# CR scattering on MHD modes

**O. Fornieri *et al.*** – **MNRAS 502, 5821–5838 (2021)**

Ottavio Fornieri
TeVPA - 成都 四川, 27 October 2021

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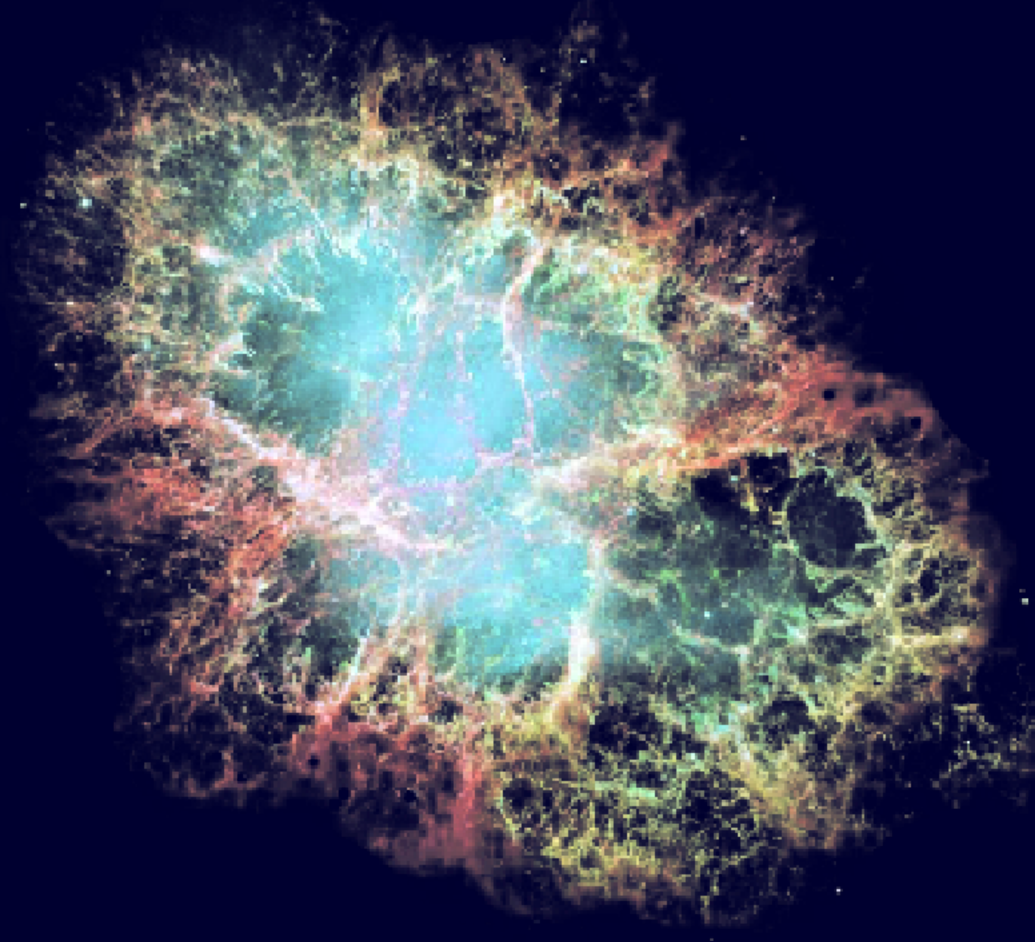
Scuola Universitaria Superiore

# Outline



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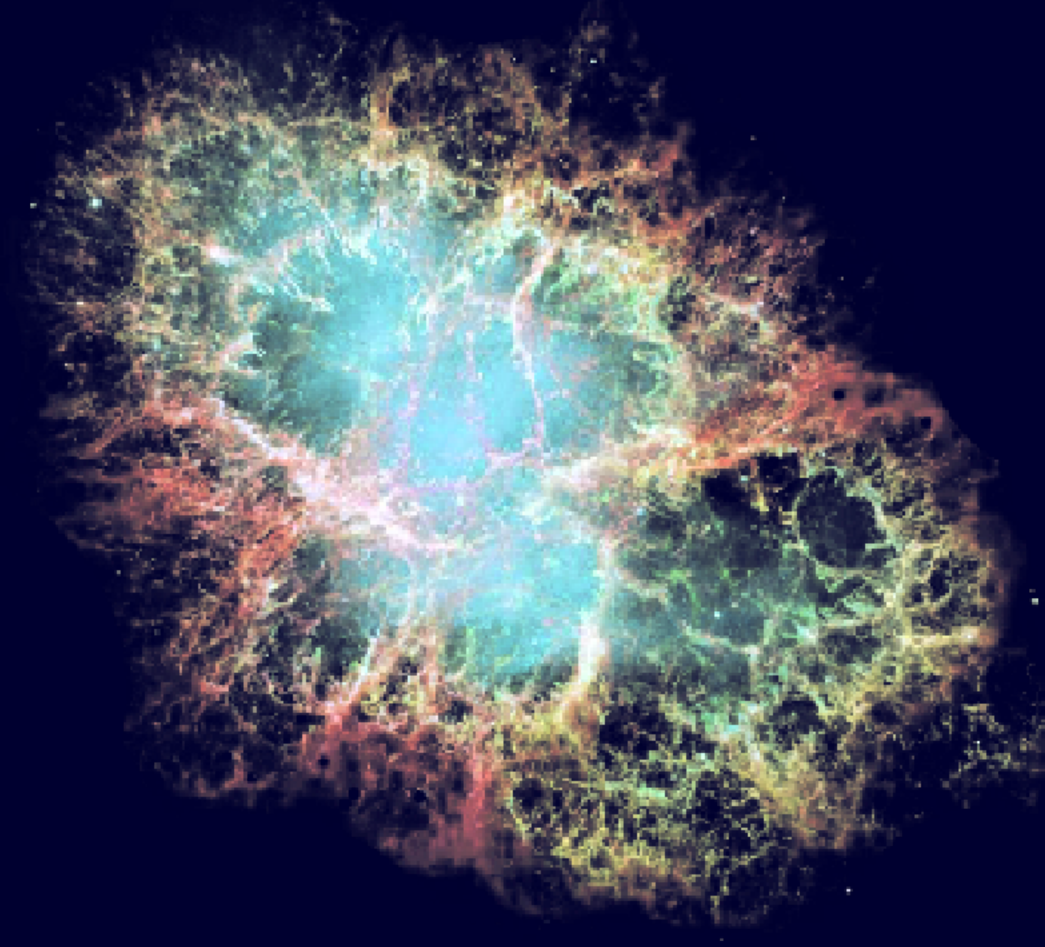
- Introduction and motivations



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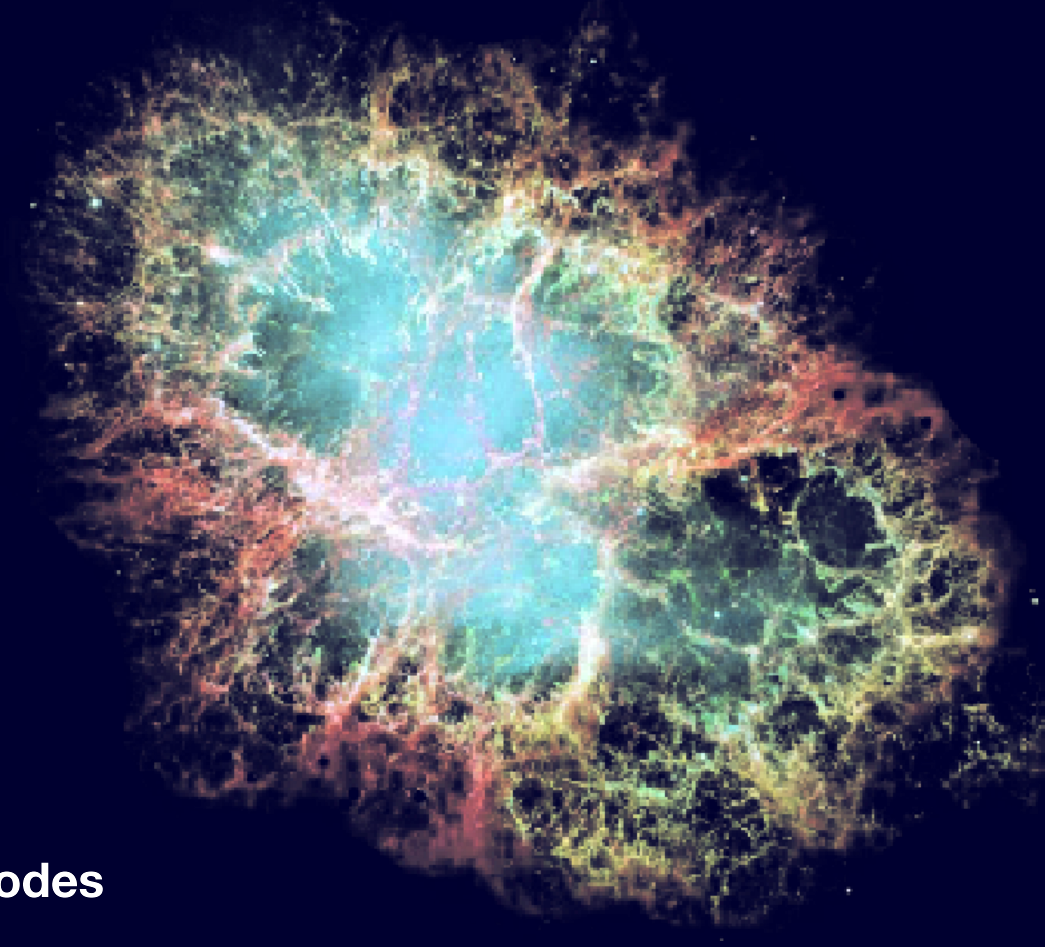

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- Change in the standard paradigm of Alfvénic CR diffusion
    - The role of the **non-linear extensions** of the QLT
    - Diffusion coefficients resulting from the **compressible modes**

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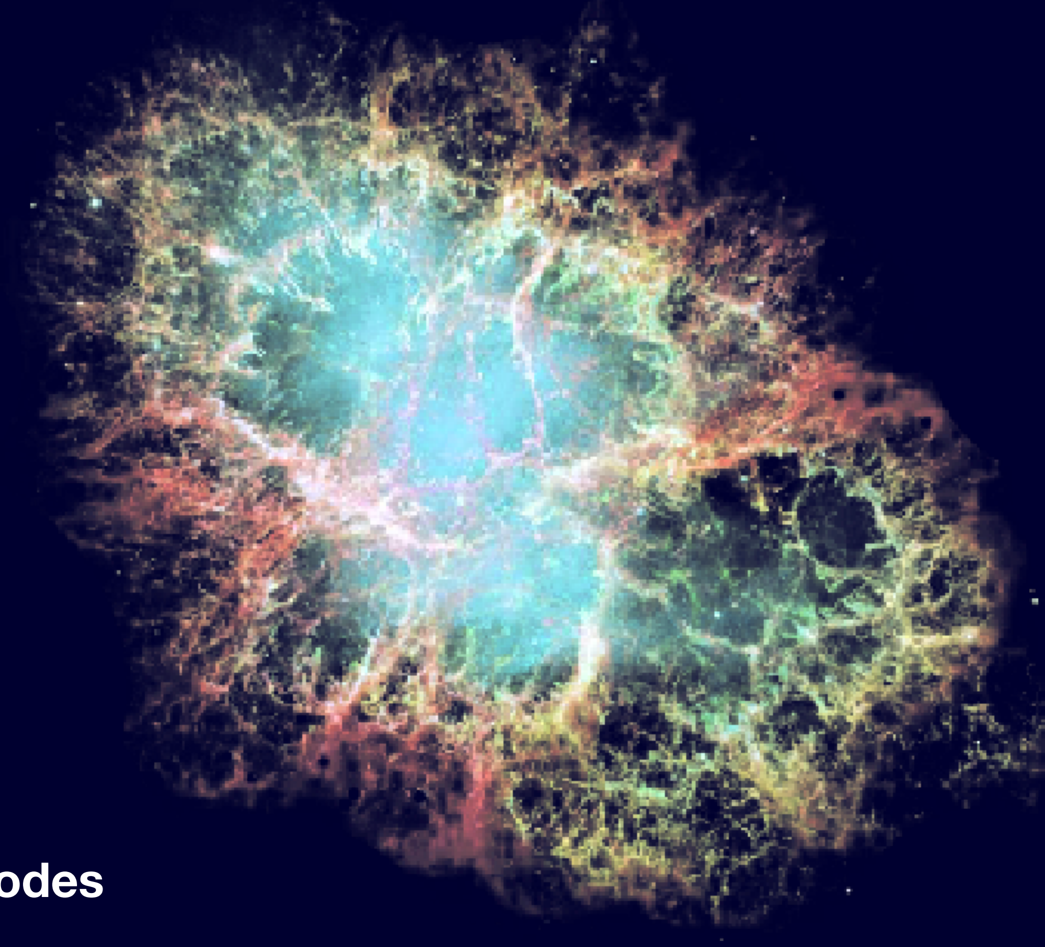


- Introduction and motivations

- Change in the standard paradigm of Alfvénic CR diffusion
  - The role of the **non-linear extensions** of the QLT
  - Diffusion coefficients resulting from the **compressible modes**

- Connecting the **micro-physics** of ISM turbulence with local CR observables
  - The role of *B/C* to constrain the **confining power** of the theory
  - A look at the hadronic species.

# CR scattering on MHD modes

[O. Fornieri *et al.* – **MNRAS 502, 5821–5838 (2021)**]

- Scattering rates of MHD modes
- Results on the observables

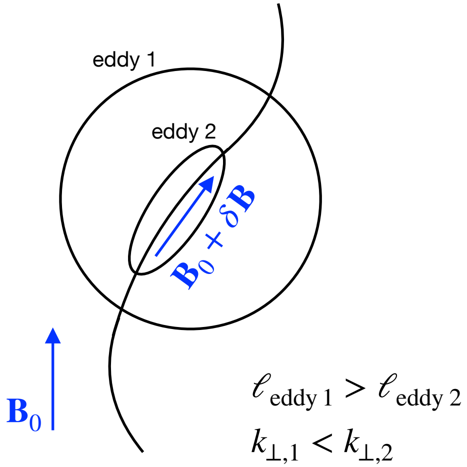
# Motivation to dig into the micro-physics

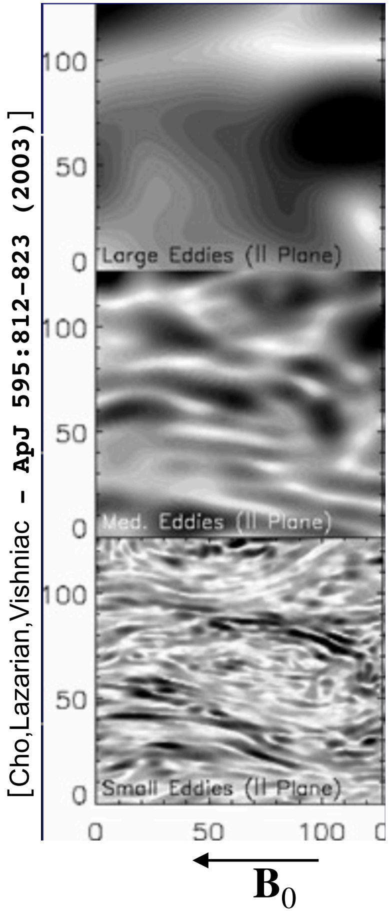
- **Conventional** diffusion models based on **QLT from slab turbulence**
  - Resonant scattering only ($\delta$-function resonance)
  - Scattering against **Alfvénic isotropic** turbulence only
  - $$D(E) = \frac{1}{3}\frac{cR_L}{kW(k)} \underset{R_L \sim E}{\sim} \frac{E}{k \cdot k^{-\alpha}} \underset{k \sim R_L^{-1}}{\sim} \frac{E}{E^{-1}E^{\alpha}} \propto E^{2-\alpha} \equiv E^{\delta}$$

- **MHD turbulence cascades in 3D** and is decomposed into **three propagating modes** (*fast* and *slow-magnetosonic*, *Alfvén*) [*e.g.* **Kulsrud05**]

- Alfvén modes are **anisotropic** $\left( (k_{\parallel} \sim \ell_{\parallel}^{-1}) \neq (k_{\perp} \sim \ell_{\perp}^{-1}) \text{ wrt } \mathbf{B}_{\rm tot} \right)$ [**Goldreich&Sridhar95, Cho&Lazarian03, Yan&Lazarian02,04,08**]

$\Longrightarrow$ highly **inefficient in confining CRs** [**Chandran00**].



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# Contribution to $D_{\mu\mu}$ from fast modes

1

Kulsrud69, Völk75, Yan&Lazarian08

$$D_{\mu\mu} = \Omega^2(1-\mu^2)\int d^3\mathbf{k} \sum_{n=-\infty}^{+\infty} \delta(k_\parallel v_\parallel - \omega + n\Omega) \left[ \frac{n^2 J_n^2(z)}{z^2} I^{\mathrm{A}}(\mathbf{k}) + \frac{k_\parallel^2}{k^2} J_n'^2(z) I^{\mathrm{M}}(\mathbf{k}) \right]$$

QLT unperturbed orbits

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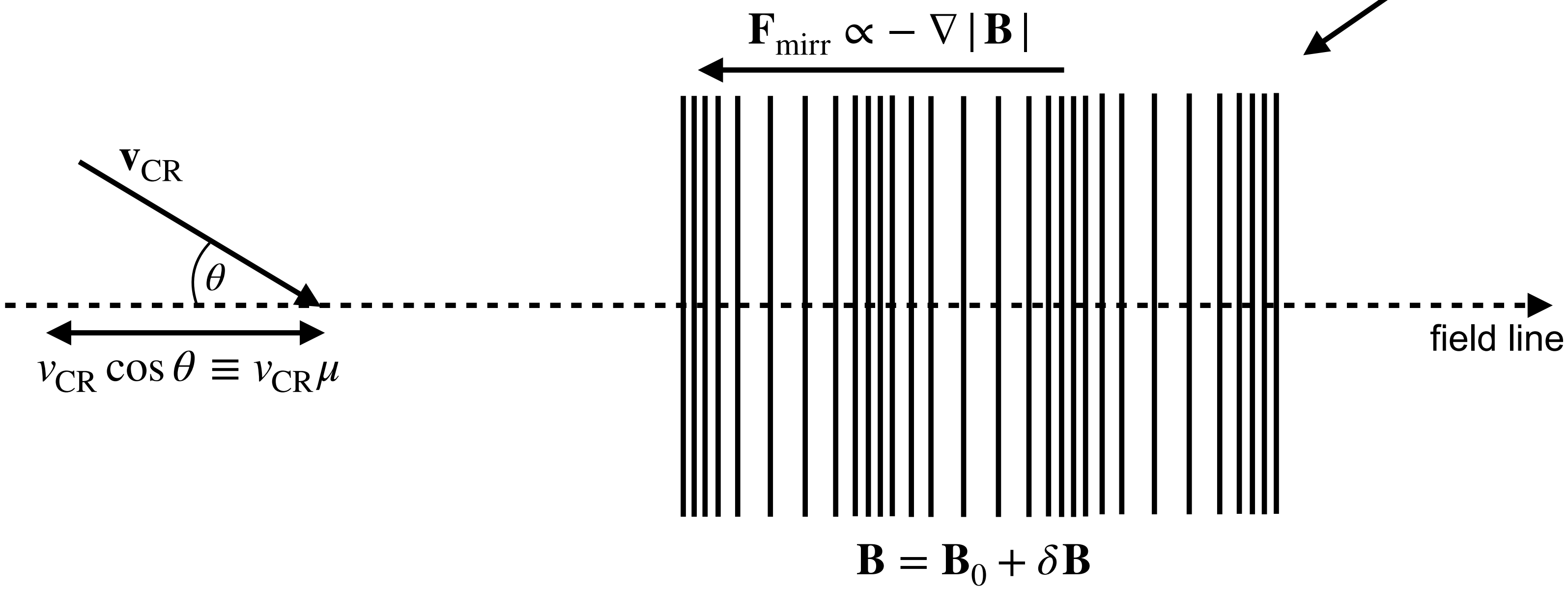


Efficient TTD interaction for

$v_{\mathrm{CR}}\mu \simeq \omega / k_\parallel$

⇩

Small $\mu$ range

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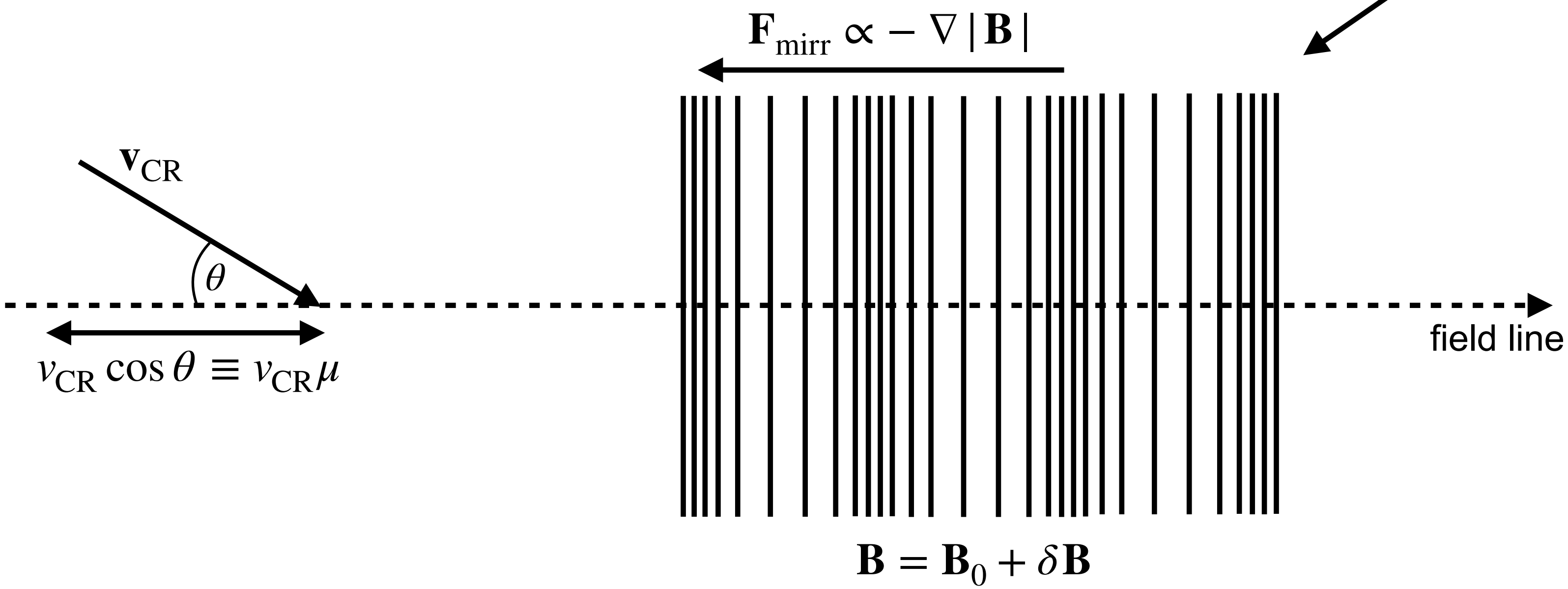


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**Magnetosonic modes are present in QLT but not efficient!**

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$$v_\perp^2 / |\mathbf{B}| \approx \text{constant}$$

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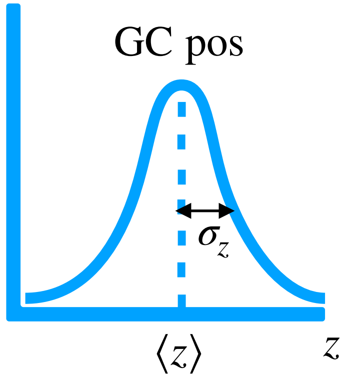
⇓ $|\mathbf{B}|$ changes

$v_\parallel$ changes ⇒ $\mu$ – range enhanced ⇒ **guiding center** perturbed




⇓

$\delta B \xrightarrow{\mathcal{F}} \widetilde{\delta B} \cdot e^{i(k_\parallel z_{pert} - \omega t)}$

$$e^{ik_\parallel z}\Big|_{pert} = \int_{-\infty}^{+\infty} dz\, e^{ik_\parallel z} \left( \frac{1}{\sqrt{2\pi\sigma_z}} e^{-\frac{(z-\langle z \rangle)^2}{2\sigma_z^2}} \right) = e^{ik_\parallel \langle z \rangle} \cdot e^{-k_\parallel^2 \frac{\sigma_z^2}{2}}$$

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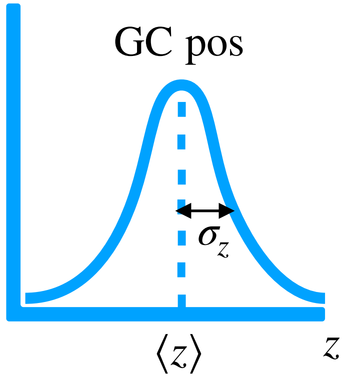


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⇓

$$R_n\left(k_\parallel v_\parallel - \omega + n\Omega\right) \equiv \text{Re}\left[\int_0^\infty dt\, e^{i\left(k_\parallel z_{\rm pert} - \omega t + n\Omega t\right)}\right] = \text{Re}\left[\int_0^\infty dt\, e^{i\left(k_\parallel v_\parallel - \omega + n\Omega\right)t - \frac{1}{2}k_\parallel^2 v_\perp^2 t^2 \left(\frac{\langle \delta B_\parallel^2 \rangle}{B_0^2}\right)^{1/2}}\right]$$

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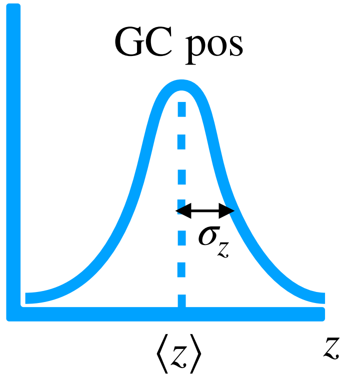


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$$k_\parallel^{\rm QLT} \sim \frac{\Omega}{v_\parallel} \xrightarrow{\theta \approx 90^\circ} +\infty$$

$$k_\parallel^{\rm NLT} \xrightarrow{\theta \approx 90^\circ} \frac{\Omega}{\Delta v_\parallel}$$

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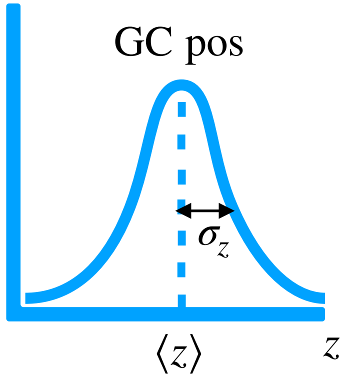
$v_\parallel$ changes $\Rightarrow$ $\mu$ – range enhanced $\Rightarrow$ **guiding center** perturbed




NLT line

$$\delta \mathrm{B} \overset{\mathcal{F}}{\rightarrow} \widetilde{\delta \mathrm{B}} \cdot e^{i(k_\parallel z_{\mathrm{pert}} - \omega t)}$$

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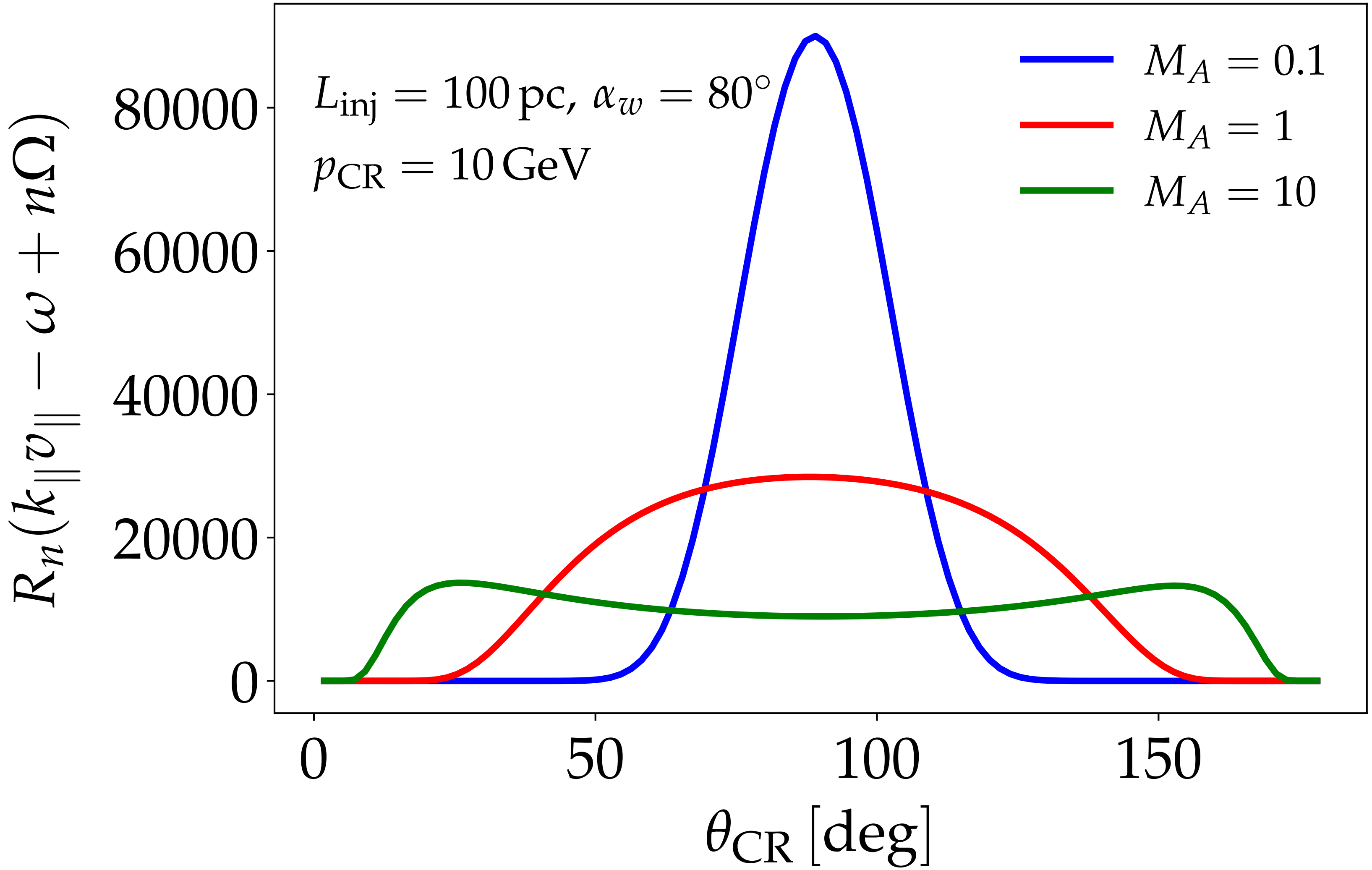
# Broadened resonance function

$$R_n(k_\parallel v_\parallel - \omega + n\Omega) = \frac{\sqrt{\pi}}{|k_\parallel| v_\perp M_\mathrm{A}^{1/2}} \cdot e^{-\frac{(k_\parallel v\mu - \omega + n\Omega)^2}{k_\parallel^2 v^2 (1-\mu^2) M_\mathrm{A}}}$$

$$v_\mathrm{CR}\mu \approx \omega / k_\parallel \sim v_A \quad \Rightarrow \quad \mu \sim 10^{-5}$$



# Turbulence spectra

**Yan&Lazarian02**

$$\left\langle \delta B_i \cdot \delta B_j \right\rangle \Big|_{\rm fast} = \frac{M_{\rm A}^2 L^{1/2}}{8\pi} J_{ij} k^{-7/2}$$

Fast modes
(isotropic)

$$\left\langle \delta B_i \cdot \delta B_j \right\rangle \Big|_{\rm A,\, sub} = \frac{M_{\rm A}^{4/3} L^{-1/3}}{6\pi} I_{ij}\, k_\perp^{-10/3} \cdot \exp\left( - \frac{L^{1/3} k_\parallel}{M_{\rm A}^{4/3} k_\perp^{2/3}} \right)$$

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Alfvén modes (anisotropic)

Evolution on the isosurfaces [GS95]

$k_\parallel \sim k_\perp^{2/3}$

# Inefficiency of Alfvén modes

**1D calculation**

$$\int dk_\parallel \, E(k_\parallel) = \int dk_\perp \, E(k_\perp)$$

$$E^{\rm GS}(k_\perp) \sim k_\perp^{-5/3}$$

$$k_\parallel \sim k_\perp^{2/3} \;\Rightarrow\; k_\parallel^{3/2} \sim k_\perp$$

$$\rightarrow \frac{dk_\perp}{dk_\parallel} = \frac{3}{2} k_\parallel^{3/2-1} = \frac{3}{2} k_\parallel^{1/2}$$

$$\Rightarrow$$

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$$\Rightarrow \quad E^{\rm GS}(k_\parallel) \sim k_\parallel^{-2}$$

But $D_{\mu\mu} \propto R_n(k_\parallel v_\parallel - \omega + n\Omega)\ \dots$

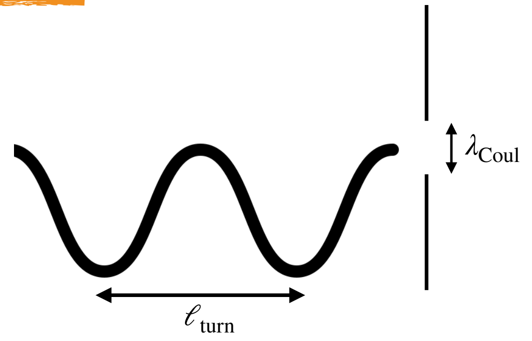
# Damping of the waves



**Thermal damping of the fast modes** [Petrosian *et al.* – ApJ 644 603 (2006)]

$$\lambda_{\rm Coul} \approx 1.3 \cdot 10^{-5} \left(\frac{\rm cm^{-3}}{n_{\rm ISM}}\right) \cdot \left(\frac{T}{10^4\,\rm K}\right)^2 \,\rm pc$$

- **Collisional** damping

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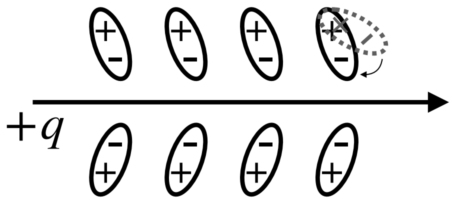


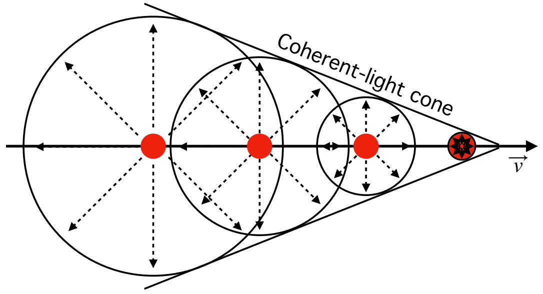

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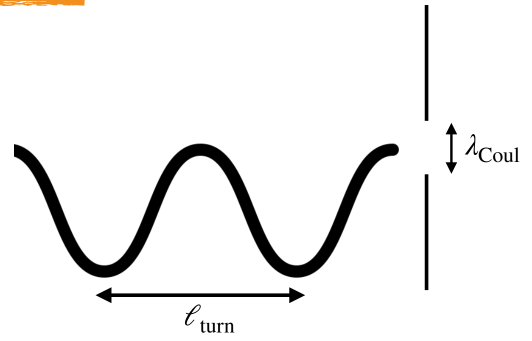


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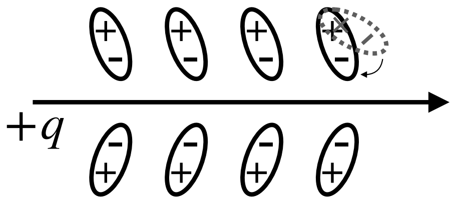


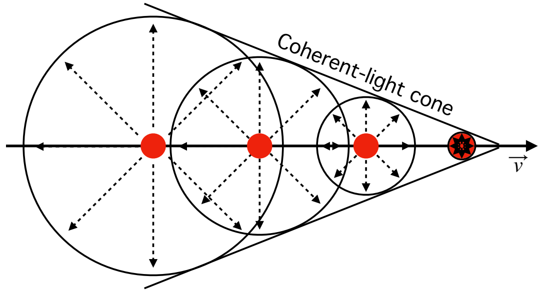


$$\lambda_{\rm Coul}^{\rm disk} \underset{\substack{n_{\rm disk} = 1\,{\rm cm^{-3}} \\ T = 10^4\,{\rm K}}}{\approx} 1.3 \cdot 10^{-5}\,{\rm pc}, \qquad \lambda_{\rm Coul}^{\rm halo} \underset{\substack{n_{\rm Halo} = 10^{-3}\,{\rm cm^{-3}} \\ T = 10^6\,{\rm K}}}{\approx} 1.3 \cdot 10^{2}\,{\rm pc} \simeq L_{\rm inj}$$

# Damping of the waves

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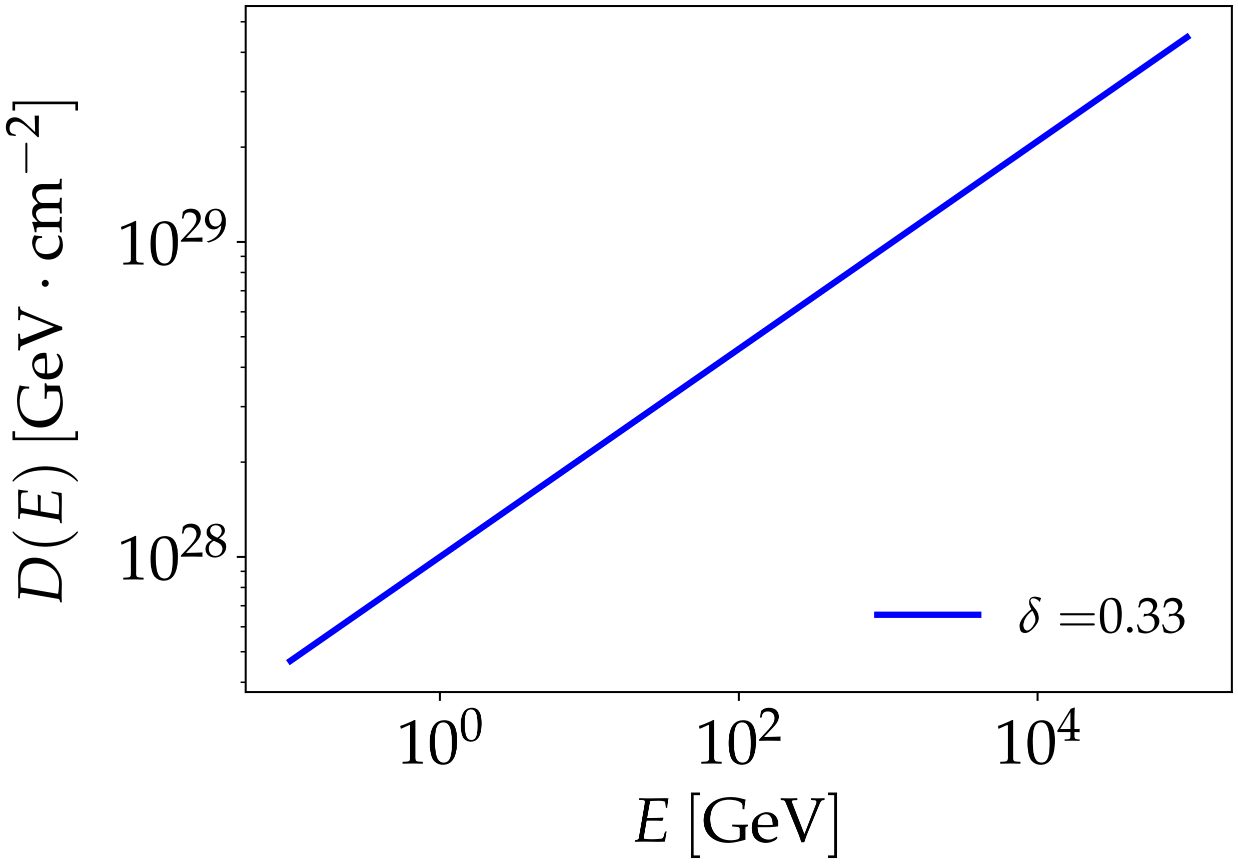
**Thermal damping of the fast modes** [Petrosian *et al.* - ApJ 644 603 (2006)]



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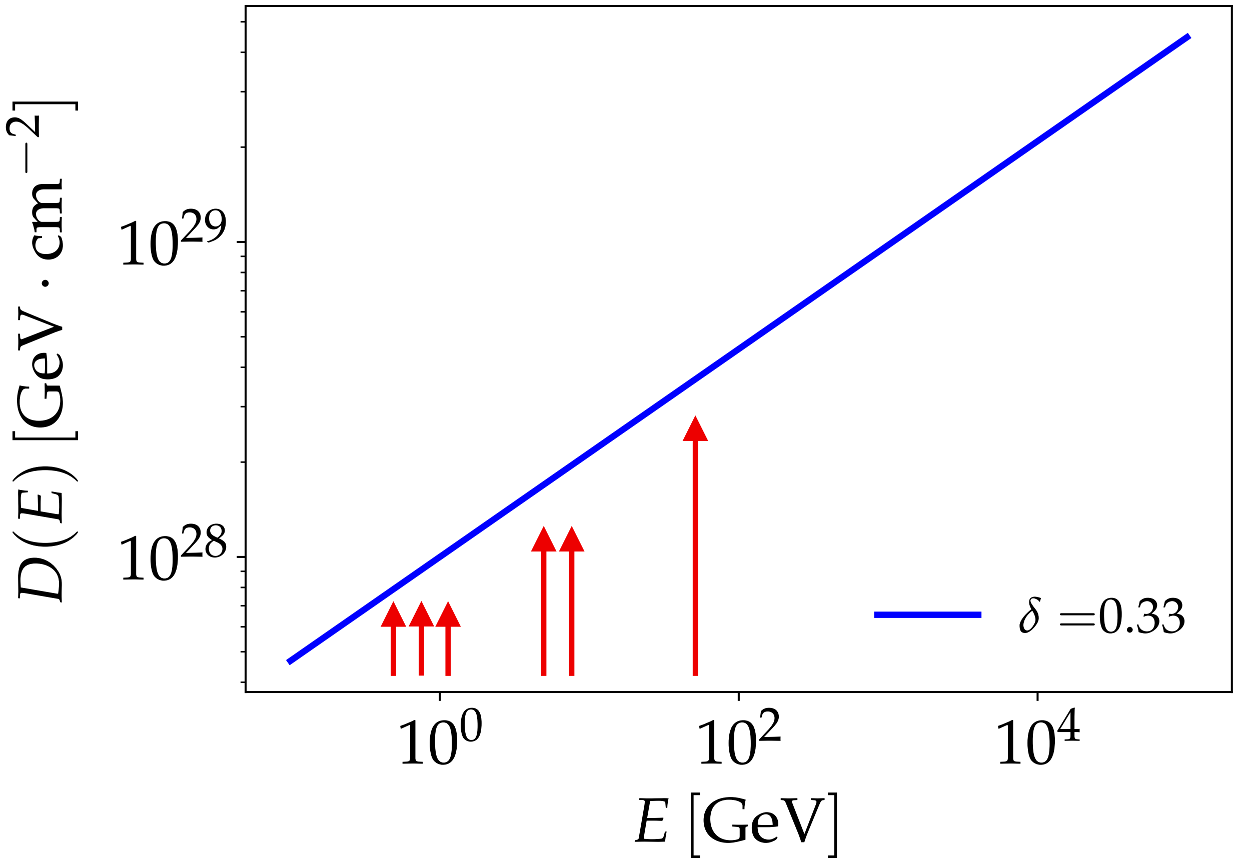

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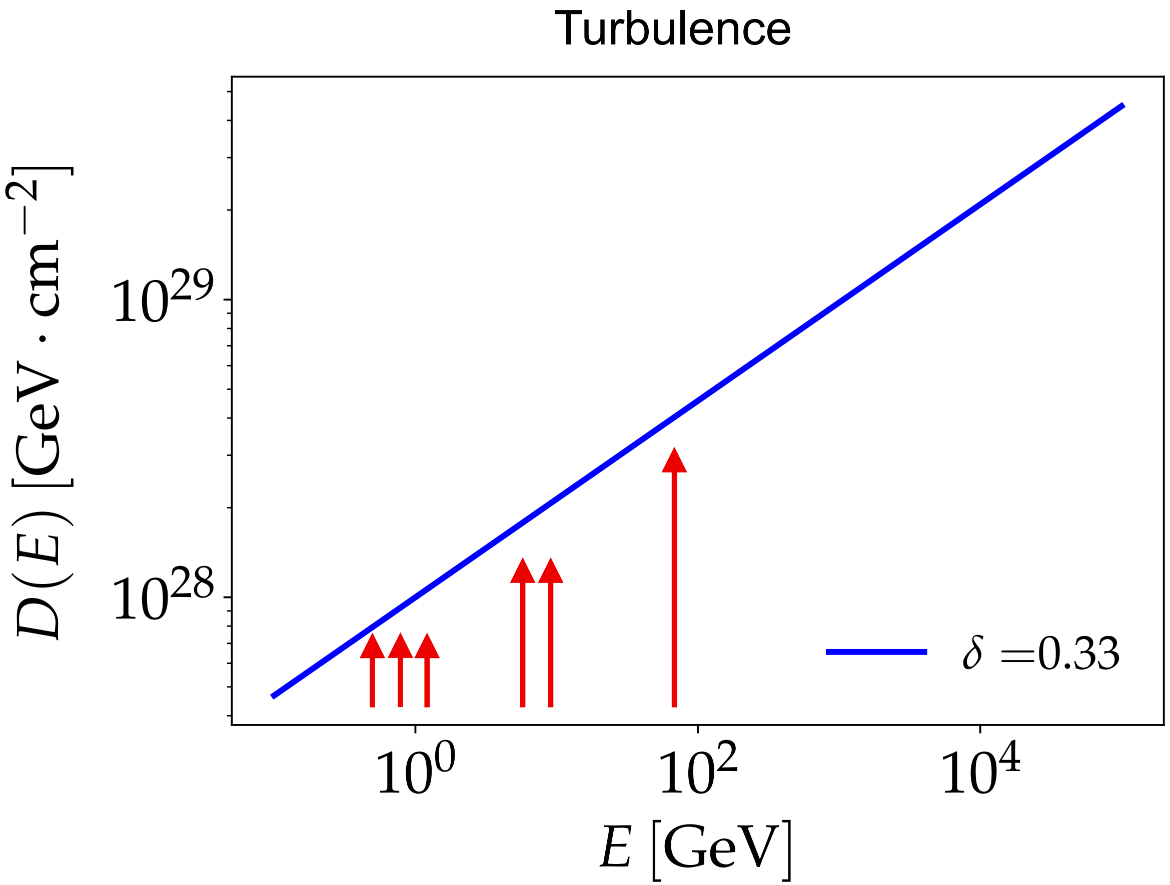
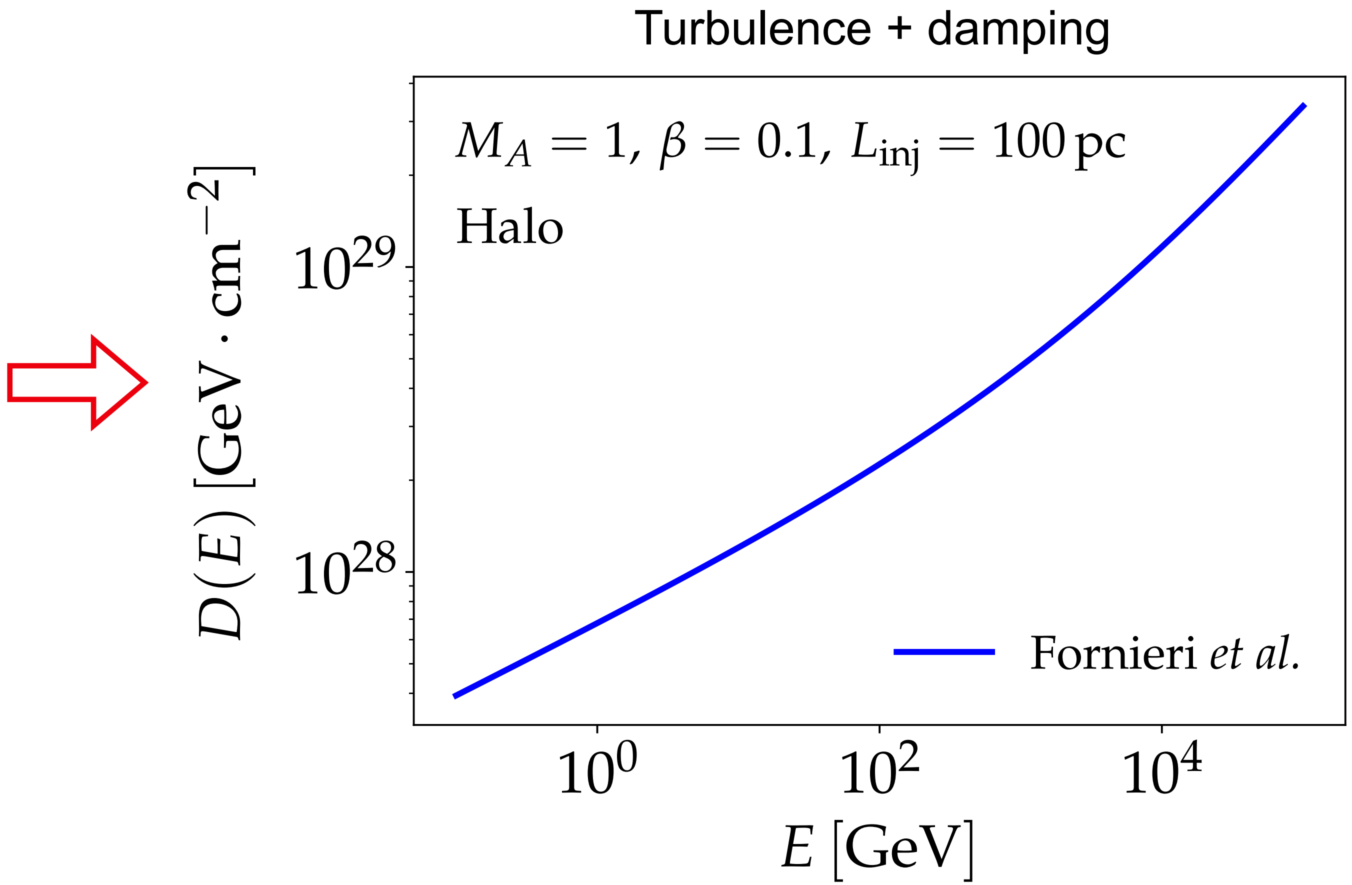
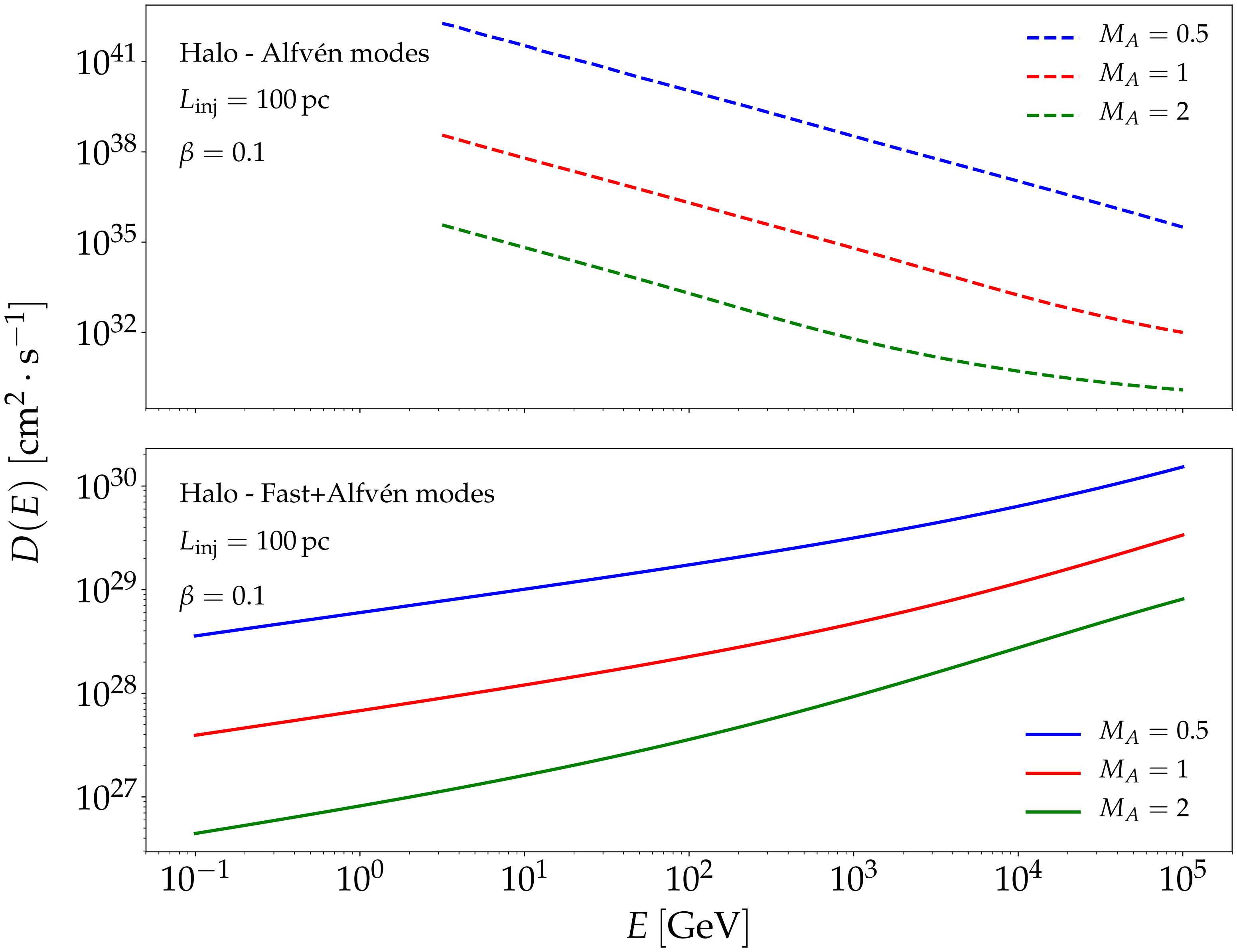


# Our result for $D(E)$



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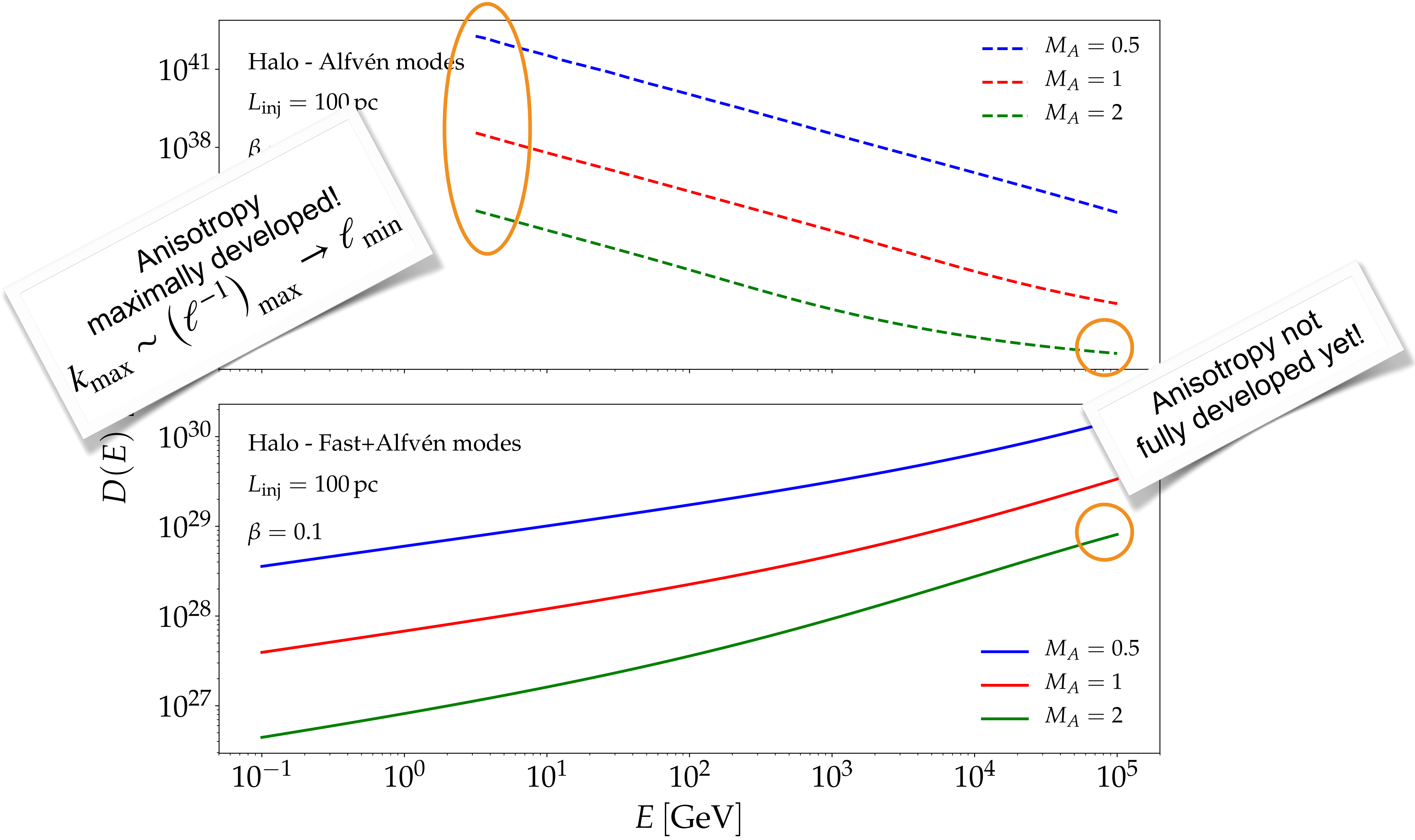
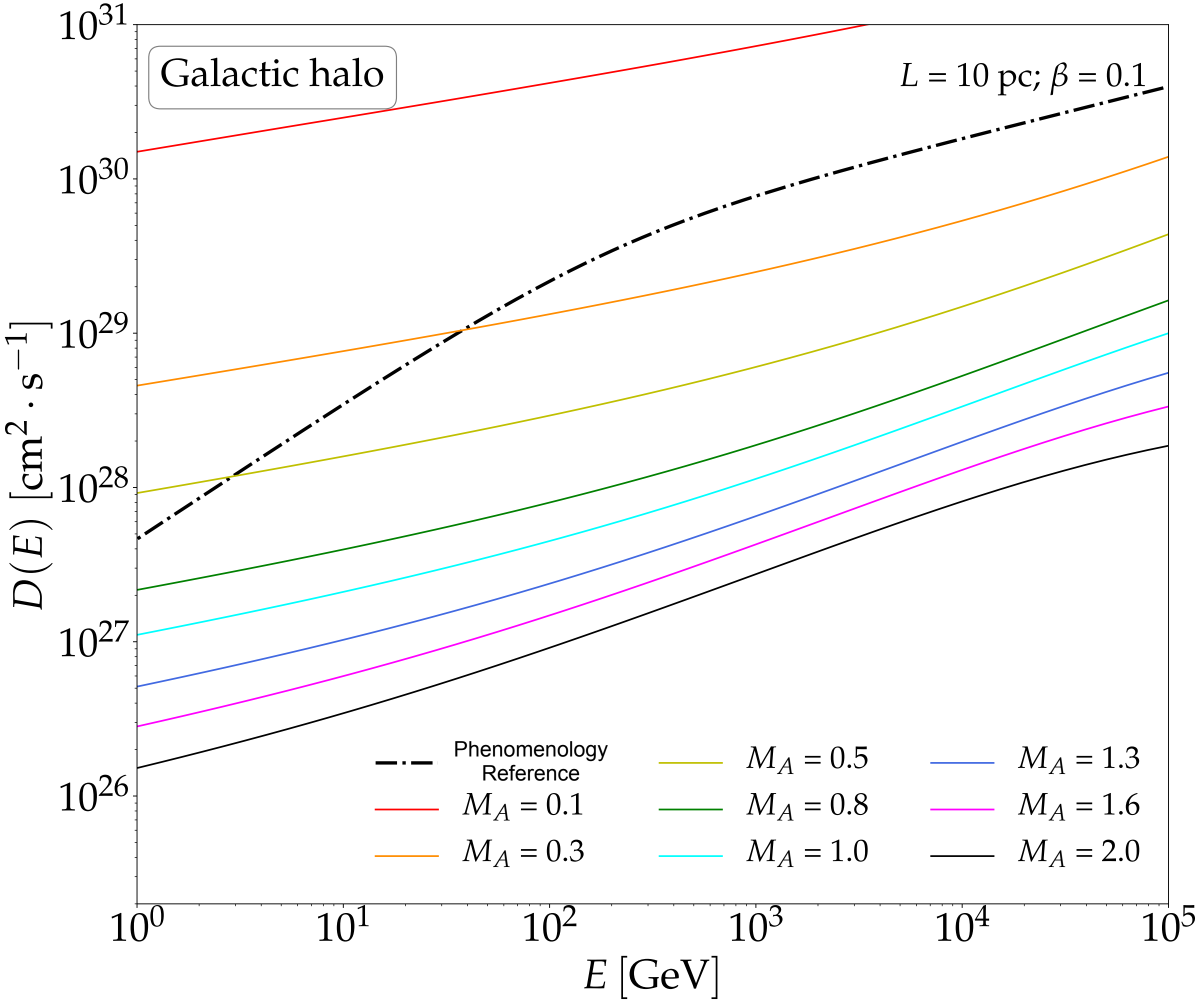
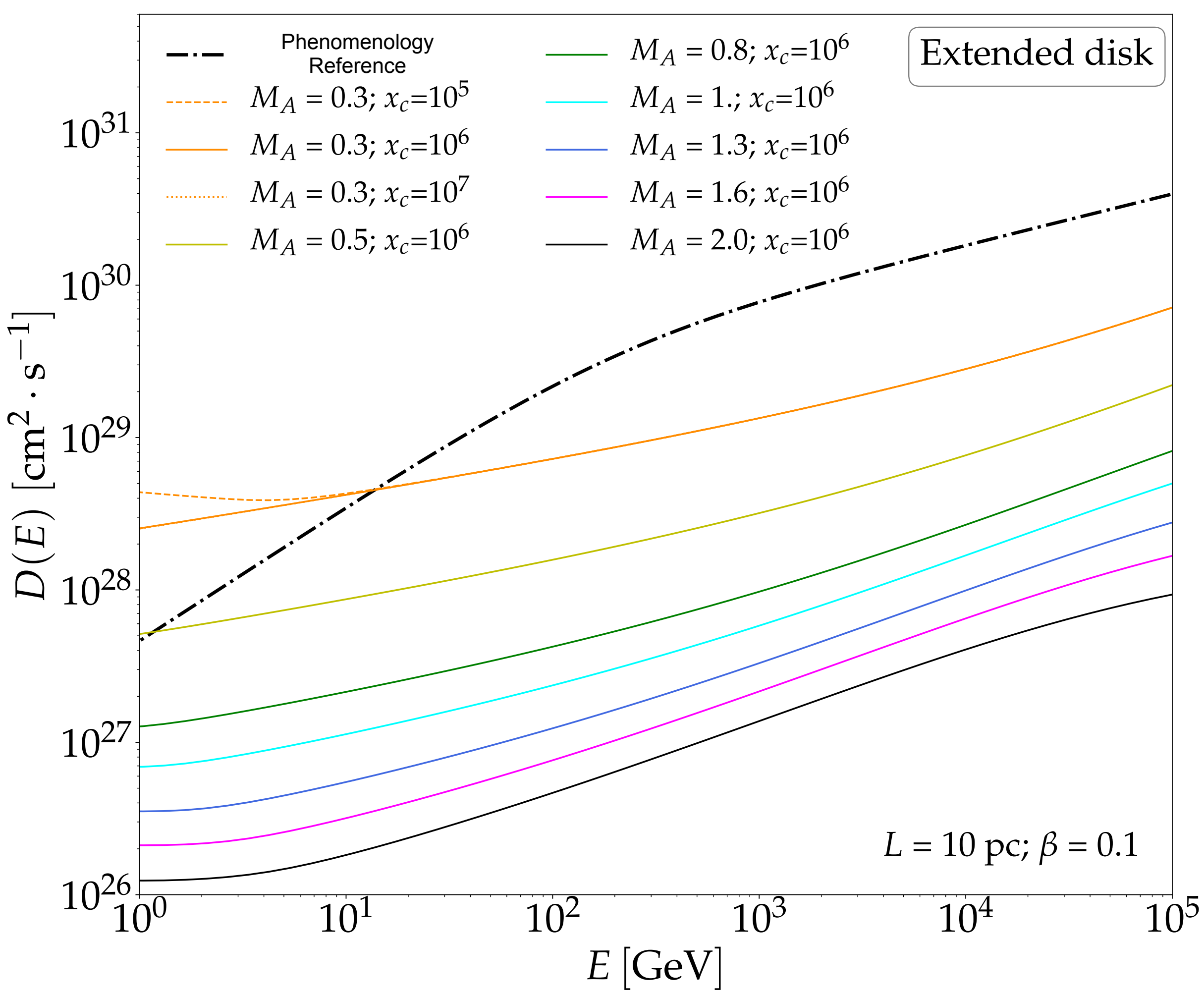


# Parametric study in the two-zone model

$D(E)$ **changing with the properties of the turbulence**



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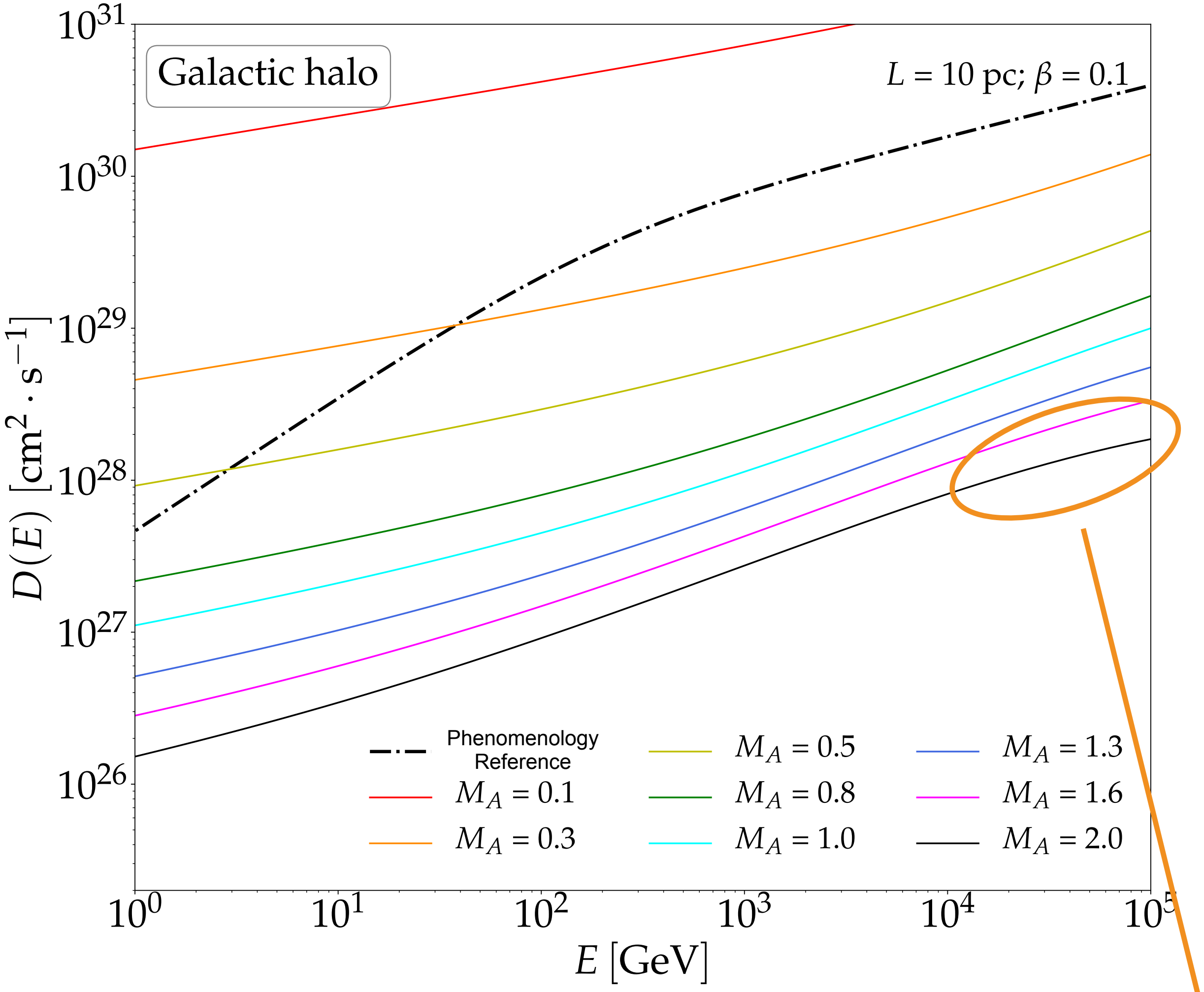
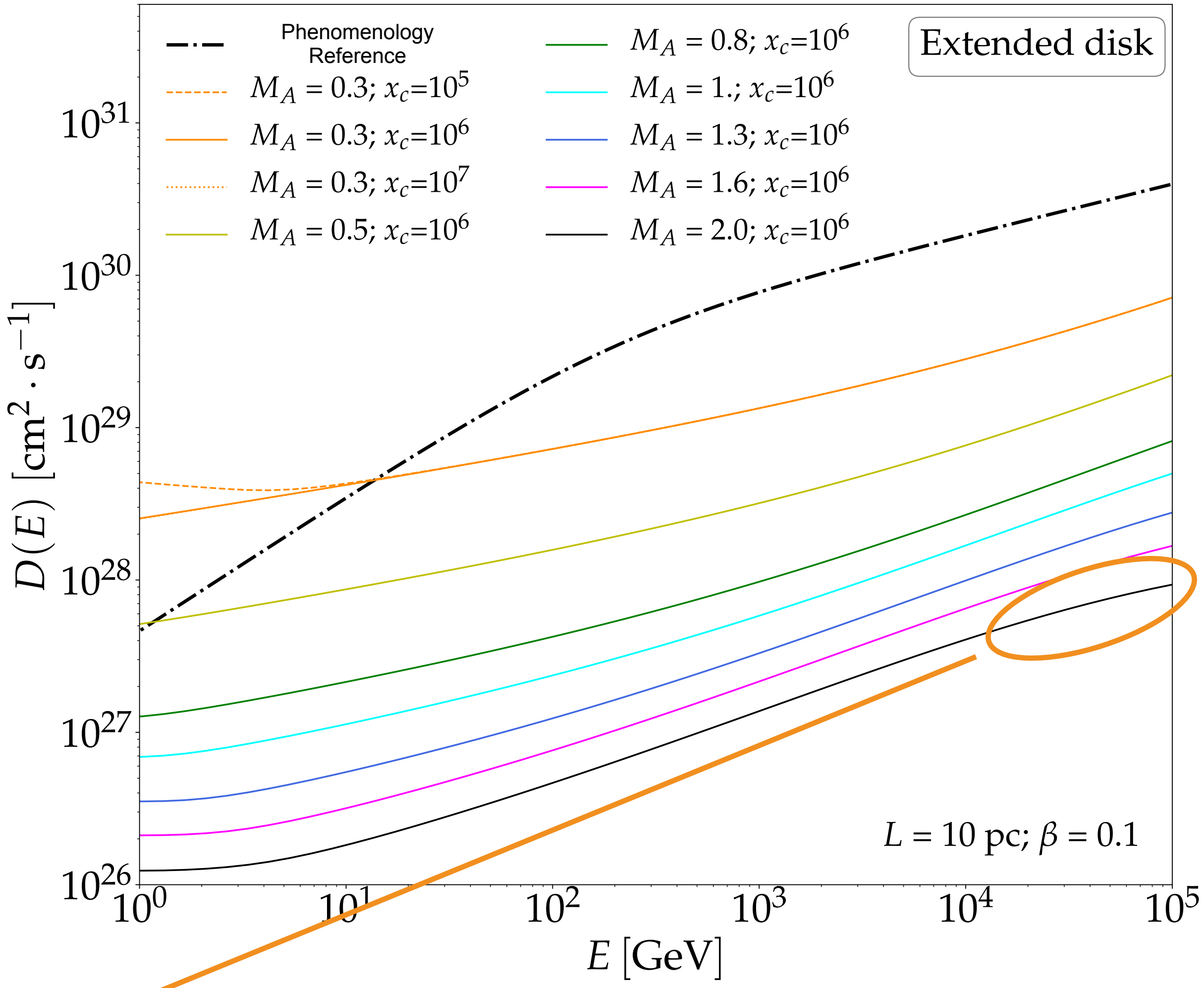
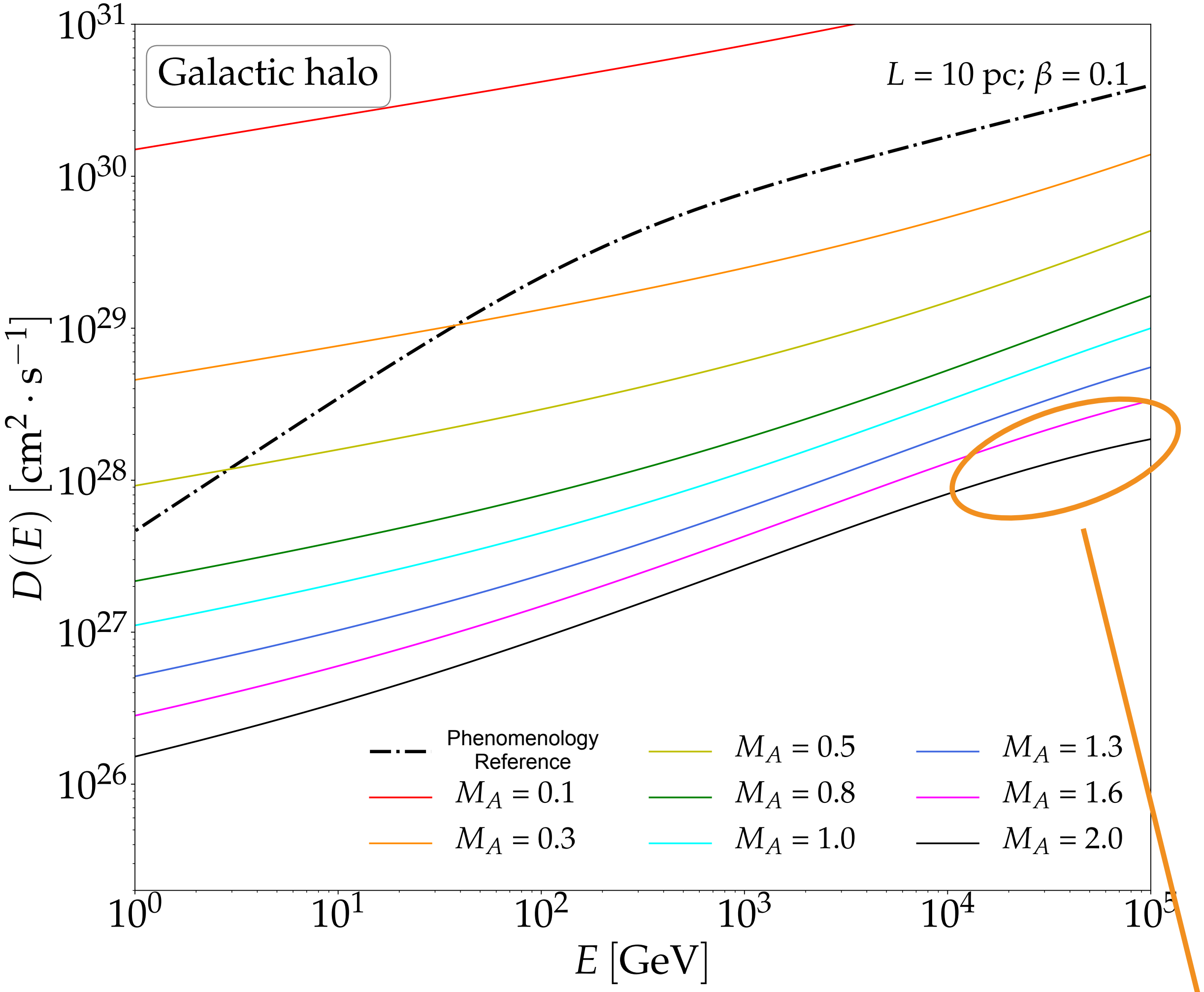


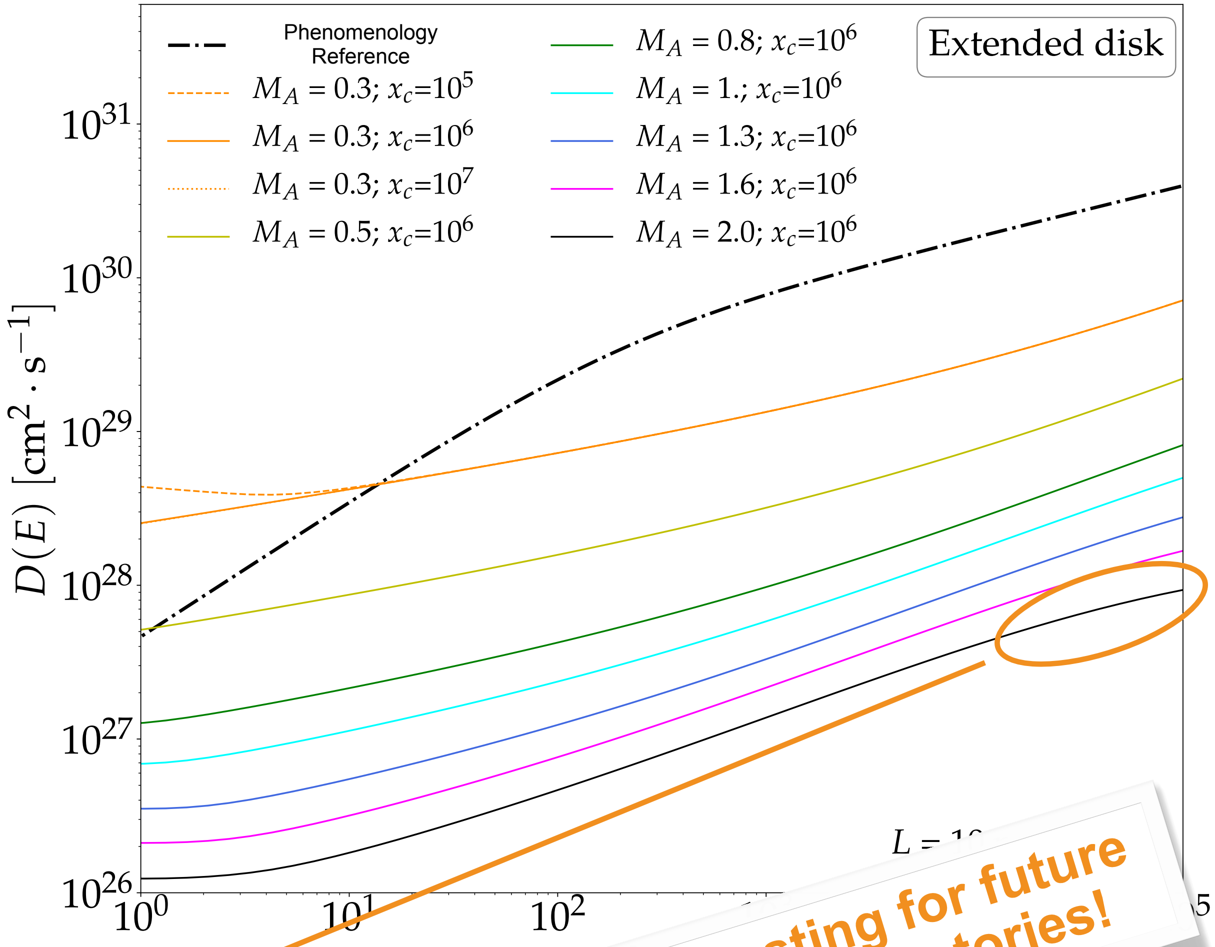




Contribution from Alfvén fluctuations

# Parametric study in the two-zone model

$D(E)$ **changing with the properties of the turbulence**




**Interesting for future CR observatories!**

Contribution from Alfvén fluctuations

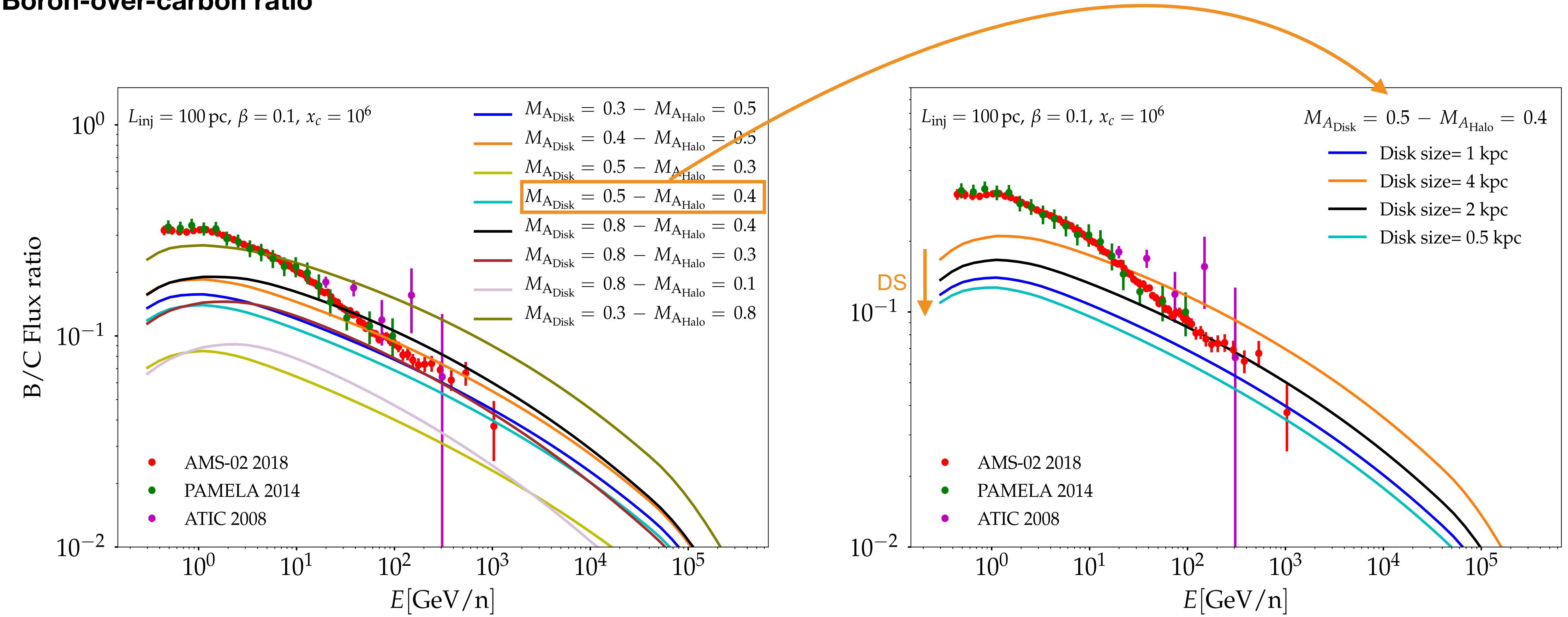
# CR scattering on MHD modes

[O. Fornieri *et al.* – MNRAS 502, 5821–5838 (2021)]

- Scattering rates of MHD modes
- Results on the observables

# Non-linear extension implemented in DRAGON2

**Boron-over-carbon ratio**




- Turbulence strength ($M_A$) & disk-size constrained by the production of boron

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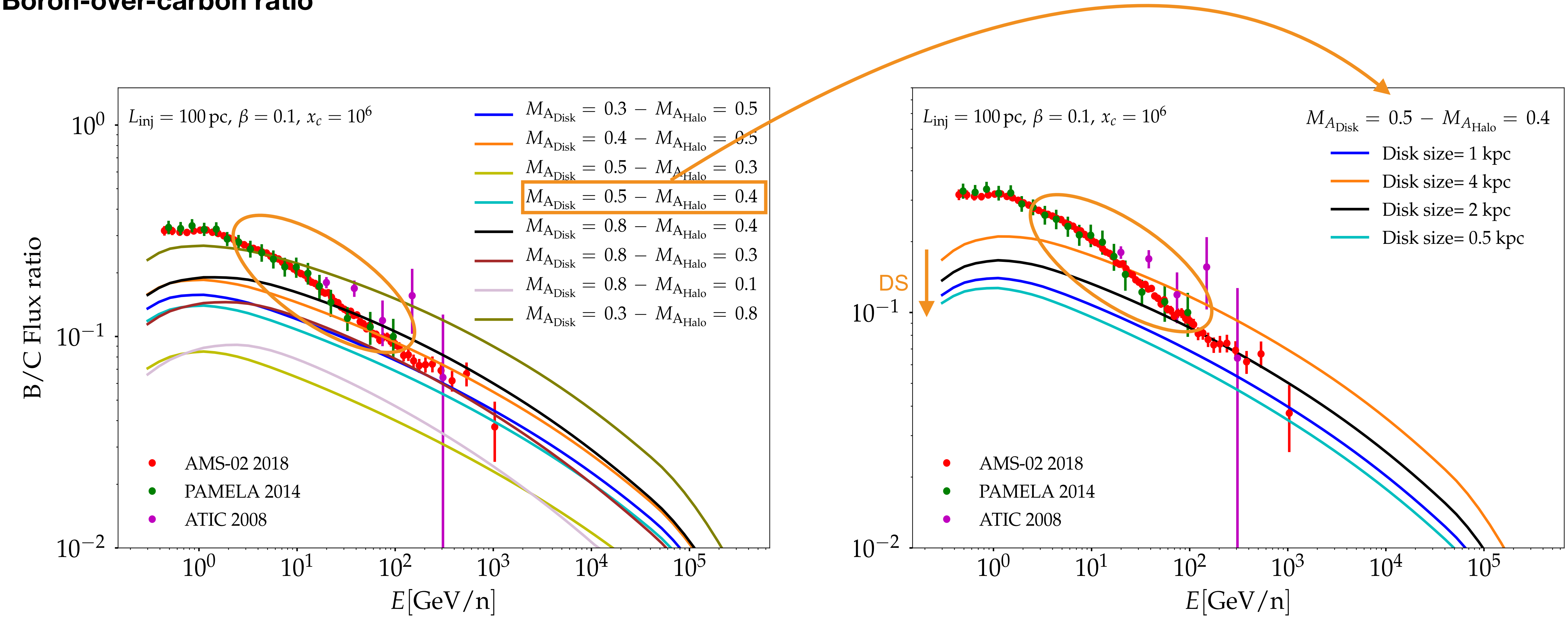
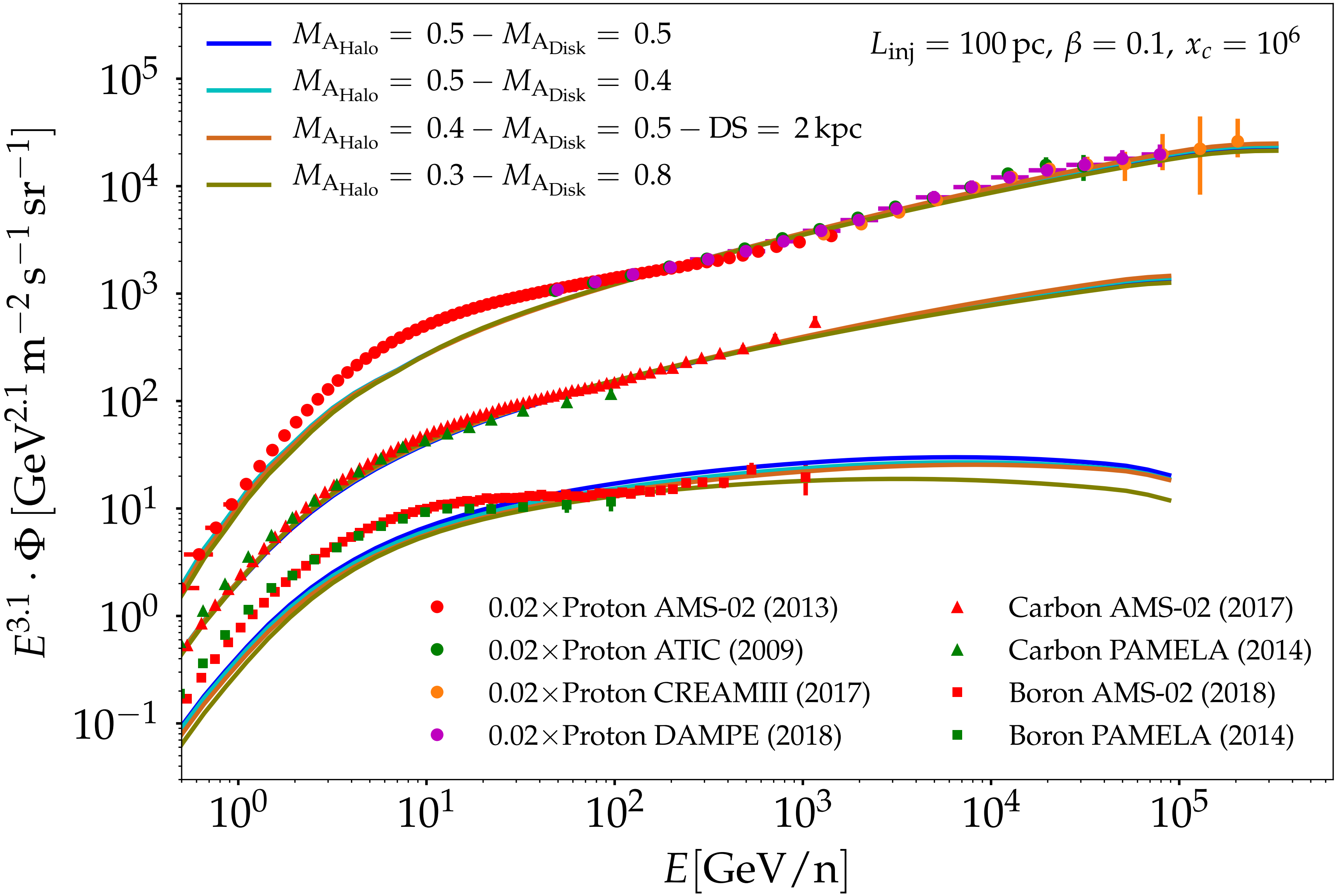



- Turbulence strength ($M_A$) & disk-size constrained by the production of boron
- High-energy slope and normalization reproduced **without *ad hoc* tuning**!
- **Iroshnikov-Kraichnan scaling of $B/C$ not reproduced** $\Rightarrow$ transition to another process!

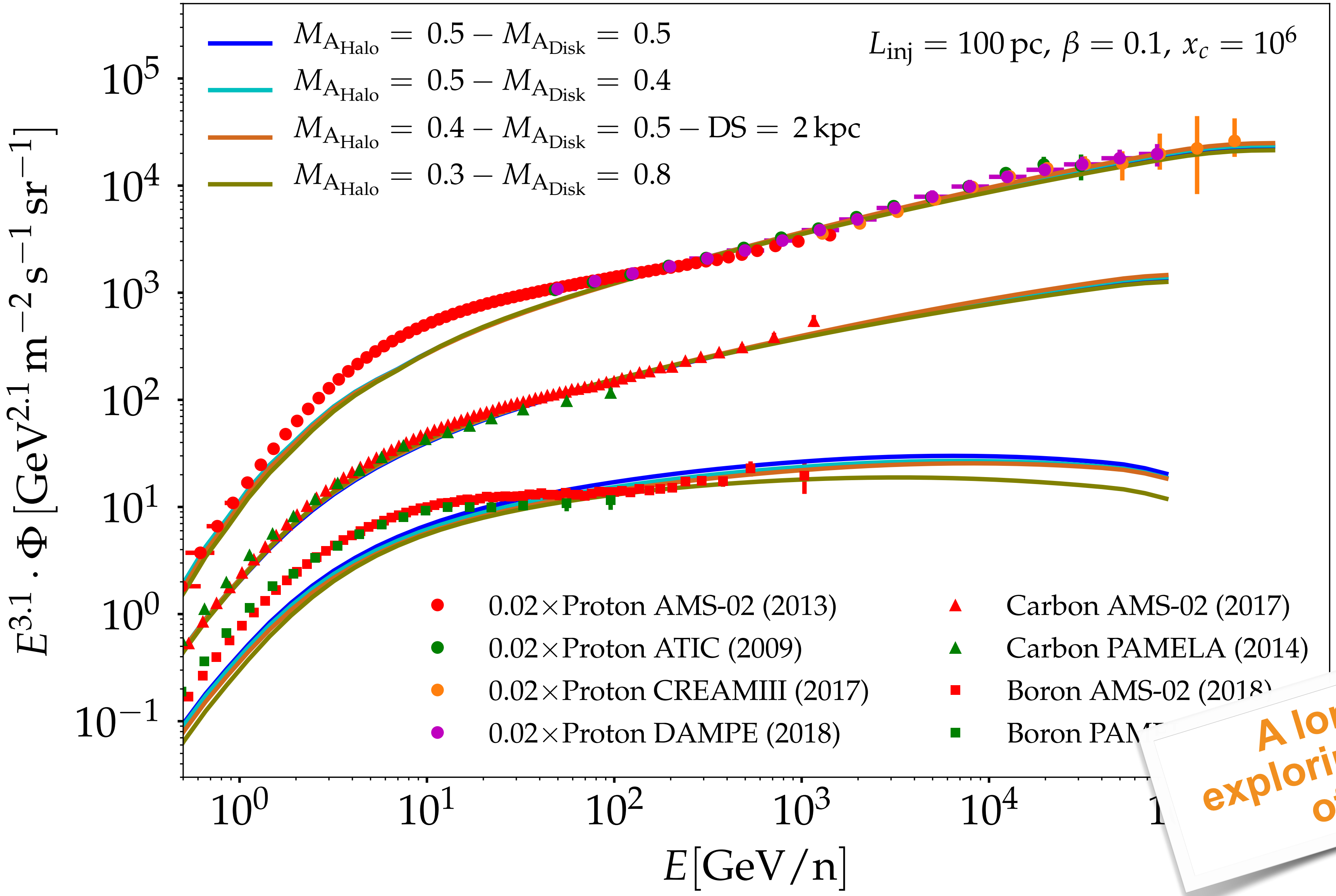
# Non-linear extension implemented in DRAGON2

**Hadronic species**



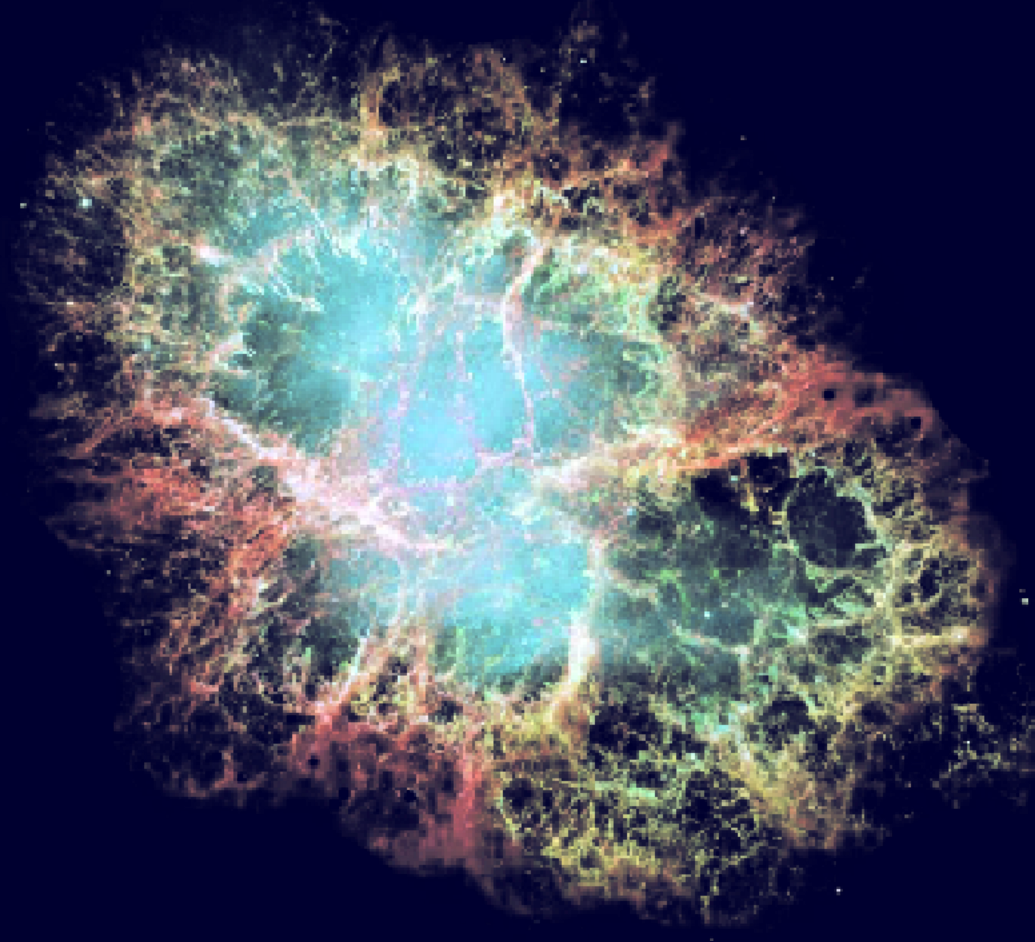
# Non-linear extension implemented in DRAGON2

**Hadronic species**




A long way to go exploring other aspects of the theory!

# Conclusions



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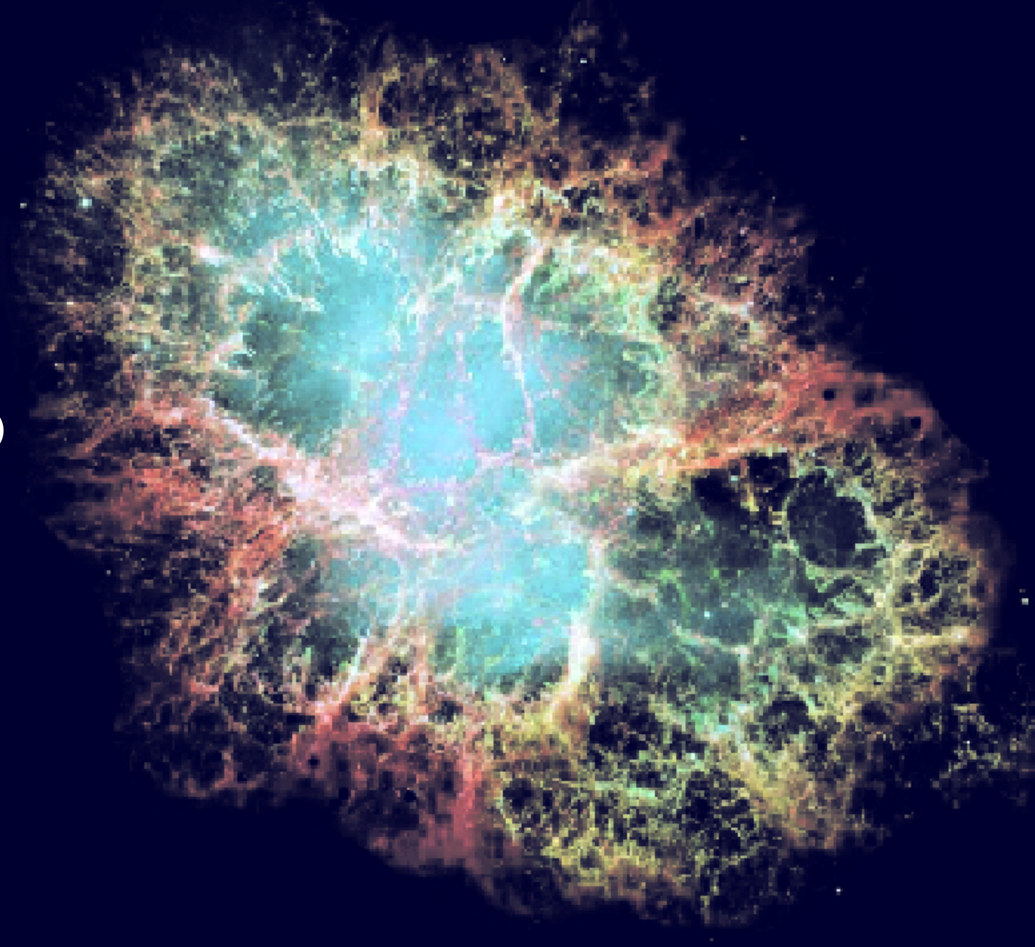
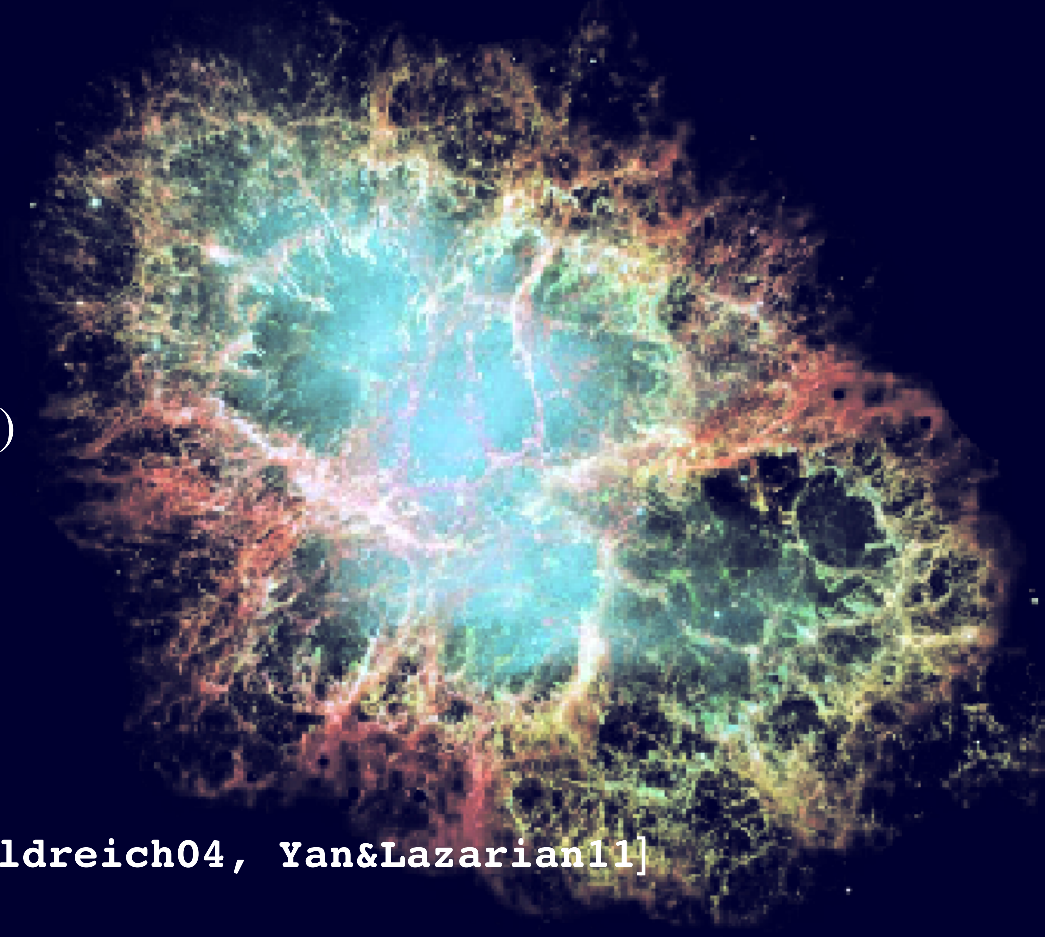


- **Alfvén modes are inefficient** plus they show the **opposite trend** for $D(E)$

# Conclusions



- **Alfvén modes are inefficient** plus they show the **opposite trend** for $D(E)$

- NLT developed in YL08 explains CR confinement **above** $\sim 200\,\mathrm{GeV}$

  - Below this energy, *streaming instabilities* may dominate [**Farmer&Goldreich04, Yan&Lazarian11**]

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- Characteristic features of such paradigm have to be investigated by the **future CR and $\gamma$-ray telescopes**.

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Thanks for your attention!

- Characteristic features of such paradigm have to be investigated by the **future CR and $\gamma$-ray telescopes**.

# Backup slides

# 3D correlation tensors from 1D scalings

$$\int E_{1D}(k)\,dk = \int E_{3D}(\mathbf{k})\,d^3\mathbf{k}$$

Iroshnikov-Kraichnan (isotropic) spectrum

$$E_{1D}^{IK}(k) \sim k^{-3/2}$$

$$\int E_{1D}(k)\,dk = \int E_{3D}(\mathbf{k})\,d^3\mathbf{k} = \int E_{3D}(k)\,4\pi k^2\,dk$$

$$\Rightarrow \quad E_{3D}^{IK}(k) \sim k^{-3/2-2} \sim k^{-7/2}$$

Goldreich-Sridhar (anisotropic) spectrum

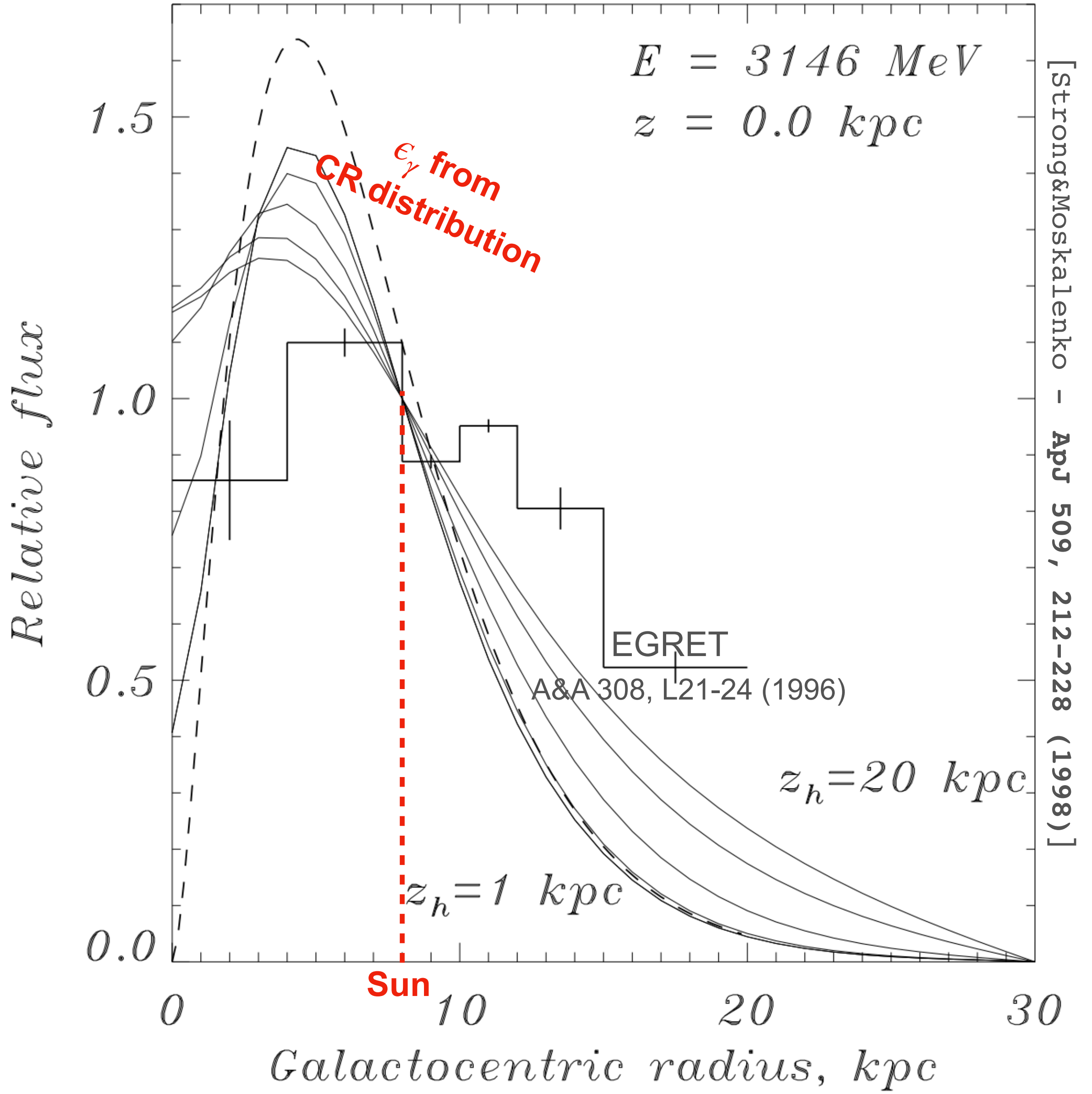
$$E_{1D}^{GS}(k) \sim k_\perp^{-5/3}$$

$$k_\parallel \sim k_\perp^{2/3}$$

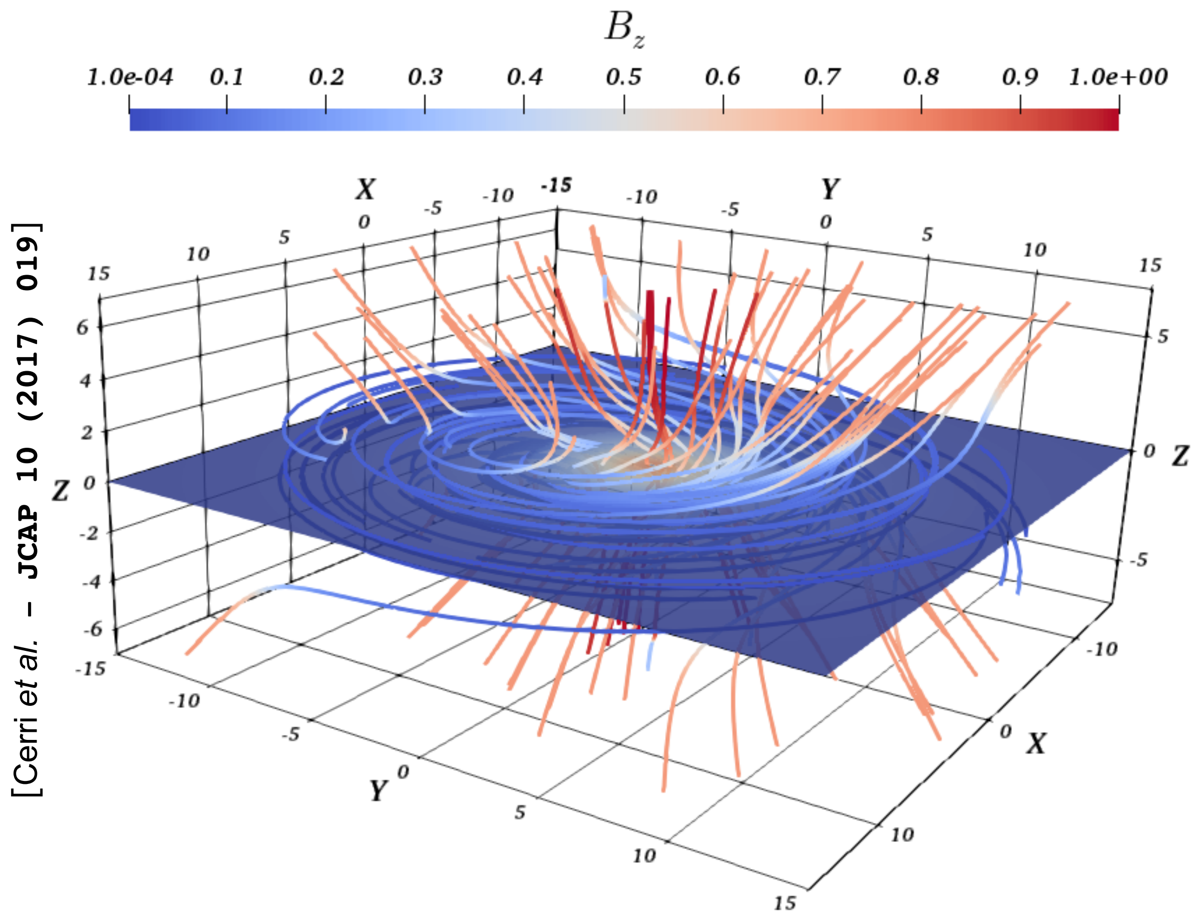
$$\begin{aligned} d^3\mathbf{k} &= dk_x \wedge dk_y \wedge dk_z = k_\perp dk_\perp \wedge dk_\parallel \wedge d\phi \\ &= k_\perp dk_\perp \wedge d(k_\perp^{2/3}) \wedge d\phi = k_\perp dk_\perp \wedge \left(\frac{d(k_\perp^{2/3})}{dk_\perp}\right) dk_\perp \wedge d\phi \\ &= \frac{2}{3} k_\perp dk_\perp \wedge k_\perp^{-1/3}(dk_\perp \hat{k}_\parallel) \wedge d\phi = \frac{2}{3} k_\perp^{2/3}\, k_\perp dk_\perp \wedge d\phi = \frac{2}{3} k_\perp^{5/3} dk_\perp \wedge d\phi \end{aligned}$$

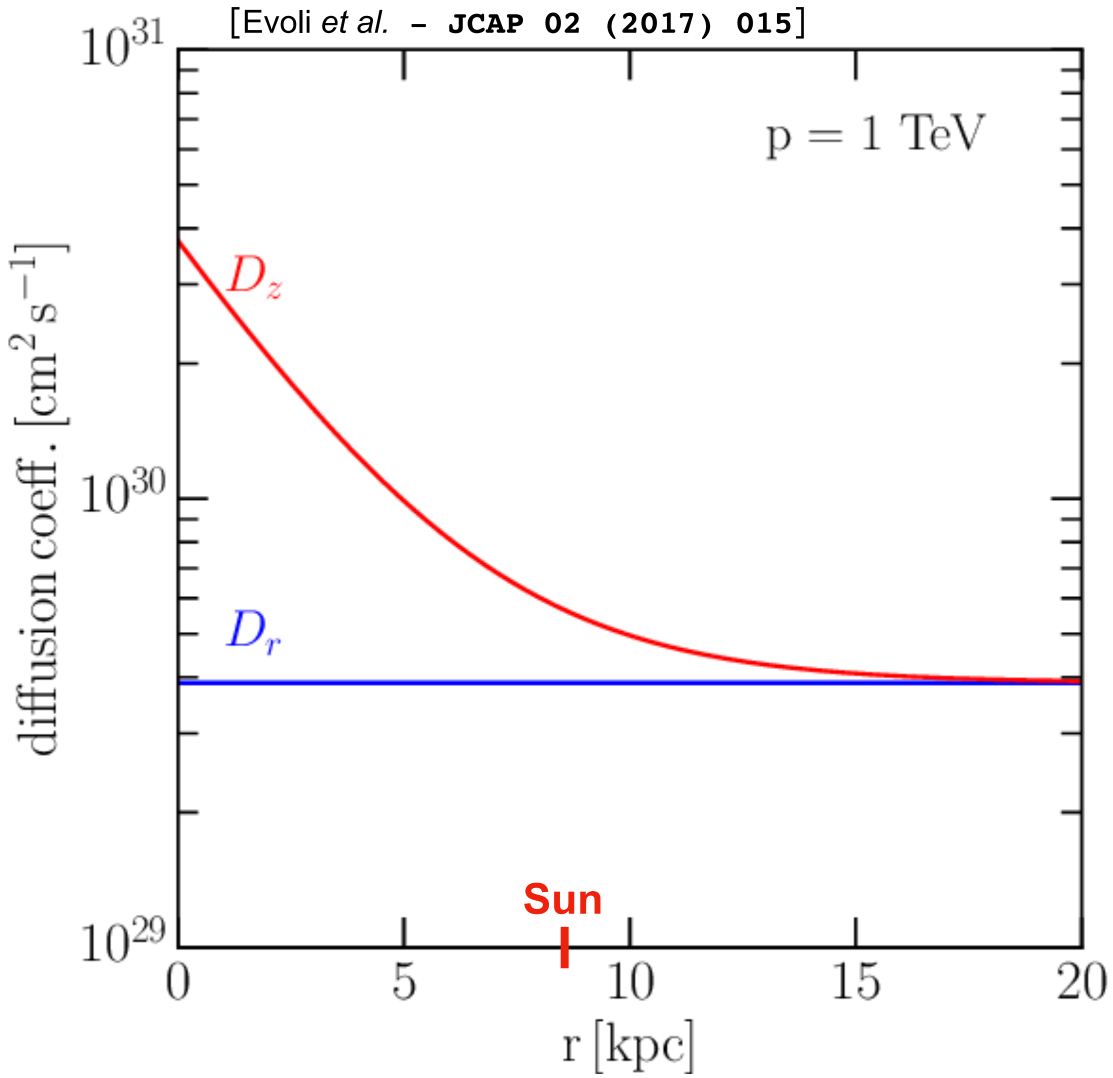
$$\Rightarrow \quad E_{3D}^{GS}(k) \sim k^{-5/3-5/3} \sim k^{-10/3}$$

# An enlightening issue?



# Connections with the propagation models

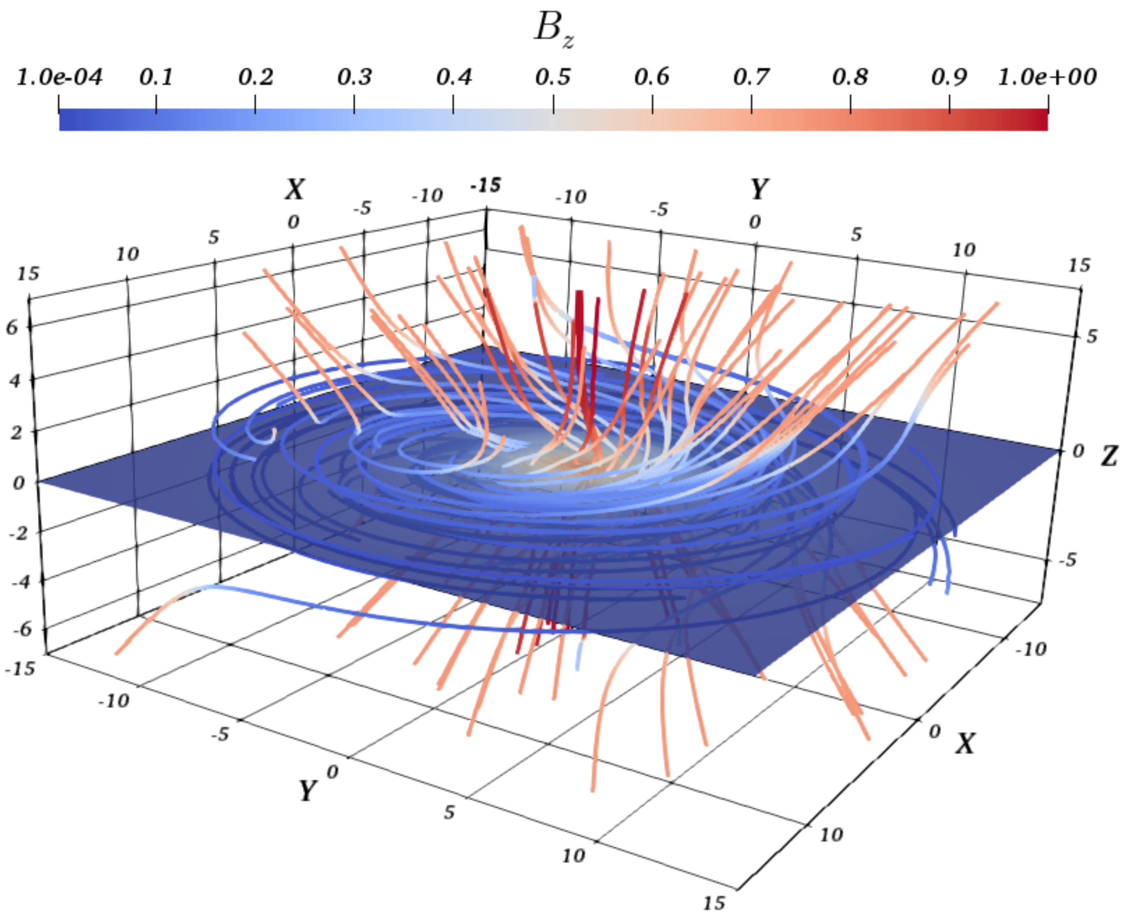


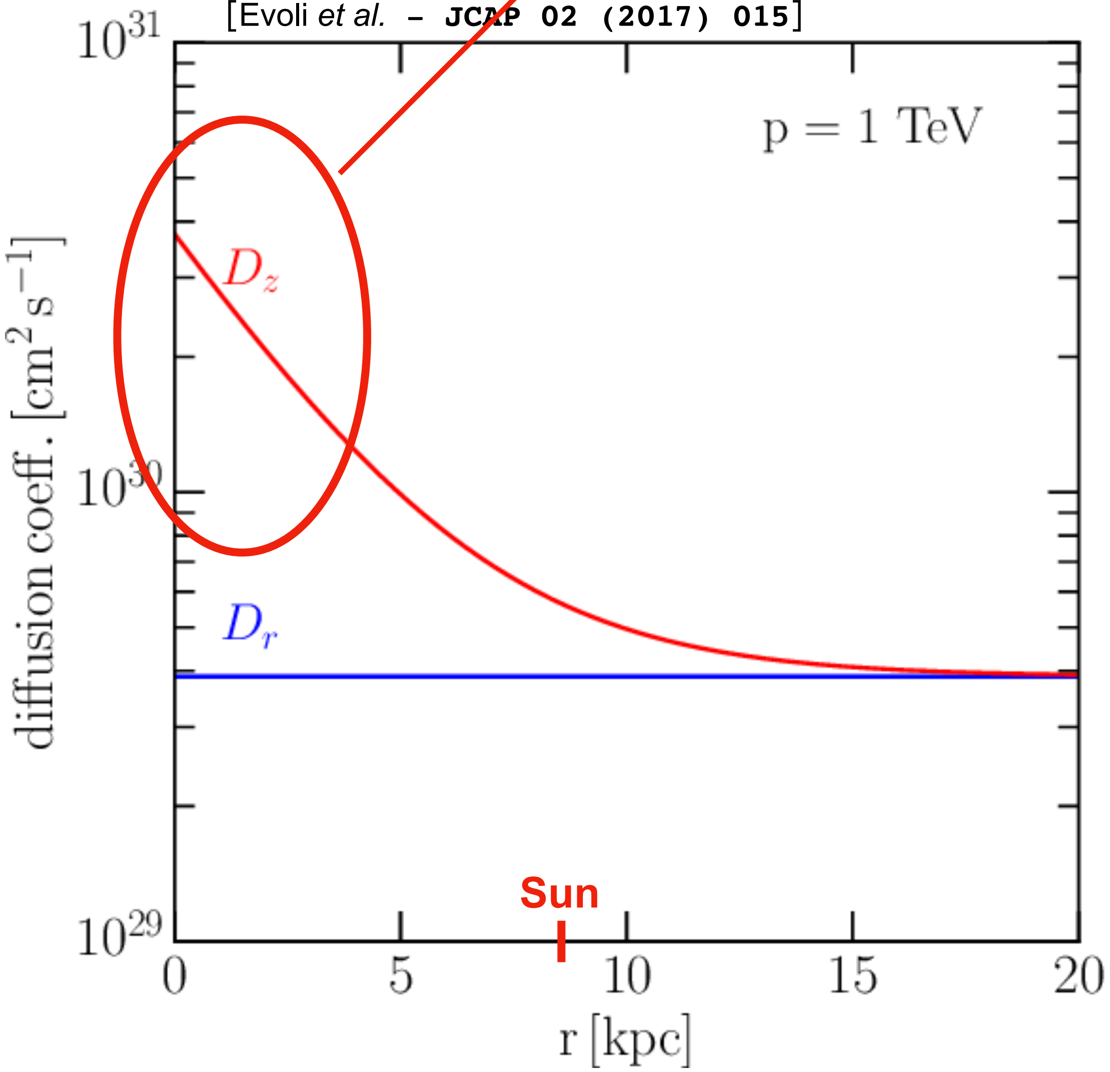
# Connections with the propagation models

Neglecting this implies longer residence time around GC

[Cerri *et al.* – JCAP 10 (2017) 019]




[Evoli *et al.* – JCAP 02 (2017) 015]



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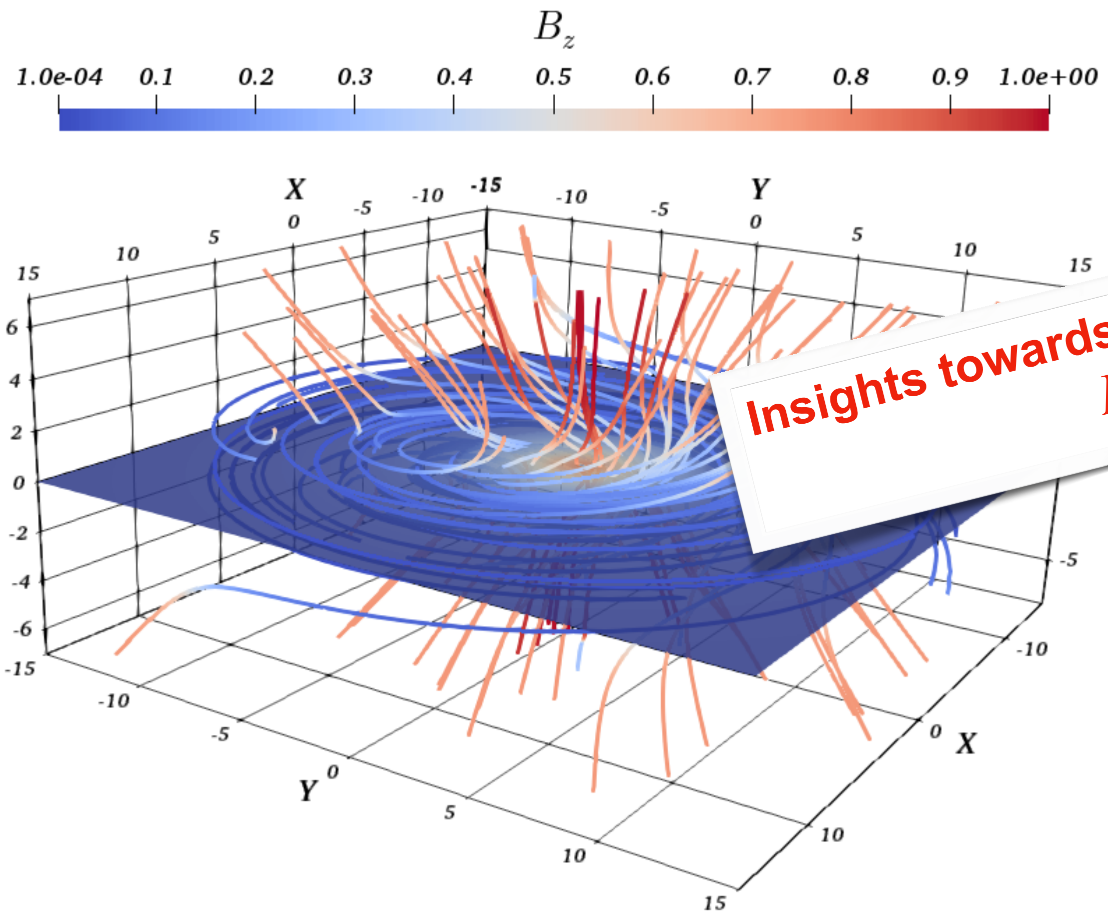


**Insights towards anisotropic transport!**
$D_{\parallel} \neq D_{\perp}$

[Evoli *et al.* – JCAP 02 (2017) 015]