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Contributions from Φ_{B2} to the $B \to PP$ decays with the QCD factorization

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概述

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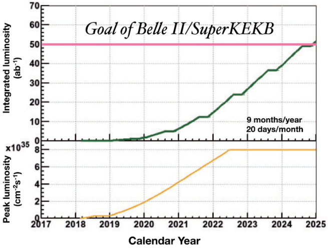
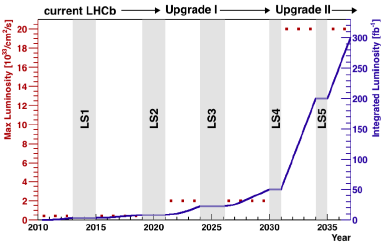
3. 理论框架
4. 总结展望

1 研究背景

1.1 实验方面





- 随着探测器的不断升级，未来BelleII探测器的亮度将会达到$50ab^{-1}$，LHCb UpgradeII探测器的亮度将会达到$300fb^{-1}$。
- 探测技术也在不断地提高，探测器的测量精度将会大大提升，并且能够收集更多关于B 介子的数据。

1.2 理论方面

- 计算B介子非轻衰变强子矩阵元的唯象方法，例如pQCD因子化方法、QCDF方法等。
- QCDF 对强子矩阵元的计算已到NNLO；幂次压低的横向振幅的贡献也已经得到计算，并且发现这些修正对提高理论精确度有很重要的贡献。
- PQCD 分支比的计算也已经超越了领头阶贡献；理论研究中也考虑了包含B介子分布振幅中幂次压低的Φ_{B2}部分的贡献。
- 为了与实验探测精度相匹配，理论计算的精确度也要不断提升。

2.1 动机一

- B介子由两个标量分布振幅（DAs）描述，按$1/m_b$ 展开到领头阶分布振幅有如下形式：

$$\langle 0|\bar{q}_\alpha(z)[...]b_\beta(0)|\bar{B}(p)\rangle = -\frac{i f_B}{4}\left\{\left(\not p + m_b\right)\gamma_5\right\}_{\beta\gamma}\int d\xi\, e^{-i\xi p_+ z_-}\left[\Phi_{B1}(\xi) + \not n_-\Phi_{B2}(\xi)\right]_{\gamma\alpha},$$

分布振幅的归一化条件为：

$$\int_0^1 d\xi\,\Phi_{B1}(\xi) = 1\,, \qquad \int_0^1 d\xi\,\Phi_{B2}(\xi) = 0\,.$$

- $\Phi_{B1} = \phi_B^+$以及$\Phi_{B2} = (\phi_B^+ - \phi_B^-)/2$，一般来讲$\phi_B^+ \neq \phi_B^-$。
- Φ_{B2}部分的贡献与Φ_{B1}相比虽然是被$\Lambda_{\mathrm{QCD}}/m_b$压低的，但是b夸克的质量是有限的，不是无限大的，$\Lambda_{\mathrm{QCD}}/m_b$是不可忽略的量

- 在文章epjc28.515和PRD 74.014027 中指出，在PQCD方法中，Φ_{B2} 对$B \to \pi$形状因子的贡献达到30%。意味着Φ_{B2}对分支比的贡献可能较大，尤其是对某些W外发射过程可能达到70%。

B to light meson transition form factors calculated in perturbative QCD approach

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Abstract. We calculate the $B \to P$, $B \to V$ (P is the light pseudoscalar meson, V the light vector meson) form factors in the large-recoil limit in the perturbative QCD approach, including both the vector (axial vector) and tensor operators. In general there are two leading components ϕ_B and $\bar{\phi}_B$ for the B meson wave functions. We consider both contributions of them. Sudakov effects ($k_\perp$ and threshold resummation) are included to regulate the soft end-point singularity. By choosing the hard scale as the maximum virtualities of the internal particles in the hard b quark decay amplitudes, Sudakov factors can effectively suppress the long-distance soft contribution. The hard contribution can be dominant in these approaches.

Table 1. B meson transition form factors at $q^2 = 0$ with the hard scale chosen in (21), and the numbers in parentheses are results without the contribution of $\bar{\phi}_B$

Process	$F_0(0) = F_1(0)$	$F_T(0)$
$B \to \pi$	0.292 ± 0.030	0.278 ± 0.028
	(0.199)	(0.189)
$B \to K$	0.321 ± 0.036	0.311 ± 0.033
	(0.231)	(0.223)

Process	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_1(0)$	$T_2(0)$	$T_3(0)$
$B \to \rho$	0.318 ± 0.032	0.366 ± 0.036	0.25 ± 0.02	0.21 ± 0.01	0.56 ± 0.05	0.013 ± 0.001	0.06 ± 0.01
	(0.226)	(0.256)	(0.17)	(0.14)	(0.41)	(0.004)	(0.05)
$B \to \omega$	0.305 ± 0.030	0.347 ± 0.036	0.24 ± 0.02	0.20 ± 0.02	0.53 ± 0.05	0.012 ± 0.001	0.06 ± 0.01
	(0.212)	(0.250)	(0.16)	(0.13)	(0.38)	(0.003)	(0.05)
$B \to K^*$	0.406 ± 0.042	0.455 ± 0.047	0.30 ± 0.03	0.24 ± 0.02	0.69 ± 0.08	0.007 ± 0.001	0.09 ± 0.01
	(0.293)	(0.336)	(0.21)	(0.16)	(0.51)	(-0.001)	(0.07)

30% contribution

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Uncertainty in the leading-order perturbative QCD calculations of B-meson decays

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Uncertainty in the perturbative quantum chromodynamics (PQCD) calculation of B decays is investigated in $B \to \pi$, $B \to D$ transition form factors and $B \to D\pi$ decay amplitudes. B-meson distribution amplitude dependence is studied by taking three kinds of distribution amplitudes suggested so far. It is found that almost the same q^2 dependence of the form factors can be obtained irrespective of the types of the B-meson distribution amplitudes by suitably choosing one parameter. The $B \to D\pi$ process shows the difference due to the distribution amplitude. The effect of the subleading component of the B-meson distribution amplitude is also studied in the three processes. The numerical results of calculations with the subleading component can be well approximated by the leading-order calculation with a suitable choice of the distribution amplitude parameters.

For a reference we show the ratio of the contribution from the $\phi_B^{\bar{n}} = (\phi_B^- - \phi_B^+)/\sqrt{2}$ component to that from all the components in Table X. The $\phi_B^{\bar{n}}$ component contribution is found to be about 30% or less.

TABLE X. Ratio of the subleading contribution to the total contribution in $F^{B\pi}(0)$.

	Gaussian	Exponential	KKQT
$F^{B\pi}_{\phi^{\bar{n}}}(0)/F^{B\pi}_{\mathrm{total}}(0)$	0.22	0.20	0.29

2.2 动机二

- $B_s \to \pi^+\pi^-$ 和$B_d \to K^+K^-$的衰变过程分支比$\mathcal{O}(10^{-6})$

	2003[1]	2005[2]	2009[3]	2012[4]	2014[5]	2021[6]
$\bar{B}_s \to \pi^+\pi^-$	$0.024^{+0.165}_{-0.024}$	$0.57^{+0.18}_{-0.16}$	$0.26^{+0.10}_{-0.09}$	$0.51^{+0.23}_{-0.19}$	0.61 ± 0.7	0.798 ± 0.092
Exp.	$-$	< 1.36		$0.98^{+0.25}_{-0.22}$	0.73 ± 0.14	0.700 ± 0.100
$\bar{B}^0 \to K^+K^-$	$0.013^{+0.088}_{-0.013}$		$0.10^{+0.04}_{-0.04}$	$0.16^{+0.06}_{-0.05}$	$0.15^{+0.03}_{-0.02}$	0.155 ± 0.027
Exp.	< 0.6			$0.13^{+0.09}_{-0.09}$	0.12 ± 0.05	0.078 ± 0.015

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3.1 QCD因子化公式

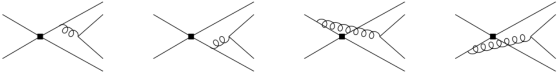
按$1/m_b$展开，有效算符$\hat{O}_i$ 的强子矩阵元写成如下形式：

$$\begin{aligned}\langle P_1 P_2|\hat{O}_i|\bar{B}\rangle &= F_0^{B\to P_1}\int_0^1 dx\, T_i^I(y)\,\phi_{P_2}(x) + F_0^{B\to P_2}\int_0^1 dy\, H_i^I(x)\,\phi_{P_1}(y)\\ &+ \int_0^1 d\xi\, dx\, dy\, T_i^{II}(\xi,x,y)\,\phi_B(\xi)\,\phi_{P_1}(y)\,\phi_{P_2}(x),\end{aligned}$$

- 第一行的两项是顶角修正的贡献，与B 介子分布振幅无关。
- 第二行的项对应于不可因子化的贡献，与B介子分布振幅相关。

- 考虑硬旁观者散射项和湮灭图贡献：

(a)



(b)

3.2 振幅分析

硬旁观者散射项$H_k^{B2}(P1P2)$的形式如下：

$$
\begin{aligned}
H_1^{B2}(P1P2) &= \pi\alpha_s \int_0^1 dx \int_0^1 dy \int_0^1 d\xi\, \Phi_{B2}(\xi) \Big\{ \Phi_{P1}(y)\, \Phi_{P2}(x)\, 2\, [\frac{\xi-\bar{x}}{\bar{x}\bar{y}^2\xi} + \frac{x-\xi}{x\bar{y}^2\xi}] \\
&\quad -\Phi_{P1}^{p}(y)\, \Phi_{P2}(x)\, r_\chi^{P1} [\frac{\bar{x}(y-\bar{y})+2\bar{y}\xi}{\bar{x}\bar{y}^2\xi} + \frac{(x-\xi)(\bar{y}-y)}{x\bar{y}^2\xi}] \Big\}, \\
H_2^{B2}(P1P2) &= \pi\alpha_s \int_0^1 dx \int_0^1 dy \int_0^1 d\xi\, \Phi_{B2}(\xi) \Big\{ \Phi_{P1}(y)\, \Phi_{P2}(x)\, 2\, [\frac{\bar{x}-\xi}{\bar{x}\bar{y}^2\xi} - \frac{x-\xi}{x\bar{y}^2\xi}] \\
&\quad +\Phi_{P1}^{p}(y)\, \Phi_{P2}(x)\, r_\chi^{P1} [\frac{(\bar{x}-\xi)(y-\bar{y})}{\bar{x}\bar{y}^2\xi} - \frac{2\xi\bar{y}-x(\bar{y}-y)}{x\bar{y}^2\xi}] \Big\}, \\
H_3^{B2}(P1P2) &= \pi\alpha_s \int_0^1 dx \int_0^1 dy \int_0^1 d\xi\, \Phi_{B2}(\xi) \Big\{ \Phi_{P1}(y)\, \Phi_{P2}(x)\, \frac{r_\chi^{P2}}{2} [\frac{2\bar{x}-\xi}{\bar{x}\bar{y}^2\xi} - \frac{2x-\xi}{x\bar{y}^2\xi}] \\
&\quad +\Phi_{P1}^{p}(y)\, \Phi_{P2}(x)\, \frac{r_\chi^{P1}\, r_\chi^{P2}}{4} [\frac{2\bar{x}-\xi}{\bar{x}\bar{y}^2\xi} - \frac{2x-\xi}{x\bar{y}^2\xi}] \Big\},
\end{aligned}
$$

- H_k^{B2}是非零的，采用渐进形式$\Phi_{P1}(y)$=6 $y\,\bar{y}$ 和$\Phi_{P1}^{p}(y)$=1 时，积分$\int_0^1 \frac{\Phi_{P1}(y)}{\bar{y}^2} dy$ 和$\int_0^1 \frac{\Phi_{P1}^{p}(y)}{\bar{y}^2} dy$ 会出现发散。
- 但是，考虑归一化条件$\int_0^1 d\xi\, \Phi_{B2}(\xi) = 0$后，剩余的项相加相消，最终$H_k^{B2}$贡献为零。

湮灭图贡献

只有不可因子化的湮灭图有贡献，$A_k^{i,B2}$形式如下：

$$A^{B2} = \pi\,\alpha_s \int_0^1 \xi\,\Phi_{B2}(\xi)\,d\xi = \pi\,\alpha_s\,\langle\xi\rangle_{B2},$$

$$A_1^{i,B2} = -A^{B2}\int_0^1 \frac{dx}{\bar{x}}\int_0^1 \frac{dy}{y}\left\{2\,\frac{\Phi_{P2}(x)\,\Phi_{P1}(y)}{1-x\,\bar{y}} - r_\chi^{P1}\,r_\chi^{P2}\,\frac{\bar{x}}{y}\,\frac{\Phi_{P2}^p(x)\,\Phi_{P1}^p(y)}{1-x\,\bar{y}}\right\},$$

$$A_2^{i,B2} = +A^{B2}\int_0^1 \frac{dx}{\bar{x}}\int_0^1 \frac{dy}{y}\left\{2\,\frac{\Phi_{P2}(x)\,\Phi_{P1}(y)}{\bar{x}\,y} - r_\chi^{P1}\,r_\chi^{P2}\Phi_{P2}^p(x)\Phi_{P1}^p(y)\left[\frac{\bar{x}}{1-x\,\bar{y}} - \frac{x}{\bar{x}\,y}\right]\right\},$$

$$A_3^{i,B2} = -A^{B2}\int_0^1 \frac{dx}{\bar{x}}\int_0^1 \frac{dy}{y}\left\{2\,r_\chi^{P2}\,\frac{x\,\Phi_{P2}^p(x)\Phi_{P1}(y)}{1-x\,\bar{y}} + r_\chi^{P1}\Phi_{P2}(x)\,\Phi_{P1}^p(y)\left[\frac{y-\bar{y}}{1-x\bar{y}} + \frac{1}{\bar{x}y}\right]\right\},$$

- Φ_{B2}对WA振幅的贡献是非零的，这是由于矩$\langle\xi\rangle_{B2}$ 是非零的。
- 采用渐进形式$\Phi_{P1}(y)$=$6\,y\,\bar{y}$ 和$\Phi_{P1}^p(y)$=1 时，会出现发散。

参数化

$$A_1^{i,B2} = -A^{B2}\left\{12\,(\pi^2-6) - r_\chi^{P1}\,r_\chi^{P2}\left[\frac{\pi^2}{6}+\frac{1}{2}\,X_A^2+X_LX_A+X_A-X_L\right]\right\},$$

$$A_2^{i,B2} = +A^{B2}\left\{72\,(X_A^2-1)^2 - r_\chi^{P1}\,r_\chi^{P2}\left[\frac{\pi^2}{6}+\frac{1}{2}\,X_A^2+X_LX_A-X_L^2\right]\right\},$$

$$A_3^{i,B2} = -A^{B2}\,6\left\{\left[\frac{\pi^2}{6}+\frac{1}{2}\,X_A^2-X_A\right](2\,r_\chi^{P2}-r_\chi^{P1}) + r_\chi^{P1}\,(X_AX_L-X_L+1)\right\}.$$

- $\int_0^1 \frac{dx}{x} \to X_A$, $\quad \int_0^1 \frac{dx}{x^2} \to X_L$,
  $\int_0^1 dx\,\frac{\ln x}{x} \to -\frac{1}{2}\,X_A^2$, $\quad \int_0^1 dx\,\frac{\ln x}{x^2} \to X_L - X_A - X_LX_A$.
- 参数X_A和X_L是包含了强相位的复数，并且一般参数化为

$$X_A = (1+\rho_A\,e^{i\phi_A})\ln\frac{m_B}{\Lambda_h},$$
$$X_L = (1+\rho_A\,e^{i\phi_A})\,\frac{m_B}{\Lambda_h}.$$

波函数（WFs）

- 采用GN-type$B$介子分布振幅[A.Grozin,M.Neubert,PRD55,272(1997)]:

$$\phi^+_{B_q}(\xi) = N^+\,\xi\exp\Big(-\frac{\xi\, m_{B_q}}{\omega_{B_q}}\Big), \qquad \phi^-_{B_q}(\xi) = N^-\exp\Big(-\frac{\xi\, m_{B_q}}{\omega_{B_q}}\Big),$$
$$\langle\xi\rangle_{B2} = \frac{1}{2}\Big(\langle\xi\rangle_+ - \langle\xi\rangle_-\Big) = \frac{\bar\Lambda}{3\, m_b},$$

其中$N^{\pm}$是归一化常数，并满足归一化条件$\int_0^1\phi^{\pm}_{B_q}(\xi)d\xi = 1$。$\langle\xi\rangle_+ = 2\langle\xi\rangle_- = \frac{4}{3}\frac{\bar\Lambda}{m_b}$ and $\bar\Lambda = m_B - m_b \approx 0.55$ GeV。

- B_s介子取$\omega_{B_s} = 0.45{\pm}0.10$ GeV，得到$\langle\xi\rangle_{B2} = 0.042{\pm}0.01$;

  [J. Sun, Z. Xiong, Y. Yang, G.Lu, Eur. Phys. J. C 73, 2437 (2013).]

- B_d介子取$\omega_{B_d} = 0.42{\pm}0.10$ GeV，得到$\langle\xi\rangle_{B2} = 0.039{\pm}0.01$。

  [T. Kurimoto, Phys. Rev. D 74, 014027 (2006).]

纯湮灭图振幅

纯湮灭过程$B_d \to K^+K^-$ 和$B_s \to \pi^+\pi^-$ 的衰变振幅写成如下形式:

$$\mathcal{A}(B_s\to\pi^+\pi^-) = i\,\frac{G_F}{\sqrt{2}}\, f_{B_s}\, f_\pi^2 \Big\{ V_{ub}^* V_{us} \Big(b_1+2\,b_4+\frac{1}{2}\,b_{4,\mathrm{EW}}\Big)+V_{cb}^* V_{cs}\Big(2\,b_4+\frac{1}{2}\,b_{4,\mathrm{EW}}\Big)\Big\},$$

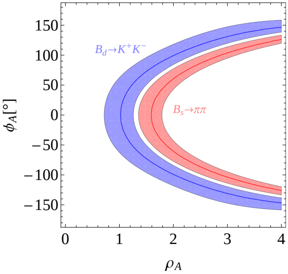
$$\mathcal{A}(B_d\to K^+K^-) = i\,\frac{G_F}{\sqrt{2}}\, f_{B_d}\, f_K^2 \Big\{ V_{ub}^* V_{ud} \Big(b_1+2\,b_4+\frac{1}{2}\,b_{4,\mathrm{EW}}\Big)+V_{cb}^* V_{cd}\Big(2\,b_4+\frac{1}{2}\,b_{4,\mathrm{EW}}\Big)\Big\}.$$

- 湮灭系数b_i定义如下所示:

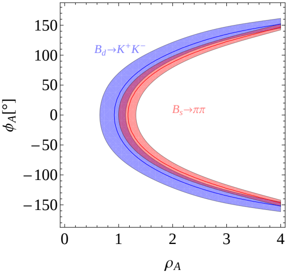
$$b_1 = \frac{C_F}{N_c^2}\, C_1\, A_1^i\,, \qquad b_4 = \frac{C_F}{N_c^2}\Big[C_4\, A_1^i + C_6\, A_2^i\Big],$$

$$b_{4,\mathrm{EW}} = \frac{C_F}{N_c^2}\Big[C_{10}\, A_1^i + C_8\, A_2^i\Big].$$

3.3 结果与分析



(c)



(d)

- 其中图(c)不含Φ_{B2}的贡献，图(d)包含了Φ_{B2} 的贡献。实线对应实验中心值，带状对应2σ误差。
- 由图(c)可知，当忽略Φ_{B2}的贡献时图(c)没有重叠区域。
- 考虑了Φ_{B2}的贡献时，图(d)有重叠区域，即$B_d \to K^+K^-$ 和$B_s \to \pi^+\pi^-$衰变过程能够取到同一组湮灭参数ρ_A 和ϕ_A。

Decay mode	This work				Beneke	
	S1		S2		S3	
	$A_k^{i,B2}=0$	$A_k^{i,B2}\neq 0$	$A_k^{i,B2}=0$	$A_k^{i,B2}\neq 0$	$A_k^{i,B2}=0$	HFLAV
$B_s\to\pi^+\pi^-$	$3.13^{+0.56}_{-0.43}$	$5.08^{+1.05}_{-0.86}$	$3.44^{+0.62}_{-0.47}$	$5.63^{+1.18}_{-0.97}$	1.49	6.7 ± 0.8
$B_d\to K^+K^-$	$0.85^{+0.17}_{-0.14}$	$1.01^{+0.20}_{-0.16}$	$0.78^{+0.15}_{-0.13}$	$0.91^{+0.18}_{-0.15}$	0.79	0.80 ± 0.15

- CP平均的分支比（以10^{-7}为单位），误差来自于非微扰输入参数。
- 方案S1：$\rho_A=1$和$\phi_A=0^\circ$，强相位ϕ_A为零是很不自然的。
- 方案S2：$\rho_A=1.2$和$\phi_A=-40^\circ$，尽量同时考虑到ρ_A 的取值和非零的ϕ_A。此外，方案S2 的取值与Beneke文章NPB675.333(2003)中S3方案的取值$\rho_A=1$ 和$\phi_A=-45^\circ$ 是比较接近的。
- 采用S2方案，Φ_{B2}分别对$\mathcal{B}(B_d\to K^+K^-)$和$\mathcal{B}(B_s\to\pi^+\pi^-)$提供了大约20%和60% 的修正。

4 总结展望

(1) 对Φ_{B2}部分的研究发现，$B \to PP$衰变过程中，Φ_{B2} 对旁观者硬散射项和可因子化的弱湮灭振幅的贡献可以忽略。

(2) Φ_{B2} 部分对纯湮灭过程$B_d \to K^+K^-$ 和$B_s \to \pi^+\pi^-$ 的贡献不可忽略，在恰当的湮灭图参数(ρ, ϕ) 空间下，上述过程的衰变分支比在一定程度上对改善理论结果与实验数据间的差异有重要意义。

(3) 期待未来实验和理论上对B介子非轻衰变更深入的研究，为我们今后理解B 介子的衰变以及反常现象等提供有意义的参考。

感谢各位老师、同学的聆听！