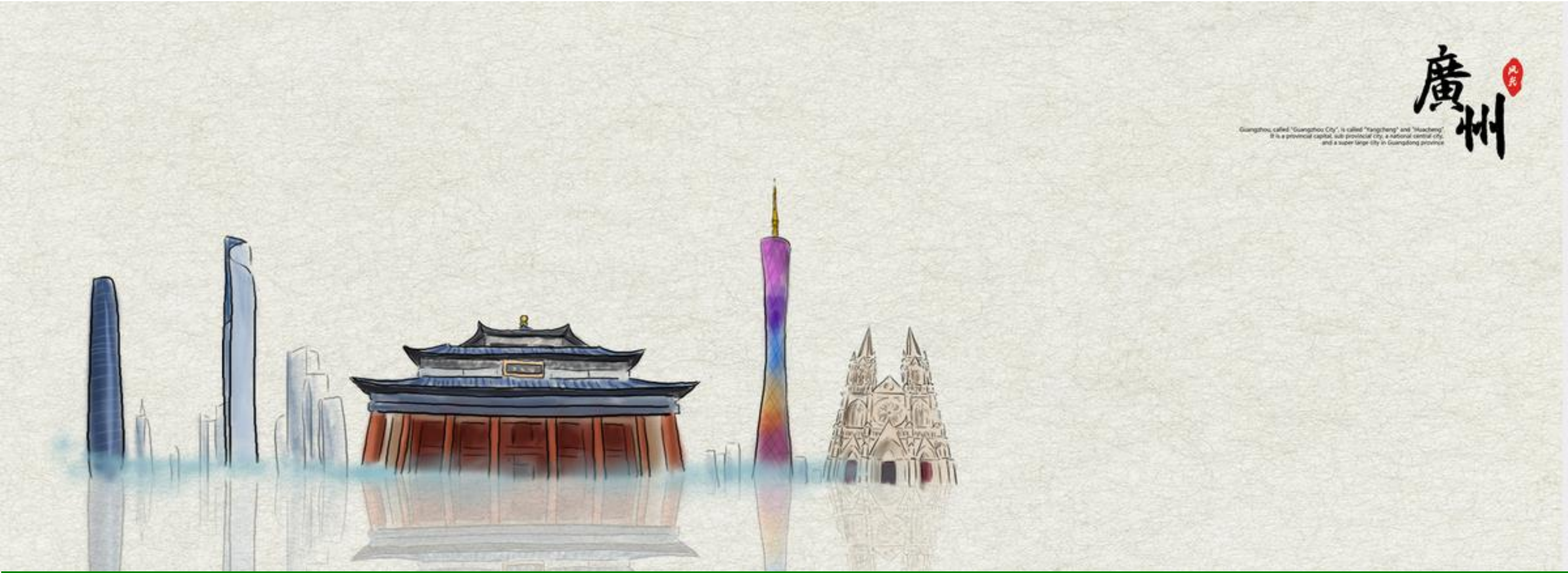
Unitarity Test of the Three Flavor Neutrino Mixing Matrix



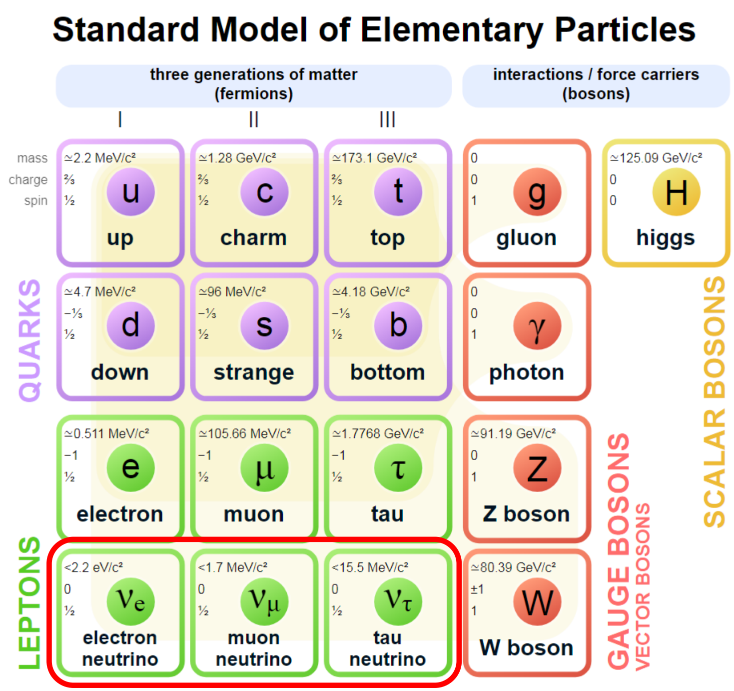
凌家杰
中山大学

HFCPV2021,

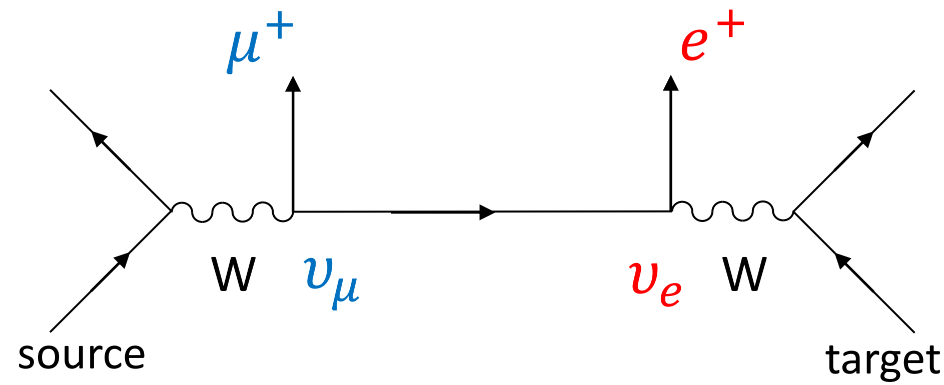
Guangzhou， 11/11/2021

Neutrino Oscillation

A phenomenon that neutrino generated with a certain lepton flavor and measured to have another lepton flavor after travelling a distance.



$$\nu_l + n = l + p, (l = e, \mu, \tau)$$



There is mixing between the flavor and mass eigenstates of neutrinos!

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Composition changes over time.

$$|\nu_l(t)\rangle = \sum_i U_{li}^* e^{-iE_i t} |\nu_i\rangle$$

PMNS Matrix

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

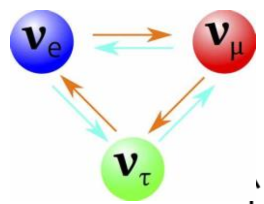
Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

(analogous to CKM matrix)

Irrelevant for oscillations

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{bmatrix}$$

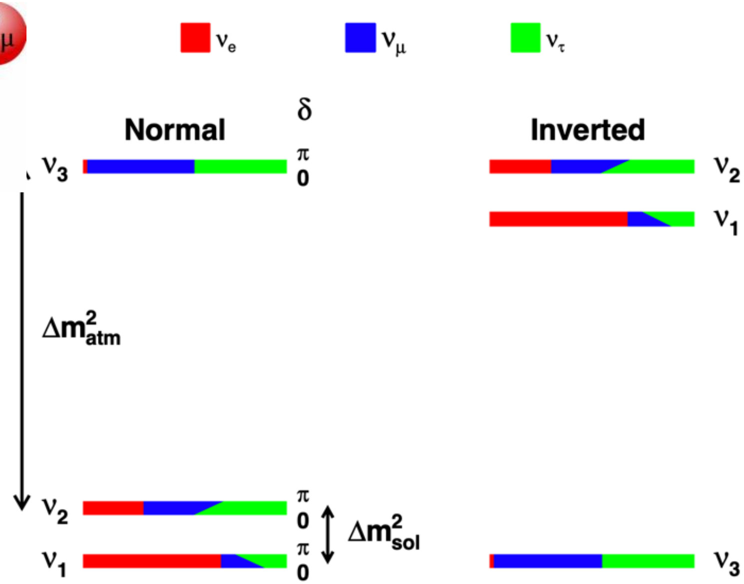
$c_{ij} = cos\theta_{ij}$ $\quad s_{ij} = sin\theta_{ij}$



Three mixing angles: θ_{23}, θ_{13}, θ_{12}

Three CP phases: δ_{CP}(Dirac), α_{21} and α_{31} (Majorana)

Three masses: m_1, Δm^2_{21}, Δm^2_{31}



Current knowledge

NUFIT *JHEP 01 (2019) 106* *JHEP 09 (2020) 178*

NuFIT 4.0	θ_{12} (°)	θ_{23} (°)	θ_{13} (°)	δ_{CP} (°)	Δm^2_{21} ($\times 10^{-5}\mathrm{eV}^2$)	Δm^2_{32} ($\times 10^{-3}\mathrm{eV}^2$)
Normal	$33.82^{+0.78}_{-0.76}$ $33.44^{+0.77}_{-0.74}$	$49.7^{+0.9}_{-1.1}$ $49.2^{+0.9}_{-1.2}$	$8.61^{+0.12}_{-0.13}$ $8.57^{+0.12}_{-0.12}$	217^{+40}_{-28} 197^{+27}_{-24}	$7.39^{+0.21}_{-0.20}$ $7.42^{+0.21}_{-0.20}$	$2.451^{+0.033}_{-0.031}$ $2.443^{+0.026}_{-0.028}$
Inverted ($\Delta\chi^2$= 9.3 → 7.1)	$33.82^{+0.78}_{-0.75}$ $33.45^{+0.78}_{-0.75}$	$49.7^{+0.9}_{-1.0}$ $49.3^{+0.9}_{-1.1}$	$8.65^{+0.12}_{-0.13}$ $8.60^{+0.12}_{-0.12}$	280^{+25}_{-28} 282^{+26}_{-30}	$7.39^{+0.21}_{-0.20}$ $7.42^{+0.21}_{-0.20}$	$-2.512^{+0.034}_{-0.031}$ $-2.498^{+0.028}_{-0.028}$
1σ precision	2.3%	2.0%	1.5%	31.3%	2.8%	1.3%

- **Well measured:**
  $\boldsymbol{\theta_{12}}$, $\boldsymbol{\theta_{13}}$, Δm^2_{21}, $|\Delta m^2_{32}|$

- **Not-so-well measured:**
  $\boldsymbol{\theta_{23}}$ Octant, $\boldsymbol{\delta_{CP}}$, neutrino mass ordering (sign of Δm^2_{32})

PMNS Matrix

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

Irrelevant for oscillations

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$c_{ij} = cos\theta_{ij}$ $\quad s_{ij} = sin\theta_{ij}$

Caution! Unitarity is assumed in this framework: $U^\dagger U = 1$

Is the PMNS matrix really unitary?

PMNS Matrix Extension

- The 3x3 PMNS matrix is assumed unitary because only 3 neutrino flavor eigenstates are found.
- Should be extended to $n \times n$ matrix if $n > 3$ flavor eigenstates. (sterile neutrinos)

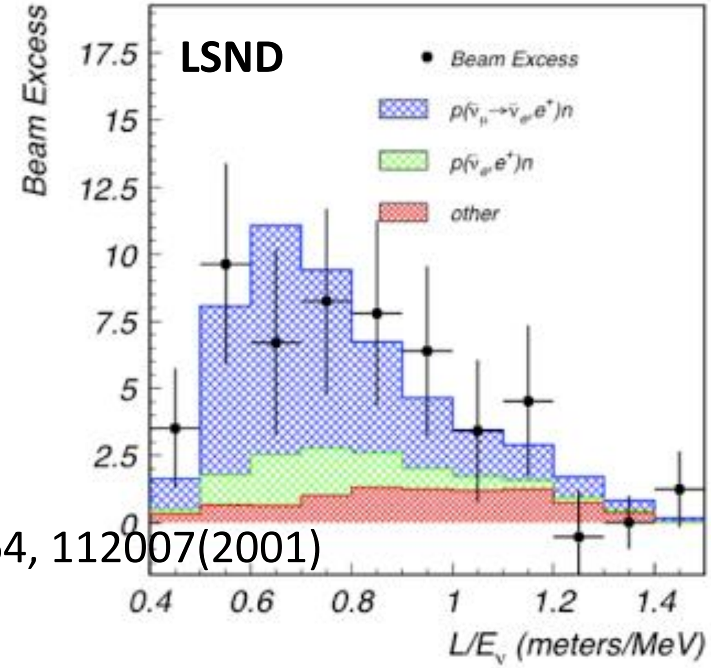
$$U_{\text{PMNS}}^{\text{Extended}} = \begin{pmatrix} \overbrace{\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{matrix}}^{U_{\text{PMNS}}^{\text{3x3}}} & \begin{matrix} \cdots & U_{en} \\ \cdots & U_{\mu n} \\ \cdots & U_{\tau n} \end{matrix} \\ \begin{matrix} \vdots & \vdots & \vdots \\ U_{s_n 1} & U_{s_n 2} & U_{s_n 3} \end{matrix} & \begin{matrix} \ddots & \vdots \\ \cdots & U_{s_n n} \end{matrix} \end{pmatrix}$$

$n \times n$ PMNS matrix
- unitary

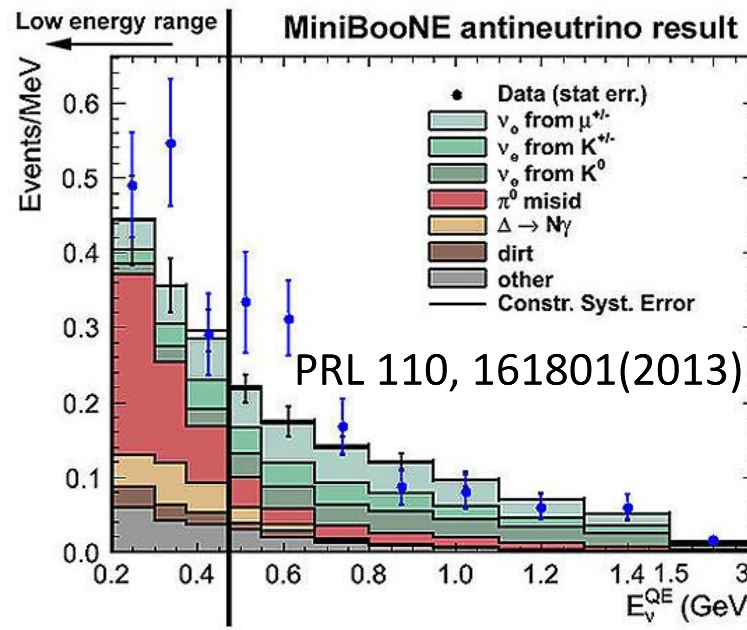
← subset

3x3 PMNS matrix
- $n = 3$, unitary
- $n > 3$, NOT unitary

Experimental “hints” of sterile neutrinos

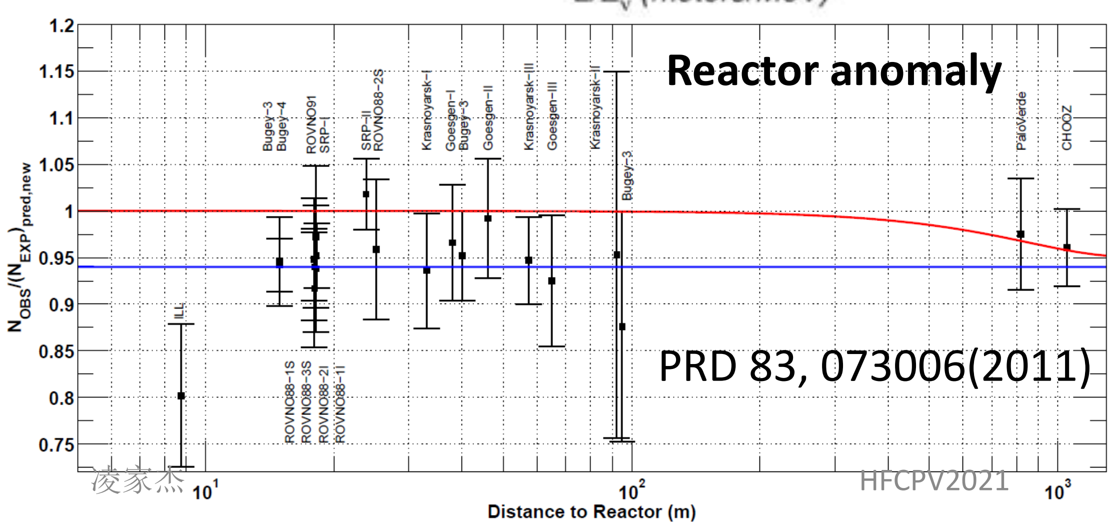


PRD 64, 112007(2001)

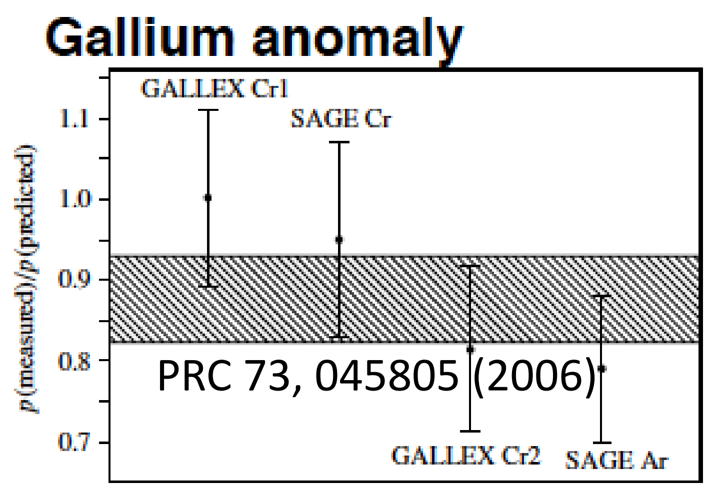


PRL 110, 161801(2013)

Note: recent results from MicroBooNE No electron excess at low energy

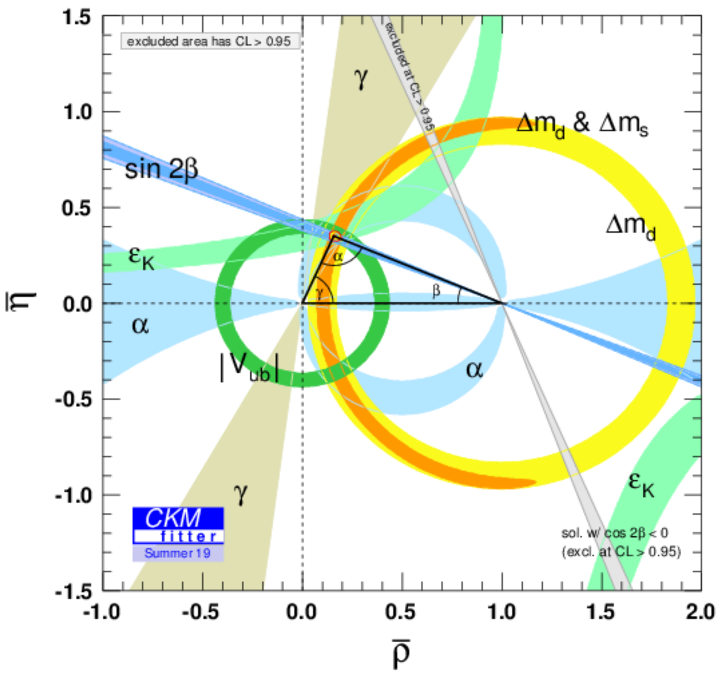


PRD 83, 073006(2011)



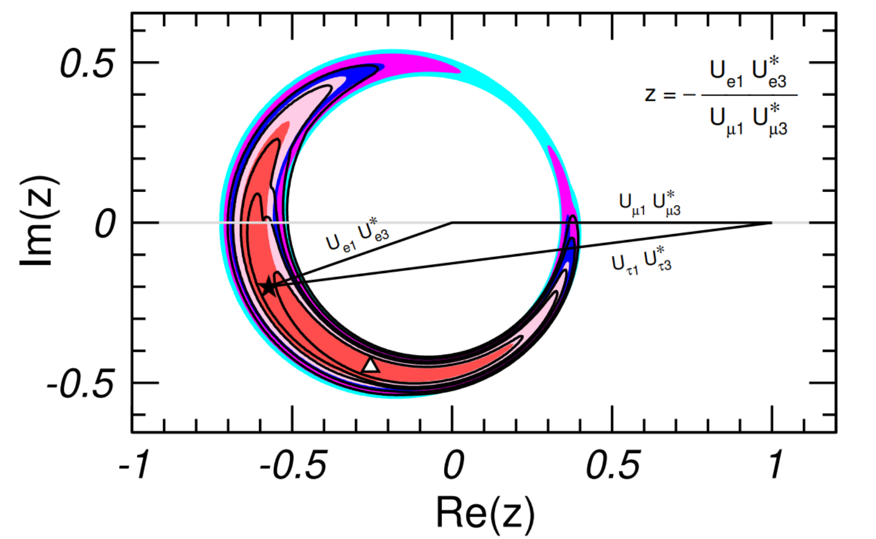
PRC 73, 045805 (2006)

CKM matrix v.s. PMNS matrix



CKM Fitter Group: Eur. Phys. J. C41, 1-131 (2005) updated for EPS-HEP 2019

$$U_{e1}U_{e3}^* + U_{\mu1}U_{\mu3}^* + U_{\tau1}U_{\mu3}^* = 0$$



NuFit Group: JHEP 09 (2020) 178 [arXiv:2007.14792]

CKM matrix	PMNS matrix
Unitarity not assumed	Unitarity assumed
$\alpha + \beta + \gamma = (179^{+7}_{-6})^\circ$	The triangle may not be closed without this assumption!

Global oscillation data analysis on the 3ν mixing without unitarity

JHEP 01 (2021) 124

Zhuojun Hu,[a] Jiajie Ling,[a] Jian Tang,[a] TseChun Wang[b,a]

[a] *School of Physics, Sun Yat-sen University, Guangzhou 510275, China*

[b] *National Center for Theoretical Sciences, Physics Division, Hsinchu, 30013, Taiwan*

E-mail: huzhj3@mail2.sysu.edu.cn, lingjj5@mail.sysu.edu.cn, tangjian5@mail.sysu.edu.cn, tsechunwang@mx.nthu.edu.tw

ABSTRACT: We present results of a combined analysis in neutrino oscillations without unitarity assumption in the 3ν mixing picture. Constraints on neutrino mixing matrix elements are based on recent data from the reactor, solar and long-baseline accelerator neutrino oscillation experiments. The current data are consistent with the standard 3ν scheme. The precision on different matrix elements can be as good as a few percent at 3σ CL, and is mainly limited by the experimental statistical uncertainty. The ν_e related elements are the most precisely measured among all sectors with the uncertainties $< 20\%$. The measured leptonic CP violation is very close to the one assuming the standard 3ν mixing. The deviations on normalization and the unitarity triangle closure are confined within $\mathcal{O}(10^{-3})$, $\mathcal{O}(10^{-2})$ and $\mathcal{O}(10^{-1})$, for ν_e, ν_μ and ν_τ sectors, respectively. We look forward to the next-generation neutrino oscillation experiments *such as* DUNE, T2HK, and JUNO, especially the precise measurements on ν_τ oscillations, to significantly improve

Unitarity Test

- Use global neutrino oscillation data to test the unitarity of the three flavor neutrino mixing matrix.

This work
Unitary NOT assumed
JHEP 01 (2021) 124

testing →

Current framework
Unitary assumed

- Without unitary assumption, the neutrino oscillation (flavor change from α to β) probability

$$P^{NU}_{\nu_\alpha\to\nu_\beta} = \left|\sum_{i=1} U^*_{\beta i} U_{\alpha i}\right|^2 - 4\sum_{i<j} \Re\left(U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}\right) \sin^2\left(\frac{\Delta m^2_{ji} L}{4E_\nu}\right)$$

$$\pm 2\sum_{i<j} \Im\left(U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}\right) \sin\left(\frac{\Delta m^2_{ji} L}{2E_\nu}\right),$$

Unitarity Conditions

- If the three flavor neutrino mixing matrix is **unitary**:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

the following **conditions** must be satisfied：

$$< v_\alpha | v_\alpha > = 1$$
$$< v_i | v_i > = 1$$
$$< v_\alpha | v_\beta > = 0$$
$$< v_i | v_j > = 0$$

➡

$$|U_{\alpha 1}^{3\nu}|^2 + |U_{\alpha 2}^{3\nu}|^2 + |U_{\alpha 3}^{3\nu}|^2 = 1, \quad \alpha = e, \mu, \tau,$$
$$|U_{ei}^{3\nu}|^2 + |U_{\mu i}^{3\nu}|^2 + |U_{\tau i}^{3\nu}|^2 = 1, \quad i = 1, 2, 3,$$
$$U_{\alpha 1}^{3\nu} U_{\beta 1}^{3\nu,*} + U_{\alpha 2}^{3\nu} U_{\beta 2}^{3\nu,*} + U_{\alpha 3}^{3\nu} U_{\beta 3}^{3\nu,*} = 0, \quad \alpha, \beta = e, \mu, \tau, \quad \alpha \neq \beta,$$
$$U_{ei}^{3\nu} U_{ej}^{3\nu,*} + U_{\mu i}^{3\nu} U_{\mu j}^{3\nu,*} + U_{\tau i}^{3\nu} U_{\tau j}^{3\nu,*} = 0, \quad i, j = 1, 2, 3, \quad i \neq j.$$

Global Neutrino Data

Various neutrino experimental data can provide constraints on the different neutrino mixing elements

- Different energy scale
- Different baseline
- Different systematics

Types	Exps	Measurements
MBL Reactor	RENO, Daya Bay Double Chooz	$4\|U_{e3}\|^2(\|U_{e1}\|^2 + \|U_{e2}\|^2)$
LBL Reactor	KamLAND	$4\|U_{e1}\|^2\|U_{e2}\|^2$
Solar	SNO	$\|U_{e2}\|^2$
LBL Accelerator $(\nu_\mu \to \nu_\mu)$	NOvA, T2K	$4\|U_{\mu 3}\|^2(\|U_{\mu 1}\|^2 + \|U_{\mu 2}\|^2)$
LBL Accelerator $(\nu_\mu \to \nu_e)$	NOvA, T2K	$4\Re[U_{e3}U^*_{\mu 3}(U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2})]$
LBL Accelerator $(\nu_\mu \to \nu_\tau)$	OPERA	$4\Re[U_{\tau 3}U^*_{\mu 3}(U_{\tau 1}U^*_{\mu 1} + U_{\tau 2}U^*_{\mu 2})]$

Global Neutrino Data

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

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LBL Accelerator ($\nu_\mu \to \nu_e$)	NOvA, T2K	$4\Re[U_{e3}U^*_{\mu 3}(U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2})]$
LBL Accelerator ($\nu_\mu \to \nu_\tau$)	OPERA	$4\Re[U_{\tau 3}U^*_{\mu 3}(U_{\tau 1}U^*_{\mu 1} + U_{\tau 2}U^*_{\mu 2})]$

Global Neutrino Data

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Types	Exps	Measurements
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LBL Accelerator $(\nu_\mu \to \nu_e)$	NOvA, T2K	$4\Re[U_{e3}U^*_{\mu 3}(U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2})]$
LBL Accelerator $(\nu_\mu \to \nu_\tau)$	OPERA	$4\Re[U_{\tau 3}U^*_{\mu 3}(U_{\tau 1}U^*_{\mu 1}+U_{\tau 2}U^*_{\mu 2})]$

Global Neutrino Data

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Types	Exps	Measurements
MBL Reactor	RENO, Daya Bay Double Chooz	$4\|U_{e3}\|^2(\|U_{e1}\|^2+\|U_{e2}\|^2)$
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Solar	SNO	$\|U_{e2}\|^2$
LBL Accelerator ($\nu_\mu \to \nu_\mu$)	NOvA, T2K	$4\|U_{\mu 3}\|^2(\|U_{\mu 1}\|^2+\|U_{\mu 2}\|^2)$
LBL Accelerator ($\nu_\mu \to \nu_e$)	NOvA, T2K	$4\Re[U_{e3}U^*_{\mu 3}(U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2})]$
LBL Accelerator ($\nu_\mu \to \nu_\tau$)	OPERA	$4\Re[U_{\tau 3}U^*_{\mu 3}(U_{\tau 1}U^*_{\mu 1}+U_{\tau 2}U^*_{\mu 2})]$

Global Neutrino Data

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Types	Exps	Measurements
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LBL Accelerator $(\nu_\mu \to \nu_e)$	NOvA, T2K	$4\Re[U_{e3}U^*_{\mu 3}(U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2})]$
LBL Accelerator $(\nu_\mu \to \nu_\tau)$	OPERA	$4\Re[U_{\tau 3}U^*_{\mu 3}(U_{\tau 1}U^*_{\mu 1} + U_{\tau 2}U^*_{\mu 2})]$

Constraints from Sterile Searches

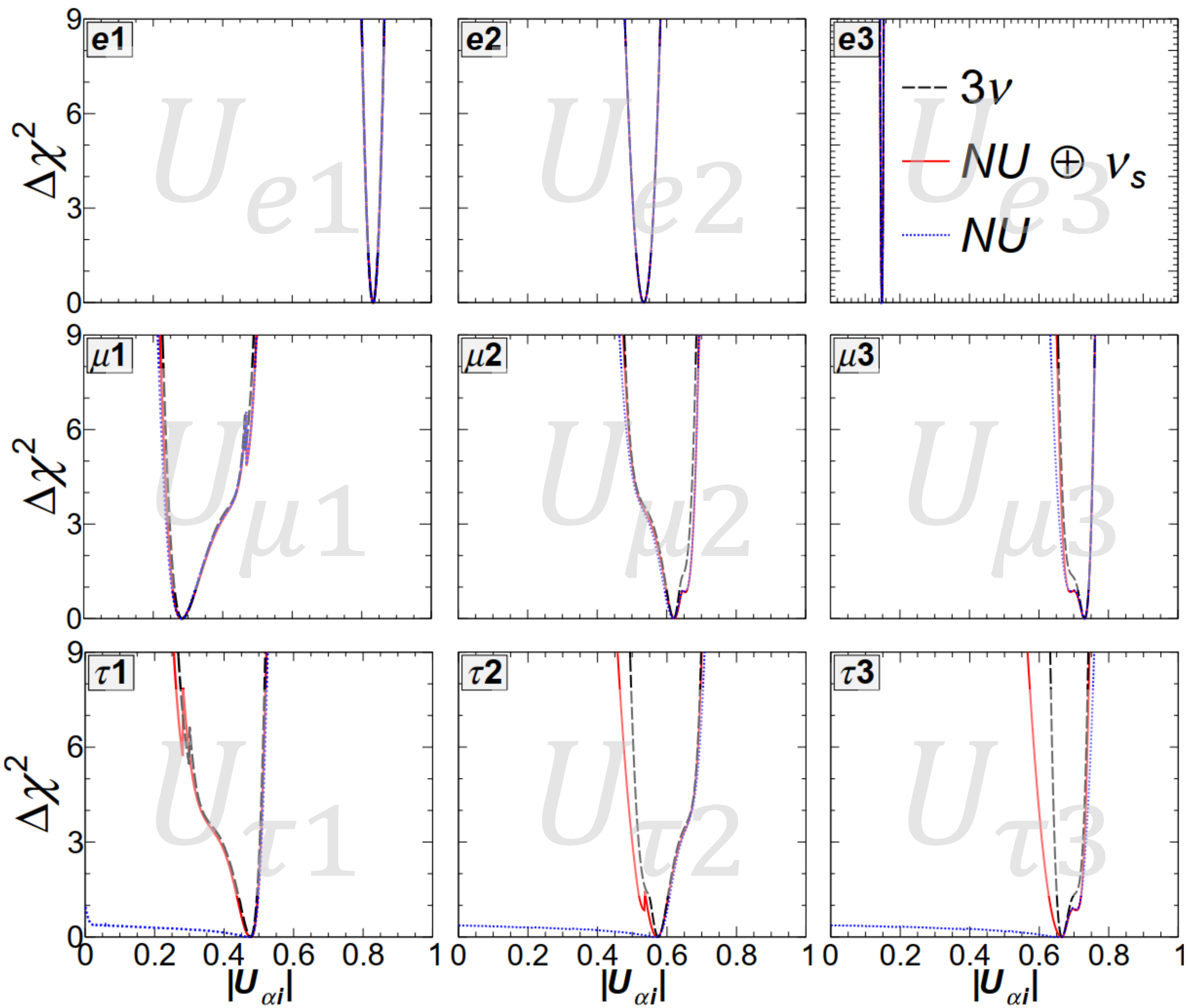
Some data provide direct constraints.

Assumed unitary
$< \upsilon_\alpha|\upsilon_\alpha > = 1, \quad < \upsilon_\alpha|\upsilon_\beta > = 0$

measurements
$1 - < \upsilon_\alpha|\upsilon_\alpha > = ?, \quad < \upsilon_e|\upsilon_\mu > = ?$

Non-unitarity	Data	Limit (1σ)
$1 - \sum_{i=1}^{3} \|U_{ei}\|^2$	SK+DC+IC	0.0589
$4\left(1 - \sum_{i=1}^{3} \|U_{ei}\|^2\right)\left(1 - \sum_{i=1}^{3} \|U_{\mu i}\|^2\right)$	OPERA(ν_e)	0.00713
$1 - \sum_{i=1}^{3} \|U_{\mu i}\|^2$	CDHSW+MNS+SB +MB+SK+DC+IC	0.0061
$1 - \sum_{i=1}^{3} \|U_{\tau i}\|^2$	CDHS+MNS+NOvA +MB+SK+DC+IC+SNO	0.0659

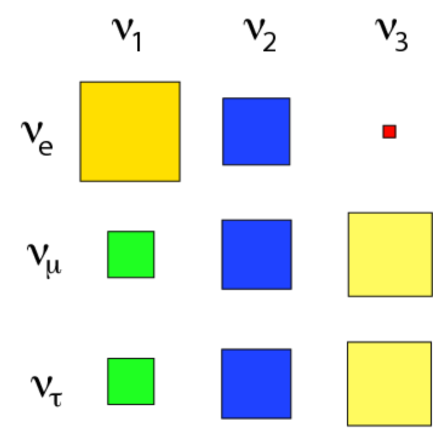
Matrix elements without unitarity



$$U_{Best-fit}$$

$$\begin{pmatrix} 0.832 & 0.535 & 0.148 \\ 0.281 & 0.622 & 0.730 \\ 0.477 & 0.572 & 0.664 \end{pmatrix}$$

PMNS



3σ Ranges

Unitary NOT assumed
3σ range

$$|U|^{\rm NU}_{3\sigma} = \begin{pmatrix} 0.800 \to 0.865 & 0.479 \to 0.582 & 0.141 \to 0.154 \\ 0.219 \to 0.497 & 0.475 \to 0.693 & 0.651 \to 0.761 \\ 0.255 \to 0.521 & 0.458 \to 0.700 & 0.566 \to 0.744 \end{pmatrix}$$

Unitary IS assumed
3σ range

$$|U|^{3\nu}_{3\sigma} = \begin{pmatrix} 0.800 \to 0.865 & 0.480 \to 0.582 & 0.143 \to 0.153 \\ 0.226 \to 0.489 & 0.479 \to 0.685 & 0.655 \to 0.761 \\ 0.268 \to 0.518 & 0.493 \to 0.698 & 0.631 \to 0.740 \end{pmatrix}$$

Without assuming unitarity,

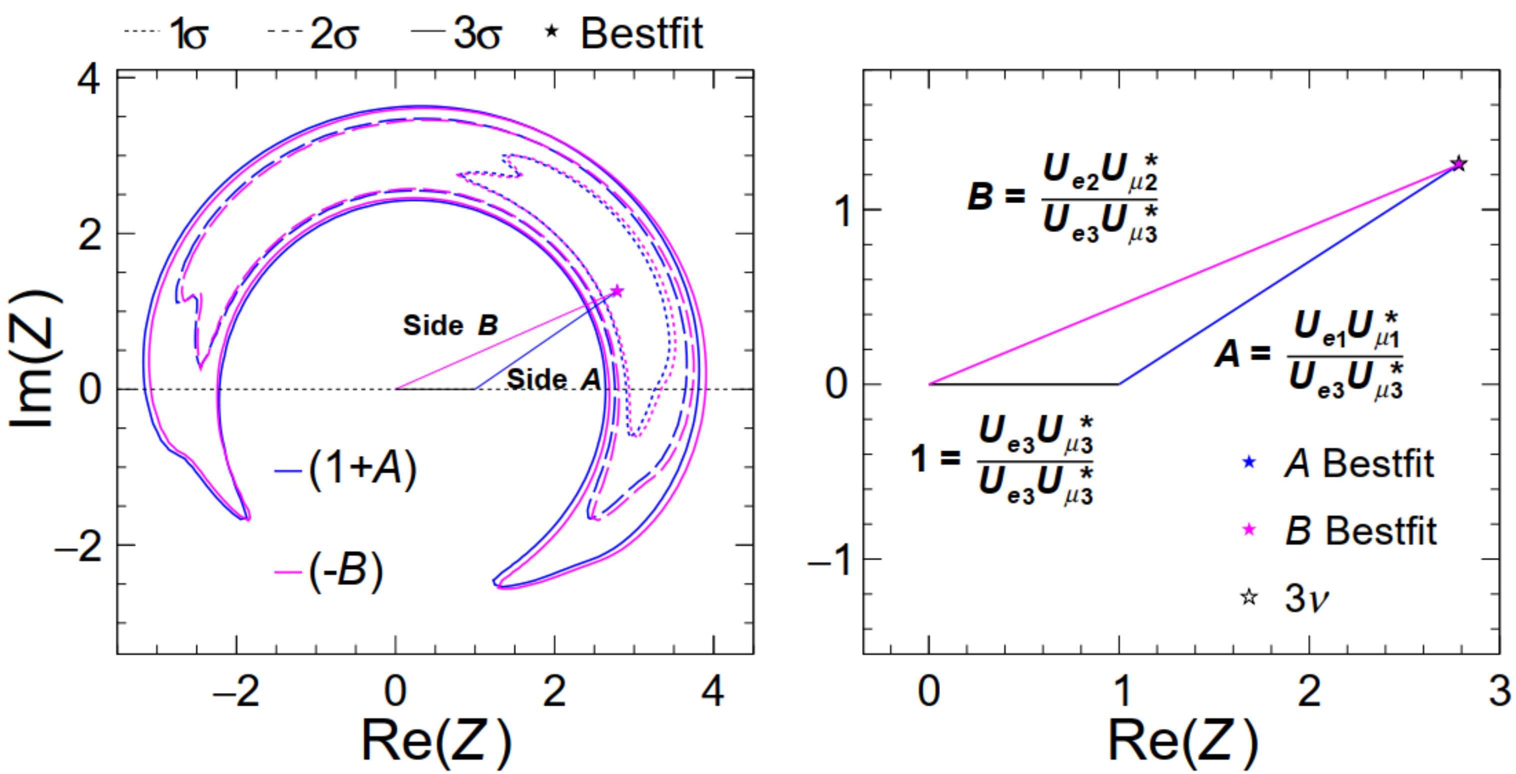
- achieved the same precision for U_{ei},
- while the precision for $U_{\mu i}$ and $U_{\tau i}$ are worse.

Unitarity Triangle

Matrix Orthogonal Test

$$U_{e1}U_{\mu1}^{*}+U_{e2}U_{\mu2}^{*}+U_{e3}U_{\mu3}^{*}=0$$

$$\frac{U_{e1}U_{\mu1}^{*}}{U_{e3}U_{\mu3}^{*}}+\frac{U_{e2}U_{\mu2}^{*}}{U_{e3}U_{\mu3}^{*}}+1=0$$



Unitarity NOT assumed

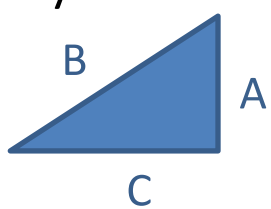
CP Violation

CP violation: $U_{3\times3} \neq U^*_{3\times3}$, Jarlskog factor $\neq 0$

Jarlskog factor:

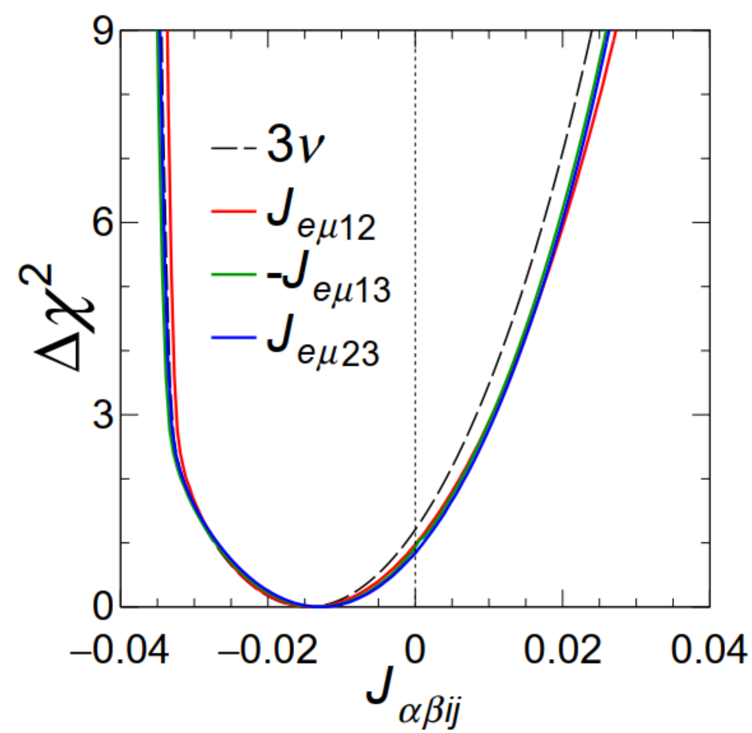
$$J = 2 \times \text{Area of Unitarity Triangle}$$
$$= \text{base} \times \text{height}$$

- Total 9 Jarlskog factors, NOT invariant when unitarity NOT assumed.



$$J = A \cdot h_A \stackrel{?}{=} B \cdot h_B \stackrel{?}{=} C \cdot h_C$$

- Tested three cases and compared to the unitary case, consistent though less precise.

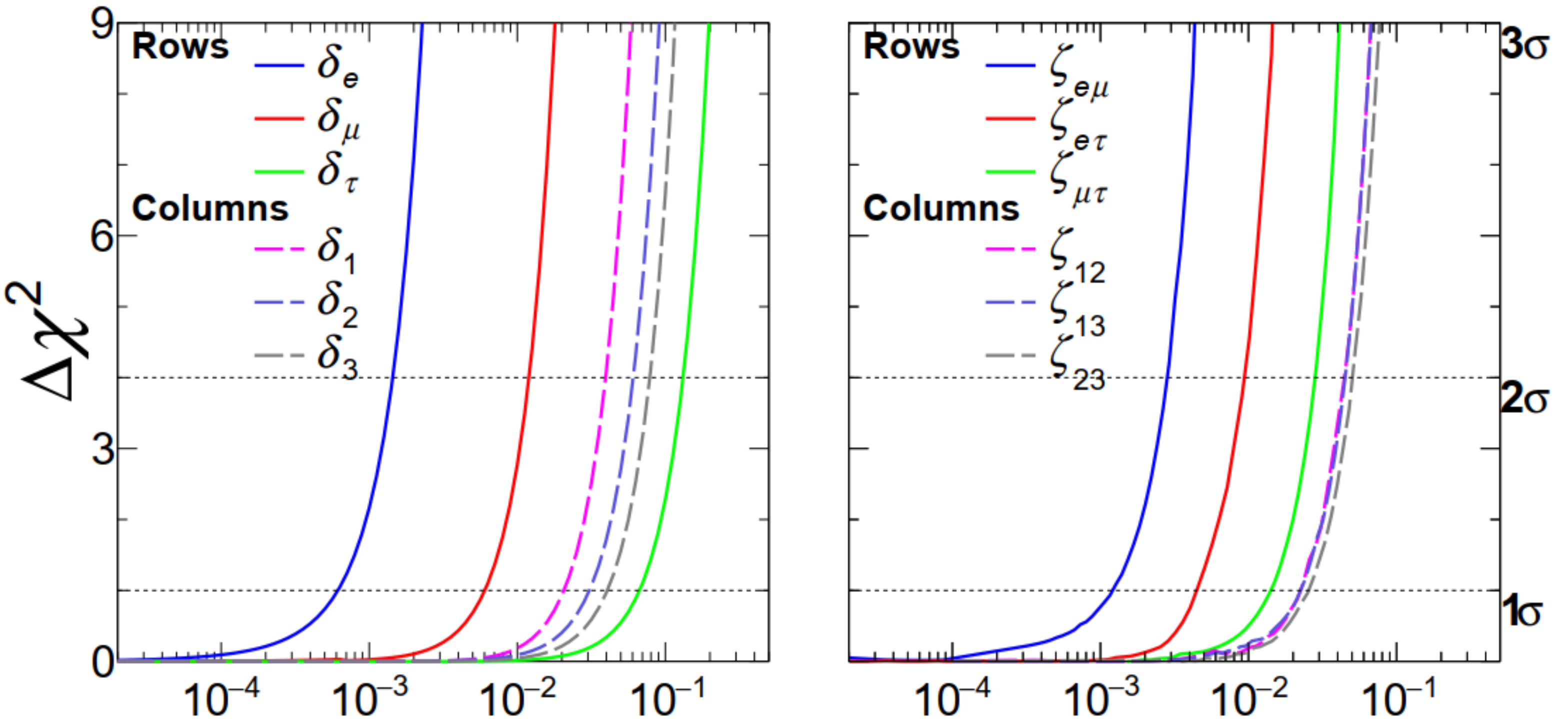


- The best fit value is ~ -0.013
- Data show a slightly hint for CP violation with $\Delta\chi^2 = 0.8$

Unitarity Test

$< \nu_\alpha|\nu_\alpha > = 1$ $< \nu_i|\nu_i > = 1$ $< \nu_\alpha|\nu_\beta > = 0$ $< \nu_i|\nu_j > = 0$

$\delta_\alpha=1-|U_{\alpha1}|^2-|U_{\alpha2}|^2-|U_{\alpha3}|^2$ $\delta_i=1-|U_{ei}|^2-|U_{\mu i}|^2-|U_{\tau i}|^2$ $\zeta_{\alpha\beta}=U_{\alpha1}U^*_{\beta1}+U_{\alpha2}U^*_{\beta2}+U_{\alpha3}U^*_{\beta3}$ $\zeta_{ij}=U_{ei}U^*_{ej}+U_{\mu i}U^*_{\mu j}+U_{\tau i}U^*_{\tau j}$

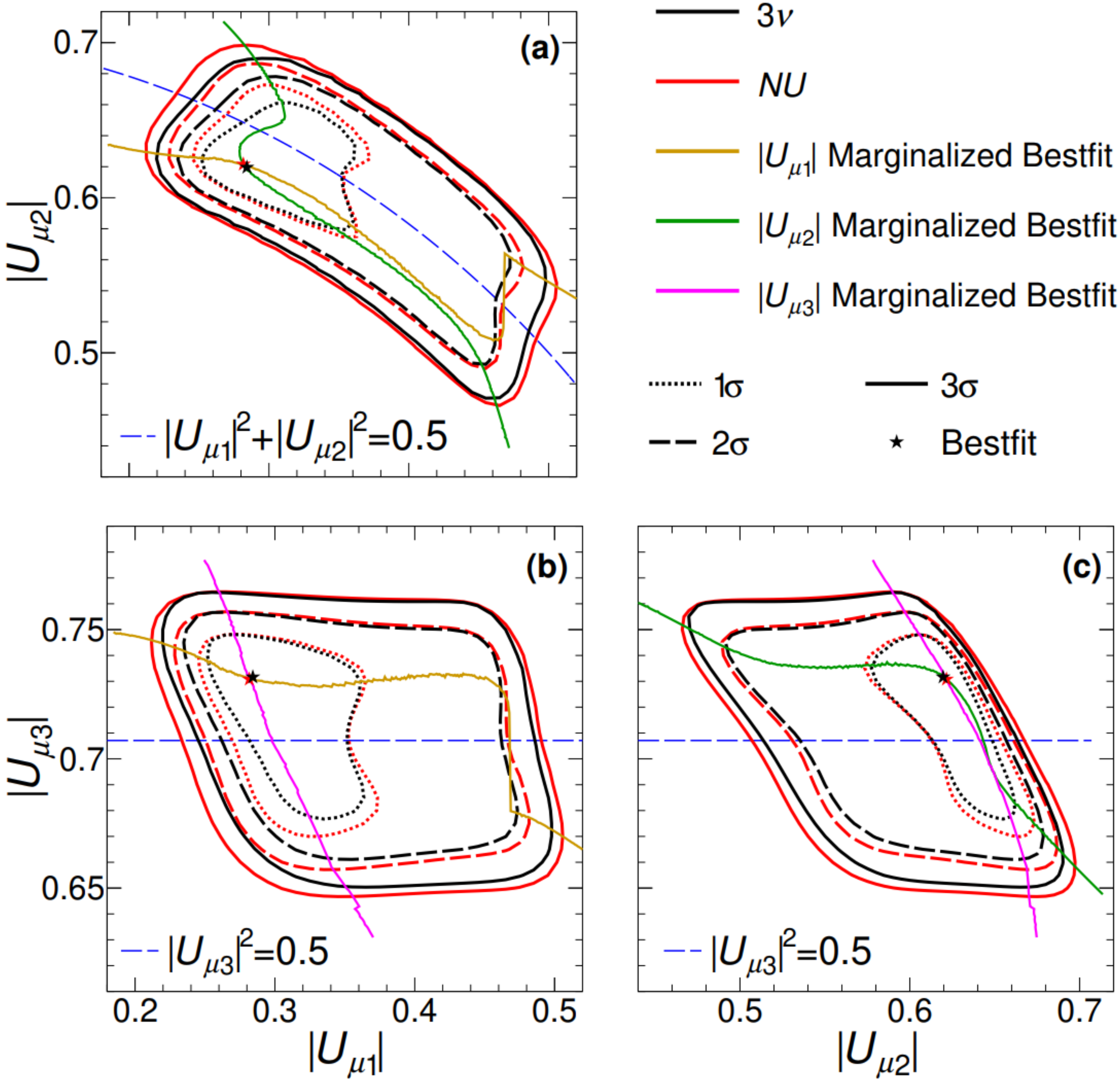


- Electron-type neutrinos have the best unitarity constraints due to large data sample
- Tau-type neutrinos are very limited by the experimental data

Conclusion

- Neutrino oscillation is the most direct evidence of physics beyond the standard model.
- Neutrino research provide an important window for new physics.
- No significant nonunitary is observed in the current PMNS matrix.
- Future precision measurement on Tau-type neutrinos sector can significantly improve the precision of matrix nonunitarity .

Thank you!



(a)
(b)
(c)
0.7
0.6
0.5
0.75
0.65
0.2
0.3
0.4
0.5
0.6
0.7
$|U_{\mu 2}|$
$|U_{\mu 3}|$
$|U_{\mu 1}|$
$|U_{\mu 1}|^2+|U_{\mu 2}|^2=0.5$
$|U_{\mu 3}|^2=0.5$
3ν
NU
$|U_{\mu 1}|$ Marginalized Bestfit
$|U_{\mu 2}|$ Marginalized Bestfit
$|U_{\mu 3}|$ Marginalized Bestfit
1σ
3σ
2σ
Bestfit

Extra Constraints

Probability conservation:

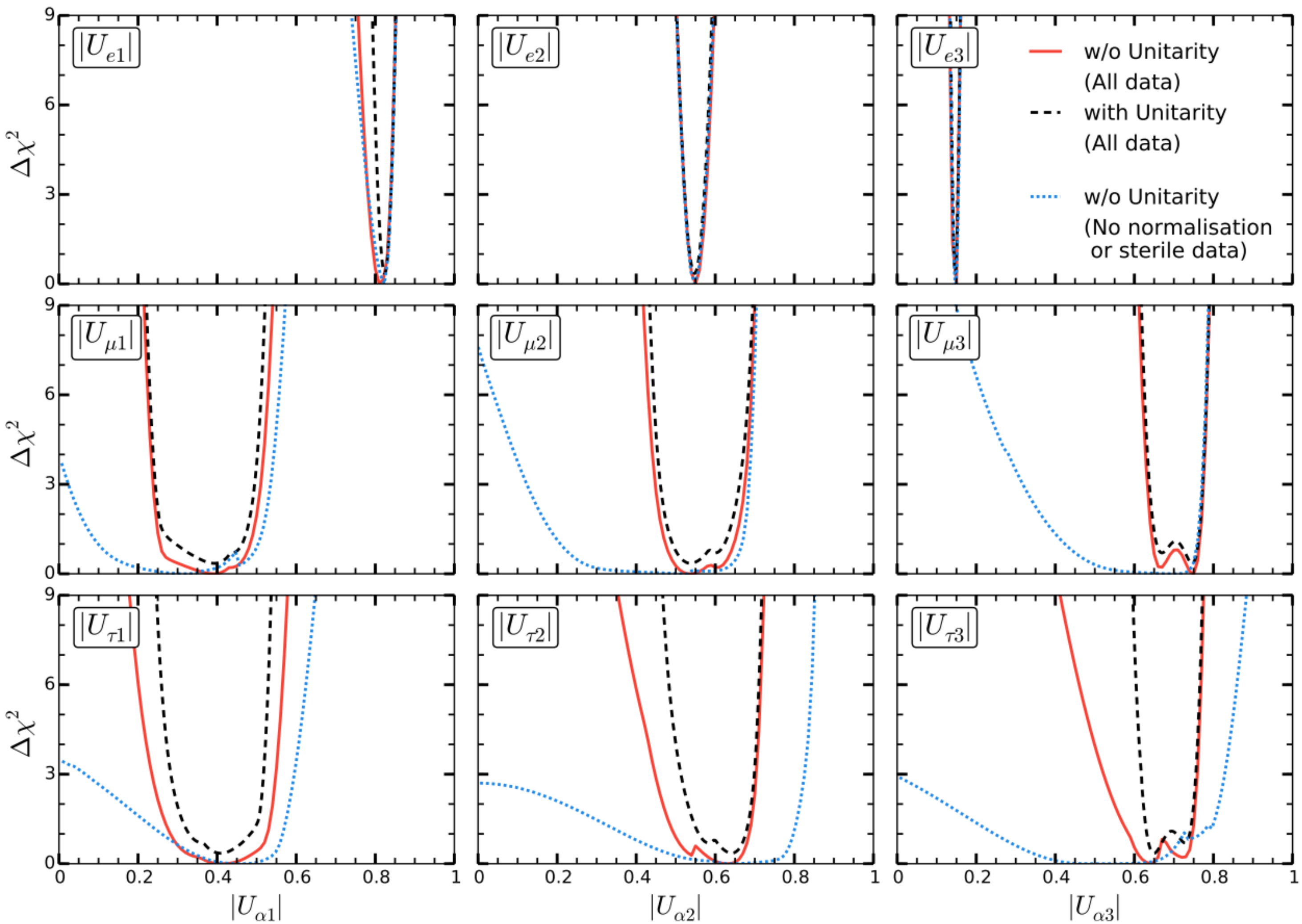
$$|U^{NU}_{\alpha 1}|^2 + |U^{NU}_{\alpha 2}|^2 + |U^{NU}_{\alpha 3}|^2 \leq 1, \quad \alpha = e, \mu, \tau;$$
$$|U^{NU}_{ei}|^2 + |U^{NU}_{\mu i}|^2 + |U^{NU}_{\tau i}|^2 \leq 1, \quad i = 1, 2, 3.$$

Cauchy-Schwartz inequalities:

$$\left| \sum_{i=1}^{3} U^{NU}_{\alpha i} U^{NU,*}_{\beta i} \right|^2 \leq \left(1 - \sum_{i=1}^{3} |U^{NU}_{\alpha i}|^2 \right) \left(1 - \sum_{i=1}^{3} |U^{NU}_{\beta i}|^2 \right), \quad \text{for } \alpha, \beta = e, \mu, \tau, \alpha \neq \beta, \tag{2.12}$$

$$\left| \sum_{\alpha=e}^{\tau} U^{NU}_{\alpha i} U^{NU,*}_{\alpha j} \right|^2 \leq \left(1 - \sum_{\alpha=e}^{\tau} |U^{NU}_{\alpha i}|^2 \right) \left(1 - \sum_{\alpha=e}^{\tau} |U^{NU}_{\alpha j}|^2 \right), \quad \text{for } i, j = 1, 2, 3, i \neq j. \tag{2.13}$$

Previous Works



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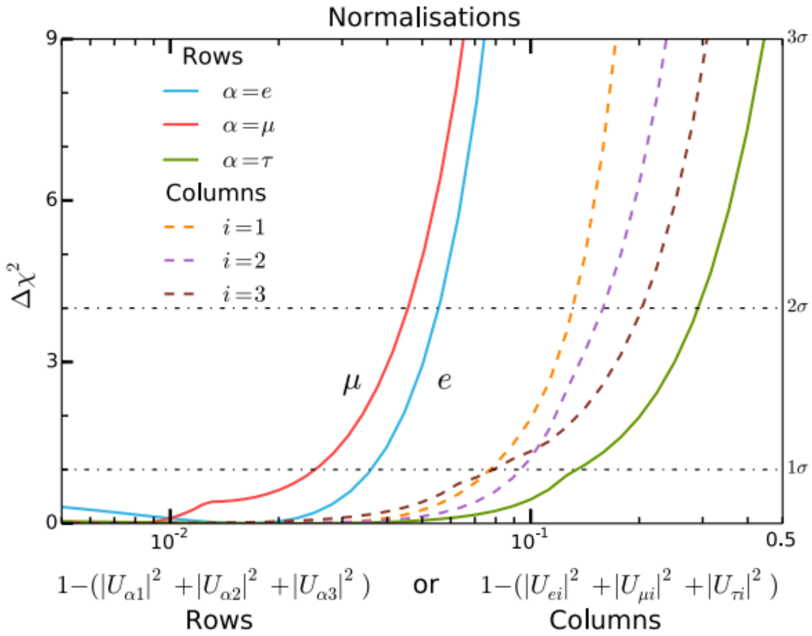
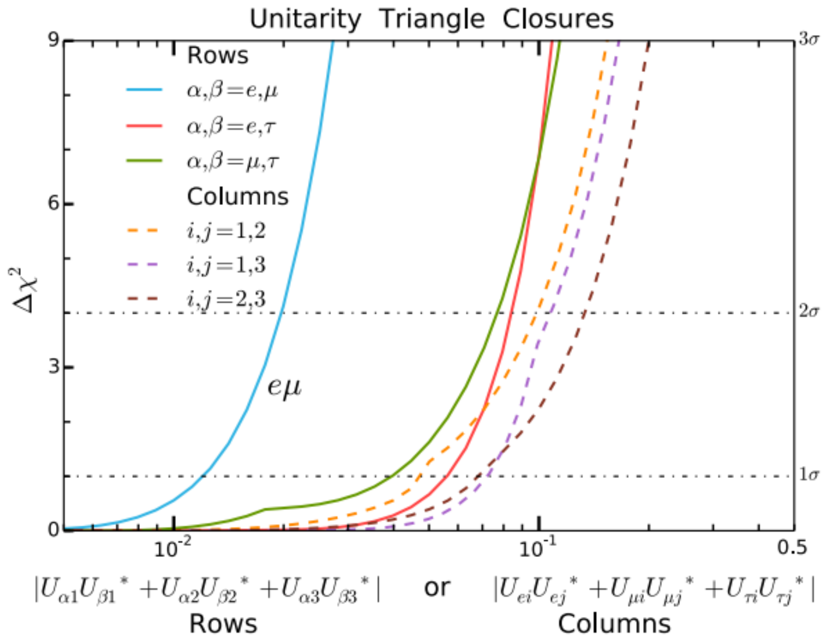
Previous Works





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