

Studying some baryon decays with the $SU(3)$ flavor symmetry

Ru-Min Wang

Ru-Min Wang, Mao-Zhi Yang, Hai-Bo Li, Xiao-Dong Cheng, Phys. Rev. D 100, 076008 (2019).

Ru-Min Wang, Yuan-Guo Xu, Chong Hua, Xiao-Dong Cheng, Phys. Rev. D 103, 013007 (2021).

JiangXi Normal University

Outline

1. Motivation

2. Hyperon nonleptonic decays

3. Baryon semileptonic decays $B_1\left(\frac{1}{2}^+\right) \rightarrow B_2\left(\frac{1}{2}^+\right) \ell^+ \ell^-$

4. Conclusion

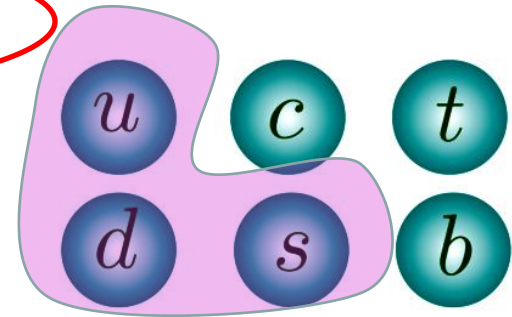
1. Motivation

- ✓ Baryon decays are **different** to the meson ones.
- ✓ Baryon decays provide the important **additional tests** of the SM predictions.
- ✓ A large number of bottomed baryons, charmed baryon and hyperons are produced at the LHC, BESIII and Belle-II.
- ✓ Theoretical calculations of the baryon transitions are not well understood.
- ✓ Symmetries (for an example, **SU(3) flavor symmetry**) provide very important information for particle physics.

SU(3) flavor symmetry approach

✓ Irreducible representation approach ^{IRA}

✓ Topological diagram approach



✓ **Advantage:** Independent of the detailed dynamics

✓ **Disadvantage:** it can not determine the sizes of the amplitudes by itself.

Meson decays with the SU(3) flavor symmetry

- ✓ Hai-Yang Cheng et. al., *JHEP* 09, 024 (2011) ; *PRD* 86, 014014 (2012); *PRD* 91, 014011 (2015) .
- ✓ Xiao-Gang He et al., *EPJC* 80, 359 (2020); *PRD* 93, 114002 (2016); *PRD* 92, 036010 (2015); *JHEP* 08, 065 (2013); *PRD* 69, 074002 (2004); *PRD* 64,034002(2001); *EPJC* 9, 443 (1999);. *PRL* 75, 1703 (1995).
- ✓ Cai-Dian Lu et al., *EPJC* 77, 125 (2017);
- ✓ M. Gronau, *PRD* 52, 6356 (1995);
- ✓ S. Shivashankara, *PRD* 91, 115003 (2015).
- ✓ D.Pirskhalava, *PLB* 712, 81 (2012).
- ✓

Baryon decays with the SU(3) flavor symmetry

baryons decays	light baryons	charmed baryons	bottom baryons
nonleptonic weak decays		① C D Lu et al., PRD 93, 056008 (2016); ② W Wang et al., EPJC 77, 800 (2017) . ③ C. Q Geng et al., PRD97,073006(2018); PLB790,225(2019);PLB776,265,(2018); EPJC,78, 593(2018). ④ X G He, et al., EPJC 80, 359 (2020). ⑤ R C Verma et al., PRD 53, 3723 (1996); PRD 55, 7067 (1997). ⑥ M.J.Savage, PLB 257, 414 (1991); PRD 42, 1527 (1990). ⑦	① Xiao-Gang He et al., PLB 750 , 82 (2015); ② Wei Wang et al., EPJC 77, 800 (2017) ③ Yu-Ji Shi et al., EPJC 78, 56 (2018); ④ A. Dery et al., JHEP 03 (2020) 165; ⑤ D. Wang, EPJC 79, 429 (2019). ⑥
$B_1 \rightarrow B_2 \ell \nu_\ell$	① N Cabibbo et al, hep-ph/0307298 ② R.F. Mendieta et al, PRD58,094028(1998). ③ T. N. Pham, PRD 87, 016002(2013). ④ T. Ledwig et. al., PRD 90, 054502(2014). ⑤ A. Kadeer et. al., EPJC 59, 27(2009). ⑥ S.Sasaki, PRD 79, 074508(2009). ⑦	① Wei Wang et al., EPJC 77, 800 (2017) ② D. Wang, EPJC 79, 429 (2019).	① D. Wang, EPJC 79, 429 (2019).
$B_1 \rightarrow B_2 \ell^+ \ell^-$			
$B_1 \rightarrow B_2 \gamma$			
EM decays			

We have studied the following baryon decays with the SU(3) flavor symmetry.

- ✓ Two-body nonleptonic weak decays of hyperons
- ✓ Semi-leptonic weak baryon decays $B_1\left(\frac{1}{2}^+\right) \rightarrow B_2\left(\frac{1}{2}^+\right)\ell^+\ell^-$
- ✓ Radiative baryon decays $B_1\left(\frac{1}{2}^+, \frac{3}{2}^+\right) \rightarrow B_2\left(\frac{1}{2}^+, \frac{3}{2}^+\right)\gamma$
- ✓ Electromagnetic baryon decays

2. Hyperon nonleptonic decays

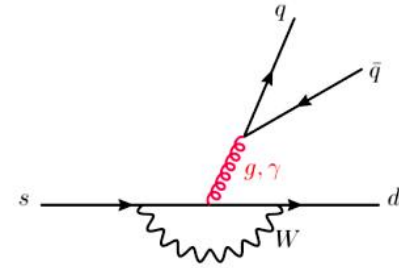
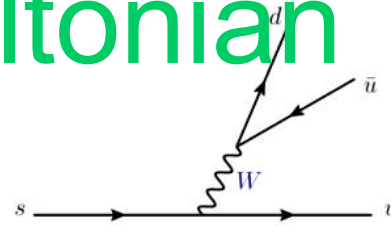
2.1 Effective Hamiltonian

2.2 $T_8 \rightarrow T_8' M_8$ weak decays

2.3 $T_{10} \rightarrow T_8 M_8$ weak decays

2.4 $T_{10} \rightarrow T_8 M_8$ ES decays

2.1 Effective Hamiltonian



$$\mathcal{H}_{eff}^{IR} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ C_+ \left[2H(4) + \frac{1}{3} \left(H(\bar{2}_t) + H(\bar{2}_p) \right) \right] + C_- \left(H(\bar{2}_t) - H(\bar{2}_p) \right) \right\},$$

dominant

$$C_{\pm} \equiv (C_2 \pm C_1)/2$$

$$C_+^2/C_-^2 \approx 13.7\%$$

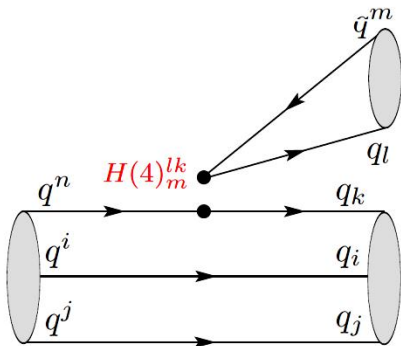
H_k^{ij} is related to $(\bar{q}_i q^k)(\bar{q}_j s)$ operators $(\bar{2} \otimes 2 \otimes \bar{2})s = (\bar{2}_p \oplus \bar{2}_t \oplus 4)s$

Non-Zero H: $H(\bar{2}_p)^2 = H(\bar{2}_t)^2 = 1, \quad H(4)_1^{12} = H(4)_1^{21} = \frac{1}{3}.$

$$A(T_{8,10} \rightarrow T_8 M_8) = \langle T_8 M_8 | \mathcal{H}_{eff}^{IR} | T_{8,10} \rangle = A(\mathcal{O}_4) + A(\mathcal{O}_{\bar{2}}).$$

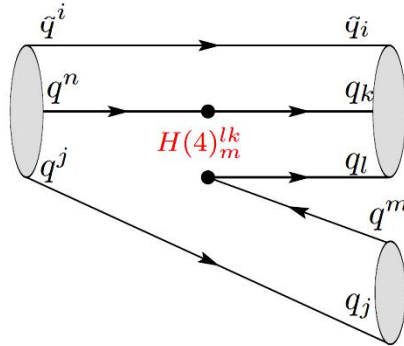
2.2 $T_8 \rightarrow T_8 M_8$ weak decays

Feynman diagrams of IRA:



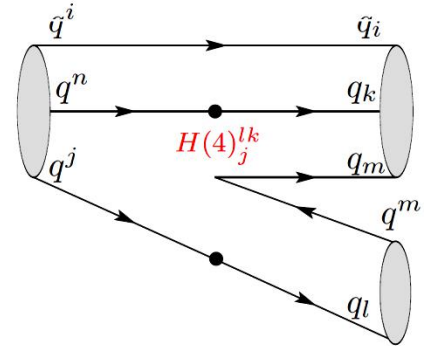
$T_8 \rightarrow T_8 M_8 : a_1, \dots, 6$

$T_{10} \rightarrow T_8 M_8 : \bar{a}_1$



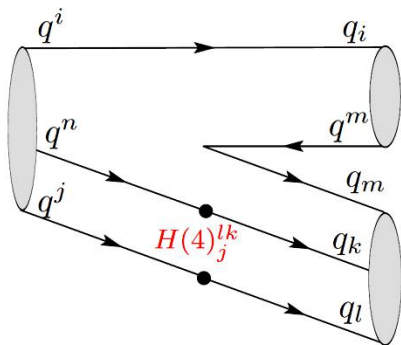
$T_8 \rightarrow T_8 M_8 : b_1, \dots, 9$

$T_{10} \rightarrow T_8 M_8 : \bar{b}_{1,2,3}$



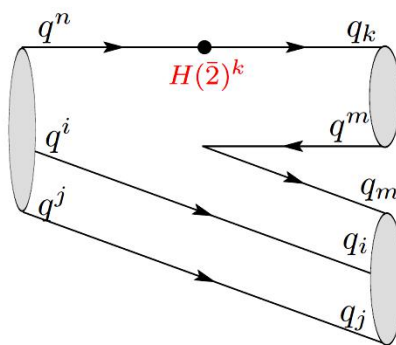
$T_8 \rightarrow T_8 M_8 : c_1, \dots, 9$

$T_{10} \rightarrow T_8 M_8 : \bar{c}_{1,2,3}$



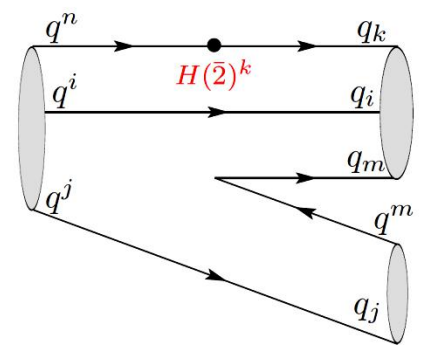
$T_8 \rightarrow T_8 M_8 : d_1, \dots, 9$

$T_{10} \rightarrow T_8 M_8 : \bar{d}_{1,2,3}$



$T_8 \rightarrow T_8 M_8 : e_1, \dots, 9$

$T_{10} \rightarrow T_8 M_8 : \bar{e}_1$



$T_8 \rightarrow T_8 M_8 : f_1, \dots, 9$

$T_{10} \rightarrow T_8 M_8 : \bar{f}_{1,2,3}$

Mesons ($q\bar{q}$)

$$P_8 = \begin{pmatrix} \frac{\eta_8}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}$$

$$V_8 = \begin{pmatrix} \frac{\omega_8}{\sqrt{6}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_8}{\sqrt{6}} - \frac{\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega_8 \end{pmatrix}$$

$$M_8 \quad (M_j^i)$$

$i = 1, 2, 3$ for u, d, s

Baryons (qqq)

$$T_8 = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}, \quad (T_8)_{[ij]k} \quad (T_{10})_{ijk}$$

$$T_{10} = \frac{1}{\sqrt{3}} \left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma^{*+} \\ \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma^{*-} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \\ \Xi^{*-} & \Xi^{*-} & \sqrt{3}\Omega^- \end{pmatrix} \right).$$

$$T_{c3} = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+),$$

$$(T_{c3})_i$$

$$T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c^{*+} \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c^{*0} \\ \frac{1}{\sqrt{2}}\Xi_c^{*+} & \frac{1}{\sqrt{2}}\Xi_c^{*0} & \Omega_c \end{pmatrix}$$

$$(T_{c6})_{ij}$$

$$T_{b3} = (\Xi_b^-, -\Xi_b^0, \Lambda_b^0),$$

$$(T_{b3})_i$$

$$T_{b6} = \begin{pmatrix} \Sigma_b^+ & \frac{1}{\sqrt{2}}\Sigma_b^0 & \frac{1}{\sqrt{2}}\Xi_b^{*0} \\ \frac{1}{\sqrt{2}}\Sigma_b^0 & \Sigma_b^- & \frac{1}{\sqrt{2}}\Xi_b^{*-} \\ \frac{1}{\sqrt{2}}\Xi_b^{*0} & \frac{1}{\sqrt{2}}\Xi_b^{*-} & \Omega_b \end{pmatrix}$$

$$(T_{b6})_{ij}$$

SU(3) IRA amplitudes:

34 terms

$$\begin{aligned} \underline{A(T_8 \rightarrow T_8 M_8)^{IRA,J}} = & a_1 H(4)_m^{lk}(T_8)^{[ij]n}(T_8)_{[ij]k}(M_8)_l^m + a_2 H(4)_m^{lk}(T_8)^{[ij]n}(T_8)_{[ik]j}(M_8)_l^m \\ & + a_4 H(4)_m^{lk}(T_8)^{[in]j}(T_8)_{[ij]k}(M_8)_l^m + a_5 H(4)_m^{lk}(T_8)^{[in]j}(T_8)_{[ik]j}(M_8)_l^m + a_6 H(4)_m^{lk}(T_8)^{[in]j}(T_8)_{[jk]i}(M_8)_l^m \\ & + b_1 H(4)_m^{lk}(T_8)^{[ij]n}(T_8)_{[il]k}(M_8)_j^m + b_4 H(4)_m^{lk}(T_8)^{[in]j}(T_8)_{[il]k}(M_8)_j^m + b_7 H(4)_m^{lk}(T_8)^{[jn]i}(T_8)_{[il]k}(M_8)_j^m \\ & + c_1 H(4)_j^{lk}(T_8)^{[ij]n}(T_8)_{[ki]m}(M_8)_l^m + c_2 H(4)_j^{lk}(T_8)^{[ij]n}(T_8)_{[km]i}(M_8)_l^m + c_3 H(4)_j^{lk}(T_8)^{[ij]n}(T_8)_{[im]k}(M_8)_l^m \\ & + c_4 H(4)_j^{lk}(T_8)^{[in]j}(T_8)_{[ki]m}(M_8)_l^m + c_5 H(4)_j^{lk}(T_8)^{[in]j}(T_8)_{[km]i}(M_8)_l^m + c_6 H(4)_j^{lk}(T_8)^{[in]j}(T_8)_{[im]k}(M_8)_l^m \\ & + c_7 H(4)_j^{lk}(T_8)^{[jn]i}(T_8)_{[ki]m}(M_8)_l^m + c_8 H(4)_j^{lk}(T_8)^{[jn]i}(T_8)_{[km]i}(M_8)_l^m + c_9 H(4)_j^{lk}(T_8)^{[jn]i}(T_8)_{[im]k}(M_8)_l^m \\ & + d_1 H(4)_j^{lk}(T_8)^{[ij]n}(T_8)_{[mk]l}(M_8)_i^m + d_4 H(4)_j^{lk}(T_8)^{[in]j}(T_8)_{[mk]l}(M_8)_i^m + d_7 H(4)_j^{lk}(T_8)^{[jn]i}(T_8)_{[mk]l}(M_8)_i^m \\ & + e_1 H(\bar{2})^k(T_8)^{[ij]n}(T_8)_{[ij]m}(M_8)_k^m + e_2 H(\bar{2})^k(T_8)^{[ij]n}(T_8)_{[im]j}(M_8)_k^m \\ & + e_4 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[ij]m}(M_8)_k^m + e_5 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[im]j}(M_8)_k^m + e_6 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[jm]i}(M_8)_k^m \\ & + f_1 H(\bar{2})^k(T_8)^{[ij]n}(T_8)_{[ki]m}(M_8)_j^m + f_2 H(\bar{2})^k(T_8)^{[ij]n}(T_8)_{[km]i}(M_8)_j^m + f_3 H(\bar{2})^k(T_8)^{[ij]n}(T_8)_{[im]k}(M_8)_j^m \\ & + f_4 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[ki]m}(M_8)_j^m + f_5 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[km]i}(M_8)_j^m + f_6 H(\bar{2})^k(T_8)^{[in]j}(T_8)_{[im]k}(M_8)_j^m \\ & + f_7 H(\bar{2})^k(T_8)^{[jn]i}(T_8)_{[ki]m}(M_8)_j^m + f_8 H(\bar{2})^k(T_8)^{[jn]i}(T_8)_{[km]i}(M_8)_j^m + f_9 H(\bar{2})^k(T_8)^{[jn]i}(T_8)_{[im]k}(M_8)_j^m. \end{aligned}$$

SU(3) IRA amplitudes of $T_8 \rightarrow T_8 P$:

C+ suppressed

Penguin
suppressed

dominant

Amplitudes	$H(4)_1^{12}$	$H(4)_2^{22}$	$H(\bar{2})^2$
$\sqrt{2}A(\Sigma^+ \rightarrow p\pi^0)$	$-2(a_5 + a_6) - 2(b_4 + b_7) + (c_4 + 2c_5 + c_6) + (c_7 + 2c_8 + c_9) - 2(d_4 + d_7)$	$2(a_5 + a_6)$	$2(e_5 + e_6) + f_4 + f_5 + f_7 + f_8$
$A(\Sigma^+ \rightarrow n\pi^+)$	$(c_4 - c_6 + c_7 - c_9) + 2(d_4 + d_7)$	$-2(b_4 + b_7)$	$f_4 - f_6 + f_7 - f_9$
$A(\Sigma^- \rightarrow n\pi^-)$	$-2(a_5 + a_6) - 2(b_4 + b_7)$	$-(c_5 + c_6 + c_8 + c_9) + 2(d_4 + d_7)$	$-2(e_5 + e_6) - (f_5 + f_6 + f_8 + f_9)$
$\sqrt{2}A(\Sigma^0 \rightarrow p\pi^-)$	$2(a_5 + a_6) - 2(b_4 + b_7) - (c_4 - c_6 + c_7 - c_9) - 2(d_4 + d_7)$	$c_4 + c_5 + c_7 + c_8$	$2(e_5 + e_6) + f_4 + f_5 + f_7 + f_8$
$2A(\Sigma^0 \rightarrow n\pi^0)$	$-2(a_5 + a_6) + 2(b_4 + b_7) + (c_4 + 2c_5 + c_6) + (c_7 + 2c_8 + c_9) - 2(d_4 + d_7)$	$2(a_5 + a_6) + 2(b_4 + b_7) - (c_4 - c_6 + c_7 - c_9) - 2(d_4 + d_7)$	$2(e_5 + e_6) - (f_4 - f_5 + f_7 - f_8)$
$\sqrt{6}A(\Lambda^0 \rightarrow p\pi^-)$	$-4(a_1 + a_2 + a_4) - 2(a_5 - a_6) - 2(2b_1 + b_4 - b_7) + (2c_1 - 2c_3 + c_4 - c_6 - c_7 + c_9) + 2(2d_1 + d_4 - d_7)$	$2c_1 + 2c_2 + c_4 + c_5 - c_7 - c_8$	$-4(e_1 + e_2 + e_4) - 2(e_5 - e_6) + (2f_1 + 2f_2 + f_4 + f_5 - f_7 - f_8)$
$2\sqrt{3}A(\Lambda^0 \rightarrow n\pi^0)$	$-4(a_1 + a_2 + a_4) - 2(a_5 - a_6) - 2(2b_1 + b_4 - b_7) - 2(c_1 + 2c_2 + c_3) - (c_4 + 2c_5 + c_6) + (c_7 + 2c_8 + c_9) + 2(2d_1 + d_4 - d_7)$	$4(a_1 + a_2 + a_4) + 2(a_5 - a_6) + 2(2b_1 + b_4 - b_7) - (2c_1 + 2c_3 - c_4 + c_6 + c_7 - c_9) - 2(2d_1 + d_4 - d_7)$	$4(e_1 + e_2 + e_4) + 2(e_5 - e_6) - (2f_1 + 2f_2 + f_4 + f_5 - f_7 - f_8)$
$\sqrt{6}A(\Xi^- \rightarrow \Lambda^0 \pi^-)$	$2(a_1 + a_2 + a_4 + 2a_5 + a_6)$	$-(c_1 + 2c_2 + c_3) + (c_7 + 2c_8 + c_9)$	$2(e_1 + e_2 + e_4 + 2e_5 + e_6) - (f_1 + 2f_2 + f_3) + (f_7 + 2f_8 + f_9)$
$2\sqrt{3}A(\Xi^0 \rightarrow \Lambda^0 \pi^0)$	$2(a_1 + a_2 + a_4 + 2a_5 + a_6) - 2(c_1 + 2c_2 + c_3) - 2(c_7 + 2c_8 + c_9)$	$-2(a_1 + a_2 + a_4 + 2a_5 + a_6)$	$-2(e_1 + e_2 + e_4 + 2e_5 + e_6) + (f_1 + 2f_2 + f_3) - (f_7 + 2f_8 + f_9)$

IRA amplitudes for $T_8 \rightarrow T_8 V$ weak decays have similar relations. 13

SU(3) IRA amplitudes of $T_8 \rightarrow T_8 P$:

C+ suppressed

penguin
suppressed

dominant

Amplitudes	$H(4)_1^{12}$	$H(4)_2^{22}$	$H(2)^2$	Simplified Amplitudes
$\sqrt{2}A(\Sigma^+ \rightarrow p\pi^0)$	$-\frac{1}{+} A_1 = 2(e_5 + e_6) + (f_4 + f_5 + f_7 + f_8),$		$2(e_5 + e_6) + f_4 + f_5 + f_7 + f_8$	A_1
$A(\Sigma^+ \rightarrow n\pi^+)$	$(\frac{1}{-} A_2 = 2(e_5 + e_6) + (f_5 + f_6 + f_8 + f_9),$		$f_4 - f_6 + f_7 - f_9$	$A_1 - A_2$
$A(\Sigma^- \rightarrow n\pi^-)$	$-\frac{1}{+} A_3 = 2(e_5 + e_6) - (f_4 - f_5 + f_7 - f_8),$		$-2(e_5 + e_6) - (f_5 + f_6 + f_8 + f_9)$	$-A_2$
$\sqrt{2}A(\Sigma^0 \rightarrow p\pi^-)$	$\frac{2}{-} A_4 = 4(e_1 + e_2 + e_4) + 2(e_5 - e_6) - (2f_1 + 2f_2 + f_4 + f_5 - f_7 - f_8),$		$2(e_5 + e_6) + f_4 + f_5 + f_7 + f_8$	A_1
$2A(\Sigma^0 \rightarrow n\pi^0)$	$-\frac{1}{+} A_5 = 2(e_1 + e_2 + e_4 + 2e_5 + e_6) - (f_1 + 2f_2 + f_3) + (f_7 + 2f_8 + f_9),$		$2(e_5 + e_6) - (f_4 - f_5 + f_7 - f_8)$	A_3
$\sqrt{6}A(\Lambda^0 \rightarrow p\pi^-)$	$\frac{-4(a_1 + a_2 + a_4) - 2(a_5 - a_6) - 2(2b_1 + b_4 - b_7)}{+(2c_1 - 2c_3 + c_4 - c_6 - c_7 + c_9) + 2(2d_1 + d_4 - d_7)}$	$2c_1 + 2c_2 + c_4 + c_5 - c_7 - c_8$	$-4(e_1 + e_2 + e_4) - 2(e_5 - e_6) + (2f_1 + 2f_2 + f_4 + f_5 - f_7 - f_8)$	$-A_4$
$2\sqrt{3}A(\Lambda^0 \rightarrow n\pi^0)$	$\frac{-4(a_1 + a_2 + a_4) - 2(a_5 - a_6) - 2(2b_1 + b_4 - b_7)}{-2(c_1 + 2c_2 + c_3) - (c_4 + 2c_5 + c_6) + (c_7 + 2c_8 + c_9) + 2(2d_1 + d_4 - d_7)}$	$\frac{4(a_1 + a_2 + a_4) + 2(a_5 - a_6) + 2(2b_1 + b_4 - b_7)}{(-2c_1 + 2c_3 - c_4 + c_6 + c_7 - c_9) - 2(2d_1 + d_4 - d_7)}$	$\frac{4(e_1 + e_2 + e_4) + 2(e_5 - e_6)}{-2(f_1 + 2f_2 + f_4 + f_5 - f_7 - f_8)}$	A_4
$\sqrt{6}A(\Xi^- \rightarrow \Lambda^0 \pi^-)$	$2(a_1 + a_2 + a_4 + 2a_5 + a_6)$	$-(c_1 + 2c_2 + c_3) + (c_7 + 2c_8 + c_9)$	$\frac{2(e_1 + e_2 + e_4 + 2e_5 + e_6)}{-(f_1 + 2f_2 + f_3) + (f_7 + 2f_8 + f_9)}$	A_5
$2\sqrt{3}A(\Xi^0 \rightarrow \Lambda^0 \pi^0)$	$\frac{2(a_1 + a_2 + a_4 + 2a_5 + a_6)}{2(c_1 + 2c_2 + c_3) - 2(c_7 + 2c_8 + c_9)}$	$-2(a_1 + a_2 + a_4 + 2a_5 + a_6)$	$\frac{-2(e_1 + e_2 + e_4 + 2e_5 + e_6)}{+(f_1 + 2f_2 + f_3) - (f_7 + 2f_8 + f_9)}$	$-A_5$

Branching ratios:

$$\begin{aligned} A(\Sigma^+ \rightarrow p\pi^0) &= -\frac{\sqrt{2}}{3} (A_{\frac{1}{2}} - A_{\frac{3}{2}}), \\ A(\Sigma^+ \rightarrow n\pi^+) &= \frac{1}{3} (2A_{\frac{1}{2}} + A_{\frac{3}{2}}), \\ A(\Sigma^- \rightarrow n\pi^-) &= -\frac{\sqrt{2}}{3} (A_{\frac{1}{2}} - A_{\frac{3}{2}}), \\ A(\Sigma^0 \rightarrow p\pi^-) &= A_{\frac{3}{2}}, \\ A(\Sigma^0 \rightarrow n\pi^0) &= \frac{1}{3} (A_{\frac{1}{2}} + 2A_{\frac{3}{2}}). \end{aligned}$$

Observables	Exp. Data [1]	SU(3) IRA	Isospin Relations
$\mathcal{B}(\Sigma^+ \rightarrow p\pi^0)(\times 10^{-2})$	51.57 ± 0.30	$51.57 \pm 0.30^\ddagger$	$51.57 \pm 0.30^\ddagger$
$\mathcal{B}(\Sigma^+ \rightarrow n\pi^+)(\times 10^{-2})$	48.31 ± 0.30	$48.31 \pm 0.30^\ddagger$	$48.31 \pm 0.30^\ddagger$
$\mathcal{B}(\Sigma^- \rightarrow n\pi^-)(\times 10^{-2})$	99.848 ± 0.005	$99.848 \pm 0.005^\ddagger$	$99.848 \pm 0.005^\ddagger$
$\mathcal{B}(\Sigma^0 \rightarrow p\pi^-)(\times 10^{-10})$	...	4.82 ± 0.49	4.82 ± 0.50
$\mathcal{B}(\Sigma^0 \rightarrow n\pi^0)(\times 10^{-10})$	...	...	2.41 ± 0.27
$\mathcal{B}(\Lambda^0 \rightarrow p\pi^-)(\times 10^{-2})$	63.9 ± 0.5	$64.19 \pm 0.21^\ddagger$	...
$\mathcal{B}(\Lambda^0 \rightarrow n\pi^0)(\times 10^{-2})$	35.8 ± 0.5	$35.42 \pm 0.12^\ddagger$	...
$\mathcal{B}(\Xi^- \rightarrow \Lambda^0\pi^-)(\times 10^{-2})$	99.887 ± 0.035	$99.887 \pm 0.035^\ddagger$	...
$\mathcal{B}(\Xi^0 \rightarrow \Lambda^0\pi^0)(\times 10^{-2})$	99.524 ± 0.012	$80.016 \pm 3.746^\otimes$	...



16% smaller than its data. Neglected C+ term and SU(3) breaking effects might give obvious contribution.

2.3 $T_{10} \rightarrow T_8 M_8$ weak decays

SU(3) IRA amplitudes:

10 terms

$$\begin{aligned} A(T_{10} \rightarrow T_8 M_8)^{IRA,J} = & \bar{a}_1 H(4)_m^{lk} (T_{10})^{nij} (T_8)_{[ik]j} (M_8)_l^m + \bar{b}_2 H(4)_m^{lk} (T_{10})^{nij} (T_8)_{[ki]l} (M_8)_j^m \\ & + \bar{c}_1 H(4)_j^{lk} (T_{10})^{nij} (T_8)_{[km]i} (M_8)_l^m + \bar{c}_2 H(4)_j^{lk} (T_{10})^{nij} (T_8)_{[ki]m} (M_8)_l^m + \bar{c}_3 H(4)_j^{lk} (T_{10})^{nij} (T_8)_{[im]k} (M_8)_j^m \\ & + \bar{d}_1 H(4)_j^{lk} (T_{10})^{nij} (T_8)_{[mk]l} (M_8)_i^m + \bar{e}_1 H(\bar{2})^k (T_{10})^{nij} (T_8)_{[im]j} (M_8)_k^m \\ & + \bar{f}_1 H(\bar{2})^k (T_{10})^{nij} (T_8)_{[ki]m} (M_8)_j^m + \bar{f}_2 H(\bar{2})^k (T_{10})^{nij} (T_8)_{[km]i} (M_8)_j^m + \bar{f}_3 H(\bar{2})^k (T_{10})^{nij} (T_8)_{[im]k} (M_8)_j^m. \end{aligned}$$

SU(3) IRA amplitudes of $T_{10} \rightarrow T_8 P$:

C+ suppressed

Penguin
suppressed

dominant

Amplitudes	$H(4)_1^{12} = \frac{1}{3}$	$H(4)_2^{22} = -\frac{1}{3}$	$H(\bar{2})^2 = 1$	Simplified Amplitudes
$A(\Omega^- \rightarrow \Xi^0 \pi^-)$	$2\bar{a}_1$		$2\bar{e}_1$	$\bar{A}_1$
$\sqrt{2}A(\Omega^- \rightarrow \Xi^- \pi^0)$	$-2\bar{a}_1$	$2\bar{a}_1$	$2\bar{e}_1$	$\bar{A}_2$
$\sqrt{6}A(\Omega^- \rightarrow \Lambda^0 K^-)$			$\bar{f}_1 + 2\bar{f}_2 + \bar{f}_3$	$\bar{A}_3$
$3\sqrt{2}A(\Xi^{*-} \rightarrow \Lambda^0 \pi^-)$	$6\bar{a}_1$	$2\bar{c}_1 + \bar{c}_2 + \bar{c}_3$	$6\bar{e}_1 + \bar{f}_1 + 2\bar{f}_2 + \bar{f}_3$	$3\bar{A}_1 + \bar{A}_3$
$\sqrt{6}A(\Xi^{*-} \rightarrow \Sigma^0 \pi^-)$	$2\bar{a}_1 - 4\bar{b}_2$	$\bar{c}_3 - \bar{c}_2$	$2\bar{e}_1 - \bar{f}_1 + \bar{f}_3$	$\bar{A}_1$
$\sqrt{6}A(\Xi^{*-} \rightarrow \Sigma^- \pi^0)$	$-2\bar{a}_1$	$2\bar{a}_1 - 2\bar{b}_2 + \bar{c}_3 - \bar{c}_2$	$2\bar{e}_1 - \bar{f}_1 + \bar{f}_3$	
$\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^+ \pi^-)$	$2\bar{a}_1 + \bar{c}_3 - \bar{c}_2$		$2\bar{e}_1$	
$\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^- \pi^+)$	$\bar{c}_2 - \bar{c}_3$	$2\bar{b}_2$	$\bar{f}_1 - \bar{f}_3$	
$6A(\Xi^{*0} \rightarrow \Lambda^0 \pi^0)$	$-6\bar{a}_1 + 2(2\bar{c}_1 + \bar{c}_2 + \bar{c}_3)$	$6\bar{a}_1$	$6\bar{e}_1 + \bar{f}_1 + 2\bar{f}_2 + \bar{f}_3$	
$2\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^0 \pi^0)$	$2\bar{a}_1 - 4\bar{b}_2$	$-2\bar{a}_1$	$-2\bar{e}_1 - \bar{f}_1 + \bar{f}_3$	$3\bar{A}_2 + \bar{A}_3$
$\sqrt{3}A(\Xi^{*-} \rightarrow n K^-)$	$2\bar{b}_2$	$2\bar{d}_1$	$-\bar{f}_2 - \bar{f}_3$	
$\sqrt{3}A(\Sigma^{*-} \rightarrow n \pi^-)$	$-2\bar{a}_1 + 2\bar{b}_2$	$-\bar{c}_1 - \bar{c}_3 + 2\bar{d}_1$	$-2\bar{e}_1 - \bar{f}_2 - \bar{f}_3$	

neglecting $H(\bar{4})_2^{22}$ terms and c_i terms in $H(4)_1^{12}$

$$\bar{A}_1 = 2(\bar{a}_1 + \bar{e}_1),$$

$$\bar{A}_2 = 2(-\bar{a}_1 + \bar{e}_1),$$

$$\bar{A}_3 = 2(\bar{f}_1 + 2\bar{f}_2 + \bar{f}_3),$$

Branching ratios:

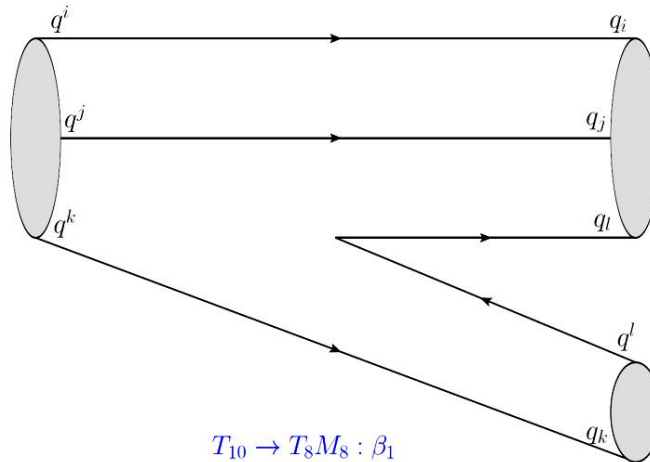
Amplitudes	Exp. data	Simplified Amplitudes
$A(\Omega^- \rightarrow \Xi^0 \pi^-)$	$\mathcal{B}(\Omega^- \rightarrow \Xi^0 \pi^-)(\times 10^{-2}) = (23.6 \pm 0.7) \times 10^{-2},$ $\mathcal{B}(\Omega^- \rightarrow \Xi^- \pi^0)(\times 10^{-2}) = (8.6 \pm 0.4) \times 10^{-2},$ $\mathcal{B}(\Omega^- \rightarrow \Lambda^0 K^-)(\times 10^{-2}) = (67.8 \pm 0.7) \times 10^{-2}.$	$\bar{A}_1$
$\sqrt{2}A(\Omega^- \rightarrow \Xi^- \pi^0)$		$\bar{A}_2$
$\sqrt{6}A(\Omega^- \rightarrow \Lambda^0 K^-)$		$\bar{A}_3$
$3\sqrt{2}A(\Xi^{*-} \rightarrow \Lambda^0 \pi^-)$	$\mathcal{B}(\Xi^{*-} \rightarrow \Lambda^0 \pi^-) = (1.06 \pm 0.90) \times 10^{-12},$ $\mathcal{B}(\Xi^{*0} \rightarrow \Sigma^+ \pi^-) = (5.96 \pm 0.58) \times 10^{-14},$ $\mathcal{B}(\Xi^{*0} \rightarrow \Lambda^0 \pi^0) = (5.02 \pm 4.06) \times 10^{-13},$	$3\bar{A}_1 + \bar{A}_3$
$\sqrt{6}A(\Xi^{*-} \rightarrow \Sigma^0 \pi^-)$		$\bar{A}_1$
$\sqrt{6}A(\Xi^{*-} \rightarrow \Sigma^- \pi^0)$		
$\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^+ \pi^-)$		$3\bar{A}_2 + \bar{A}_3$
$\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^- \pi^+)$		
$6A(\Xi^{*0} \rightarrow \Lambda^0 \pi^0)$		
$2\sqrt{3}A(\Xi^{*0} \rightarrow \Sigma^0 \pi^0)$		
$\sqrt{3}A(\Xi^{*-} \rightarrow n K^-)$		
$\sqrt{3}A(\Sigma^{*-} \rightarrow n \pi^-)$		
$\sqrt{6}A(\Sigma^{*0} \rightarrow p \pi^-)$		
$2\sqrt{3}A(\Sigma^{*0} \rightarrow n \pi^0)$		
$\sqrt{6}A(\Sigma^{*+} \rightarrow p \pi^0)$		

The life times of $\Xi^{*0,-}, \Sigma^{*0,-}, \Delta^{0,-}$ baryons are very small.¹⁸

2.4 $T_{10} \rightarrow T_8 M_8$ electromagnetic or strong decays

($T_8 \rightarrow T_8 M$, $T_{10} \rightarrow T_8 K$ ES decays are not allowed by the phase space.)

Feynman diagrams of IRA:



SU(3) IRA amplitudes:

$$A(T_{10} \rightarrow T_8 M_8)^{ES,IRA} = \beta_1 (T_{10})^{ijk} (T_8)_{[il]j} (M_8)_k^l$$

SU(3) IRA amplitudes:

14 decay modes

Amplitudes	SU(3) IRA amplitudes
$\sqrt{6}A(\Sigma^{*+} \rightarrow \Sigma^0 \pi^+)$	β_1
$\sqrt{6}A(\Sigma^{*+} \rightarrow \Sigma^+ \pi^0)$	β_1
$2\sqrt{6}A(\Sigma^{*0} \rightarrow \Sigma^0 \pi^0)$	0
$\sqrt{6}A(\Sigma^{*0} \rightarrow \Sigma^+ \pi^-)$	β_1
$\sqrt{6}A(\Sigma^{*0} \rightarrow \Sigma^- \pi^+)$	$-\beta_1$
$\sqrt{6}A(\Sigma^{*-} \rightarrow \Sigma^- \pi^0)$	β_1
$\sqrt{6}A(\Sigma^{*-} \rightarrow \Sigma^0 \pi^-)$	β_1
$3\sqrt{2}A(\Sigma^{*+} \rightarrow \Lambda^0 \pi^+)$	$-3\beta_1$
$6\sqrt{2}A(\Sigma^{*0} \rightarrow \Lambda^0 \pi^0)$	$6\beta_1$
$3\sqrt{2}A(\Sigma^{*-} \rightarrow \Lambda^0 \pi^-)$	$3\beta_1$
$\sqrt{6}A(\Xi^{*0} \rightarrow \Xi^0 \pi^0)$	β_1
$\sqrt{3}A(\Xi^{*0} \rightarrow \Xi^- \pi^+)$	$-\beta_1$
$\sqrt{6}A(\Xi^{*-} \rightarrow \Xi^- \pi^0)$	β_1
$\sqrt{3}A(\Xi^{*-} \rightarrow \Xi^0 \pi^-)$	β_1

Branching ratios:

Branching ratios	Exp. data	SU(3) IRA
$\mathcal{B}(\Sigma^{*+} \rightarrow \Sigma^0 \pi^+)(\times 10^{-2})$	...	5.34 ± 0.50
$\mathcal{B}(\Sigma^{*+} \rightarrow \Sigma^+ \pi^0)(\times 10^{-2})$	...	6.59 ± 0.61
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^0 \pi^0)(\times 10^{-2})$	...	0
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^+ \pi^-)(\times 10^{-2})$	...	6.20 ± 0.78
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^- \pi^+)(\times 10^{-2})$	...	4.71 ± 0.59
$\mathcal{B}(\Sigma^{*-} \rightarrow \Sigma^- \pi^0)(\times 10^{-2})$	...	5.40 ± 0.60
$\mathcal{B}(\Sigma^{*-} \rightarrow \Sigma^0 \pi^-)(\times 10^{-2})$	...	5.66 ± 0.63
$\mathcal{B}(\Sigma^* \rightarrow \Sigma \pi)(\times 10^{-2})$	11.7 ± 1.5	11.24 ± 0.28
$\mathcal{B}(\Sigma^{*+} \rightarrow \Lambda^0 \pi^+)(\times 10^{-2})$	...	86.14 ± 7.62
$\mathcal{B}(\Sigma^{*0} \rightarrow \Lambda^0 \pi^0)(\times 10^{-2})$	...	91.68 ± 11.36
$\mathcal{B}(\Sigma^{*-} \rightarrow \Lambda^0 \pi^-)(\times 10^{-2})$	...	84.44 ± 8.96
$\mathcal{B}(\Sigma^* \rightarrow \Lambda^0 \pi)(\times 10^{-2})$	87.0 ± 1.5	$87.00 \pm 1.50^\dagger$
$\mathcal{B}(\Xi^{*0} \rightarrow \Xi^0 \pi^0)(\times 10^{-2})$	...	48.22 ± 6.55
$\mathcal{B}(\Xi^{*0} \rightarrow \Xi^- \pi^+)(\times 10^{-2})$	...	76.23 ± 10.32
$\mathcal{B}(\Xi^{*-} \rightarrow \Xi^- \pi^0)(\times 10^{-2})$	...	43.05 ± 11.01
$\mathcal{B}(\Xi^{*-} \rightarrow \Xi^0 \pi^-)(\times 10^{-2})$	...	94.33 ± 24.12
$\mathcal{B}(\Xi^* \rightarrow \Xi \pi)(\times 10^{-2})$	1.3σ 100	$131.01 \pm 24.40^\otimes$

effective
constraints

The SU(3) breaking effects
could give visible contributions
to $\Xi^* \rightarrow \Xi \pi$.

Branching ratios:

Branching ratios	Exp. data	SU(3) IRA
$\mathcal{B}(\Sigma^{*+} \rightarrow \Sigma^0 \pi^+)(\times 10^{-2})$	...	5.34 ± 0.50
$\mathcal{B}(\Sigma^{*+} \rightarrow \Sigma^+ \pi^0)(\times 10^{-2})$	...	6.59 ± 0.61
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^0 \pi^0)(\times 10^{-2})$	...	0
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^+ \pi^-)(\times 10^{-2})$	...	6.20 ± 0.78
$\mathcal{B}(\Sigma^{*0} \rightarrow \Sigma^- \pi^+)(\times 10^{-2})$	...	4.71 ± 0.59
$\mathcal{B}(\Sigma^{*-} \rightarrow \Sigma^- \pi^0)(\times 10^{-2})$	...	5.40 ± 0.60
$\mathcal{B}(\Sigma^{*-} \rightarrow \Sigma^0 \pi^-)(\times 10^{-2})$	...	5.66 ± 0.63
$\mathcal{B}(\Sigma^* \rightarrow \Sigma \pi)(\times 10^{-2})$	11.7 ± 1.5	11.24 ± 0.28
$\mathcal{B}(\Sigma^{*+} \rightarrow \Lambda^0 \pi^+)(\times 10^{-2})$	...	86.14 ± 7.62
$\mathcal{B}(\Sigma^{*0} \rightarrow \Lambda^0 \pi^0)(\times 10^{-2})$	...	91.68 ± 11.36
$\mathcal{B}(\Sigma^{*-} \rightarrow \Lambda^0 \pi^-)(\times 10^{-2})$	...	84.44 ± 8.96
$\mathcal{B}(\Sigma^* \rightarrow \Lambda^0 \pi)(\times 10^{-2})$	87.0 ± 1.5	$87.00 \pm 1.50^\ddagger$
$\mathcal{B}(\Xi^{*0} \rightarrow \Xi^0 \pi^0)(\times 10^{-2})$	...	48.22 ± 6.55
$\mathcal{B}(\Xi^{*0} \rightarrow \Xi^- \pi^+)(\times 10^{-2})$	...	76.23 ± 10.32
$\mathcal{B}(\Xi^{*-} \rightarrow \Xi^- \pi^0)(\times 10^{-2})$	...	43.05 ± 11.01
$\mathcal{B}(\Xi^{*-} \rightarrow \Xi^0 \pi^-)(\times 10^{-2})$	...	94.33 ± 24.12
$\mathcal{B}(\Xi^* \rightarrow \Xi \pi)(\times 10^{-2})$	1.3σ 100	$131.01 \pm 24.40^\otimes$

effective
constraints

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the chiral quark soliton model
considering SU(3) breaking effects

Decay modes	Γ	$\Gamma(\text{Exp.})[2]$
$\Delta \rightarrow N\pi$	± 1.31	$116 - 120$
$\Sigma^{*+} \rightarrow \Sigma^0 \pi^+$		
$\Sigma^{*+} \rightarrow \Sigma^+ \pi^0$	36.25 ± 0.42	36.0 ± 0.7
$\Sigma^{*+} \rightarrow \Lambda \pi^+$		
$\Sigma^{*0} \rightarrow \Sigma^0 \pi^0$		
$\Sigma^{*0} \rightarrow \Sigma^+ \pi^-$	37.21 ± 0.69	36 ± 5
$\Sigma^{*0} \rightarrow \Sigma^- \pi^+$		
$\Sigma^{*0} \rightarrow \Lambda \pi^0$		
$\Sigma^{*-} \rightarrow \Sigma^- \pi^0$		
$\Sigma^{*-} \rightarrow \Sigma^0 \pi^-$	38.18 ± 0.48	39.4 ± 2.1
$\Sigma^{*-} \rightarrow \Lambda \pi^-$		
$\Xi^{*0} \rightarrow \Xi^0 \pi^0$	11.26 ± 0.17	9.1 ± 0.5
$\Xi^{*0} \rightarrow \Xi^- \pi^+$		
$\Xi^{*-} \rightarrow \Xi^- \pi^0$	13.01 ± 0.21	$9.9^{+1.7}_{-1.9}$
$\Xi^{*-} \rightarrow \Xi^0 \pi^-$		

3. Baryon semileptonic decays

$$B_1\left(\frac{1}{2}^+\right) \rightarrow B_2\left(\frac{1}{2}^+\right) \ell^+ \ell^-$$

$$3.1 \quad T_{b3} \rightarrow T_8 \ell^+ \ell^-$$

$$3.2 \quad T_{c3} \rightarrow T_8 \ell^+ \ell^-$$

$$3.3 \quad T_8 \rightarrow T'_8 \ell^+ \ell^-$$

3.1 $T_{b3} \rightarrow T_8 \ell^+ \ell^-$

$$H(T_{b3} \rightarrow T_8 \ell^+ \ell^-)_{VA,\lambda}^{L(R),s_p,s_k} = e_1(T_{b3})^{[ij]}T(\bar{3})^k(T_8)_{[ij]k} + e_2(T_{b3})^{[ij]}T(3)^k(T_8)_{[ik]j},$$

Decay modes	$A(T_{b3} \rightarrow T_8 \ell^+ \ell^-)$	
$T_{b3} \rightarrow T_8 \ell^+ \ell^-$ via the $b \rightarrow s \ell^+ \ell^-$ transition:		
$\Lambda_b^0 \rightarrow \Lambda^0 \ell^+ \ell^-$	$-2\lambda_{bs} E/\sqrt{6}$	$E \equiv e_1 + e_2$
$\Lambda_b^0 \rightarrow \Sigma^0 \ell^+ \ell^-$	0	
$\Xi_b^0 \rightarrow \Xi^0 \ell^+ \ell^-$	$-\lambda_{bs} E$	
$\Xi_b^- \rightarrow \Xi^- \ell^+ \ell^-$	$\lambda_{bs} E$	
$T_{b3} \rightarrow T_8 \ell^+ \ell^-$ via the $b \rightarrow d \ell^+ \ell^-$ transition:		
$\Lambda_b^0 \rightarrow n \ell^+ \ell^-$	$\lambda_{bd} E$	
$\Xi_b^0 \rightarrow \Lambda^0 \ell^+ \ell^-$	$-\lambda_{bd} E/\sqrt{6}$	
$\Xi_b^0 \rightarrow \Sigma^0 \ell^+ \ell^-$	$-\lambda_{bd} E/\sqrt{2}$	
$\Xi_b^- \rightarrow \Sigma^- \ell^+ \ell^-$	$\lambda_{bd} E$	

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Branching ratios:

Decay modes	Experimental data [41]	Our results in S_1	Our results in S_2	Other predictions
$b \rightarrow s\ell^+\ell^- :$				
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)(\times 10^{-6})$	1.08 ± 0.28	1.08 ± 0.28	1.08 ± 0.28	1.05 [47]
$\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 \mu^+ \mu^-)(\times 10^{-6})$	...	$1.55^{+0.45}_{-0.43}$	$1.77^{+0.49}_{-0.53}$	relativistic quark model
$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-)(\times 10^{-6})$	...	$1.65^{+0.49}_{-0.46}$	$1.87^{+0.56}_{-0.54}$	
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)(\times 10^{-7})$	...	2.30 ± 0.60	$2.74^{+0.85}_{-0.71}$	2.60 [47]
$\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 \tau^+ \tau^-)(\times 10^{-7})$	...	$3.23^{+0.94}_{-0.89}$	$4.42^{+1.36}_{-1.21}$	Bethe-Salpeter equation approach
$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \tau^+ \tau^-)(\times 10^{-7})$	...	$3.42^{+1.01}_{-0.95}$	$4.76^{+1.44}_{-1.36}$	
$b \rightarrow d\ell^+\ell^- :$				
$\mathcal{B}(\Lambda_b^0 \rightarrow n\mu^+ \mu^-)(\times 10^{-8})$	...	$8.15^{+2.44}_{-2.30}$	$7.77^{+2.42}_{-2.28}$	$\begin{pmatrix} 4.1^{+5.4} \\ -1.2 \\ 3.75 \end{pmatrix}$ [47, 48]
$\mathcal{B}(\Xi_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)(\times 10^{-8})$	...	$1.34^{+0.43}_{-0.39}$	$1.45^{+0.44}_{-0.45}$	Bethe-Salpeter equation approach
$\mathcal{B}(\Xi_b^0 \rightarrow \Sigma^0 \mu^+ \mu^-)(\times 10^{-8})$	...	$3.77^{+1.22}_{-1.10}$	$4.13^{+1.36}_{-1.24}$	
$\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- \mu^+ \mu^-)(\times 10^{-8})$	...	$8.00^{+2.56}_{-2.40}$	$8.61^{+3.06}_{-2.52}$	
$\mathcal{B}(\Lambda_b^0 \rightarrow n\tau^+ \tau^-)(\times 10^{-8})$	...	$2.07^{+0.62}_{-0.58}$	$2.46^{+0.78}_{-0.70}$	$\begin{pmatrix} 2.9^{+3.7} \\ -0.8 \\ 1.21 \end{pmatrix}$ [47, 48]
$\mathcal{B}(\Xi_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)(\times 10^{-9})$	...	$3.42^{+1.11}_{-1.00}$	$4.63^{+1.71}_{-1.35}$	Bethe-Salpeter equation approach
$\mathcal{B}(\Xi_b^0 \rightarrow \Sigma^0 \tau^+ \tau^-)(\times 10^{-9})$	...	$8.97^{+2.91}_{-2.62}$	$12.23^{+4.12}_{-3.62}$	
$\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- \tau^+ \tau^-)(\times 10^{-8})$	...	$1.91^{+0.61}_{-0.57}$	$2.60^{+0.87}_{-0.77}$	

Branching ratios in different q^2 bins:

TABLE III: Branching ratios for $T_{b3} \rightarrow T_8 \mu^+ \mu^-$ weak decays in different q^2 bins with 1σ error in S_1 and S_2 cases (in unit of 10^{-7}).

$[q_{min}^2, q_{max}^2](\text{GeV}^2)$	[0.1, 2.0]	[2.0, 4.3]	[0.1, 4.3]	[4.0, 6.0]	[1.0, 6.0]	[6.0, 8.0]	[4.3, 8.68]	[10.09, 12.86]	[14.18, 16.0]	[0.1, 16.0]	[18.0, 20.0]	[15.0, 20.0]
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)^{Exp.}$	0.71 ± 0.27	$0.28^{+0.28}_{-0.21}$	2.7 ± 2.7	$0.04^{+0.18}_{-0.02}$	$0.47^{+0.31}_{-0.27}$	$0.50^{+0.26}_{-0.24}$	0.5 ± 0.7	2.2 ± 0.6	1.7 ± 0.5	7.0 ± 2.9	2.44 ± 0.57	6.0 ± 1.3
$\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 \mu^+ \mu^-)_{S_1}$	$1.03^{+0.42}_{-0.41}$	$0.41^{+0.43}_{-0.31}$	$3.91^{+4.00}_{-3.85}$	$0.058^{+0.270}_{-0.030}$	$0.68^{+0.47}_{-0.40}$	$0.73^{+0.40}_{-0.36}$	$0.73^{+1.06}_{-1.02}$	$3.21^{+0.97}_{-0.92}$	$2.47^{+0.80}_{-0.77}$	$10.18^{+4.53}_{-4.35}$	$3.23^{+0.85}_{-0.81}$	$8.46^{+2.12}_{-2.01}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 \mu^+ \mu^-)_{S_2}$	$1.14^{+0.49}_{-0.46}$	$0.45^{+0.48}_{-0.34}$	$4.39^{+4.58}_{-4.32}$	$0.065^{+0.302}_{-0.033}$	$0.76^{+0.54}_{-0.45}$	$0.81^{+0.45}_{-0.39}$	$0.81^{+1.20}_{-0.81}$	$3.60^{+1.14}_{-1.06}$	$2.83^{+0.91}_{-0.89}$	$11.46^{+5.29}_{-4.96}$	$3.82^{+1.01}_{-1.03}$	$9.81^{+2.45}_{-2.55}$
$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-)_{S_1}$	$1.09^{+0.46}_{-0.43}$	$0.43^{+0.45}_{-0.32}$	$4.15^{+4.28}_{-4.08}$	$0.062^{+0.289}_{-0.032}$	$0.72^{+0.51}_{-0.42}$	$0.77^{+0.43}_{-0.38}$	$0.77^{+1.13}_{-1.09}$	$3.40^{+1.04}_{-0.99}$	$2.62^{+0.86}_{-0.82}$	$10.80^{+4.89}_{-4.65}$	$3.40^{+0.91}_{-0.87}$	$8.94^{+2.30}_{-2.15}$
$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \mu^+ \mu^-)_{S_2}$	$1.23^{+0.52}_{-0.49}$	$0.48^{+0.51}_{-0.37}$	$4.57^{+4.92}_{-4.50}$	$0.069^{+0.324}_{-0.035}$	$0.81^{+0.58}_{-0.47}$	$0.86^{+0.50}_{-0.42}$	$0.86^{+1.28}_{-0.86}$	$3.84^{+1.17}_{-1.13}$	$3.00^{+0.99}_{-0.94}$	$12.22^{+5.56}_{-5.38}$	$3.98^{+1.14}_{-1.03}$	$10.30^{+2.77}_{-2.56}$
$\mathcal{B}(\Lambda_b^0 \rightarrow n \mu^+ \mu^-)_{S_1}$	$0.047^{+0.020}_{-0.019}$	$0.019^{+0.020}_{-0.014}$	$0.180^{+0.189}_{-0.177}$	$0.0027^{+0.0127}_{-0.0014}$	$0.031^{+0.022}_{-0.019}$	$0.034^{+0.019}_{-0.017}$	$0.034^{+0.050}_{-0.047}$	$0.152^{+0.048}_{-0.045}$	$0.123^{+0.041}_{-0.039}$	$0.482^{+0.225}_{-0.208}$	$0.236^{+0.065}_{-0.062}$	$0.491^{+0.128}_{-0.121}$
$\mathcal{B}(\Lambda_b^0 \rightarrow n \mu^+ \mu^-)_{S_2}$	$0.044^{+0.020}_{-0.018}$	$0.017^{+0.019}_{-0.013}$	$0.169^{+0.176}_{-0.166}$	$0.0025^{+0.0116}_{-0.0012}$	$0.029^{+0.021}_{-0.017}$	$0.030^{+0.017}_{-0.015}$	$0.031^{+0.045}_{-0.031}$	$0.132^{+0.042}_{-0.040}$	$0.107^{+0.036}_{-0.034}$	$0.423^{+0.204}_{-0.181}$	$0.215^{+0.059}_{-0.056}$	$0.445^{+0.117}_{-0.114}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)_{S_1}$	0.008 ± 0.003	0.003 ± 0.003	$0.029^{+0.031}_{-0.029}$	$0.0000^{+0.0025}_{-0.0000}$	$0.005^{+0.004}_{-0.003}$	0.005 ± 0.003	0.005 ± 0.008	0.025 ± 0.008	0.020 ± 0.007	$0.079^{+0.038}_{-0.035}$	$0.039^{+0.012}_{-0.011}$	$0.081^{+0.023}_{-0.021}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)_{S_2}$	$0.008^{+0.004}_{-0.003}$	$0.003^{+0.003}_{-0.002}$	$0.030^{+0.034}_{-0.029}$	$0.0004^{+0.0021}_{-0.0002}$	$0.005^{+0.004}_{-0.003}$	0.005 ± 0.003	$0.005^{+0.008}_{-0.005}$	0.024 ± 0.008	$0.020^{+0.007}_{-0.006}$	$0.077^{+0.039}_{-0.034}$	$0.041^{+0.012}_{-0.011}$	$0.083^{+0.024}_{-0.022}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Sigma^0 \mu^+ \mu^-)_{S_1}$	$0.023^{+0.010}_{-0.009}$	$0.009^{+0.010}_{-0.007}$	$0.087^{+0.093}_{-0.085}$	$0.0013^{+0.0062}_{-0.0013}$	$0.015^{+0.011}_{-0.009}$	$0.016^{+0.010}_{-0.008}$	$0.016^{+0.025}_{-0.023}$	$0.073^{+0.024}_{-0.023}$	$0.058^{+0.022}_{-0.019}$	$0.231^{+0.113}_{-0.103}$	$0.103^{+0.031}_{-0.028}$	$0.225^{+0.063}_{-0.057}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Sigma^0 \mu^+ \mu^-)_{S_2}$	$0.024^{+0.011}_{-0.010}$	$0.010^{+0.011}_{-0.007}$	$0.093^{+0.101}_{-0.092}$	$0.0014^{+0.0065}_{-0.0007}$	$0.016^{+0.012}_{-0.009}$	$0.017^{+0.010}_{-0.009}$	$0.017^{+0.025}_{-0.017}$	$0.075^{+0.025}_{-0.023}$	$0.060^{+0.023}_{-0.020}$	$0.237^{+0.120}_{-0.103}$	$0.114^{+0.034}_{-0.032}$	$0.245^{+0.069}_{-0.064}$
$\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- \mu^+ \mu^-)_{S_1}$	$0.048^{+0.022}_{-0.020}$	$0.019^{+0.021}_{-0.014}$	$0.184^{+0.200}_{-0.181}$	$0.0027^{+0.0133}_{-0.0014}$	$0.032^{+0.024}_{-0.019}$	$0.034^{+0.021}_{-0.017}$	$0.034^{+0.053}_{-0.049}$	$0.155^{+0.053}_{-0.047}$	$0.124^{+0.045}_{-0.041}$	$0.491^{+0.240}_{-0.218}$	$0.220^{+0.066}_{-0.061}$	$0.478^{+0.140}_{-0.122}$
$\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- \mu^+ \mu^-)_{S_2}$	$0.051^{+0.024}_{-0.020}$	$0.020^{+0.023}_{-0.015}$	$0.196^{+0.213}_{-0.193}$	$0.0029^{+0.0140}_{-0.0015}$	$0.034^{+0.026}_{-0.020}$	$0.036^{+0.021}_{-0.018}$	$0.036^{+0.054}_{-0.036}$	$0.159^{+0.056}_{-0.049}$	$0.129^{+0.047}_{-0.043}$	$0.508^{+0.260}_{-0.225}$	$0.243^{+0.077}_{-0.066}$	$0.515^{+0.151}_{-0.130}$

TABLE IV: Branching ratios for $T_{b3} \rightarrow T_8 \tau^+ \tau^-$ weak decays in different q^2 bins with 1σ error in S_1 and S_2 cases (in unit of 10^{-7}).

$[q_{min}^2, q_{max}^2](\text{GeV}^2)$	[14.18, 16.0] in S_1	[14.18, 16.0] in S_2	[18.0, 20.0] in S_1	[18.0, 20.0] in S_2	[15.0, 20.0] in S_1	[15.0, 20.0] in S_2
$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$	0.83 ± 0.25	$0.84^{+0.27}_{-0.25}$	1.51 ± 0.36	1.52 ± 0.37	3.41 ± 0.76	$3.44^{+0.86}_{-0.80}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 \tau^+ \tau^-)$	$1.21^{+0.39}_{-0.38}$	$1.40^{+0.46}_{-0.45}$	$2.00^{+0.53}_{-0.50}$	$2.32^{+0.66}_{-0.59}$	$4.79^{+1.20}_{-1.14}$	$5.65^{+1.43}_{-1.44}$
$\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \tau^+ \tau^-)$	$1.29^{+0.42}_{-0.40}$	$1.49^{+0.50}_{-0.48}$	$2.11^{+0.57}_{-0.54}$	2.50 ± 0.67	$5.07^{+1.30}_{-1.22}$	$5.92^{+1.58}_{-1.48}$
$\mathcal{B}(\Lambda_b^0 \rightarrow n \tau^+ \tau^-)$	$0.060^{+0.020}_{-0.019}$	0.053 ± 0.018	$0.147^{+0.041}_{-0.039}$	$0.133^{+0.038}_{-0.034}$	$0.282^{+0.074}_{-0.070}$	$0.257^{+0.068}_{-0.064}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$	$0.010^{+0.004}_{-0.003}$	$0.0095^{+0.0039}_{-0.0030}$	0.024 ± 0.007	0.026 ± 0.007	$0.047^{+0.013}_{-0.012}$	$0.048^{+0.014}_{-0.013}$
$\mathcal{B}(\Xi_b^0 \rightarrow \Sigma^0 \tau^+ \tau^-)$	$0.029^{+0.011}_{-0.009}$	$0.030^{+0.011}_{-0.010}$	$0.064^{+0.019}_{-0.017}$	$0.071^{+0.021}_{-0.019}$	$0.129^{+0.036}_{-0.033}$	$0.140^{+0.041}_{-0.037}$
$\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- \tau^+ \tau^-)$	$0.061^{+0.022}_{-0.020}$	$0.063^{+0.023}_{-0.021}$	$0.137^{+0.041}_{-0.038}$	$0.152^{+0.047}_{-0.041}$	$0.273^{+0.080}_{-0.070}$	$0.301^{+0.086}_{-0.078}$

Longitudinal polarization fractions and forward-backward asymmetries :

$$F_L(q^2) = \left(|H_{VA,0}^{L,+\frac{1}{2}+\frac{1}{2}}|^2 + |H_{VA,0}^{L,-\frac{1}{2}-\frac{1}{2}}|^2 + |H_{VA,0}^{R,+\frac{1}{2}+\frac{1}{2}}|^2 + |H_{VA,0}^{R,-\frac{1}{2}-\frac{1}{2}}|^2 \right) [H_M(q^2)]^{-1}.$$

$$A_{FB}^\ell(q^2) = \frac{3}{4} \left[\left(|H_{VA,-}^{R,+\frac{1}{2}-\frac{1}{2}}|^2 + |H_{VA,+}^{L,-\frac{1}{2}+\frac{1}{2}}|^2 \right) - \left(|H_{VA,-}^{L,+\frac{1}{2}-\frac{1}{2}}|^2 + |H_{VA,+}^{R,-\frac{1}{2}+\frac{1}{2}}|^2 \right) \right] [H_M(q^2)]^{-1}.$$

TABLE V: Longitudinal polarization fractions and forward-backward asymmetries for $\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-$, $\Lambda^0 \tau^+ \tau^-$ decays in different q^2 bins with 1σ error in S_2 case.

$[q_{min}^2, q_{max}^2](\text{GeV}^2)$	[0.1, 2.0]	[2.0, 4.3]	[0.1, 4.3]	[4.0, 6.0]	[1.0, 6.0]	[6.0, 8.0]	[4.3, 8.68]	[10.09, 12.86]	[14.18, 16.0]	[0.1, 16.0]	[18.0, 20.0]	[15.0, 20.0]	whole q^2 regions
$\overline{f_L}(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)$	$0.64^{+0.04}_{-0.01}$	0.86 ± 0.01	$0.77^{+0.06}_{-0.03}$	0.81 ± 0.01	$0.83^{+0.02}_{-0.01}$	0.73 ± 0.01	0.77 ± 0.02	$0.57^{+0.00}_{-0.02}$	0.46 ± 0.01	$0.66^{+0.02}_{-0.03}$	0.36 ± 0.01	$0.39^{+0.02}_{-0.01}$	0.60 ± 0.02
$\langle f_L \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)$	$0.36^{+0.04}_{-0.02}$	$0.86^{+0.06}_{-0.04}$	$0.42^{+0.04}_{-0.02}$	$0.81^{+0.03}_{-0.02}$	$0.82^{+0.05}_{-0.02}$	0.73 ± 0.01	0.75 ± 0.03	0.56 ± 0.01	0.45 ± 0.01	$0.47^{+0.04}_{-0.02}$	0.36 ± 0.01	$0.40^{+0.01}_{-0.02}$	$0.34^{+0.03}_{-0.02}$
$\overline{A_{FB}^\ell}(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)$	0.12 ± 0.01	$0.05^{+0.00}_{-0.01}$	$0.08^{+0.00}_{-0.01}$	$-0.05^{+0.00}_{-0.01}$	$0.03^{+0.00}_{-0.01}$	$-0.15^{+0.00}_{-0.01}$	$-0.12^{+0.00}_{-0.01}$	-0.29 ± 0.01	-0.36 ± 0.00	-0.15 ± 0.01	$-0.31^{+0.01}_{-0.00}$	$-0.35^{+0.01}_{-0.02}$	$-0.19^{+0.00}_{-0.01}$
$\langle A_{FB}^\ell \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)$	$0.08^{+0.00}_{-0.01}$	$0.06^{+0.00}_{-0.01}$	0.07 ± 0.00	$-0.05^{+0.00}_{-0.01}$	$0.06^{+0.00}_{-0.01}$	$-0.15^{+0.00}_{-0.01}$	$-0.12^{+0.00}_{-0.01}$	$-0.29^{+0.01}_{-0.02}$	$-0.37^{+0.02}_{-0.01}$	$-0.03^{+0.00}_{-0.01}$	-0.30 ± 0.01	$-0.34^{+0.01}_{-0.02}$	$-0.04^{+0.00}_{-0.01}$
$\overline{f_L}(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$									$0.46^{+0.00}_{-0.01}$		$0.36^{+0.01}_{-0.00}$	$0.39^{+0.02}_{-0.01}$	0.42 ± 0.01
$\langle f_L \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$									0.46 ± 0.01		0.36 ± 0.01	$0.39^{+0.02}_{-0.01}$	$0.40^{+0.03}_{-0.01}$
$\overline{A_{FB}^\ell}(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$									-0.36 ± 0.00		-0.31 ± 0.01	$-0.34^{+0.01}_{-0.02}$	-0.34 ± 0.01
$\langle A_{FB}^\ell \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-)$									$-0.36^{+0.01}_{-0.02}$		-0.30 ± 0.01	$-0.34^{+0.01}_{-0.02}$	$-0.33^{+0.02}_{-0.01}$

LHCb data are not used:

$$\langle f_L \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)_{[15,20]} = 0.61^{+0.11}_{-0.14},$$

$$\langle A_{FB}^\ell \rangle(\Lambda_b^0 \rightarrow \Lambda^0 \mu^+ \mu^-)_{[15,20]} = -0.39 \pm 0.04 \pm 0.01.$$

1.5 σ

1 σ

Lepton flavor universality :

Lepton flavor universality of $T_{b3} \rightarrow T_8 \ell^+ \ell^-$ baryon weak decays in different q^2 bins with 1σ error in the S_2 case.

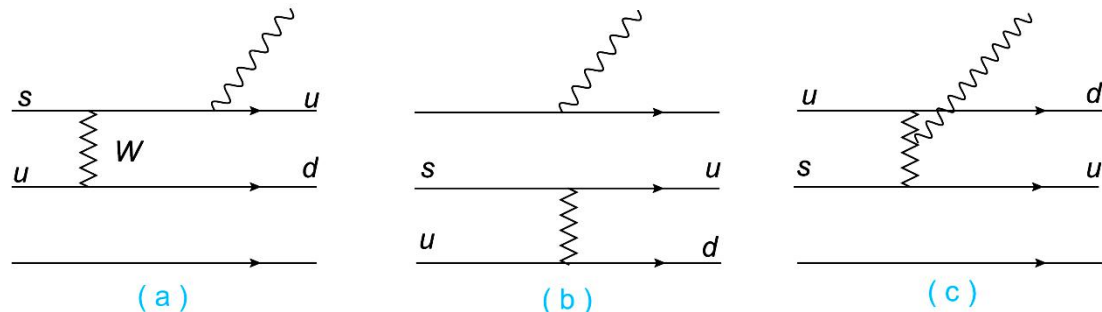
$[q_{min}^2, q_{max}^2](\text{GeV}^2)$	$[1, 6]$	$[0.1, 16.0]$	$[15.0, 20.0]$
$R_{\Lambda_b^0 \rightarrow \Lambda^0}$	$0.99^{+0.05}_{-0.03}$	$0.98^{+0.07}_{-0.04}$	$1.02^{+0.04}_{-0.07}$
$R_{\Xi_b^0 \rightarrow \Xi^0}$	0.99 ± 0.04	$0.99^{+0.06}_{-0.05}$	$0.99^{+0.08}_{-0.06}$
$R_{\Xi_b^- \rightarrow \Xi^-}$	0.99 ± 0.04	$0.99^{+0.06}_{-0.05}$	$1.03^{+0.03}_{-0.09}$
$R_{\Lambda_b^0 \rightarrow n}$	1.00 ± 0.03	$0.98^{+0.07}_{-0.04}$	$1.01^{+0.03}_{-0.05}$
$R_{\Xi_b^0 \rightarrow \Lambda^0}$	$1.01^{+0.02}_{-0.05}$	$1.01^{+0.04}_{-0.08}$	1.00 ± 0.04
$R_{\Xi_b^0 \rightarrow \Sigma^0}$	$0.98^{+0.04}_{-0.03}$	$0.98^{+0.07}_{-0.04}$	1.00 ± 0.03
$R_{\Xi_b^- \rightarrow \Sigma^-}$	$1.00^{+0.03}_{-0.04}$	$1.03^{+0.02}_{-0.09}$	$1.00^{+0.03}_{-0.02}$

3.2 $T_{c3} \rightarrow T_8 \ell^+ \ell^-$

$$\begin{aligned}
 H(T_{c3} \rightarrow T_8 \ell^+ \ell^-)_{VA,\lambda}^{L(R),s_p,s_k} = & f_1(T_{c3})^{[ij]} T'(\bar{3})^k (T_8)_{[ij]k} + f_2(T_{c3})^{[ij]} T'(3)^k (T_8)_{[ik]j} \\
 & + \left(\tilde{f}_1 H(\bar{6})_j^{lk} + \tilde{f}_4 H(15)_j^{lk} \right) (T_{c3})^{[ij]} (T_8)_{[il]k} \\
 & + \left(\tilde{f}_2 H(\bar{6})_j^{lk} + \tilde{f}_5 H(15)_j^{lk} \right) (T_{c3})^{[ij]} (T_8)_{[ik]l} \\
 & + \left(\tilde{f}_3 H(\bar{6})_j^{lk} + \tilde{f}_6 H(15)_j^{lk} \right) (T_{c3})^{[ij]} (T_8)_{[lk]i},
 \end{aligned}$$

W-exchange contributions of the two-quark and three-quark transitions.

$$\tilde{f}_2 = -\tilde{f}_1, \quad \tilde{f}_5 = \tilde{f}_4, \quad \tilde{f}_6 = 0,$$



$d + c \rightarrow u + s + \ell^+ \ell^-$, Cabibbo allowed
 $d + c \rightarrow u + d + \ell^+ \ell^-$ ($s + c \rightarrow u + s + \ell^+ \ell^-$), Singly Cabibbo suppressed
 $s + c \rightarrow u + d + \ell^+ \ell^-$, Doubly Cabibbo suppressed

TABLE VII: The SU(3) IRA amplitudes of the $T_{c3} \rightarrow T_8 \ell^+ \ell^-$ weak decays in the S_1 case, $F_1 \equiv f_1 + f_2$, $\tilde{F}_1 \equiv \tilde{f}_1 - \tilde{f}_3 + \tilde{f}_4$, $\tilde{F}_2 \equiv \tilde{f}_1 - \tilde{f}_3 - \tilde{f}_4$, and $\tilde{F} \equiv \tilde{f}_1 - \tilde{f}_3$.

Decay modes	$A(T_{c3} \rightarrow T_8 \ell^+ \ell^-)$	Approximative $A(T_{c3} \rightarrow T_8 \ell^+ \ell^-)$
Cabibbo allowed $T_{c3} \rightarrow T_8 \ell^+ \ell^-$:		
$\Lambda_c^+ \rightarrow \Sigma^+ \ell^+ \ell^-$	$-\tilde{F}_1$	$-\tilde{F}$
$\Xi_c^0 \rightarrow \Xi^0 \ell^+ \ell^-$	$-\tilde{F}_2$	$-\tilde{F}$
singly Cabibbo suppressed $T_{c3} \rightarrow T_8 \ell^+ \ell^-$:		
$\Lambda_c^+ \rightarrow p \ell^+ \ell^-$	$\left[F_1 - \left(\frac{5}{8} \tilde{F}_1 - \frac{1}{8} \tilde{F}_2 \right) \right] s_c$	$\left[F_1 - \frac{1}{2} \tilde{F} \right] s_c$
$\Xi_c^+ \rightarrow \Sigma^+ \ell^+ \ell^-$	$\left[-F_1 - \left(\frac{5}{8} \tilde{F}_1 - \frac{1}{8} \tilde{F}_2 \right) \right] s_c$	$- \left[F_1 + \frac{1}{2} \tilde{F} \right] s_c$
$\Xi_c^0 \rightarrow \Lambda^0 \ell^+ \ell^-$	$\left[F_1 + 3 \left(\frac{1}{8} \tilde{F}_1 - \frac{5}{8} \tilde{F}_2 \right) \right] s_c / \sqrt{6}$	$\left[F_1 - \frac{3}{2} \tilde{F} \right] s_c / \sqrt{6}$
$\Xi_c^0 \rightarrow \Sigma^0 \ell^+ \ell^-$	$\left[-F_1 + \left(\frac{1}{8} \tilde{F}_1 - \frac{5}{8} \tilde{F}_2 \right) \right] s_c / \sqrt{2}$	$- \left[F_1 + \frac{1}{2} \tilde{F} \right] s_c / \sqrt{2}$
doubly Cabibbo suppressed $T_{c3} \rightarrow T_8 \ell^+ \ell^-$:		
$\Xi_c^+ \rightarrow p \ell^+ \ell^-$	$\tilde{F}_1 s_c^2$	$\tilde{F} s_c^2$
$\Xi_c^0 \rightarrow n \ell^+ \ell^-$	$\tilde{F}_2 s_c^2$	$\tilde{F} s_c^2$

Ignoring the Wilson Coefficient suppressed H(15) term contributions, there are only two parameters.

the contribution of $H(\bar{6})$ to the decay branching ratio is about 5.5 times larger than one of $H(15)$

S_1 : Only considering the W -exchange contributions by setting $F_1 = 0$

S_2 : Only considering the single-quark transition contributions by setting $\tilde{F} = 0$ 30

TABLE VIII: Branching ratios of $T_{c3} \rightarrow T_8 \ell^+ \ell^-$ decays within 1σ theoretical error in S_1 case.

Decay modes	Exp. UL [41]	Our predictions without F_1	Others without LD	Others with LD
$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ e^+ e^-)(\times 10^{-6})$	...	≤ 2.63		
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 e^+ e^-)(\times 10^{-6})$	...	≤ 2.35		
$\mathcal{B}(\Lambda_c^+ \rightarrow p e^+ e^-)(\times 10^{-8})$	≤ 550	≤ 7.95	$(3.8 \pm 0.5) \times 10^{-4}$ $(4.05 \pm 2.37) \times 10^{-6}$ [20, 21]	37 ± 8 420 ± 73 [20, 21]
$\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ e^+ e^-)(\times 10^{-7})$	...	≤ 1.29		
$\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 e^+ e^-)(\times 10^{-8})$	...	≤ 8.69		
$\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 e^+ e^-)(\times 10^{-8})$	...	≤ 2.22		
$\mathcal{B}(\Xi_c^+ \rightarrow p e^+ e^-)(\times 10^{-8})$	...	≤ 5.55		
$\mathcal{B}(\Xi_c^0 \rightarrow n e^+ e^-)(\times 10^{-8})$	...	≤ 1.92		
$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \mu^+ \mu^-)(\times 10^{-6})$	...	≤ 2.50		
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \mu^+ \mu^-)(\times 10^{-6})$	...	≤ 2.25		
$\mathcal{B}(\Lambda_c^+ \rightarrow p \mu^+ \mu^-)(\times 10^{-8})$	≤ 7.7	≤ 7.7	$(2.8 \pm 0.4) \times 10^{-4}$ $(3.77 \pm 2.28) \times 10^{-6}$ [20, 21]	37 ± 8 230 ± 66 [20, 21]
$\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \mu^+ \mu^-)(\times 10^{-7})$	...	≤ 1.25		
$\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 8.42		
$\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 2.15		
$\mathcal{B}(\Xi_c^+ \rightarrow p \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 5.41		
$\mathcal{B}(\Xi_c^0 \rightarrow n \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 1.87		

TABLE VIII: Branching ratios of $T_{c3} \rightarrow T_8 \ell^+ \ell^-$ decays within 1σ theoretical error in S_1 case.

Decay modes	Exp. UL [41]	Our predictions without F_1		Others without LD	Others with LD
$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ e^+ e^-)(\times 10^{-6})$	...	≤ 2.63			
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 e^+ e^-)(\times 10^{-6})$	...	≤ 2.35			
$\mathcal{B}(\Lambda_c^+ \rightarrow p e^+ e^-)(\times 10^{-8})$	≤ 550	≤ 7.95	S2	$(3.8 \pm 0.5) \times 10^{-4}$ $(4.05 \pm 2.37) \times 10^{-6}$ [20, 21]	37 ± 8 420 ± 73 [20, 21]
$\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ e^+ e^-)(\times 10^{-7})$	...	≤ 1.29			
$\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 e^+ e^-)(\times 10^{-8})$	...	≤ 8.69	0.97	Only considering single quark contributions	
$\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 e^+ e^-)(\times 10^{-8})$	...	≤ 2.22			
$\mathcal{B}(\Xi_c^+ \rightarrow p e^+ e^-)(\times 10^{-8})$	...	≤ 5.55			
$\mathcal{B}(\Xi_c^0 \rightarrow n e^+ e^-)(\times 10^{-8})$	...	≤ 1.92			
$\mathcal{B}(\Lambda_c^+ \rightarrow \Sigma^+ \mu^+ \mu^-)(\times 10^{-6})$	...	≤ 2.50			
$\mathcal{B}(\Xi_c^0 \rightarrow \Xi^0 \mu^+ \mu^-)(\times 10^{-6})$	...	≤ 2.25			
$\mathcal{B}(\Lambda_c^+ \rightarrow p \mu^+ \mu^-)(\times 10^{-8})$	≤ 7.7	≤ 7.7		$(2.8 \pm 0.4) \times 10^{-4}$ $(3.77 \pm 2.28) \times 10^{-6}$ [20, 21]	37 ± 8 230 ± 66 [20, 21]
$\mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \mu^+ \mu^-)(\times 10^{-7})$	...	≤ 1.25			
$\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 8.42	0.94		
$\mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 2.15			
$\mathcal{B}(\Xi_c^+ \rightarrow p \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 5.41			
$\mathcal{B}(\Xi_c^0 \rightarrow n \mu^+ \mu^-)(\times 10^{-8})$	...	≤ 1.87			

3.3 $T_8 \rightarrow T_8' \ell^+ \ell^-$ weak decays

$$\begin{aligned}
 H(T_8 \rightarrow T_8' \ell^+ \ell^-)_{VA,\lambda}^{L(R),s_p,s_k} = & g_1(T_8)^{[ij]n} T''(\bar{3})^k (T_8')_{[ij]k} + g_2(T_8)^{[ij]n} T''(3)^k (T_8)_{[ik]j} \\
 & + g_3(T_8)^{[in]j} T''(\bar{3})^k (T_8')_{[ij]k} + g_4(T_8)^{[in]j} T''(\bar{3})^k (T_8')_{[ik]j} \\
 & + g_5(T_8)^{[in]j} T''(\bar{3})^k (T_8')_{[jk]i} + \tilde{g}_1(T_8)^{[ij]n} (T_8')_{[il]k} H(4)_j^{lk} \\
 & + \tilde{g}_2(T_8)^{[in]j} (T_8')_{[il]k} H(4)_j^{lk} + \tilde{g}_3(T_8)^{[jn]i} (T_8')_{[il]k} H(4)_j^{lk},
 \end{aligned}$$

W exchange contributions

TABLE IX: The SU(3) IRA amplitudes of the $T_8 \rightarrow T'_8 \ell^+ \ell^-$ weak decays, $G_1 \equiv g_1 + g_2 + g_3 - g_5$ and $G_2 \equiv g_4 + g_5$, $\tilde{G}_A \equiv \tilde{g}_1 - \tilde{g}_3$, $\tilde{G}_B \equiv \tilde{g}_2 + \tilde{g}_3$.

Decay modes	$A(T_8 \rightarrow T'_8 \ell^+ \ell^-)$
$\Xi^- \rightarrow \Sigma^- \ell^+ \ell^-$	G_1
$\Xi^0 \rightarrow \Lambda^0 \ell^+ \ell^-$	$(G_1 + 2G_2)/\sqrt{6}$
$\Xi^0 \rightarrow \Sigma^0 \ell^+ \ell^-$	$(G_1 + 2\tilde{G}_A)/\sqrt{2}$
$\Lambda^0 \rightarrow n \ell^+ \ell^-$	$-[(G_1 + 2G_2) + (C_1 + 2\tilde{G}_A) - (G_2 - \tilde{G}_B)]/\sqrt{6}$
$\Sigma^0 \rightarrow n \ell^+ \ell^-$	$-(G_2 - \tilde{G}_B)/\sqrt{2}$
$\Sigma^+ \rightarrow p \ell^+ \ell^-$	$-(G_2 + \tilde{G}_B)$

TABLE X: Branching ratios of $T_8 \rightarrow T'_8 \ell^+ \ell^-$ decays within 1σ theoretical error in S_1 case.

Decay modes	Experimental data [41]	Our predictions without $\tilde{G}_{A,B}$ in S_1
$\mathcal{B}(\Xi^- \rightarrow \Sigma^- e^+ e^-)(\times 10^{-6})$	...	$2.49^{+0.31}_{-0.29}$
$\mathcal{B}(\Xi^0 \rightarrow \Lambda^0 e^+ e^-)(\times 10^{-6})$	7.6 ± 0.6	7.6 ± 0.6
$\mathcal{B}(\Xi^0 \rightarrow \Sigma^0 e^+ e^-)(\times 10^{-6})$	...	2.05 ± 0.17
$\mathcal{B}(\Lambda^0 \rightarrow n e^+ e^-)(\times 10^{-5})$	...	$2.06^{+0.25}_{-0.23}$
$\mathcal{B}(\Sigma^0 \rightarrow n e^+ e^-)(\times 10^{-17})$	...	$7.61^{+6.59}_{-4.44}$
$\mathcal{B}(\Sigma^+ \rightarrow p e^+ e^-)(\times 10^{-7})$	< 70	$1.60^{+1.14}_{-0.87}$
$\mathcal{B}(\Sigma^0 \rightarrow n \mu^+ \mu^-)(\times 10^{-17})$	...	$1.22^{+1.05}_{-0.71}$
$\mathcal{B}(\Sigma^+ \rightarrow p \mu^+ \mu^-)(\times 10^{-8})$	$2.4^{+1.7}_{-1.3}$	$2.40^{+1.70}_{-1.30}$

$|\tilde{G}_B| \gg G_2$ $\tilde{G}_B \approx G_2$ $\tilde{G}_B \approx -G_2$

...
...
0

4. Conclusion

- ✓ Testing SU(3) Flavor Symmetry in nonleptonic two-body decays of hyperons and semileptonic baryon weak decays.
- ✓ Predicted the not-yet-measured observables by using the constrained irreducible representation amplitudes from relevant data, and many of them are obtained for the first time.
- ✓ Some modes could be measured at BESIII, LHCb and Belle-II.

$$B(\Lambda_b^0 \rightarrow \Lambda^0 \tau^+ \tau^-), B(\Xi_b^0 \rightarrow \Xi^0 \ell^+ \ell^-), B(\Xi_b^- \rightarrow \Xi^- \ell^+ \ell^-), \dots$$

A blue scroll graphic with a white border. The scroll is unrolled, showing a light blue background. The text "Thank you!" is written in a dark blue, cursive font with a white outline and a slight shadow effect. The scroll has a small blue circle at the top right corner and a small blue circle at the bottom left corner.

Thank you!