CP asymmetry in the angular distributions of $\tau \to K_S \pi \nu_\tau$ decays

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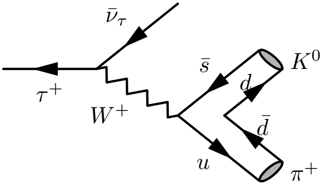
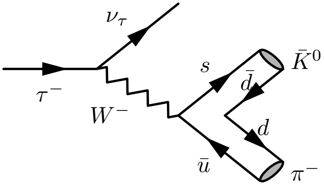
Outline

$\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau$ vs. $\tau^- \to \bar{K}^0 \pi^- \nu_\tau$ in the SM ($\Delta S = \Delta Q$ rule)





$$A_{\rm CP}^{\rm rate} \equiv \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"} \pi^+ \bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"} \pi^- \nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"} \pi^+ \bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"} \pi^- \nu_\tau)} \quad (1)$$

I. I. Bigi and A. I. Sanda PLB 625 (2005) 4752

2.8 σ discrepancy

$$\begin{cases} A_{\rm CP}^{\rm Exp} = (-3.6 \pm 2.5) \times 10^{-3} & \text{PRD 85 (2012) 031102} \\ A_{\rm CP}^{\rm SM} = (3.6 \pm 0.1) \times 10^{-3} & \text{PLB 625 (2005) 4752} \end{cases} \quad (2)$$

- CP Violation in $\tau \to \nu \pi K_S$ and $D \to \pi K_S$: The Importance of K_S-K_L Interference.
  Y. Grossman and Y. Nir, JHEP 04 (2012) 002.

- Can the observed CP asymmetry in $\tau \to K \pi \nu_\tau$ be due to nonstandard tensor interactions?
  H.Z. Devi *at al*, PRD 90 (2014) 013016.

- A no-go theorem for non-standard explanations of the $\tau \to K_S \pi \nu_\tau$ CP asymmetry.
  V. Cirigliano *at al*, PRL 120 (2018) 141803

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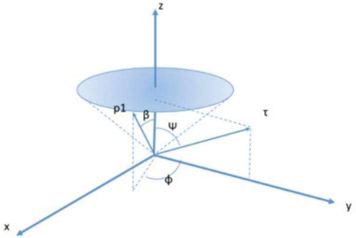
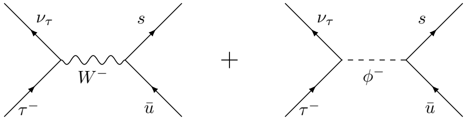




$$\cos\alpha = \cos\beta\cos\psi + \sin\beta\sin\psi\cos\phi$$

No CP asymmetry in the angular distributions

$$A^i_{CP} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} - \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d^2\Gamma(\tau^- \to K_S\pi^-\nu_\tau)}{ds\, d\cos\alpha} + \frac{d^2\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}_\tau)}{ds\, d\cos\alpha}\right] ds\, d\cos\alpha} \quad (3)$$

Belle Collaboration, PRL 107 (2011) 131801

Motivation

- Indirect CPV in $K^0 - \bar{K}^0$ mixing in the angular distributions of $\tau \to K_S \pi \nu_\tau$ decays.

- Producing direct CPV: $\delta_1^{\rm w} - \delta_2^{\rm w} \neq 0$ && $\delta_1^{\rm s} - \delta_2^{\rm s} \neq 0$

$$\begin{aligned} \mathcal{A}_i &= |\mathcal{A}_i|\, e^{i\delta_i^s} e^{i\delta_i^w}, \quad i \in \{1,2\} \\ A_{CP} &\propto |\mathcal{A}_1 + \mathcal{A}_2|^2 - \left|\overline{\mathcal{A}}_1 + \overline{\mathcal{A}}_2\right|^2 \\ &= -4\,|\mathcal{A}_1|\,|\mathcal{A}_2| \sin[\delta_1^{\rm s} - \delta_2^{\rm s}] \sin[\delta_1^{\rm w} - \delta_2^{\rm w}] \end{aligned} \tag{4}$$

$$\begin{aligned} A_{\rm CP}^{\rm rate} &\propto \sin[\delta_V^{\rm s} - \delta_T^{\rm s}] \sin[\delta_V^{\rm w} - \delta_T^{\rm w}] \\ A_{CP}^{i} &\propto \sin[\delta_V^{\rm s} - \delta_S^{\rm s}] \sin[\delta_V^{\rm w} - \delta_S^{\rm w}] \quad \text{or} \quad \sin[\delta_T^{\rm s} - \delta_S^{\rm s}] \sin[\delta_T^{\rm w} - \delta_S^{\rm w}] \end{aligned}$$

- A generally model-independent analysis is still missing.

- Mass basis vs. Flavor basis

$$|K_{S,L}\rangle = p|K^0\rangle \pm q|\bar{K}^0\rangle \qquad (|p|^2 + |q|^2 = 1) \tag{5}$$

- Reciprocal basis ($\langle\tilde{K}_{S,L}| \neq \langle K_{S,L}|$)

$$\langle\tilde{K}_{S,L}| = \frac{1}{2}\left(p^{-1}\langle K^0| \pm q^{-1}\langle\bar{K}^0|\right) \tag{6}$$

$$\langle\tilde{K}_S|K_S\rangle = \langle\tilde{K}_L|K_L\rangle = 1\,, \quad \langle\tilde{K}_S|K_L\rangle = \langle\tilde{K}_L|K_S\rangle = 0\,,$$

$$|K_S\rangle\langle\tilde{K}_S| + |K_L\rangle\langle\tilde{K}_L| = 1$$

J. P. Silva, PRD 62 (2000) 116008

- The time-evolution operator for $K^0 - \bar{K}^0$ system

$$\exp(-i\mathbf{H}t) = e^{-i\mu_S t}|K_S\rangle\langle\tilde{K}_S| + e^{-i\mu_L t}|K_L\rangle\langle\tilde{K}_L| \tag{7}$$

- The time-evolution amplitudes

$$\mathcal{A}^+ = \langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle\tilde{K}_S|T|\tau^+\rangle + \langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle\tilde{K}_L|T|\tau^+\rangle$$

$$= \frac{1}{2p}\left[\langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} + \langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right]\langle K^0|T|\tau^+\rangle \quad (8)$$

$$\mathcal{A}^- = \langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle\tilde{K}_S|T|\tau^-\rangle + \langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle\tilde{K}_L|T|\tau^-\rangle$$

$$= \frac{1}{2q}\left[\langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t} - \langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\right]\langle \bar{K}^0|T|\tau^-\rangle \quad (9)$$

- CP asymmetry in the angular distribution ($d\omega = ds\, dcos\alpha$)

$$A^i_{CP}(t_1,t_2) = \frac{\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\cos\alpha\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt - \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}{\frac{1}{2}\int_{s_{1,i}}^{s_{2,i}}\int_{-1}^{1}\left[\frac{d\Gamma^{\tau^-}}{d\omega}\int_{t_1}^{t_2}F(t)\bar{\Gamma}_{\pi^+\pi^-}(t)\,dt + \frac{d\Gamma^{\tau^+}}{d\omega}\int_{t_1}^{t_2}F(t)\Gamma_{\pi^+\pi^-}(t)\,dt\right]d\omega}$$

$$= \frac{\left(\langle\cos\alpha\rangle_i^{\tau^-} - \langle\cos\alpha\rangle_i^{\tau^+}\right) - \left(\langle\cos\alpha\rangle_i^{\tau^-} + \langle\cos\alpha\rangle_i^{\tau^+}\right)A^K_{CP}(t_1,t_2)}{1 - A^K_{CP}(t_1,t_2)\cdot A^{\tau,i}_{CP}}, \quad (10)$$

$$\langle \cos\alpha \rangle_i^{\tau^-} + \langle \cos\alpha \rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}, \quad (11)$$

$$\langle \cos\alpha \rangle_i^{\tau^-} - \langle \cos\alpha \rangle_i^{\tau^+} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \cos\alpha \left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}{\frac{1}{2} \int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}, \quad (12)$$

$$A_{CP}^{\tau,i} = \frac{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d\Gamma^{\tau^-}}{d\omega} - \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}{\int_{s_{1,i}}^{s_{2,i}} \int_{-1}^{1} \left[\frac{d\Gamma^{\tau^-}}{d\omega} + \frac{d\Gamma^{\tau^+}}{d\omega} \right] d\omega}, \quad (13)$$

$$A_{CP}^{K}(t_1, t_2) = \frac{\int_{t_1}^{t_2} dt\, F(t) \left[\Gamma(K^0(t) \to \pi^+\pi^-) - \Gamma(\bar{K}^0(t) \to \pi^+\pi^-) \right]}{\int_{t_1}^{t_2} dt\, F(t) \left[\Gamma(K^0(t) \to \pi^+\pi^-) + \Gamma(\bar{K}^0(t) \to \pi^+\pi^-) \right]} \quad (14)$$

The SM prediction

- $\mathcal{A}(\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0 \pi^- \nu_\tau)$

$$\frac{d\Gamma^{\tau^+}}{d\omega} = \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A^{CP}_{\tau,i} = 0\,, \quad \langle \cos\alpha \rangle_i^{\tau^-} = \langle \cos\alpha \rangle_i^{\tau^+}$$

$$\Downarrow$$

$$A_i^{CP}(t_1, t_2) = -2\,\langle \cos\alpha \rangle_i^{\tau^-} A_K^{CP}(t_1, t_2) \tag{15}$$

- $A_K^{CP}(t_1, t_2)$: CPV in $K^0 - \bar{K}^0$ mixing

$$A_K^{CP}(t_1 \ll \Gamma_S^{-1}, \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}) \approx 2\Re e(\epsilon_K) = 3.32 \pm 0.06 \times 10^{-3}$$

$$F(t) = \begin{cases} 1 & t_1 < t < t_2 \\ 0 & \text{otherwise.} \end{cases}$$

Y. Grossman and Y. Nir, JHEP 04 (2012) 002

- $\langle \cos\alpha \rangle^{\tau^-} = \frac{2}{3} \mathrm{A}_{\mathrm{FB}}^{\tau^-}$ L. Beldjoudi and T. N. Truong, PLB 351 (1995) 357368

$$A_{\mathrm{FB}}^{\tau^-}(s) = \frac{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha - \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha}{\int_0^1 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha + \int_{-1}^0 \frac{d^2\Gamma^{\tau^-}}{ds\, d\cos\alpha} d\cos\alpha} \tag{16}$$

The NP prediction

- $\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) \neq \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau)$

$$\frac{d\Gamma^{\tau^+}}{d\omega} \neq \frac{d\Gamma^{\tau^-}}{d\omega} \Longrightarrow A^{CP}_{\tau,i} \neq 0\,, \quad \langle \cos\alpha\rangle_i^{\tau^-} \neq \langle \cos\alpha\rangle_i^{\tau^+}$$

$$\Downarrow$$

$$A^i_{CP}(t_1,t_2) \simeq \left(\langle \cos\alpha\rangle_i^{\tau^-} - \langle \cos\alpha\rangle_i^{\tau^+}\right) - \left(\langle \cos\alpha\rangle_i^{\tau^-} + \langle \cos\alpha\rangle_i^{\tau^+}\right) A^K_{CP}(t_1,t_2) \quad (17)$$

Model-independent analysis

- $SU(3)_C \times U(1)_{em}$ invariant low-energy effective Lagrangian

$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{us}}{\sqrt{2}} \Big\{ \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\,[\gamma^\mu - (1-2\,\hat{\epsilon}_R)\gamma^\mu\gamma_5]\,s$$
$$+\,\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}\,[\hat{\epsilon}_S - \hat{\epsilon}_P\gamma_5]\,s + 2\,\hat{\epsilon}_T\,\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}s\Big\} + \mathrm{h.c.}\,, \quad (18)$$

- The $\tau^- \to \overline{K}^0\pi^-\nu_\tau$ decay amplitude

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_S + \mathcal{M}_T$$
$$= \frac{G_F V_{us}}{\sqrt{2}}\left[L_\mu H^\mu + \hat{\epsilon}_S^* LH + 2\hat{\epsilon}_T^* L_{\mu\nu} H^{\mu\nu}\right] \quad (19)$$

- Leptonic currents and hadronic matrix elements

$$L = \overline{u}_{\nu_\tau}\left(p'\right)(1+\gamma_5)\,u_\tau(p)$$
$$L_\mu = \overline{u}_{\nu_\tau}\left(p'\right)\gamma_\mu\,(1-\gamma_5)\,u_\tau(p)$$
$$L_{\mu\nu} = \overline{u}_{\nu_\tau}\left(p'\right)\sigma_{\mu\nu}\,(1+\gamma_5)\,u_\tau(p)$$
$$H = \langle \pi^-\overline{K}^0 \mid \bar{s}u \mid 0\rangle = F_S(s)$$
$$H^\mu = \langle \pi^-\overline{K}^0 \mid \bar{s}\gamma^\mu u \mid 0\rangle = Q^\mu F_+(s) + \frac{\Delta_{K\pi}}{s} q^\mu F_0(s)$$
$$H^{\mu\nu} = \langle \pi^-\overline{K}^0 \mid \bar{s}\sigma^{\mu\nu} u \mid 0\rangle = iF_T(s)\left(p_K^\mu p_\pi^\nu - p_\pi^\mu p_K^\nu\right)$$

Form factors

- Vector form factor
  D.R. Boito, R. Escribano and M. Jamin, Eur. Phys. J. C59 (2009) 821

$$F_+(s) = \exp\left\{\lambda'_+ \frac{s}{M_{\pi^-}^2} + \frac{1}{2}(\lambda''_+ - \lambda'^2_+)\frac{s^2}{M_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{cut}} ds' \frac{\delta_+(s')}{(s')^3(s'-s-i\epsilon)}\right\},$$

- Scalar form factor
  M. Jamin, J.A. Oller and A. Pich, Nucl. Phys. B622 (2002) 279

$$F_0^1(s) = \frac{1}{\pi}\sum_{j=1}^{3}\int_{s_j}^{\infty} ds' \frac{\sigma_j(s') F_0^j(s') t_0^{1\to j}(s')^*}{s'-s-i\epsilon}, (\, 1 \equiv K\pi,\ 2 \equiv K\eta,\ \text{and } 3 \equiv K\eta')$$

- Tensor form factor
  F.-Z. Chen, X.-Q. Li, Y.-D. Yang and X. Zhang, Phys. Rev. D100 (2019) 113006

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s'-s-i\epsilon)}\right\},$$

$$A^i_{CP} \simeq \Delta_{K\pi}\, S_{\rm EW}\, \frac{N_s}{n_i} \int_{s_{1,i}}^{s_{2,i}} \Bigg\{ -\frac{{\rm Im}[\hat{\epsilon}_S]}{m_\tau(m_s - m_u)}\, {\rm Im}\left[F_+(s)F_0^*(s)\right] - \frac{2{\rm Im}[\hat{\epsilon}_T]}{m_\tau}\, {\rm Im}\left[F_T(s)F_0^*(s)\right]$$
$$+ \left[\left(\frac{1}{s} + \frac{{\rm Re}[\hat{\epsilon}_S]}{m_\tau(m_s - m_u)}\right) {\rm Re}\left[F_+(s)F_0^*(s)\right] - \frac{2{\rm Re}[\hat{\epsilon}_T]}{m_\tau}\, {\rm Re}[F_T(s)F_0^*(s)]\right] A^{CP}_K \Bigg\} C(s)\, ds\,. \tag{20}$$

${\rm Re}[\hat{\epsilon}_S] = (0.8^{+0.8}_{-0.9} \pm 0.3)\%$, ${\rm Re}[\hat{\epsilon}_T] = (0.9 \pm 0.7 \pm 0.4)\%$

S. Gonzàlez-Solís, A. Miranda, J. Rendón and P. Roig, Phys. Lett. B 804 (2020) 135371

Table: The SM predictions vs. the Belle measurements

$\sqrt{s}$ [GeV]	$A^{CP}_{\mathrm{SM},i}$ $[10^{-3}]$	$A^{CP}_{\mathrm{exp},i}$ $[10^{-3}]$	n_i/N_s [%]
$0.625 - 0.890$	0.39 ± 0.01	$7.9 \pm 3.0 \pm 2.8$	36.53 ± 0.14
$0.890 - 1.110$	0.04 ± 0.01	$1.8 \pm 2.1 \pm 1.4$	57.85 ± 0.15
$1.110 - 1.420$	0.12 ± 0.02	$-4.6 \pm 7.2 \pm 1.7$	4.87 ± 0.04
$1.420 - 1.775$	0.27 ± 0.05	$-2.3 \pm 19.1 \pm 5.5$	0.75 ± 0.02

$\chi^2_{\min} = 4.20, \quad \mathrm{Im}[\hat{\epsilon}_S] = -0.008 \pm 0.027, \quad \mathrm{Im}[\hat{\epsilon}_T] = 0.03 \pm 0.12$

$$\chi^2 = \sum_{i=1}^{4} \left(\frac{A^{CP}_{\exp,i} - A^{CP}_{\mathrm{th},i}}{\sigma_i} \right)^2 + \left(\frac{\mathcal{B}^{\tau^-}_{\exp} - \mathcal{B}^{\tau^-}_{\mathrm{th}}}{\sigma_{\mathcal{B}}} \right)^2 , \tag{21}$$

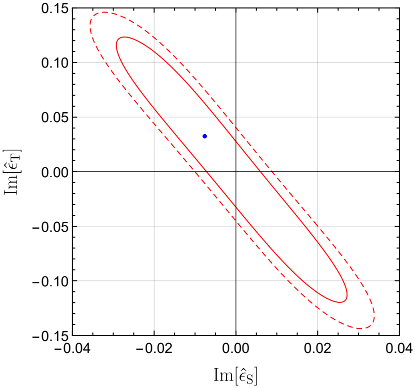


Figure: The allowed region for $\mathrm{Im}[\hat{\epsilon}_\mathrm{S}]$ and $\mathrm{Im}[\hat{\epsilon}_\mathrm{T}]$ at 68% C.L. (solid) and 90% C.L. (dashed).

SM vs. NP

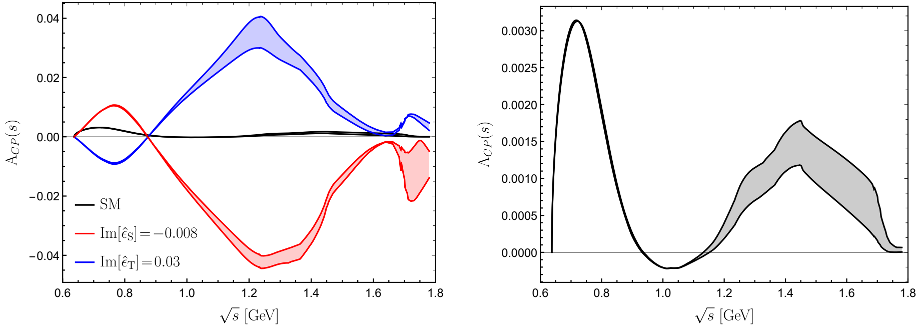


Figure: Left: SM prediction (gray band), $\mathrm{Im}[\hat{\epsilon}_S] = -0.008$ (red band), and $\mathrm{Im}[\hat{\epsilon}_T] = 0.03$ (blue band). Right: The zoomed-in version of the SM prediction.

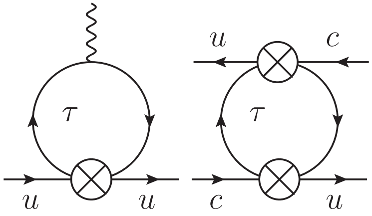
Other bounds on $\mathrm{Im}[\hat{\epsilon}_T]$

- $SU(3) \times SU(2) \times U(1)$ gauge-invariant SMEFT operator:

$$\mathcal{L}_T = [C^{(3)}_{\ell equ}]_{klmn} (\bar{\ell}^i_{Lk} \sigma_{\mu\nu} e_{Rl}) \epsilon^{ij} (\bar{q}^j_{Lm} \sigma^{\mu\nu} u_{Rn}) + \mathrm{h.c.}\,, \tag{22}$$

  In the mass basis

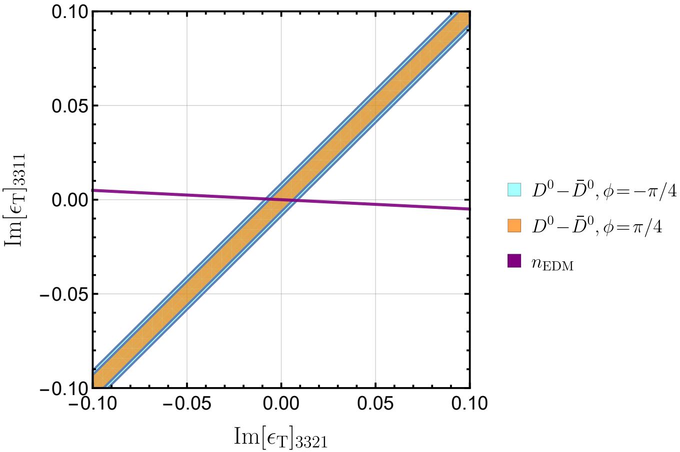
$$\mathcal{L}'_T = [C^{(3)}_{\ell equ}]_{klmn} \left[(\bar{\nu}_k \sigma_{\mu\nu} e_{Rl})(\bar{d}_{Lm} \sigma^{\mu\nu} u_{Rn}) - V_{am} (\bar{e}_{Lk} \sigma_{\mu\nu} e_{Rl})(\bar{u}_{La} \sigma^{\mu\nu} u_{Rn}) \right] + \mathrm{h.c.}\,,$$



- One-operator-at-a-time constraint from neutron EDM
  V. Cirigliano *at al*, PRL 120 (2018) 141803

$$2|\mathrm{Im}[\hat{\epsilon}_T]| < 10^{-5} \Longrightarrow A^i_{CP}|_{\mathrm{Max}} \sim \mathcal{O}(10^{-6})$$

- Cancellations occur in combinations $V_{ud}[C^{(3)}_{\ell equ}]_{3311} + V_{us}[C^{(3)}_{\ell equ}]_{3321}$ and $V_{cd}[C^{(3)}_{\ell equ}]_{3311} + V_{cs}[C^{(3)}_{\ell equ}]_{3321}$



Other bounds on $\mathrm{Im}[\hat{\epsilon}_S]$

- $SU(3) \times SU(2) \times U(1)$ gauge-invariant SMEFT operator:

$$\mathcal{L}_S = [C^{(1)}_{\ell equ}]_{klmn}(\bar{\ell}^i_{Lk} e_{Rl})\epsilon^{ij}(\bar{q}^j_{Lm} u_{Rn}) + [C_{\ell edq}]_{klmn}(\bar{\ell}^i_{Lk} e_{Rl})(\bar{d}^i_{Rm} q_{Ln}) + \mathrm{h.c.}\,, \tag{23}$$

In the mass basis

$$\begin{aligned}\mathcal{L}'_S = [C^{(1)}_{\ell equ}]_{klmn} &\left[(\bar{\nu}_k e_{Rl})(\bar{d}_{Lm} u_{Rn}) - V_{am}(\bar{e}_{Lk} e_{Rl})(\bar{u}_{La} u_{Rn})\right] \\ + [C_{\ell edq}]_{klmn} &\left[V^*_{an}(\bar{\nu}_k e_{Rl})(\bar{d}_{Rm} u_{La}) + (\bar{e}_{Lk} e_{Rl})(\bar{d}_{Rm} d_{Ln})\right] + \mathrm{h.c.}\,,\end{aligned}$$

- One-operator-at-a-time constraint from $D^0 - \bar{D}^0$ mixing

$$\mathrm{Im}[\hat{\epsilon}_S] \in [-3.1, 1.6] \times 10^{-4} \Longrightarrow A^i_{CP}|_{\mathrm{Max}} \sim \mathcal{O}(10^{-3}) \sim A^{CP}_{SM} \tag{24}$$

Summary

- CP asymmetry in the angular distributions of $\tau \to K_S \pi \nu_\tau$ decays can be induced by CPV in $K^0 - \bar{K}^0$ mixing, the SM predictions are of $\mathcal{O}(10^{-3})$;
- A model-independent analysis suggests that either (nonstandard) scalar or tensor interaction could produce CPV to $\tau \to K_S \pi \nu_\tau$ decays;
- It is difficult for NP contributions to play significant effects to the CP asymmetry in the angular distribution of $\tau \to K_S \pi \nu_\tau$ decays, unless there exists cancellation effects in the combined contributions.

(Interested readers are referred to JHEP 05 (2020) 151 and arXiv: hep-ph/2107.12310)