Energy budget of cosmological first-order phase transition

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October 29th , 2021, TeV PA 2021@Chengdu

Based on PRD 103 (2021) 10, 103520

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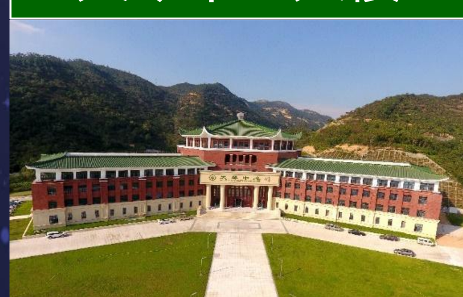

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Outline

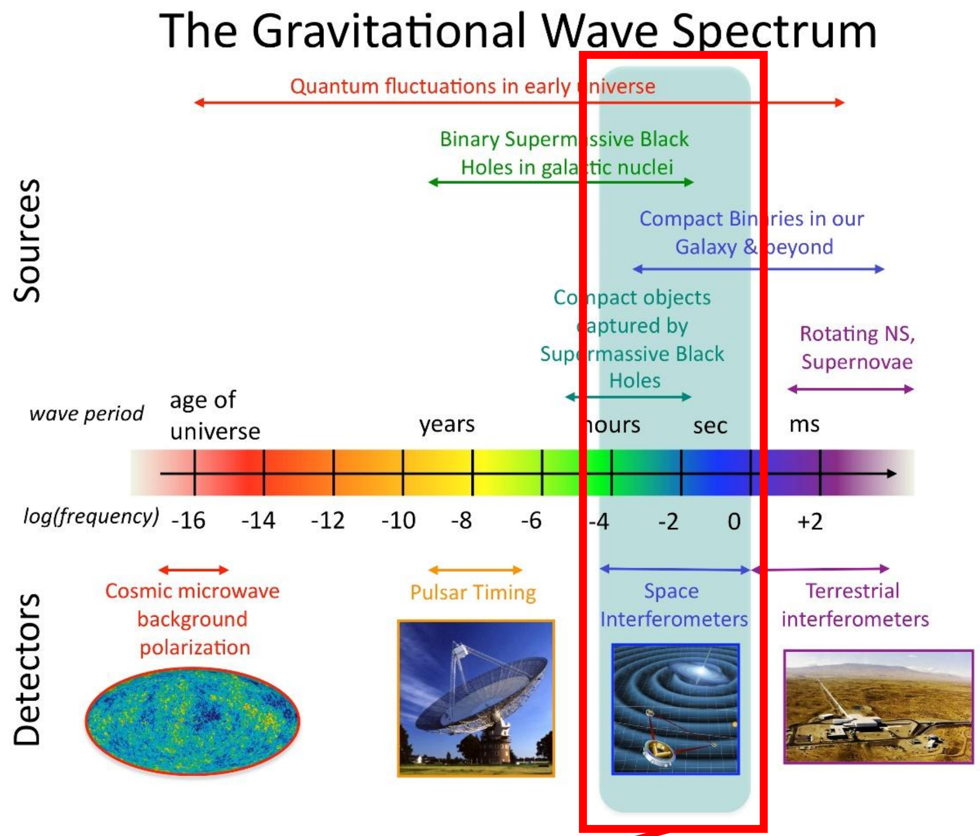
- Introduction
- Energy Budget
- Gravitational Wave
- Summary

Introduction

After the observation of the gravitational wave by LIGO, the GW detector provides a new technique to study the fundamental physics.

Sources of GWs:

- *Astrophysical origin*: black hole, neutron star, etc.
- *Cosmological origin*: inflation, **FOPT, etc.**



EW FOPT

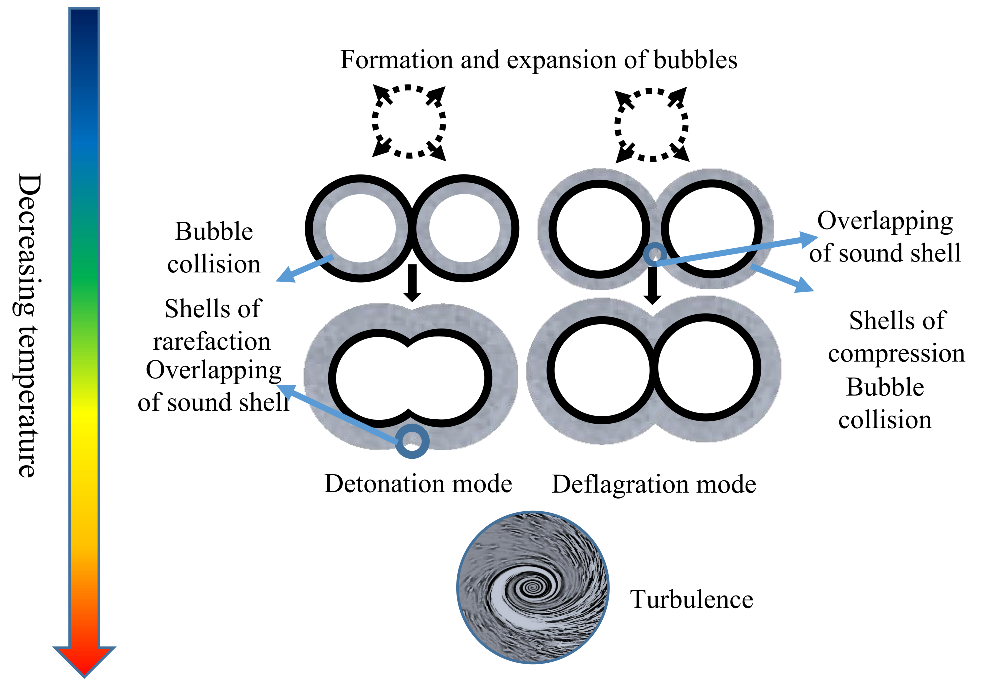
Introduction

GWs sources of FOPT:

- Bubble collisions
- Sound wave
- Turbulence

The amplitude and shape of GW spectrum are strongly related to PT dynamics.

JCAP 0809, 022 (2008); PRL112, 041301 (2014); PRD92, no. 12,123009 (2015); PRD96, no. 10,103520 (2017); Phys. Rev. D 66, 024030 (2002), Phys. Rev. D 76 (2007) 083002, JCAP 0912, 024 (2009)



$$h^2\Omega_{\rm co}(f) \simeq 1.67\times 10^{-5}\left(\frac{H_*R_*}{(8\pi)^{1/3}}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} \frac{0.11 v_w}{0.42+v_w^2} \frac{3.8(f/f_{\rm co})^{2.8}}{1+2.8(f/f_{\rm co})^{3.8}}$$

Bubble collision

$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times 10^{-6}(H_*\tau_{\rm sw})(H_*R_*)\left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{1/3} (f/f_{\rm sw})^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

Sound wave

$$h^2\Omega_{\rm turb}(f) \simeq 1.14\times 10^{-}H_*R_*\left(\frac{\kappa_{\rm turb} \alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*}\right)^{1/3} \frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}(1+8\pi f/H_*)}$$

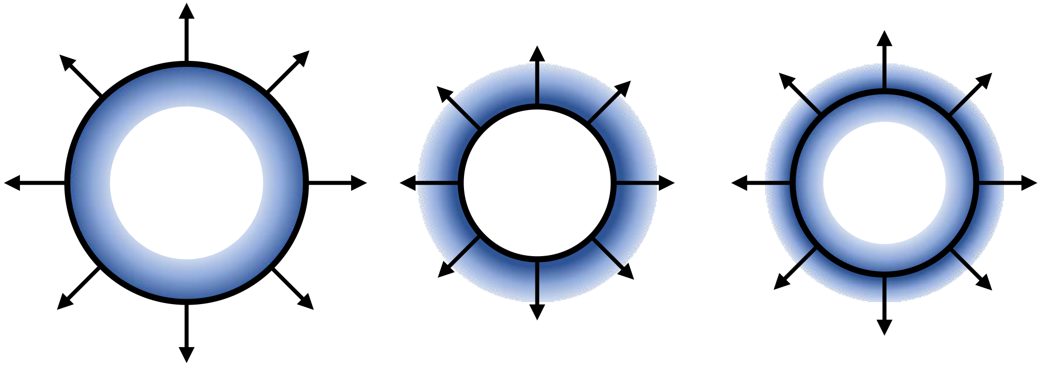
Turbulence

Energy Budget

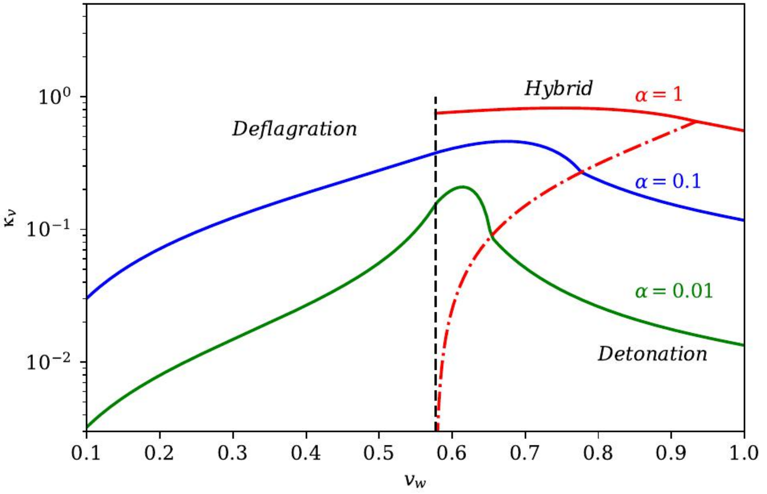
Energy budget (to measure the efficiency of the energy released by PT convert to the kinetic energy of sounding plasma)

Kinetic energy fraction: $K \equiv \frac{\rho_{\rm fl}}{e_+} = \frac{3}{e v_w^3} \int w(\xi)\, v^2 \gamma^2 \xi^2 d\xi \,, \quad \rho_{\rm fl} = \frac{3}{v_w^3} \int \xi^2 v^2 \gamma^2 w d\xi$

Efficiency parameter: $\kappa_v = \frac{3}{\epsilon v_w^3} \int w(\xi) v^2 \gamma^2 \xi^2 d\xi$



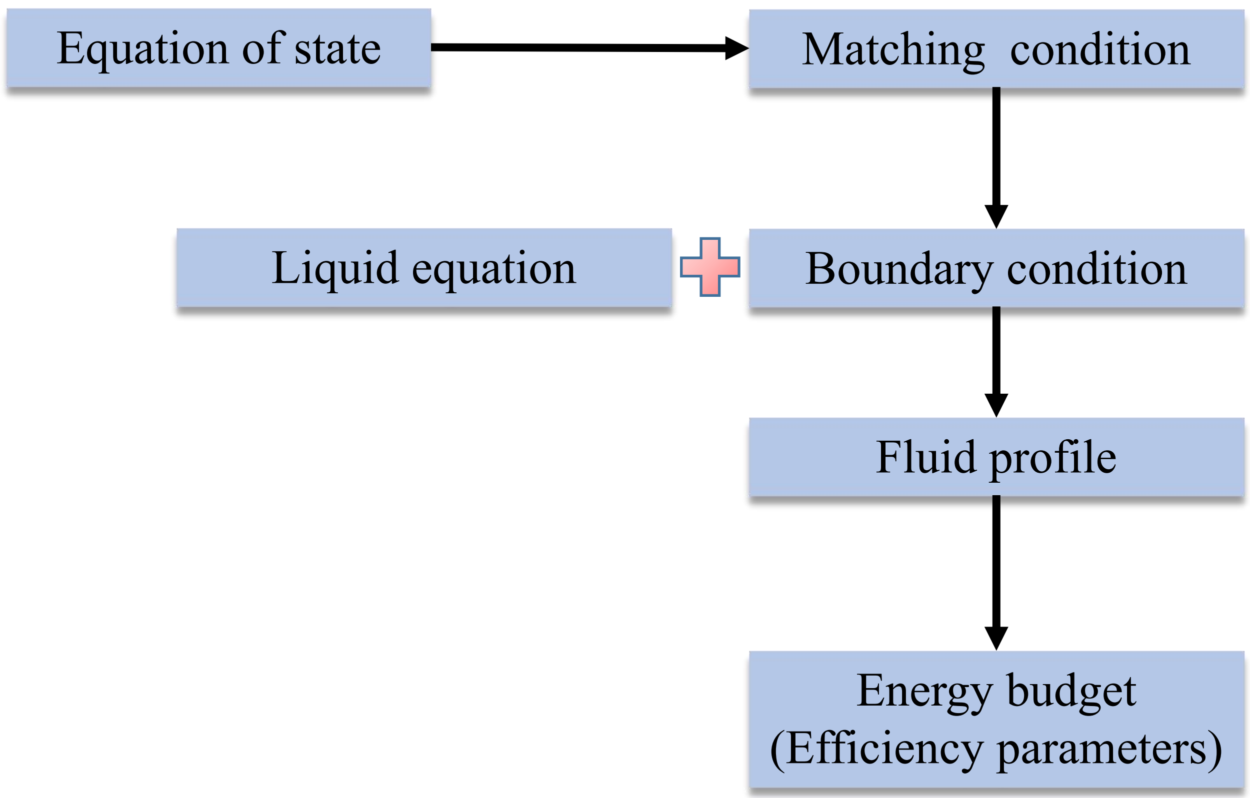
Energy budget is also strongly related to the bubble wall velocity



$h^2\Omega_{GW} \propto \kappa^2$ or $\kappa^{\frac{3}{2}}$

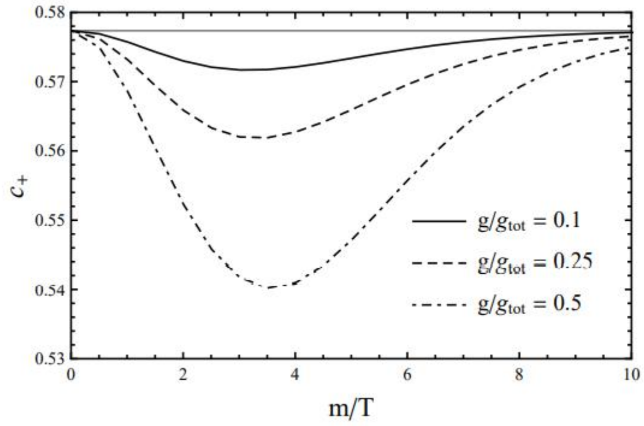
Energy Budget

A particle physics model independent analysis for energy budget:



Energy Budget

- Most of previous studies of the efficiency parameter are based on bag EoS, which assume the sound velocity is $1/\sqrt{3}$ in both phases. For a realistic FOPT, particle can obtain the mass in broken phase. Hence, the sound velocity can deviate from pure radiation phase.



L. Leitao and A. Megevand, Nucl. Phys. B **891**, 159-199 (2015)

Energy Budget

Bag EoS

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \;,$$

$$p_- = \frac{1}{3}a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \;.$$

$$\alpha_\theta = \frac{4}{3}\frac{\Delta\epsilon}{w_+}, \quad \epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$$

Strength parameter

$$\partial p/\partial e = c_s^2 = \text{constant}$$

EoS with different sound velocity (DSVM)

$$p_+ = c_+^2 a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \;,$$

$$p_- = c_-^2 a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \;,$$

$$\alpha_{\bar\theta} = \frac{\Delta\bar\theta}{3w_+}, \quad \bar\theta = e - p/c_-^2$$

Strength parameter

Energy Budget

- Energy momentum conservation derives fluid equations:

$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + \gamma^2(1 - v\xi)\partial_\xi v \ ,$$

$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v \ .$$

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v$$

Velocity profile

$$\frac{\partial_\xi w}{w} = \left(1 + \frac{1}{c_s^2}\right)\mu\gamma^2\partial_\xi v$$

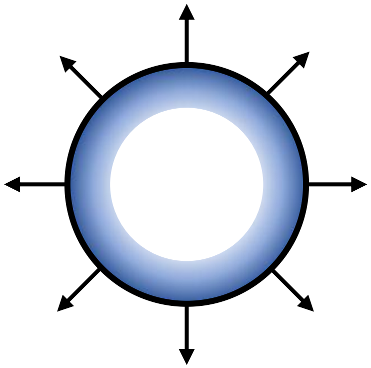
Enthalpy profile

$$\frac{\partial_\xi T}{T} = \gamma^2\mu\partial_\xi v$$

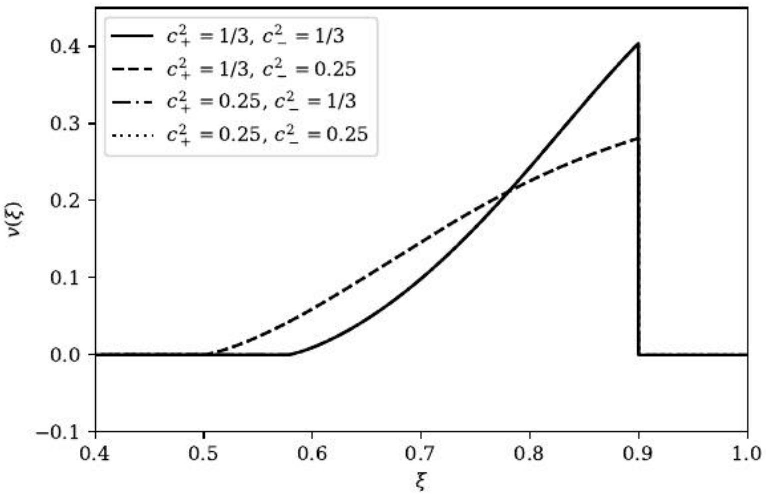
Temperature profile

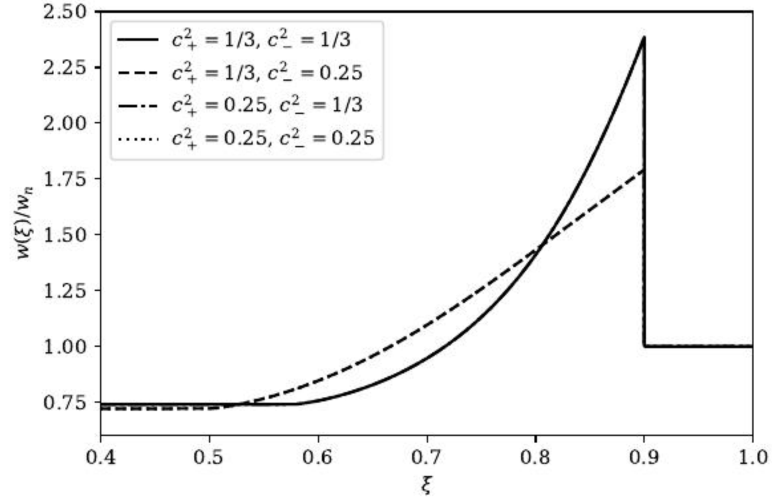
Different boundary conditions give different hydrodynamical modes.

✓ Boundary conditions of Detonation and the relevant velocity and enthalpy profiles:



$$\tilde{v}_+ = 0, \quad v_+ = v_w, \quad v_- = v_-(\alpha_{\bar{\theta}+}, v_+), \quad v(v_w) = \tilde{v}_- = \mu(v_w, v_-)$$





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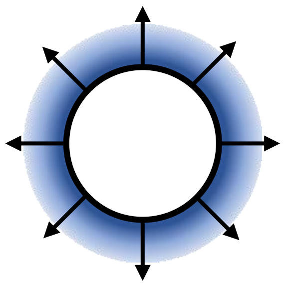
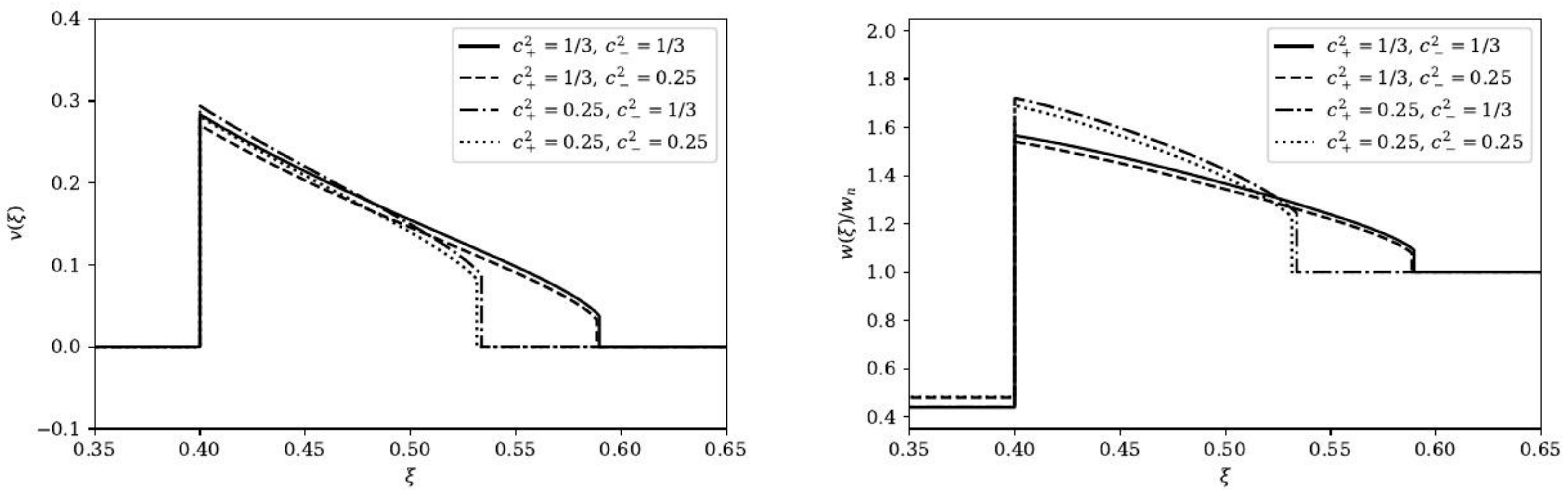
Energy Budget

✓ Boundary conditions of Deflagration and the relevant velocity and enthalpy profiles:



$$\tilde{v}_- = 0, \quad v_- = v_w, \quad v_+ = v_+(\alpha_{\bar{\theta}+}, v_-), \quad \tilde{v}_+ = \mu(v_w, v_+)$$



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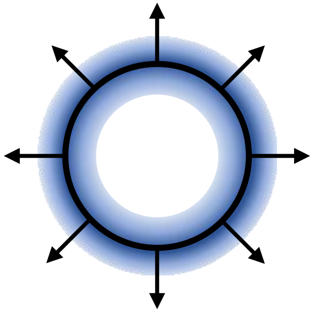
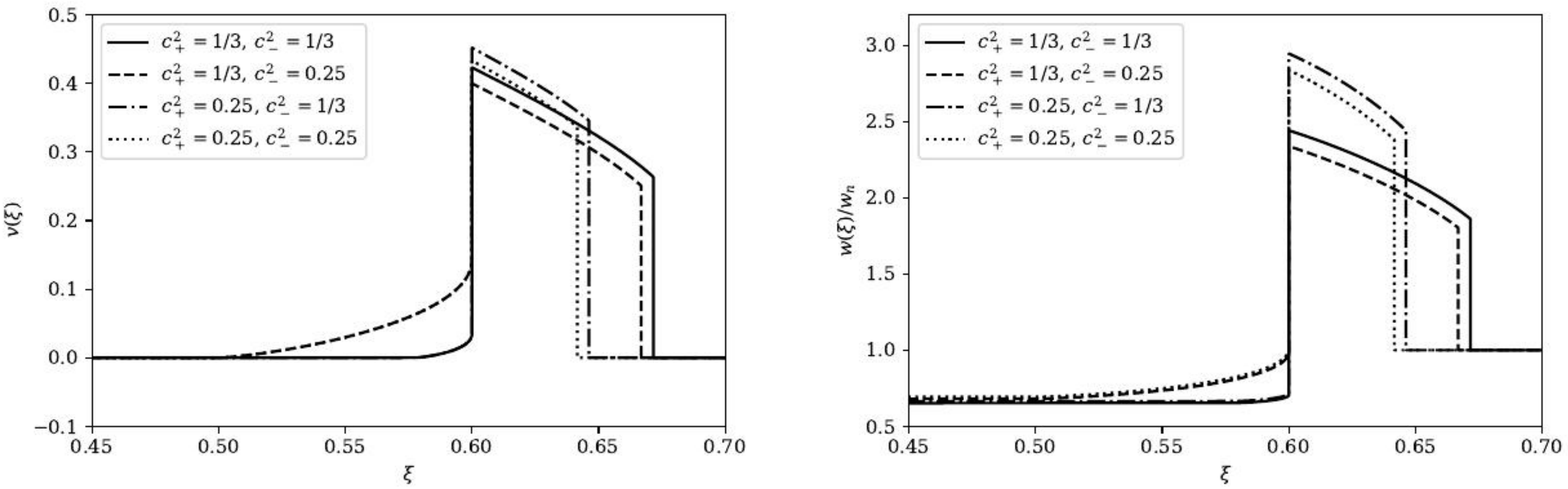
Energy Budget

✓ Boundary conditions of Hybrid and the relevant velocity and enthalpy profiles:



$$v_- = c_-, \quad \tilde{v}_- = \mu(v_w, v_-), \quad v_+ = v_J^{\rm def}(\alpha_{\bar{\theta}+}), \quad \tilde{v}_+ = \mu(v_w, v_+)$$



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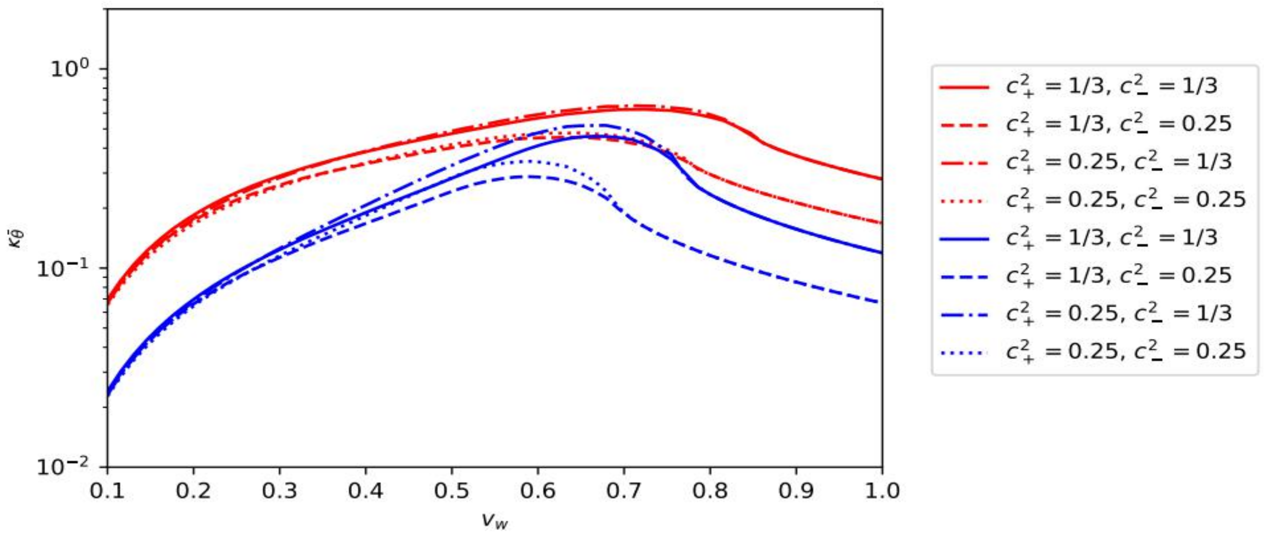
Energy Budget

The efficiency parameters of DSVM：

$$\rho_{\rm fl} = \frac{3}{v_w^3}\int \xi^2 v^2 \gamma^2 w d\xi \quad \kappa_{\bar{\theta}} = \frac{4\rho_{\rm fl}}{\Delta\bar{\theta}}$$



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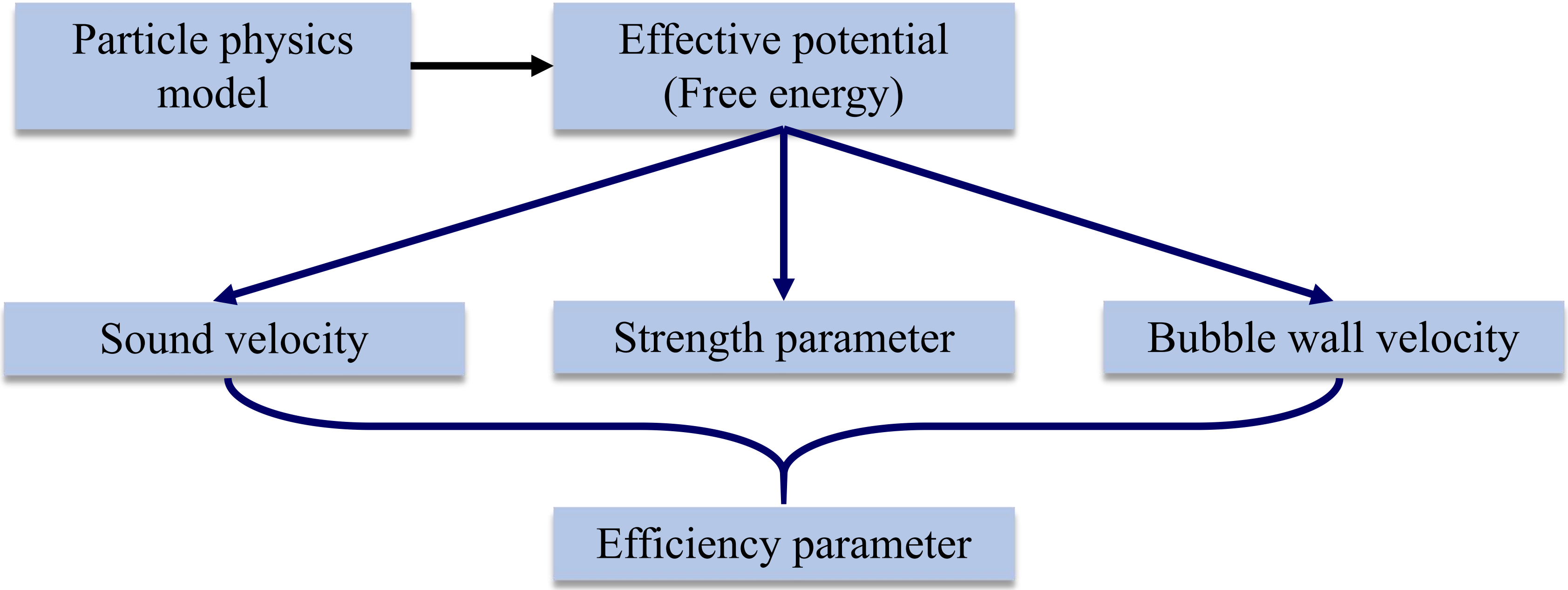
Gravitational Wave

- Mapping a particle physics model on the DSVM to get efficiency parameter:



Gravitational Wave

- The sound velocity of broken phase and symmetric phase in Dim-6 effective model:

$$\mathcal{F}(\phi,T) \approx -\frac{a_\pm}{3}T^4 + \frac{\mu^2 + cT^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\kappa}{8\Lambda^2}\phi^6$$

Sound velocity:

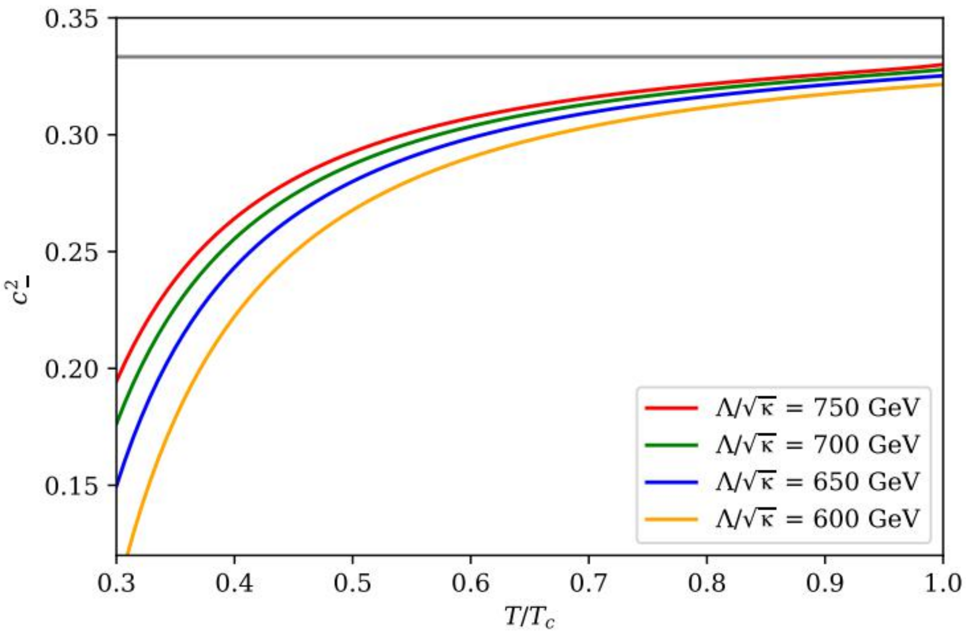
$$c_s^2 = \frac{4a_\pm T^3 - 3cT\phi^2}{12a_\pm T^3 - 3cT\phi^2}$$

$$c_+^2 = \frac{4a_+T^3}{12a_+^2T^3} = \frac{1}{3}$$

$$c_-^2 = \frac{4a_-T^3 - 3cT\phi_{\rm true}^2}{12a_-T^3 - 3cT\phi_{\rm true}^2}$$

Strength parameter:

$$\alpha_{\bar{\theta}n} = \frac{(1+1/c_-^2)\Delta V_{\rm eff} - T\frac{\partial \Delta V_{\rm eff}}{\partial T}}{3(1+c_+^2)\rho_{\rm R}}, \quad \rho_{\rm R} = a_+T_+^4$$



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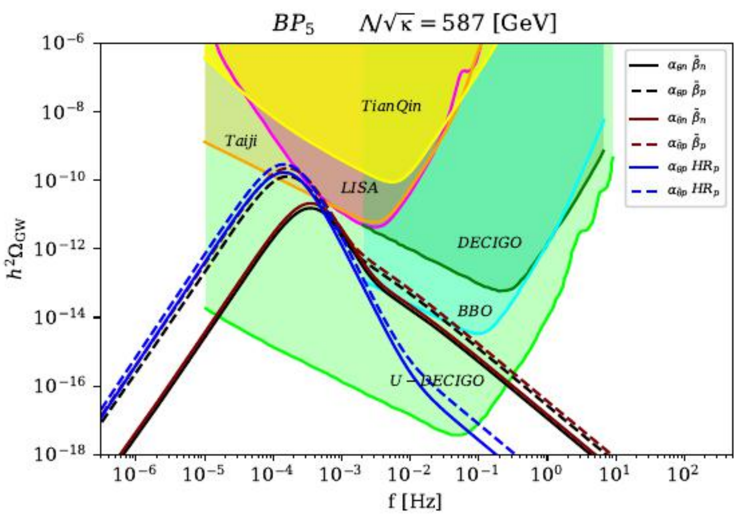
Gravitational Wave

GW spectrum and SNR for different EoS with different parameter combination:



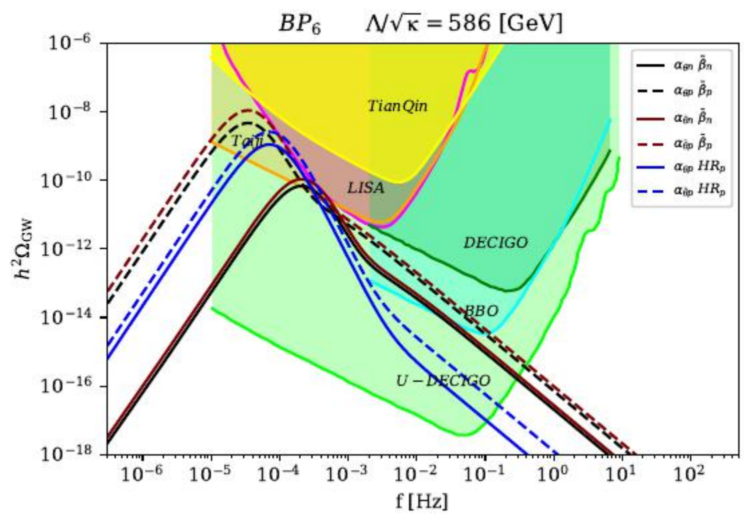


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde\beta_n$	$\alpha_{\theta p}\,\tilde\beta_p$	$\alpha_{\bar\theta n}\,\tilde\beta_n$	$\alpha_{\bar\theta p}\,\tilde\beta_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar\theta p}\,HR_p$
$SNR_{(LISA)}$	7.949	16.930	10.913	28.836	16.009	27.468
$SNR_{(Taiji)}$	14.760	58.607	20.271	100.343	66.216	113.609
$SNR_{(TianQin)}$	0.452	1.506	0.620	2.576	1.629	2.794

TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde\beta_n$	$\alpha_{\theta p}\,\tilde\beta_p$	$\alpha_{\bar\theta n}\,\tilde\beta_n$	$\alpha_{\bar\theta p}\,\tilde\beta_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar\theta p}\,HR_p$
$SNR_{(LISA)}$	14.230	15.368	22.470	26.382	17.367	40.816
$SNR_{(Taiji)}$	38.666	427.813	61.208	1000.501	213.123	500.668
$SNR_{(TianQin)}$	1.060	5.569	1.678	12.934	3.973	9.333

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Summary

- The different sound velocity in different phase can give a non negligible correction to energy budget of phase transition; hence it can affect the strength of GW signal;
- To get a precise prediction of phase transition GWs, a more valid calculation of energy budget is crucial.
- The effect of reheating phenomena are not well incorporated in calculations;
- Comparisons with a full particle physics model dependent calculation deserves a further study.

Thank you!!!