \times 10^{-10}$$

Outline

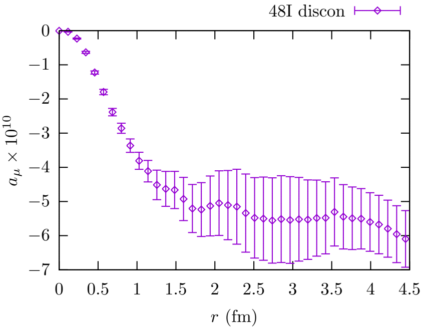
- Introduction
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- **Hybrid continuum limit**
- Systematic error estimation
- Conclusion and outlook: infinite volume QED

T. Blum et al. 2020 (PRL 124, 132002)

$$\frac{a_\mu}{m_\mu}\bar{u}_{s'}(\vec{0})\frac{\Sigma}{2}u_s(\vec{0}) = \sum_{r=x-y}\sum_{z}\sum_{x_{\rm op}}\frac{1}{2}(\vec{x}_{\rm op}-\vec{x}_{\rm ref})\times\bar{u}_{s'}(\vec{0})i\vec{\mathcal{F}}^C(\vec{0};x,y,z,x_{\rm op})u_s(\vec{0})$$



Connected diagrams

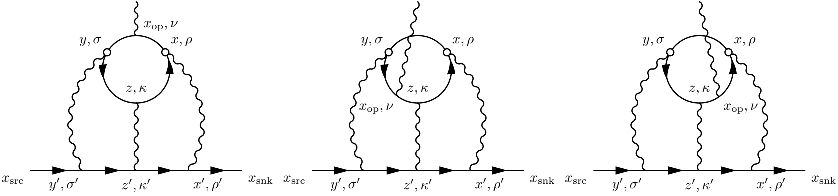


Disconnected diagrams

Partial sum is plotted above. Full sum is the right most data point.

Contribution to the connected diagrams mostly from small r ($r < 1$ fm).

Reorder the summation



- The three internal vertex attached to the quark loop are equivalent (all permutations are included).

- We can pick the closer two points as the point sources x, y.

$$\sum_{x,y,z} \to \sum_{x,y,z} \begin{cases} 3 & \text{if } |x-y| < |x-z| \text{ and } |x-y| < |y-z| \\ 3/2 & \text{if } |x-y| = |x-z| < |y-z| \\ 3/2 & \text{if } |x-y| = |y-z| < |x-z| \\ 1 & \text{if } |x-y| = |y-z| = |x-z| \\ 0 & \text{others} \end{cases}$$

QED_L: Hybrid continuum

Split the a_μ^{con} into two parts:

$$a_\mu^{\text{con}} = a_\mu^{\text{con,short}} + a_\mu^{\text{con,long}}$$

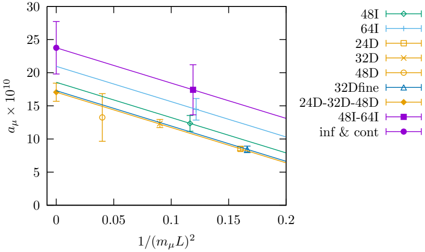
- $a_\mu^{\text{con,short}} = a_\mu^{\text{con}}(r \leq 1\text{fm})$:
  most of the contribution, small statistical error.
- $a_\mu^{\text{con,long}} = a_\mu^{\text{con}}(r > 1\text{fm})$:
  small contribution, large statistical error.

Perform continuum extrapolation for short and long parts separately.

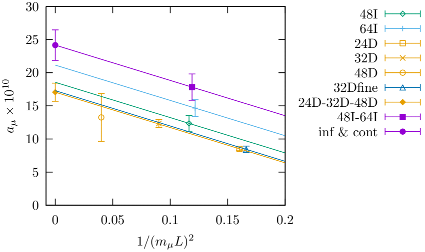
- $a_\mu^{\text{con,short}}$: conventional a^2 fitting.
- $a_\mu^{\text{con,long}}$: simply use 48I value.
  Conservatively estimate the relative $\mathcal{O}(a^2)$ error: it may be as large as for $a_\mu^{\text{con,short}}$ from 48I.

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

I-DSDR and Iwasaki ensembles have different $\mathcal{O}(a^2)$ coefficients.



Conventional continuum limit



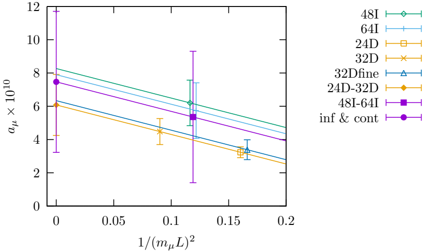
Hybrid continuum limit

$$a_\mu = 23.76(3.96)_{\mathrm{stat}} \times 10^{-10} \to 24.16(2.30)_{\mathrm{stat}}(0.20)_{\mathrm{sys},a^2} \times 10^{-10}$$

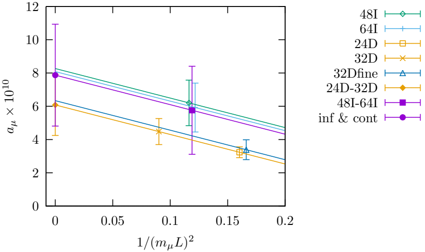
Inf vol & hybrid continuum for total

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

I-DSDR and Iwasaki ensembles have different $\mathcal{O}(a^2)$ coefficients.



Conventional continuum limit



Hybrid continuum limit

$$a_\mu = 7.47(4.24)_{\mathrm{stat}} \times 10^{-10} \to 7.87(3.06)_{\mathrm{stat}}(0.20)_{\mathrm{sys},a^2} \times 10^{-10}$$

Outline

- Introduction
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- **Systematic error estimation**
- Conclusion and outlook: infinite volume QED

T. Blum et al. 2020 (PRL 124, 132002)

Systematic error summary

	con	discon	tot
a_μ	24.16(2.30)	-16.45(2.13)	7.87(3.06)
sys hybrid $\mathcal{O}(a^2)$	0.20(0.45)	0	0.20(0.45)
sys $\mathcal{O}(1/L^3)$	2.34(0.41)	1.72(0.32)	0.83(0.56)
sys $\mathcal{O}(a^4)$	0.88(0.31)	0.71(0.28)	0.95(0.92)
sys $\mathcal{O}(a^2 \log(a^2))$	0.23(0.08)	0.25(0.09)	0.02(0.11)
sys $\mathcal{O}(a^2/L)$	4.43(1.38)	3.49(1.37)	1.08(1.57)
sys strange con	0.30	0	0.30
sys sub-discon	0	0.50	0.50
sys all	5.11(1.32)	3.99(1.29)	1.77(1.13)

- Systematic error has some cancellation between the connected and disconnected diagrams.

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

$\mathcal{O}(1/L^3)$

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} + \frac{b_2}{(m_\mu L)^3} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

$\mathcal{O}(a^2 \log(a^2))$

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\left(1 - \frac{b_2}{(m_\mu L)^2} - \Big(c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 + c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 - c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big) \times\Big(1 - \frac{\alpha_S}{\pi}\log\big((a\ \mathrm{GeV})^2\big)\Big)\right)$$

Sys error from difference of fits

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

$\mathcal{O}(a^4)$ (maximum of the following two)

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2(a\ \mathrm{GeV})^4\Big)$$

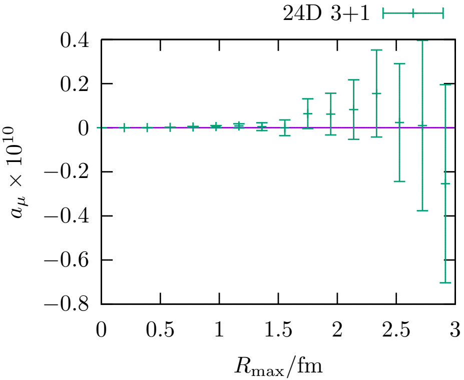
$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1(a\ \mathrm{GeV})^2 + c_2^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^4 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

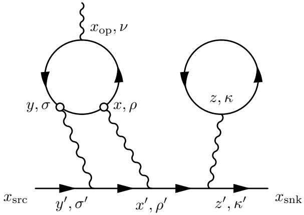
$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2} - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

$\mathcal{O}(a^2/L)$ (maximum of the following two)

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\left(1 - \frac{b_2}{(m_\mu L)^2} - \Big(c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 + c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 - c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)\Big(1 - \frac{1}{m_\mu L}\Big)\right)$$

$$a_\mu(L, a^{\mathrm{I}}, a^{\mathrm{D}}) = a_\mu\Big(1 - \frac{b_2}{(m_\mu L)^2}\Big) \times\Big(1 - c_1^{\mathrm{I}}(a^{\mathrm{I}}\ \mathrm{GeV})^2 - c_1^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^2 + c_2^{\mathrm{D}}(a^{\mathrm{D}}\ \mathrm{GeV})^4\Big)$$

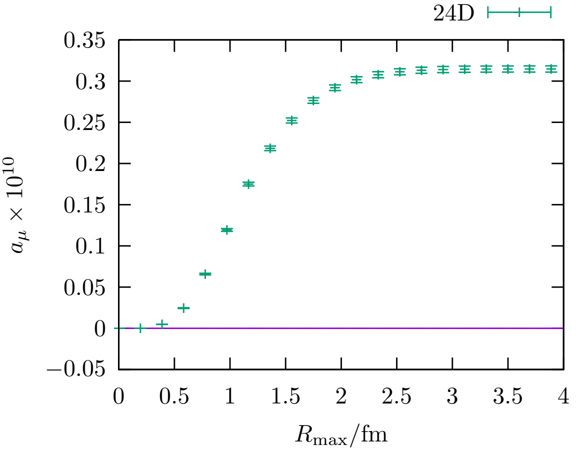


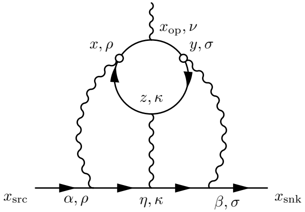


- 24D: $24^3 \times 64$
  $L = 4.8$ fm
- $a^{-1} = 1.015$ GeV
  $M_\pi = 142$ MeV
  $M_K = 512$ MeV

- Partial sum upto $R_{\max}$
  $R_{\max} = \max(|x - y|, |x - z|, |y - z|)$
- The tadpole part comes from C. Lehner et al. 2016 (PRL 116, 232002)
- Systematic error (subdiscon): 0.5×10^{-10}

Strange connected at physical point





- 24D: $24^3 \times 64$
  $L = 4.8$ fm
- $a^{-1} = 1.015$ GeV
  $M_\pi = 142$ MeV
  $M_K = 512$ MeV

- Partial sum upto $R_{\max}$
  $R_{\max} = \max(|x - y|, |x - z|, |y - z|)$
- Systematic error (strange con): 0.3×10^{-10}

Outline

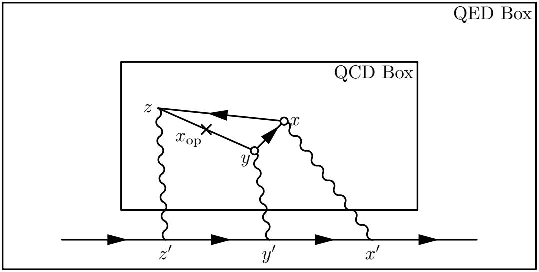
- Introduction
- Connected diagrams: exact photon propagator, the moment method
- Disconnected diagrams
- Results @ $L = 5.5\text{fm}, 1/a = 1.73\text{GeV}$
- Continuum and infinite volume limit
- Hybrid continuum limit
- Systematic error estimation
- **Conclusion and outlook: infinite volume QED**

T. Blum et al. 2017 (PRD 96, 034515)

Conclusion and outlook

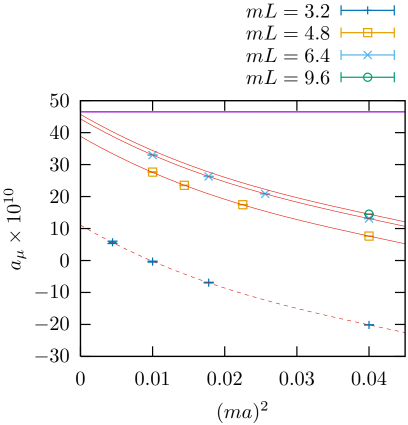
- $a_\mu = 7.87(3.06)_{\text{stat}}(1.77)_{\text{sys}} \times 10^{-10}$.
- Consistent with hadronic model estimate: $10.3(2.9) \times 10^{-10}$ (compiled by Fred Jegerlehner 2017).
- Leaves little room for the HLbL contribution to explain the difference between the Standard Model and the BNL experiment.
- Better accuracy is desired to compare with the on-going Fermilab muon $g-2$ experiments. Initial experimental result (using portion of the statistics) is expected to release later this year.
- Plan to invest in the infinite volume QED approach.

- Mainz group initially proposed the idea of calculating QED part of the process in infinite volume.
  N. Asmussen, J. Green, H. Meyer, A. Nyffeler 2016
- Motivated by Mainz group, we have also started to work on this approach.
  T. Blum et al, PRD 96, 034515

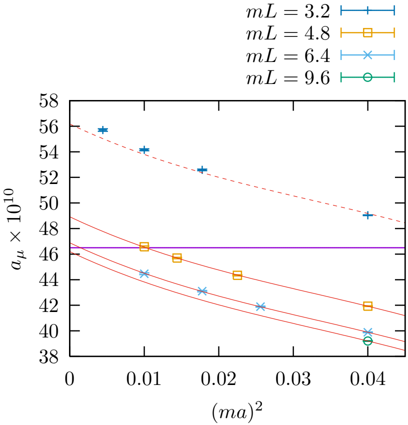


QED$_\infty$: Muon leptonic LbL

- Compare the two $\mathfrak{G}_{\rho,\sigma,\kappa}(x, y, z)$ in **pure QED computation**.



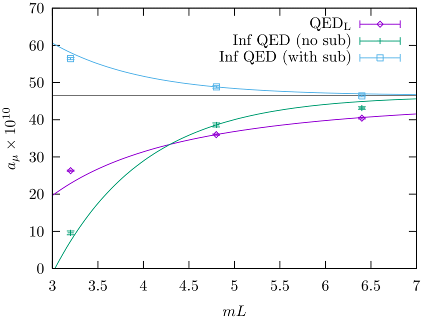
Original QED kernel



Subtracted QED kernel

- Notice the vertical scales in the two plots are different.

- Compare the finite volume effects in different approaches in **pure QED computation**,



- QED$_L$: $\mathcal{O}(1/L^2)$ finite volume effect, because the photons are emitted from a conserved loop. Phys.Rev. D93 (2016) 1, 014503.
- Inf QED (no sub): $\mathcal{O}(e^{-mL})$ finite volume effect. Everything except the four-point-correlation function is evaluated in infinite volume. arXiv:1705.01067.
- Inf QED (with sub): smaller $\mathcal{O}(e^{-mL})$ finite volume effect. arXiv:1705.01067.

QED$_\infty$: Mainz @ $m_\pi = m_K = 420$ MeV

- En-Hung Chao, Antoine Gerardin, Jeremy R. Green, Renwick J. Hudspith, and Harvey B. Meyer. arXiv:2006.16224
- Connected diagram: $a_\mu = 9.89(25) \times 10^{-10}$.
- Disconnected diagram: $a_\mu = -3.35(42) \times 10^{-10}$.
- Total: $a_\mu = 6.54(49)(66)_{\text{sys-cont}} \times 10^{-10}$.
- Adjust to physical pion/kaon mass: $a_\mu = 10.41(91) \times 10^{-10}$. Subtracting the π^0-pole contribution in this unphysical setup and add back the physical π^0-pole contribution.


IBM
MIRA
muon

Thank You!