

## Track and Vertex reconstruction

## A. Salzburger, CERN

## What is track reconstruction?

- Track reconstruction is finding sets of measurements coming from one charged particle and building the associated trajectory through the detector. Tracks are generally used as input to higher level reconstruction objects.
- set of measurements from charged particles

Part 1 - basics \& principle of tracking and tracking detectors

- interaction of particles with (sensitive or not sensitive) detector material
- finding associated measurements

Part 2 - track finding strategies, global and local pattern recognition algorithms

- trajectory estimation \& track cleaning
- track fitting, fake and efficiency estimation
- adaptive, multi-variant and specialised methods
- tracks as input to higher level reconstruction and analysis

Part 3 - primary and secondary vertex reconstruction

- analysis usage
- the reality


## Part I - Basics \& Tracking Detectors



## Boring - Definitions

- Let's get them out of the way ...
- coordinate systems are right-handed global : ( $x, y, z$ )
local: ( $\left(x, l_{y}, l_{z}\right)$
- $\phi$ measured in transverse plane in $[-\pi,+\pi)$ (azimuthal angle)
- $\theta$ is measured from $z$ axis in $[0, \pi]$ (polar angle)

$-\lambda=\pi / 2-\theta$
- $\eta=-\ln [\tan (\theta / 2)]$ is the pseudo-rapidity (rapidity of a massless particle)


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= Andreas


## Track parameterisation

- When bound to a surface, a trajectory of a charged particle needs in a magnetic field five parameters to be defined



## Track parameterisation

- there is a certain level of freedom in the actual parameterisation
- general feature:

2 local* parameters bound to the surface
3 global* parameters combining the momentum and charge

CDF $\quad \mathbf{q}^{\prime \prime}=\left(l_{1}, l_{2}, \phi, \cot (\theta), C\right)$
CMS $\quad \mathbf{q}^{\prime}=\left(l_{1}, l_{2}, \phi, \lambda, q / p\right)$
ATLAS $\quad \mathbf{q}=\left(l_{1}, l_{2}, \phi, \theta, q / p\right)$

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$\operatorname{LHCb} \quad \mathbf{q}^{\prime \prime \prime}=\left(x, y, t_{x}, t_{y}, q / p\right)$

$$
t_{x(y)}=\frac{\partial p}{\partial x(y)}
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## Track parameterisation with uncertainties

- When bound to a surface, a trajectory of a charged particle needs in a magnetic field five parameters $(\mathbf{q})$ to be defined: track parameters

$\mathbf{C}=\left(\begin{array}{cccccl}\sigma^{2}\left(l_{1}\right) & \operatorname{cov}\left(l_{1}, l_{2}\right) & \operatorname{Cov}\left(l_{1}, \phi\right) & \operatorname{cov}\left(l_{1}, \theta\right) & \operatorname{cov}\left(l_{1}, q / p\right) \\ \cdot & \sigma^{2}\left(l_{2}\right) & \operatorname{cov}\left(l_{2}, \phi\right) & \operatorname{cov}\left(l_{2}, \theta\right) & \operatorname{cov}\left(l_{2}, q / p\right) \\ \cdot & \cdot & \sigma^{2}(\phi) & \operatorname{cov}(\phi, \theta) & \operatorname{cov}(\phi, q / p) \\ \cdot & \cdot & \cdot & \sigma^{2}(\theta) & \operatorname{cov}(\theta / q / p) \\ \cdot & \cdot & \cdot & \cdot & \sigma^{2}(q / p)\end{array}\right) \quad \begin{aligned} & \text { loment position on surface } \\ & \text { charge } \\ & \\ & \end{aligned}$


## The special one: the Perigee

- Perigee representation
- parameterisation of closest approach to a reference line:
transverse $\left(d_{0}\right)$ and longitudinal $\left(z_{0}\right)$ impact parameter



## The special one: the Perigee with uncertainties

- Perigee representation



## Part 1 - Tracking Detectors

- Track reconstruction in central tracking devices
- high granular detectors as close as possible to the beam-beam interaction region usually hermetic detector design (although dependent on experimental setup)
- objective is to measure a precise localisation of the charged particle on a certain detection device, e.g.
- planar detectors, e.g.
semiconductor based pixels, strip
- panar drift detector. e.g. micromegas
- drift tube detectors
- time projection chamber (TPC)


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## Semiconductor based detectors

- LHC innermost tracking devices are planar silicon detectors:
- exist as pixel and strip detectors (they need a local pattern recognition to find clusters of connected pixels/strip)
- ionisation of the silicon through charged particle (primary and secondary ionisation)
- drift of deposited charge to reädout surface ūing ān electric field (E)
- when embedded in magnetic field (B), drift deflection by Lorentz angle $\theta_{\mathrm{L}}$



## Planar detectors: cluster finding

- LHC innermost tracking devices are planar silicon detectors:
- either pixel or strip technology with binary (on/off) or non-binary readout (e.g. charge collected by time over readout threshold)

- more than one pixel/strip can be traversed by one particle: clustering needed usually performed with a connected component analysis (4-cell, 8-cell connectivity)
- example of connected component labelling with 8-cell connectivity:



## Planar detectors: cluster building

- multiple cells hit can be used to increase measurement precision



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the charge-weighted approach :



## Planar detectors: cluster building

- multiple cells hit can be used to increase measurement precision
the charge-weighted approach :

which one is better?
let's measure it using the residuum

$$
\mathbf{r}=\mathbf{m}-\mathbf{s}
$$


salzburg\$ ipython -i --matplotlib=osx PixelClustering.py
In [1]: fig, plots = buildPixels()
In [2]: shoot(fig, plots, 1000 )

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tails from single pixel clusters
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- what can we say about the uncertainty of the measurement ?
pitch size $a$

binary case, single pixel cluster


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$$
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binary case, $\underline{n}$-pixel cluster

## MC Toy: Dixel clustering

- what can we say about the uncertainty of the measurement?
pitch size $a$ most lucky cases: $\mathbf{r}=\mathbf{m}-\mathbf{s}=0$

binary case, n-pixel cluster
usually, a minimum path length is required to deposit enough charge, turns the biggest error into $<a / 2$
- what's the variance of a uniform distribution between $-a / 2$ and $a / 2$ ?

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$$
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[>>] mu = 0.00814352769061 | sigma = 0.70764581917
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## A classic planar detector design

- Planar modules arranged in cylinders \& discs
- highest granularity in innermost layers
- barrel structure around the interaction region
- end-cap disk structure at higher pseudo rapidity
- overlap of modules to guarantee hermetic coverage (e.g. overlaps in $\phi$, and along $z$ in general)
- stereo angle technique for strip detectors
(two- or double sided modules)




## Drift tube detectors

- Gas-filled tubes with a central wire
- inoisation of gas by traversing charged particle
- charge drift to wire through electric field (E), in case of embedding in magnetic field also some Lorentz force drift effects
- measurement is a drift time measurement



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- charge drift to wire through electric field (E), in case of embedding in magnetic field also some Lorentz force drift effects
- measurement is a drift time measurement
- Track reconstruction with drift measurements
- drift time converted into drift radius

- remaining left-right ambiguity that needs to be resolved usually done in the pattern recognition when already having some idea about the track direction


## Drift tube detectors

- ATLAS Transition Radiation Tracker
- used to do particle identification (PID)
- needs a dedicated detector design:
material with rapidly changing dielectric constant
-> transition radiation creates additional ionisation, e.g. higher signal
-> transition radiation is strongly dependent on Lorentz factor



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ATLAS Testbeam results

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QATLAS

Run: 203602
Event: 82614360 Date: 2012-e5-18 Time: 20:28:11 CES

## Time projection chamber (TPC)

- TPCs allow to build huge tracking devices to relative moderate cost
- precise track reconstruction

a gas filled vessel (ionisable)
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charge drift to the readout chambers
measurements:
$(x, y)$ from readout segmentation
(z) from drift time


Pb+Pb @ sqrt(s) = 2.76 ATeV
2010-11-08 11:30:46
Fill : 1482
Run: 137124
Event : 0x00000000D3BBE693


## Enemy No. 1: material

- Unfortunately there a difference between how we'd like an ideal detector to be and the reality
- Let's face it: the reality is always more messy ...
- General aim in the construction of tracking detectors:
- build them as light as possible material interactions disturb the measurement in the tracker itself tracker is usually before the calorimeter (material disturbs the calorimeter measurement)


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## Multiple (Coulomb) scattering

- A charged particle undergoes random deflection
- mainly caused by multiple (Coulomb) scattering off the core of atoms
- additional component from single large (Rutherford) scattering


$$
\sigma_{m s}^{p r o j}=\frac{13.6 \mathrm{MeV}}{\beta c p} Z \sqrt{t / \mathrm{X}_{0}}\left[1+0.038 \ln \left(t / \mathrm{X}_{0}\right)\right]
$$




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## MC Toy: multiple scattering



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## MC Toy: multiple scattering

in the presence of multiple coulomb scattering and single large Rutherford scattering



```
salzburg$ ipython -i --matplotlib=osx MultipleScattering.py
In [1]: fig, plots = buildPixels()
In [2]: shoot(fig, plots, 5000, sfraction = 0.01 )
```


## Energy loss

## - charged particle loses energy when traversing material

- ionisation loss : Bethe-Bloch


$$
\begin{gathered}
(d E / d x) \text { ion }=\alpha^{2} 2 \pi N_{a} \lambda_{e}^{2} \frac{Z m_{e}}{A \beta^{2}}\left[\ln \frac{2 m_{e} \beta^{2} \gamma^{2} E_{m}^{\prime}}{I^{2}(Z)}-2 \beta^{2}+1 / 4 \frac{E_{m}^{\prime 2}}{E^{2}}-\delta\right] \\
\hline N_{a}= \\
Z, A \\
m, m_{e} \\
\beta=
\end{gathered}=\begin{aligned}
& 6.023 \cdot 10^{23}, \text { Avogadro's number } \\
& \text { atomic number and weight of the traversed medium } \\
& \gamma= \\
& r_{e}= \\
& p / E, \text { where p is the particle momentum } \\
& E / m \\
& I(Z) \\
& E_{m}^{\prime} \\
& 3.8616 \cdot 10^{-11} \mathrm{~cm} \text { is the Compton wavelength of the electron } \\
& \text { the mean ionisation potential of the medium, } \\
& \text { the maximum energy transferable to the electrons of the medium with } \\
& E_{m}^{\prime}=2 m_{e} \frac{p^{2}}{m_{e}^{2}+m^{2}+2 m_{e} \sqrt{p^{2}+m^{2}}} \\
& \text { density correction. }
\end{aligned}
$$

- bremsstrahlung: Bethe-Heitler


$$
\begin{aligned}
& (d E / d x)_{r a d}=4 \alpha N_{A} \frac{z^{2} Z^{2}}{A}\left(\frac{1}{4 \pi \epsilon_{0}} \frac{e^{2}}{m c^{2}}\right)^{2} E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^{2}} \\
& (d E / d x)_{r a d}=-E_{i} / X_{0}
\end{aligned}
$$

$$
X_{0}=\frac{A}{4 \alpha N_{A} Z^{2} r_{e}^{2} \ln \frac{183}{Z^{\frac{1}{3}}}}
$$

## Energy loss

## - A charged particle loses energy when traversing material

- ionisation loss: Bethe-Heitler

- bremsstrahlung: Bethe-Bloch


Landau distribution with most probable value, mean value and Landau tail

For Tracking detectors with rather little material:
$\Delta \mathrm{E} \ll \mathrm{E}$ in


Very long tail with high probability to lose significant fraction of the particle energy

## Hadronic interaction

- Hadrons can undergo nuclear interaction with the detector material
- leads usually to the destruction of the particle (as much as it concern us)

- there are many different processes that can happen in hadron-nucleus interactions
- resulting shower has hadronic, but also EM shower components
- nuclear interaction length defined as the mean path length $\Lambda_{0}$ by which the number of charged particles is traversing through matter is reduced by $1 / \mathrm{e}$
- Unfortunately most our charged particles are hadrons
- this is the main source of track reconstruction inefficiency (if you wrote you algorithms correctly)


# Summary - particle interaction with matter 

| Type | particles | fund. parameter | characteristics | effect |
| :---: | :---: | :---: | :---: | :---: |
| Multiple Scattering | all charged particle | radiation length $X$ | almost gaussian <br> average effect 0 <br> depends $\sim 1 /$ p | deflects particles, increases measurement uncertainty |
| Ionisation loss | all charged particle | effective density $A / Z^{*} \rho$ | small effect in tracker, small dependence on p | increases momentum uncertainty |
| Bremsstrahlung | all charged particle, dominant for e | radiation length $X$ | highly nongaussian, depends | introduces measurement bias |
| Hadronic Int. | all hadronic particles | nuclear interaction length $\Lambda$ | destroys particle, rather constant effect in p | main source of track reconstruction inefficiency |

## Detector material

- general aim in the construction of tracking detectors:
- build them as light as possible
material interactions disturb the measurement in the tracker itself
tracker is usually before the calorimeter (material disturbs the calorimeter measurement)
- two fundamental measures: radiation length $X_{0}$ and nuclear interaction length $\Lambda_{0}$




## MC Toy: detector materia




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In [1]: fig, plots = buildFrame()
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## The magnetic field

- A magnetic field is essential to bend the charged particles in order to measure their momenta
- in a perfect homogenous field : circle in transverse direction
- yields a helical track in a solenoidal field
keep transverse \& longitudinal components independent
$\odot \odot \odot \odot \odot \odot \quad$ В


$$
\frac{d^{2} \mathbf{r}}{d s^{2}}=\frac{q}{p}\left[\frac{d \mathbf{r}}{d s} \times \mathbf{B}(\mathbf{r})\right]
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$\odot \odot \odot \odot \odot \odot$
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$\odot \odot \odot \quad \odot \quad \odot \quad \mathrm{cms}^{\prime}=\left(l_{1}, l_{2}, \phi, \lambda, q / p\right)$
ATLAS

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$$

## Realistic magnetic fields: CMS \& ATLAS

these are not homogeneous magnetic fields !


## Track propagation

## - problems to solve

- transport of track parameters through the magnetic field

$$
\frac{d^{2} \mathbf{r}}{d s^{2}}=\frac{q}{p}\left[\frac{d \mathbf{r}}{d s} \times \mathbf{B}(\mathbf{r})\right]
$$

- application of material effects according to the detector material

$$
\frac{d^{2} \mathbf{r}}{d s^{2}}=\frac{q}{p}\left[\frac{d \mathbf{r}}{d s} \times \mathbf{B}(\mathbf{r})\right]+g(p, \mathbf{r}) \frac{d \mathbf{r}}{d s} \begin{aligned}
& \text { deterministic energy loss } \\
& \text { treatment }
\end{aligned}
$$

solve this for any $\mathbf{B}(\mathbf{r})$
we need a numerical integration method!

## Numerical integration

- Re-formulate the equation of motion as a movement along $z$

$$
\begin{gathered}
\frac{d^{2} \mathbf{r}}{d s^{2}}=\frac{q}{p}\left[\frac{d \mathbf{r}}{d s} \times \mathbf{B}(\mathbf{r})\right] \\
\frac{d^{2} x}{d z^{2}}=\frac{q}{p} R\left[\frac{d x}{d z} \frac{d y}{d z} B_{x}-\left(1+\left(\frac{d x}{d z}\right)^{2}\right) B_{y}+\frac{d y}{d z} B_{z}\right] \\
\frac{d^{2} y}{d z^{2}}=\frac{q}{p} R\left[\left(1+\left(\frac{d y}{d z}\right)^{2}\right) B_{x}-\frac{d x}{d z} \frac{d y}{d z} B_{y}-\frac{d x}{d z} B_{z}\right]
\end{gathered}
$$

- Integrate to solve for $x(z)$ and $y(z)$ :
- Numerical integration methods:
- Euler's method
- Midpoint method
- Runge-Kutta integration


## Track propagation

- Many components of the track reconstruction need the expression of the track at different places (i.e. surfaces) in the detector



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## Recap for today



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## Summary - Tracking detectors

|  | ALICE | ATLAS | CMS |
| :---: | :---: | :---: | :---: |
| R inner | 3.9 cm | 5.0 cm | 4.4 cm |
| R outer | 3.7 m | 1.1 m | 1.1 m |
| Length | 5 m | 5.4 m | 5.8 m |
| $\|\eta\|$ range | 0.9 | 2.5 | 2.5 |
| B field | 0.5 T | 2 T | 4 T |
| Total $X_{0}$ near $\eta=0$ | $\begin{aligned} & 0.08 \text { (ITS) } \\ & +0.035 \text { (TPC) } \\ & +0.234 \text { (TRD) } \end{aligned}$ | 0.3 | 0.4 |
| Power | 6 kW (ITS) | 70 kW | 60 kW |
| r $\phi$ resolution near outer radius | $\begin{aligned} & \sim 800 \mu \mathrm{~m} \text { TPC } \\ & \sim 500 \mu \mathrm{~m} \text { TRD } \end{aligned}$ | $130 \mu \mathrm{~m}$ per TRT straw | $35 \mu \mathrm{~m}$ per strip layer |
| $\mathrm{p}_{\mathrm{T}}$ resolution at 1 GeV and at $\quad 100 \mathrm{GeV}$ | $\begin{aligned} & 0.7 \% \\ & 3 \% \text { (in pp) } \end{aligned}$ | $\begin{aligned} & 1.3 \% \\ & 3.8 \% \end{aligned}$ | $\begin{aligned} & 0.7 \% \\ & 1.5 \% \end{aligned}$ |

## Numerical integration in a nutshell $\quad \partial y / \partial x=f(x, y)$

- Euler method with start values $x_{n}, y_{n}$
- what is the function value at $y_{n+1}$ at $x_{n+1}=x_{n}+h$ ?


Accuracy: 1st order

## Numerical integration in a nutshell $\partial y / \partial x=f(x, y)$

- Midpoint method with start values $x_{n}, y_{n}$
- what is the function value at $y_{n+1}$ at $x_{n+1}=x_{n}+h$ ?

- on the step to $x_{n+1}=x_{n}+h$ you stop at the midpoint and take this derivate for the evaluation of your final value from the full step


## Numerical integration in a nutshell

- Runge-Kutta method with start values


Fourth-order Runge-Kutta method. In each step the derivative is evaluated four times: once at the initial point, twice at trial midpoints, and once at a trial endpoint.
From these derivatives the final function value (shown as a filled dot) is calculated.

$$
\begin{aligned}
k_{1} & =h f\left(x_{n}, y_{n}\right) \\
k_{2} & =h f\left(x_{n}+\frac{h}{2}, y_{n}+\frac{k_{1}}{2}\right) \\
k_{3} & =h f\left(x_{n}+\frac{h}{2}, y_{n}+\frac{k_{2}}{2}\right) \\
k_{4} & =h f\left(x_{n}+h, y_{n}+k_{3}\right) \\
y_{n+1} & =y_{n}+\frac{k_{1}}{6}+\frac{k_{2}}{3}+\frac{k_{3}}{3}+\frac{k_{4}}{6}+O\left(h^{5}\right)
\end{aligned}
$$

## Some food for thoughts



- How can a non-binary readout work?
-How would you "measure" the Lorentz angle?
- Why were if off with our pull distribution?

- Think of a great positive feature of such double sided modules
- Can we do PID with the silicon detector/TPC ?


