HFCPV 2021

Production of Fully-Heavy Tetraquarks Using NRQCD Factorization

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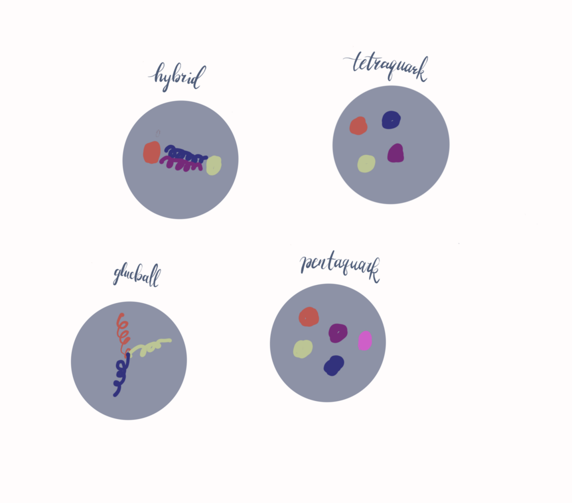
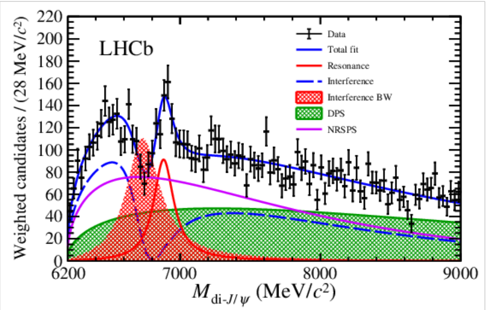
Contents

Motivation

Exotic Hadrons



Discovery of $X(6900)$



Invariant mass spectrum of J/ψ-pair candidates (LHCb, 2020)

- First fully-charm tetraquark candidate
- Strong decay to two J/ψ, $C = +$

Fully-heavy Tetraquark

- Theoretical investigations on the fully heavy tetraquarks date back to late 1970s (Iwasaki, 1976; Chao, 1981).
- Phenomenological studies of spectra and decay properties: Badalian et al., 1987; et al., 2006; Wang, 2017,2020; W. Chen et al., 2017,2018; Wu et al., 2018; Liu et al., 2019; Wang, Di, 2019; H.-X. Chen et al., 2020; Jin et al., 2020; Guo, Oller, 2020....
- Search for the fully-bottom tetraquark on Lattice NRQCD: found no indication of any states below $2\eta_b$ threshold in the $0^{++}, 1^{+-}$ and 2^{++} channels (Hughes et al., 2018).

Production Mechanism

- **Duality relations:** Berezhnoy et al., 2011, 2012; Kaliner et al., 2017
- **Color evaporation model:** Carvalho et al., 2016; Maciuła et al., 2020
- $\gamma\gamma$ **interactions:** Gonçalves, Moreira, 2021

Production Mechanism

- **Duality relations:** Berezhnoy et al., 2011, 2012; Kaliner et al., 2017
- **Color evaporation model:** Carvalho et al., 2016; Maciuła et al., 2020
- **NRQCD-inspired factorization:** Y.-Q. Ma, Zhang, 2020; Feng et al., 2020,2021; R.-L. Zhu, 2020
- **$\gamma\gamma$ interactions:** Gonçalves, Moreira, 2021

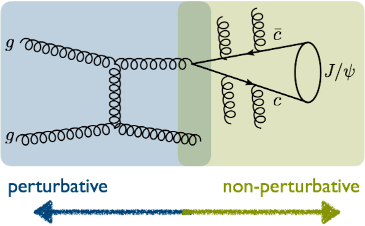
NRQCD Factorization

QCD Factorization Theorem

The inclusive production of high-$P_\perp$ hadrons in the high-energy hadron collision experiments is dominated by the fragmentation mechanism (Collins et al., 1989).

$$\mathrm{d}\sigma\left[A+B\longrightarrow H\left(P_{\mathsf{T}}\right)+X\right]$$

$$=\sum_i \mathrm{d}\hat{\sigma}\left[A+B\longrightarrow i\left(\frac{P_{\mathsf{T}}}{z}\right)+X\right]\otimes \boldsymbol{D_{i\to H}}\left(z,\mu\right)+\mathcal{O}\left(\frac{1}{P_{\mathsf{T}}^2}\right)$$



PARTICLEBITES, 2016

$\hat{\sigma}$: partonic cross section,
$D_{i\to H}\left(z,\mu\right)$: fragmentation function

Fragmentation Function

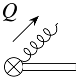
Gauge-invariant operator definition for the gluon fragmentation function (Collins, Soper, 1982)

$$D_{g\to H}(z,\mu_\Lambda) = \frac{-g_{\mu\nu} z^{d-3}}{2\pi k^+ \left(N_c^2-1\right)(d-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+x^-} \sum_X \left\langle 0 \middle| G_c^{+\mu}(0)\mathcal{E}^\dagger(0,0,\mathbf{0}_\perp)_{cb} \middle| H(P)+X \right\rangle \left\langle H(P)+X \middle| \mathcal{E}(0,x^-,\mathbf{0}_\perp)_{ba} G_a^{+\nu}(0,x^-,\mathbf{0}_\perp) \middle| 0 \right\rangle$$

G: field-strength tensor of gluons,
k: momentum of G,
$\mathcal{E}$: gauge link,
$d := 4 - 2\epsilon$: spacetime dimension,
$z := P^+/k^+$

Feynman Rules

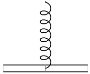
- Creation of a gluon with by the operator $G_a^{+\nu}(k)$



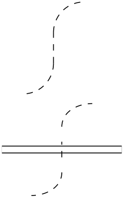
$$= +\mathrm{i}\left(g^{\nu\alpha}k^{+} - Q^{\nu}n^{\alpha}\right)\delta_{ab},$$

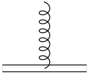
where α is vector indices the created gluon, and b is the color index of the eikonal line.

- Eikonal propagators and vertices

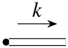


$$= -\mathrm{i}g_s t^a n^{\mu};$$

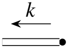




$$= \mathrm{i}g_s t^a n^{\mu};$$



$$= \frac{\mathrm{i}}{n\cdot k + \mathrm{i}\epsilon};$$



$$= \frac{\mathrm{i}}{n\cdot k + \mathrm{i}\epsilon};$$

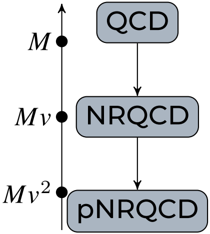
$2\pi\delta(n\cdot k)$

Fragmentation Function

Fragmentation function follows Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation

$$\mu \frac{\partial}{\partial \mu} D_{g \to T_{4c}}(z, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_{j \in \{g,c\}} \int_z^1 \frac{\mathrm{d}y}{y} P_{g \leftarrow j}\left(\frac{z}{y}, \mu\right) D_{g \to T_{4c}}(y, \mu)$$

$P_{g \leftarrow j}$ is the splitting kernel.

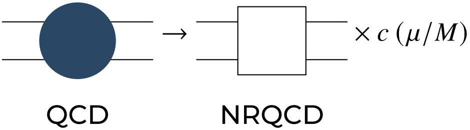


Vairo,
Hadron 2011

- Quarkonium energy scale (Braaten, 1997)

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

- Integrate out the heavy($\sim M$) degrees of freedom



Qiu, 2011

NRQCD Factorization

- For inclusive processes, we use the vacuum-saturation approximation.
- Factorization formula for the fragmentation function $D_{g\to T_{4c}}$

$$
\begin{aligned}
D_{g\to T_{4c}}(z,\mu_\Lambda) =& \frac{d_{3\times3}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\right|^2 \\
&+\frac{d_{6\times6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)}\right|0\right\rangle\right|^2 \\
&+\frac{d_{3\times6}\left[g\to cc\bar{c}\bar{c}^{(J)}\right]}{m^9}2\mathrm{Re}\left[\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\left\langle 0\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}\right|T_{4c}^{(J)}\right\rangle\right]
\end{aligned}
$$

NRQCD Factorization

- For the exclusive production at B factory

$$\sigma(e^+e^- \to T_{4c}^J + \gamma) = \frac{F_{3,3}^{[J]}}{m_c^8}(2M_{T_{4c}})\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\right|^2 + \frac{F_{6,6}^{[J]}}{m_c^8}(2M_{T_{4c}})\left|\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)}\right|0\right\rangle\right|^2 + \frac{F_{3,6}^{[J]}}{m_c^8}(2M_{T_{4c}})2\mathrm{Re}\left[\left\langle T_{4c}^{(J)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\right|0\right\rangle\left\langle 0\left|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)\dagger}\right|T_{4c}^{(J)}\right\rangle\right]$$

- For the inclusive production at B factory

$$\sigma(e^+e^- \to T_{4c}^{1+-} + X) = \frac{F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{m_c^8}(2M_{T_{4c}})\left|\left\langle T_{4c}^{(1)}\left|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(1)}\right|0\right\rangle\right|^2$$

NRQCD Operators

We construct the NRQCD local operators for the S-wave tetraquark with $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$

$$\mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} = -\frac{1}{\sqrt{3}}\left(\psi_a^\dagger \sigma^i \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^t \mathrm{i}\sigma^2 \sigma^i \chi_d\right)\ \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}},$$

$$\mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} = \left(\psi_a^\dagger \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^t \mathrm{i}\sigma^2 \chi_d\right)\ \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}},$$

$$\mathcal{O}^{(1)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(m_j) = \varepsilon^i(m_j)\frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger \sigma^j \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^t \mathrm{i}\sigma^2 \sigma^k \chi_d\right)\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$\mathcal{O}^{(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(m_j) = \varepsilon^{kl}(m_j)\left(\psi_a^\dagger \sigma^m \mathrm{i}\sigma^2 \psi_b^*\right)\left(\chi_c^t \mathrm{i}\sigma^2 \sigma^n \chi_d\right)\ \Gamma^{kl;mn}\ \mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} := \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}), \quad \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} := \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

$$\Gamma^{kl;mn} := \frac{1}{2}\left(\delta^{km}\delta^{ln} + \delta^{kn}\delta^{lm} - \frac{2}{3}\delta^{kl}\delta^{mn}\right)$$

NRQCD Operators

- The operators manifest the correct C-parity under the charge conjugation transformations

$$\psi \rightarrow \mathrm{i}\left(\chi^{\dagger}\sigma^{2}\right)^{t}, \quad \chi \rightarrow -\mathrm{i}\left(\psi^{\dagger}\sigma^{2}\right)^{t}$$

- We use the basis in which the quark and anti-quark pairs in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.
- These NRQCD operators in can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents(H.-X. Chen et al.,2020).

Perturbative Matching

We use the perturbative matching procedure to determine the short-distance coefficients(SDCs).

- Replace the physical tetraquark state T_{4c}^{J} with a free 4-quark state
- Calculate both sides of factorization formula in perturbative QCD and perturbative NRQCD
- Solving the factorization formula to determine the SDCs.

The SDCs are insensitive to the long-distance physics.

Four-Quark States

For convenience, we use the eigenstates of the angular momentum, manifesting the same quantum numbers as the physical tetraquark states.

$$\left|\mathcal{T}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{J,m_j}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle$$

$$C_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle$$

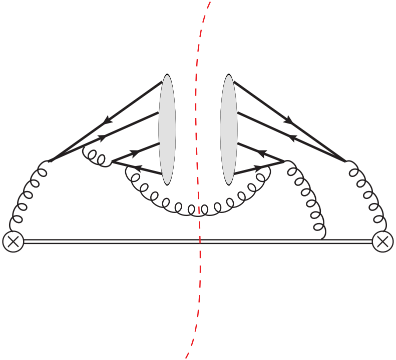
$$\left|\mathcal{T}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{0,0}(Q)\right\rangle = \frac{1}{2}\sum_{\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|00\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|00\right\rangle$$

$$C_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{ab;cd}\left|c_a^{\lambda_1}(q_1)c_b^{\lambda_2}(P-q_1)\bar{c}_c^{\lambda_3}(q_2)\bar{c}_d^{\lambda_4}(Q-P-q_2)\right\rangle,$$

Fragmentation Production of T_{4c} at LHC

Feynman Diagrams

- We employ the self-written program `HepLib`, which employ `Qgraf` and `GiNaC` internally to generate the Feynman diagrams (Feng et al., 2021).
- There are about 100 diagrams for the amplitude.



Perturbative QCD Calculation

To project the $QQ\bar{Q}\bar{Q}$ into correct spin/color quantum number of tetraquark, we use the following projector

$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_\mu)^{ij} (\Pi_\nu C)^{lk} C^{abcd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} J^{\mu\nu}_{0,1,2}$$
$$\bar{u}_i^a \bar{u}_j^b v_k^c v_l^d \to (C\Pi_0)^{ij} (\Pi_0 C)^{lk} C^{abcd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}$$

C is the charge conjugate operator, $\Pi_\mu(\Pi_0)$ is the spin-triplet(singlet) projector of quarks (Petrelli et al., 1997), $J^{\mu\nu}_{0,1,2}$ are the spin projectors of quark and anti-quark pairs (Braaten, Lee, 2003).

$$J_0^{\mu\nu} = \frac{1}{\sqrt{3}} \eta^{\mu\nu}(P), \quad J_1^{\mu\nu}(\epsilon) = -\frac{i}{\sqrt{2P^2}} \varepsilon^{\mu\nu\rho\sigma} \epsilon_\rho P_\sigma,$$
$$J_2^{\mu\nu}(\epsilon) = \epsilon_{\rho\sigma} \left\{ \frac{1}{2} \left[\eta^{\mu\rho}(P)\eta^{\nu\sigma}(P) + \eta^{\mu\sigma}(P)\eta^{\nu\rho}(P) \right] - \frac{1}{3} \eta^{\mu\nu}(P)\eta^{\rho\sigma}(P) \right\}$$

Perturbative NRQCD Calculation

To match the SDCs, we need to compute the matrix element on the right hand side of the factorization formula in perturbative NRQCD.

$$\left\langle \mathcal{T}^{0}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \middle| \mathcal{O}^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \middle| 0 \right\rangle = 4$$

$$\left\langle \mathcal{T}^{0}_{\mathbf{6}\otimes\bar{\mathbf{6}}} \middle| \mathcal{O}^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} \middle| 0 \right\rangle = 4$$

$$\left\langle \mathcal{T}^{2,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \middle| \mathcal{O}^{(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(m_j) \middle| 0 \right\rangle = 4$$

SDCs

$$
\begin{aligned}
d_{3\times3}\left(g\to 0^{++}\right) =& \frac{\pi^2\alpha_s^4}{497664z(2-z)^2(3-z)}\Big[186624-430272z+511072z^2-425814z^3\\
&+217337z^4-61915z^5+7466z^6+42(1-z)(2-z)(3-z)(-144+634z\\
&-385z^2+70z^3)\log(1-z)+36(2-z)(3-z)\left(144-634z+749z^2-364z^3\right.\\
&\left.+74z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(72-362z+361z^2-136z^3+23z^4\right)\\
&\times\log\left(1-\frac{z}{3}\right)\Big].
\end{aligned}
$$

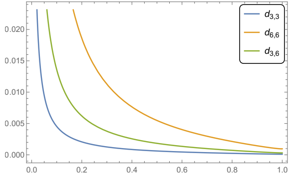
$$
\begin{aligned}
d_{6\times6}\left(g\to 0^{++}\right) =& \frac{\pi^2\alpha_s^4}{55296z(2-z)^2(3-z)}\Big[186624-430272z+617824z^2-634902z^3\\
&+374489z^4-115387z^5+14378z^6-6(1-z)(2-z)(3-z)(-144-2166z\\
&+1015z^2+70z^3)\log(1-z)-156(2-z)(3-z)\left(144-1242z+1693z^2-876z^3\right.\\
&\left.+170z^4\right)\log\left(1-\frac{z}{2}\right)+300(2-z)(3-z)\left(72-714z+953z^2-472z^3+87z^4\right)\\
&\times\log\left(1-\frac{z}{3}\right)\Big].
\end{aligned}
$$

$$
\begin{aligned}
d_{3\times6}\left(g\to 0^{++}\right) =& -\frac{\pi^2\alpha_s^4}{165888z(2-z)^2(3-z)}\Big[186624-430272z+490720z^2-394422z^3\\
&+199529z^4-57547z^5+7082z^6+6(1-z)(2-z)(3-z)(-432+3302z\\
&-1855z^2+210z^3)\log(1-z)-12(2-z)(3-z)\left(720-2258z+2329z^2-1052z^3\right.\\
&\left.+226z^4\right)\log\left(1-\frac{z}{2}\right)+12(2-z)(3-z)\left(936-4882z+4989z^2-1936z^3+331z^4\right)\\
&\times\log\left(1-\frac{z}{3}\right)\Big].
\end{aligned}
$$

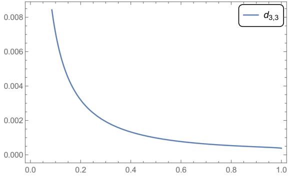
SDCs

$$
\begin{aligned}
d_{3\times 3}\left(g \to 2^{++}\right) =& \frac{\pi^2 \alpha_s^4}{622080 z^2 (2-z)^2 (3-z)} \left[2\left(46656 - 490536 z + 1162552 z^2\right.\right. \\
& -1156308 z^3 + 595421 z^4 - 170578 z^5 + 21212 z^6\big) z \\
& +3(1-z)(2-z)(3-z)(-20304 - 31788 z)(1296 + 1044 z \\
& +73036 z^2 - 36574 z^3 + 7975 z^4\big) \log(1-z)\Big] + 33(2-z)(3-z)(1296 + 25) \\
& -9224 z^2 + 9598 z^3 - 3943 z^4 + 725 z^5\big) \log\left(1 - \frac{z}{3}\right)\Big], \\
d_{6\times 6}\left(g \to 2^{++}\right) =& d_{3\times 6}\left(g \to 2^{++}\right) = 0.
\end{aligned}
$$

- There is NO IR divergence.



SDCs for T_{4c}^{0++}



SDC for T_{4c}^{2++}

Long-Distance Matrix Elements

- The NRQCD Long-Distance Matrix Elements(LDMEs) should be calculated in lattice QCD in principle since they are non-perturbative.
- We use three phenomenological four-quark models to calculate the LDMEs. The results are proportional to the wave functions at the origin.

$$\left\langle T_{4c}^{J}\left|\mathcal{O}_{\text{color}}^{J}\right|0\right\rangle_{\text{I}} = \frac{1}{\pi^2}\sqrt{\frac{105}{2}}R_{\text{I, color}}^{J}(0)$$

$$\left\langle T_{4c}^{J}\left|\mathcal{O}_{\text{color}}^{J}\right|0\right\rangle_{\text{II}} = 4\psi_{\text{color}}^{J}(0)$$

$$\left\langle T_{4c}^{J}\left|\mathcal{O}_{\text{color}}^{J}\right|0\right\rangle_{\text{III}} = \frac{1}{2\pi^{3/2}}R_{\text{III, color}}^{J}(0)$$

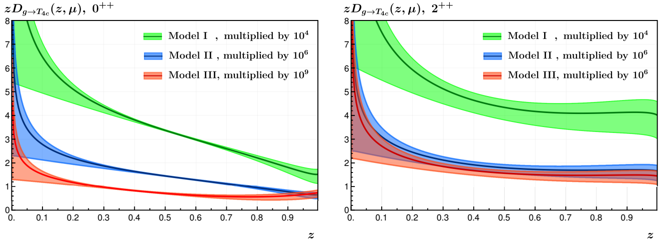
- The numerical values are: ($\mathrm{GeV}^{9/2}$)

Model	Ref.	$\left\langle T_{4c}^{0}\left\|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\right\|0\right\rangle$	$\left\langle T_{4c}^{0}\left\|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)}\right\|0\right\rangle$	$\left\langle T_{4c}^{2}\left\|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(2)}\right\|0\right\rangle$
I	Zhao et al., 2020	2.402	2.085	1.865
II	Lü et al., 2020	−0.1864	−0.1132	0.1200
III	Liu et al., 2020	−0.136737	0.117944	0.112084

Evolution of Fragmentation Function

Since the process is gluon dominance, the leading order splitting kernels read (n_f: number of active light quark flavors):

$$P_{g\leftarrow g}(z) = 6\left[\frac{(1-z)}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_\mathrm{f}}{18}\right)\delta(1-z)\right]$$

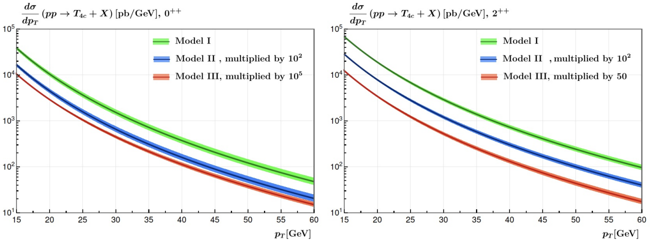


Evolution of $g \to T_{4c}$ fragmentation functions

Phenomenology at LHC

- Parameters: $\sqrt{s} = 13\,\mathrm{TeV}$; CTEQ14 PDF sets; factorization scale $\mu \in [p_T/2, 2p_T]$
  $m_c = 1.5\,\mathrm{GeV}$; $\quad p_T \in [15, 60]\,\mathrm{GeV}$

	0^{++}		2^{++}	
Model	σ/pb	N_{events}	σ/pb	N_{events}
I	1.6×10^5	4.8×10^{11}	2.9×10^5	8.7×10^{11}
II	6.9×10^2	2.1×10^9	1.2×10^3	3.61×10^9
III	0.466	1.3×10^6	1.1×10^3	3.15×10^9



p_{T} distribution of T_{4c} production on LHC

Production of T_{4c} at B Factory

Exclusive production of C-even T_{4c}

The differential cross section can be expressed in terms of the differential decay rate of a virtual photon.($\lambda := \lambda_1 - \lambda_2$)

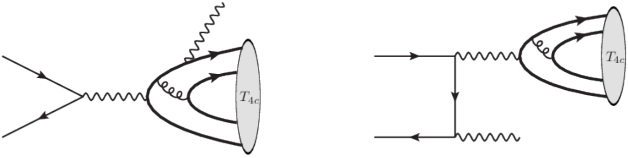
$$\frac{\mathrm{d}\sigma\left[e^+e^- \to \gamma(\lambda_1) + T_{4c}^J(\lambda_2)\right]}{\mathrm{d}\cos\theta} = \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{\mathrm{d}\Gamma\left[\gamma^*(S_z) \to \gamma(\lambda_1) + T_{4c}^J(\lambda_2)\right]}{\mathrm{d}\cos\theta}$$

$$= \frac{2\pi\alpha}{s^{3/2}} \sum_{S_z=\pm 1} \frac{\left|\mathbf{p}_f\right|}{16\pi s} \frac{3}{4\pi} \left|\mathcal{M}_{\lambda_1,\lambda_2}^J\right|^2 \left|d_{S_z,\lambda}^1(\theta)\right|^2$$

The factorization holds true at the helicity amplitude level for the exclusive process.

$$\mathcal{M}_{\lambda_1,\lambda_2}^J = \frac{\mathcal{A}_{\lambda_1,\lambda_2}^{3[J]}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T_{4c}^J | \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)} | 0 \rangle + \frac{\mathcal{A}_{\lambda_1,\lambda_2}^{6[J]}}{m_c^4} \sqrt{2M_{T_{4c}}} \langle T_{4c}^J | \mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(J)} | 0 \rangle$$

- There are roughly 40 s-channel diagrams in total.
- Due to C-parity conservation, the t-channel process in $b)$ does not contribute.



$a)$ $b)$

SDCs

- The SDCs are ($r := 16m_c^2/s$):

$$\mathcal{A}_{1,0}^{3[0]} = \mathcal{A}_{-1,0}^{3[0]} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-17r+9r^2\right)}{27\sqrt{3}(3-r)(2-r)},$$

$$\mathcal{A}_{1,0}^{6[0]} = \mathcal{A}_{-1,0}^{6[0]} = -\frac{16\pi^{5/2}\alpha\alpha_s\left(10-9r+r^2\right)}{27\sqrt{2}(3-r)(2-r)},$$

$$\mathcal{A}_{1,0}^{3[2]} = \mathcal{A}_{-1,0}^{3[2]} = \frac{128\pi^{5/2}\alpha\alpha_s}{27\sqrt{6}(3-r)},$$

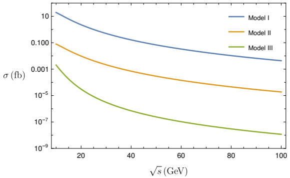
$$\mathcal{A}_{1,1}^{3[2]} = \mathcal{A}_{-1,-1}^{3[2]} = \frac{512\pi^{5/2}\alpha\alpha_s}{27\sqrt{2}(3-r)}\left(\frac{m_c}{s^{1/2}}\right),$$

$$\mathcal{A}_{1,2}^{3[2]} = \mathcal{A}_{-1,-2}^{3[2]} = \frac{2048\pi^{5/2}\alpha\alpha_s}{27(3-r)}\left(\frac{m_c}{s^{1/2}}\right)^2.$$

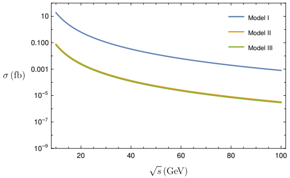
- At large $\sqrt{s}$ limit, the polarized cross section scales as $\sigma \propto s^{-2-|\lambda|}$, which is compatible with the helicity selection rule.

Phenomenology

- The center-of-mass energy is $\sqrt{s} = 10.58\,\mathrm{GeV}$, and the designed luminosity is about $50\,\mathrm{ab}^{-1}$.
- The total cross sections decline quite fast with increasing $\sqrt{s}$.



Total cross section(0^{++})



Total cross section(2^{++})

- The cross sections and the event numbers are

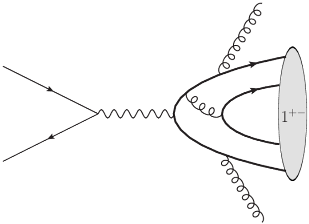
	0^{++}		2^{++}	
Model	σ/fb	$N_{\mathrm{events}}/10^4$	σ/fb	$N_{\mathrm{events}}/10^4$
I	17	86	14	70
II	0.070	0.35	0.058	0.29
III	0.0015	0.0075	0.050	0.25

Inclusive production of C-odd T_{4c}

- Factorization formula for the inclusive production at B factory:

$$\sigma(e^+e^- \to T_{4c}^{1^{+-}} + X) = \frac{F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{m_c^8}(2M_{T_{4c}}) \left| \left\langle T_{4c}^{(1)} \left| \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(1)} \right| 0 \right\rangle \right|^2$$

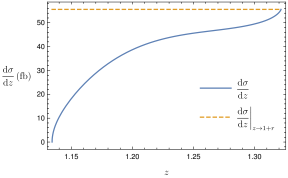
- There are roughly 400 Feynman diagrams in total.
- C-parity conservation requires that two additional gluons emitted accompanied with C-odd T_{4c}.



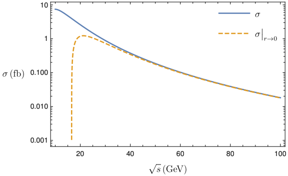
The full analytical expression is too lengthy to be presented here. We choose to present its limiting value near the upper endpoint:

$$\left.\frac{\mathrm{d}F_{\bar{\mathbf{3}}\otimes\mathbf{3}}}{\mathrm{d}z}\right|_{z\to 1+r} = \frac{2^2\pi^3\alpha^2\alpha_s^4}{3^8 s^2(3-r)^2(2-r)^2(3+r)(6+r)}$$
$$\times \Big(550800 + 482112\ln 2 - 803628r - 183168r\ln 2 + 275616r\ln r$$
$$+ 27\left(17856 - 16992r - 844r^2 + 4764r^3 - 779r^4 - 336r^5 + 70r^6 + r^7\right)\ln(2-r)$$
$$+ 16\left(-30132 + 11448r - 3897r^2 + 8403r^3 - 2489r^4 - 475r^5 + 166r^6\right)\ln(3-r)$$
$$+ 235854r^2 + 62352r^2\ln 2 + 85140r^2\ln r + 62742r^3 - 134448r^3\ln 2 - 263076r^3\ln r$$
$$- 50316r^4 + 39824r^4\ln 2 + 60857r^4\ln r + 2706r^5 + 7600r^5\ln 2 + 16672r^5\ln r$$
$$+ 1842r^6 - 2656r^6\ln 2 - 4546r^6\ln r - 27r^7\ln r\Big).$$

- We also adopt three phenomenological models to estimate the LDME.



Energy Distribution(Model I)



Total cross section(Model I)

- The cross sections and the event numbers are ($\sqrt{s} = 10.58\,\text{GeV}$, designed luminosity $\approx 50\,\text{ab}^{-1}$)

Model	$\left\langle T^{(1)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(m_j) \middle| \mathcal{O}^{(1)}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(m_j) \middle| 0 \right\rangle$	σ/fb	$N_{\text{events}}/10^4$
I	$2.31684\,\text{GeV}^{9/2}$	7.3	37
II	$-0.1612\,\text{GeV}^{9/2}$	0.035	0.18
III	$0.126437\,\text{GeV}^{9/2}$	0.022	0.11

Summary

- We propose a model-independent approach to study the production of fully heavy tetraquark, based on NRQCD factorization.
- We analytically calculate the SDCs for S-wave T_{4c} production at LHC and B factory, while the LDMEs should be determined by nonperturbative methods.
- The LDMEs calculated from different models lead to diverse predictions.
- Future experimental search may help to identify the model that is more favorable.