Fully-heavy tetraquarks

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低质量多轻子末态研讨会

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Based on: M. N. Anwar, J. Ferretti, FKG, E. Santopinto, B.-S. Zou, EPJC78(2018)647

Introduction

- Experimental motivation: CMS
- Does Nature allow fully-heavy tetraquarks?
- $QQ\bar{Q}\bar{Q}$ systems are similar to the di-positronium molecule (Ps_2, $e^+e^+e^-e^-$)
- Ps_2 was predicted long ago — Wheeler (1946)
- Binding energy predicted to be 0.4 eV — Ore, Hylleraas (1947)
- Produced experimentally in 2007 — Cassidy, Mills, Nature 449 (2007) 195

EFT at leading order

- Consider the Cornell potential for heavy quarkonium
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$
- For bottomonium, size $\sim \frac{1}{m_b \alpha_s} \sim \frac{1}{m_b v} \sim \frac{1}{1.5\ \text{GeV}}$, $\alpha_s(1.5\ \text{GeV}) \approx 0.31$, then
$$\frac{4}{3}\frac{\alpha_s}{r} \sim 0.6\ \text{GeV} \gg \sigma r \sim 0.12\ \text{GeV}$$
- Fully-bottom systems can be approximately treated by considering only the color-Coulomb part (leading order pNRQCD)
- For $m_Q v \gg \Lambda_{\text{QCD}}$, dynamics dominated by the short-range interaction (perturbative gluon-exchange potential), weakly coupled systems   Pineda, Yndurain (1998); Brambilla et al (1999)
- Examples: ground state bottomonium, B_c, doubly- and triply-heavy baryons

Titard, Yndurain (1994); Brambilla et al. (2001, 2005); Y. Jia, JHEP0610, 073

- Similar to the hydrogen binding energy $E_H = \frac{m_e \alpha^2}{2}$, b.e. for 1S $b\bar{b}$: $-\frac{4}{9}\alpha_s^2 m_b$

$$M_{b\bar{b}(1S)} = 2m_b\left(1 - \frac{2}{9}\alpha_s^2\right) = \frac{1}{4}[3M_{\Upsilon(1S)} + M_{\eta_b(1S)}] \quad \text{gives} \quad m_b = 4.82\ \text{GeV}$$

Approximations

- For $bb\bar{b}\bar{b}$, to leading order approximation, we consider

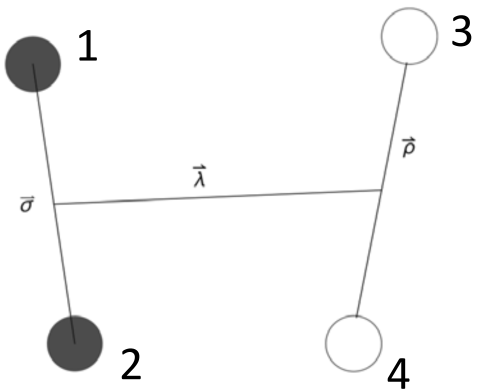
$$\mathcal{H}^{\rm NR} = \sum_{i=1}^{4} T_i + \sum_{i<j} V_{\rm SI}(\mathbf{r}_{ij}), \quad (1)$$

where $T_i = m_i + \mathbf{p}_i^2/(2m_i)$, $\mathbf{r}_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$, and the second term is the spin-independent OGE color Coulomb potential,

$$V_{\rm SI}(\mathbf{r}_{ij}) = \sum_{i<j} \frac{\boldsymbol{\lambda}_i}{2} \cdot \frac{\boldsymbol{\lambda}_j}{2} \frac{\alpha_s}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad (2)$$

- Color wave function:

$$|\psi_c\rangle = \alpha|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}; \mathbf{1}_{1234}\rangle + \beta|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}; \mathbf{1}_{1234}\rangle,$$



- Ground state, S-wave

bb in color anti-triplet, spin = 1, attractive

bb in color sextet, spin = 0, repulsive

Thus, mixing of the two components is suppressed,

the ground state should be mainly $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}; \mathbf{1}_{1234}\rangle$

the same approximation also in, e.g., K.T.-Chao, ZPC7(1981)317; Z. Liu, E. Eichten, 1709.09605

Result

On the basis of the $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34};\,\mathbf{1}_{1234}\rangle$ color configuration, the kinetic energy matrix elements can be written as

$$T = \frac{\mathbf{p}_\sigma^2}{2m_1} + \frac{\mathbf{p}_\rho^2}{2m_2} + \frac{\mathbf{p}_\lambda^2}{2m_3} = \frac{1}{m_b}(\mathbf{p}_\sigma^2 + \mathbf{p}_\rho^2 + 2\mathbf{p}_\lambda^2), \tag{6}$$

where m_1, m_2 and m_3 are the reduced masses of Q_1Q_2, $\bar{Q}_3\bar{Q}_4$ and $Q_1Q_2 - \bar{Q}_3\bar{Q}_4$, respectively. The spatial trial wave function is written in terms of the Jacobi coordinates of Eq. (4) and Fig. 1,

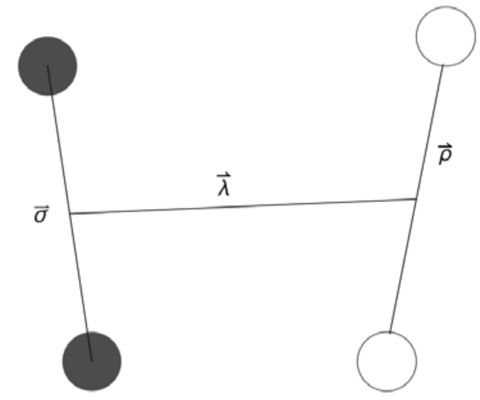
$$\psi(\boldsymbol{\sigma}, \boldsymbol{\rho}, \boldsymbol{\lambda})_{\text{spatial}} = \mathcal{N} \prod_{i=1}^{3} \exp\left[-\frac{1}{2}\beta_i^2 \boldsymbol{\xi}_i^2\right], \tag{7}$$

where $(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3) \equiv (\boldsymbol{\sigma}, \boldsymbol{\rho}, \boldsymbol{\lambda})$, β_i are the oscillatory (variational) parameters, and

$$\mathcal{N} = \left(\frac{1}{\sqrt{\pi}}\right)^{9/2} \prod_i \beta_i^{3/2}, \tag{8}$$

is the overall normalization constant. It has been recently

$$E_{bb\bar{b}\bar{b}}(1S) \equiv 4m_b - M_{bb\bar{b}\bar{b}}^{\text{gs,NR}} = (0.56 \pm 0.02)\ \text{GeV}.$$



References	Mass (GeV)	Δ (GeV) $M-2M_{\eta_b}$
This work (NR)	18.72 ± 0.02	-0.08 ± 0.02
This work (REL)	18.75	-0.05
Karliner et al. [47]	18.862 ± 0.025	0.06 ± 0.03
Bai et al. [45]	18.69 ± 0.03	-0.11 ± 0.03
Berezhnoy et al. [48]	18.754	-0.04
Chen et al. [51]	18.462 ± 0.15	-0.34 ± 0.15
Wu et al. [49]	18.462–18.568	-0.28 ± 0.05
Wang [52]	18.84 ± 0.09	0.04 ± 0.09

Inequality

For $m_Q \to \infty$, the color w.f. of $QQ\bar{Q}\bar{Q}$ can be approximated by $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34};\mathbf{1}_{1234}\rangle$

$$H_4(Q_1Q_2\bar{Q}_3\bar{Q}_4) = \sum_{i=1}^{4} T_i(Q_i) + V_{Q_1Q_2} + V_{\bar{Q}_3\bar{Q}_4} + V_{Q_1\bar{Q}_3} + V_{Q_1\bar{Q}_4} + V_{Q_2\bar{Q}_3} + V_{Q_2\bar{Q}_4}. \quad (16)$$

The color-antitriplet potential $V_{QQ}^{(\bar{3})}$ between any quark (or antiquark) pair can be related to the color-singlet quark–antiquark potential, $V_{Q\bar{Q}}^{(1)}$ [88], via

$$V_{QQ}^{(\bar{3})} = \frac{1}{[N_c-1]} V_{Q\bar{Q}}^{(1)}, \quad (17)$$

where $1/[N_c-1]$ is the overall ratio of the color coefficient in the leading SU(N_c) group. The above relation can also be verified by using the eigenvalues of the Casimir invariants from the Table 2, and it holds when the QQ and $Q\bar{Q}$ pairs are in the same spin and orbital quantum states. With the use of $V_{QQ}^{(\bar{3})} = \frac{1}{2}V_{Q\bar{Q}}^{(1)}$ for SU(3), Eq. (16) can be rearranged as follows

$$H_4(Q_1Q_2\bar{Q}_3\bar{Q}_4) = \sum_{i=1}^{4} T_i(Q_i) + \frac{1}{2}\left(V_{12}^{(1)} + V_{34}^{(1)}\right) + \sum_{i=1,2;\,j=3,4} V_{Q_i\bar{Q}_j}, \quad (18)$$

$$\sum_{i<j} V_{ij} = \frac{1}{2}(V_{12}^{(1)} + V_{34}^{(1)}) + \frac{1}{4}(V_{13}^{(1)} + V_{14}^{(1)} + V_{23}^{(1)} + V_{24}^{(1)}). \quad (22)$$

This means that under the approximation of one-gluon exchange and that the two quarks are in color anti-triplet, the four-quark Hamiltonian [Eq. (18)] can be expressed in the following form,

$$H_4(Q_1Q_2\bar{Q}_3\bar{Q}_4) = \frac{1}{2}(H_{12} + H_{34}) + \frac{1}{4}(H_{13} + H_{14} + H_{23} + H_{24}), \quad (23)$$

where $H_{ij} = T_i + T_j + V_{ij}^{(1)}(\mathbf{r}_{ij})$ is the quarkonium Hamil-

In heavy quark limit, there should be

$$M_{Q_1Q_1\bar{Q}_2\bar{Q}_2} \le 2M_{Q_1\bar{Q}_2(1S)}$$

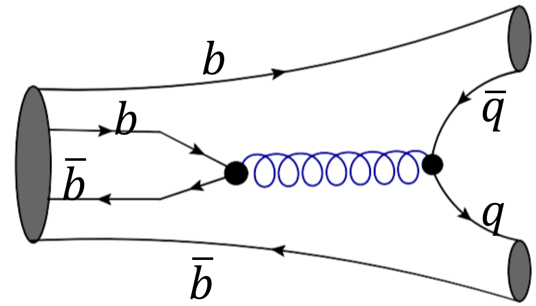
$$M_{bb\bar{b}\bar{b}} \lesssim 2M_{b\bar{b}(1S)} = 18.89 \text{ GeV}$$

Decays

- If below the two-η_b threshold, the dominant decay channels:

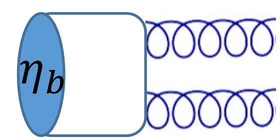
  hadronic final states with a bottom and an anti-bottom hadron
  (through annihilating $Q\bar{Q}$ into a gluon)

$$\Gamma(X_{bb\bar{b}\bar{b}} \to h_1 h_2 \ldots) \propto \alpha_s(m_b)\, |R_{b\bar{b}_{(8)}}(0)|^2$$



- Rough estimate:

$$\Gamma(X_{bb\bar{b}\bar{b}} \to h_1 h_2 \ldots) \simeq \frac{1}{\alpha_s(m_b)} \Gamma(\eta_b \to \text{hadrons})$$



with $\Gamma(\eta_b) = 10^{+5}_{-4}$ MeV and $\alpha_s(m_b) \approx 0.22$,

$$\Gamma(X_{bb\bar{b}\bar{b}}) = \mathcal{O}(50\ \text{MeV}).$$

- Partial width of $cc\bar{c}\bar{c}$ into final states with a pair of charmed hadrons:

$$\Gamma(X_{cc\bar{c}\bar{c}} \to c\bar{c}g) \simeq \frac{\Gamma(\eta_c)}{\alpha_s(m_c)} = \mathcal{O}(100\ \text{MeV}).$$

Similar to result in K.-T. Chao, ZPC7(1981)317

Decays

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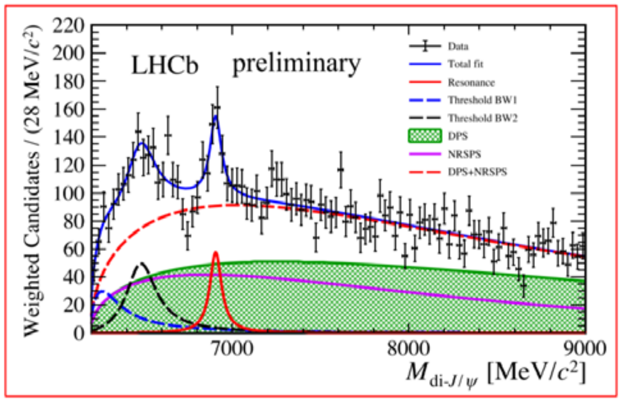
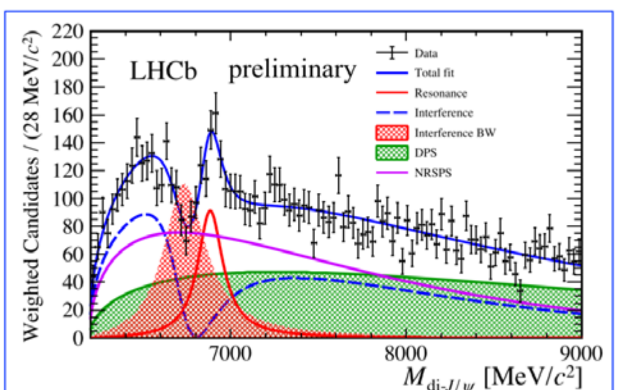
- The LHCb di-J/ψ structure: Liupan An, LHC Seminar on 16/06/2020



preliminary

No-interference fit:
✓Fit p-value: 4.6%
✓Parameters for 6.9 GeV/c^2 peak
$M = 6905 \pm 11(\text{stat})$ MeV/c^2
$\Gamma = 80 \pm 19(\text{stat})$ MeV/c^2
$N_{\text{signal}} = 252 \pm 63$

[LHCb-PAPER-2020-011]



preliminary

A simple model with interference:
✓Fit p-value: 15.5%
✓Parameters for 6.9 GeV/c^2 peak
$M = 6886 \pm 11(\text{stat})$ MeV/c^2
$\Gamma = 168 \pm 33(\text{stat})$ MeV/c^2
$N_{\text{signal}} = 784 \pm 148$

[LHCb-PAPER-2020-011]
in preparation

6/16/20 Liupan An 43/49

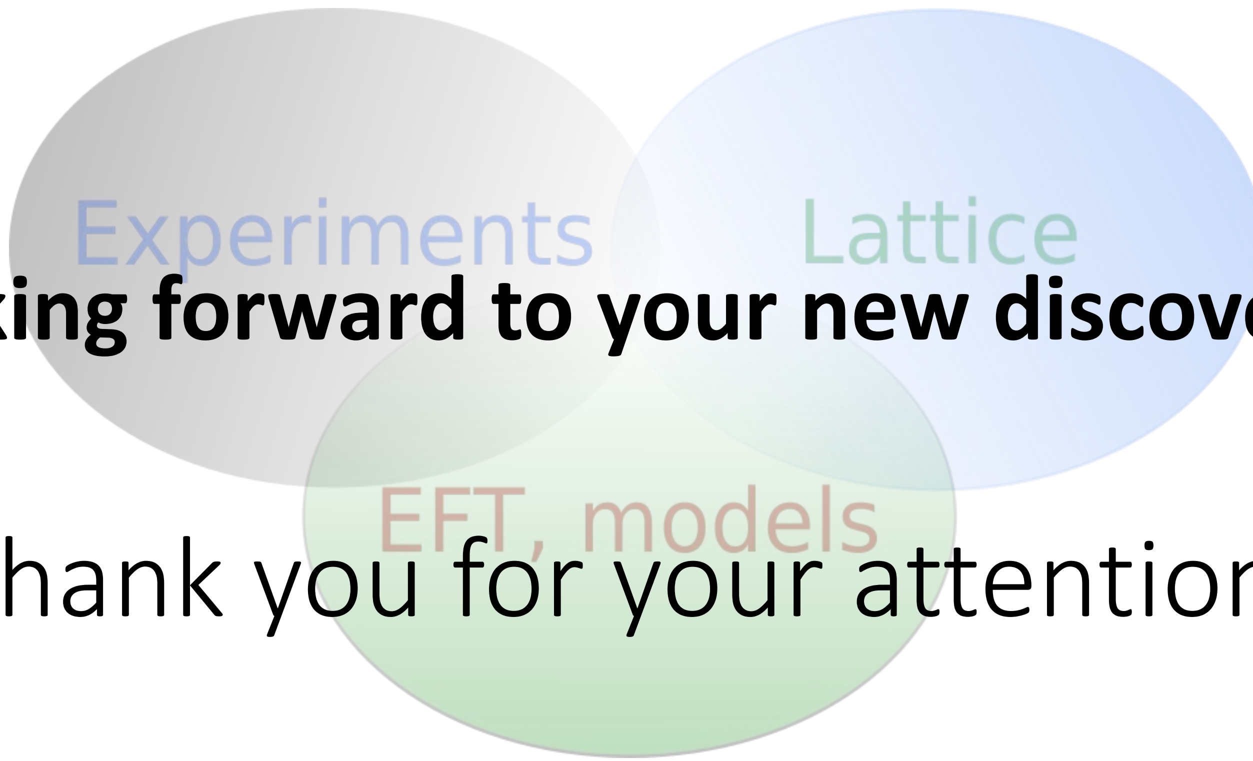
Table 1a. The quantum numbers and masses for the $(cc)_{\bar{3}^*}-(\bar{c}\bar{c})_3$ states (without spin-dependent forces between two clusters)

L	S	J^{PC}	Mass (GeV)
1	0	1^{--}	6.55
	1	$0^{-+}, 1^{-+}, 2^{-+}$	
	2	$1^{--}, 2^{--}, 3^{--}$	
2	0	2^{++}	6.78
	1	$1^{+-}, 2^{+-}, 3^{+-}$	
	2	$0^{++}, 1^{++}, 2^{++}, 3^{++}, 4^{++}$	
3	0	3^{--}	6.98
	1	$2^{-+}, 3^{-+}, 4^{-+}$	
	2	$1^{--}, 2^{--}, 3^{--}, 4^{--}, 5^{--}$	

K.-T. Chao, ZPC7(1981)317

Conclusion

- The LO calculation in the pNRQCD approach supports the existence of a fully-bottom tetraquark below the two-η_b threshold
- In heavy quark limit, the ground state $M_{Q_1Q_1\bar{Q}_2\bar{Q}_2} \leq 2M_{Q_1\bar{Q}_2(1S)}$
- Width: $\Gamma(X_{bb\bar{b}\bar{b}}) = \mathcal{O}(50\ \mathrm{MeV})$.



Looking forward to your new discoveries!

Thank you for your attention!