CP violation phase in BSM amplitudes

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Outline

- Introduction
- BSM amplitudes in SMEFT and their decomposition relations
- BSM amplitudes from on-shell scattering amplitudes
- Summary

CP violation in SM

$$\mathcal{L}_{\text{Yukawa}} = \lambda_d^{ij}(\bar{q}_i\, H)d_j + \lambda_u^{ij}(\bar{q}_i\, \tilde{H})u_j + \lambda_e^{ij}(\bar{l}_i\, H)e_j + \text{h.c.}$$

M.E. Peskin, D.V. Schroeder, p721-723
Introduction to quantum field theory

$U(3)^5$ $\quad q^i \to U_q^{ij} q^j\,,\ u^i \to U_u^{ij} u^j\,,\ d^i \to U_d^{ij} d^j\,,\ l^i \to U_l^{ij} l^j\,,\ e^i \to U_e^{ij} e^j$

$$\mathcal{L}_{\text{Yukawa}} = Y_d^{ij}(\bar{q}_i\, H)d_j + Y_u^{ij}(\bar{q}_i\, \tilde{H})u_j + Y_e^{ij}(\bar{l}_i\, H)e_j + \text{h.c.}\,.$$

Spurions $\quad Y_u \to U_q\, Y_u\, U_u^\dagger \quad , \quad Y_d \to U_q\, Y_d\, U_d^\dagger \quad , \quad Y_e \to U_l\, Y_e\, U_e^\dagger\,,$

R.S. Chivukula, H.Georgi.
PLB 188, 99 (1987)

J.Ellis, M.Madigan, K. Mimasu,
V. Sanz, T. You, 2012.02779

$$\langle Y_d\rangle^{ij} = y_d^{ij} \propto m_d^{ij},\ \langle Y_e\rangle^{ij} = y_e^{ij} \propto m_e^{ij},\ \langle Y_u\rangle^{ij} = (V^\dagger y_u)^{ij} \propto (V^\dagger)^{ik} m_u^{kj}$$

$$u_L^i \to (V^\dagger)^{ik} u_L^k$$ **CKM matrix**

CKM matrix originates from Yukawa couplings,
so as weak CP violation.

CP violation in SM

$$L_\theta = \theta \frac{g^2}{32\pi^2} F_a^{\mu\nu} \tilde{F}_{a\mu\nu}.$$

$$L_{\text{Mass}} = \bar{q}_{iR} M_{ij} q_{jL} + h.c.$$

below 1 GeV

Chiral rotation $q_f \to e^{i\alpha\gamma_5/2} q_f$

$$\bar{\theta} = \theta + Arg\ detM$$

R. D. Peccei, hep-ph/0607269

Stong CP phase relates to Yukawa coupling.

CP violation in beyond SM

- In new physics beyond SM, a CP-violating Higgs is common.

e.g. Two-Higgs-Doublet Model, MSSM Model

$$\Phi_i = \begin{pmatrix} \varphi_i^+ \\ (v_i + \eta_i + i\chi_i)/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix}$$

$$\langle \phi_1 \rangle_{vac} = 2^{-1/2} \rho_1 e^{i\theta}$$

T.D. Lee, PRD8,1226

and

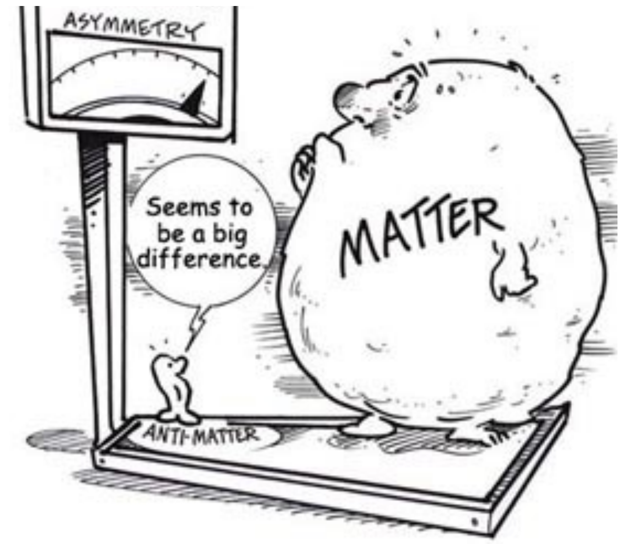
$$\langle \phi_2 \rangle_{vac} = 2^{-1/2} \rho_2 \, .$$

Spontaneous CP violation

Baryogenesis

- Matter-antimatter asymmetry.

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$$



- Sakharov's conditions:

1. **Baryon number violation.**
2. **C and CP violation.**
3. **Interactions out of thermal equilibrium.**

$$e_L^- \overset{CP}{\longleftrightarrow} e_R^+$$

$$A_{\rm CP} = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)J$$

$$J = \mathrm{Im}(V_{ub}V_{cb}V_{ub}^* V_{cd}^*) \simeq s_{12}s_{23}s_{13}\sin\delta_{\rm KM} \simeq 3\times 10^{-5}$$

C. Jarlskog, Z.Phys. C29 (1985) 491-497

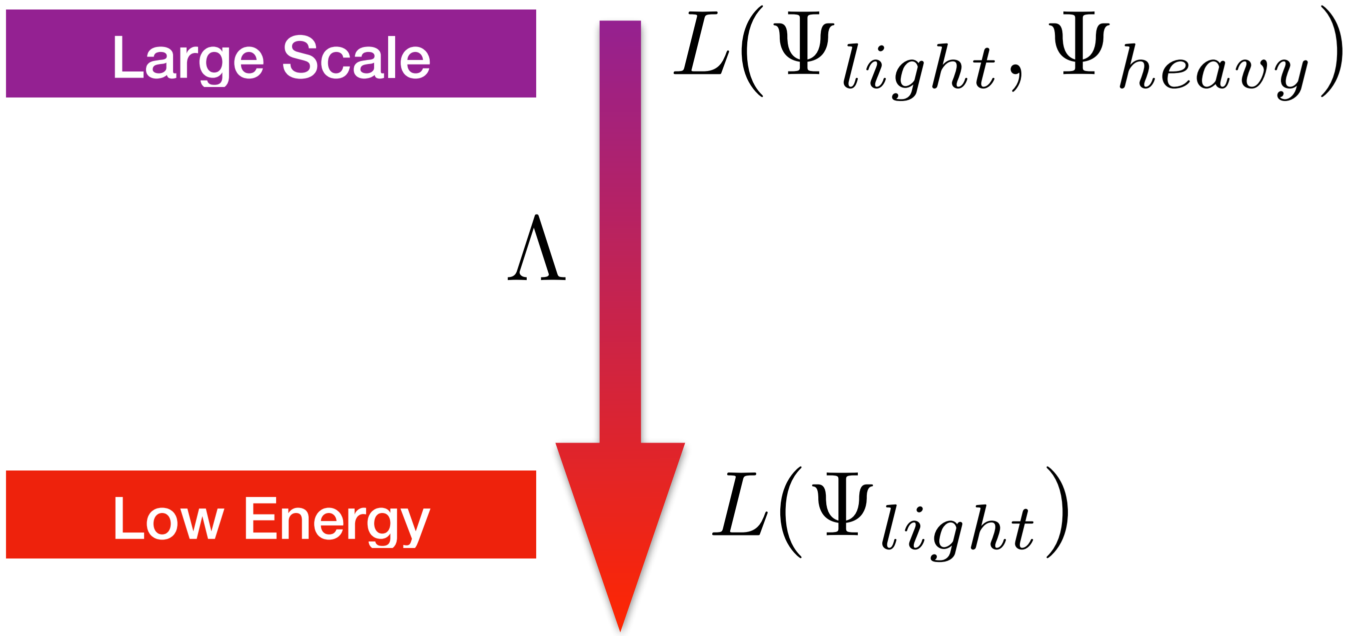
$$\delta_{\rm CP} \sim \frac{A_{\rm CP}}{T_C^{12}} \sim 10^{-20}$$

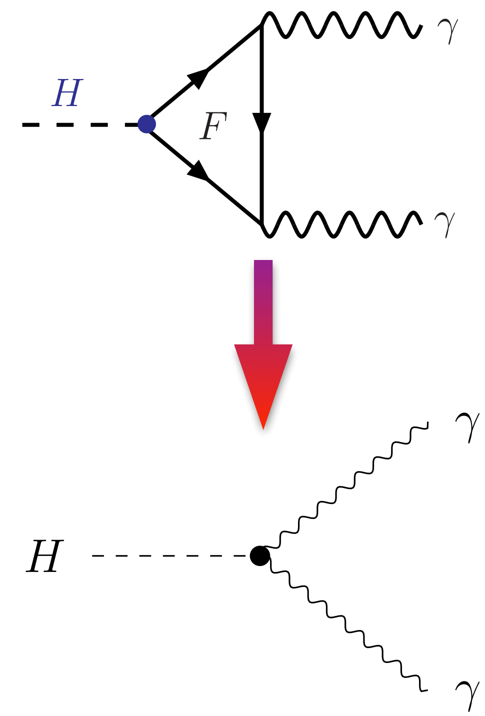
Needs new CP violation source.

M.E. Shaposhnikov, Nucl.Phys. B287 (1987) 757

Model independent ways

- Standard Model Effective Field Theory (SMEFT).





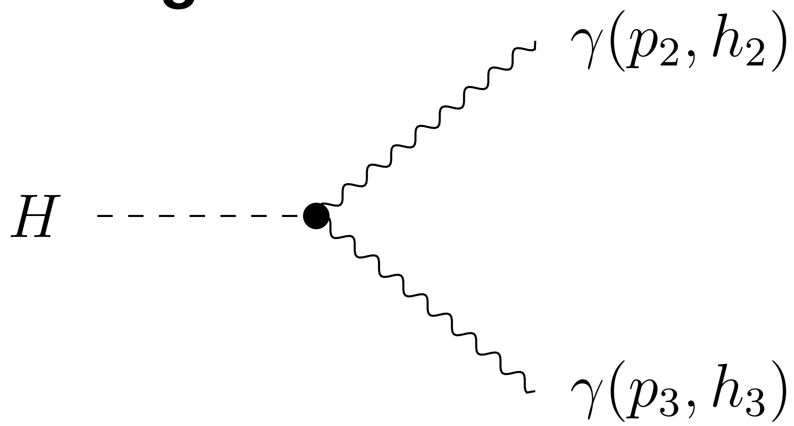
- On-shell scattering amplitude.

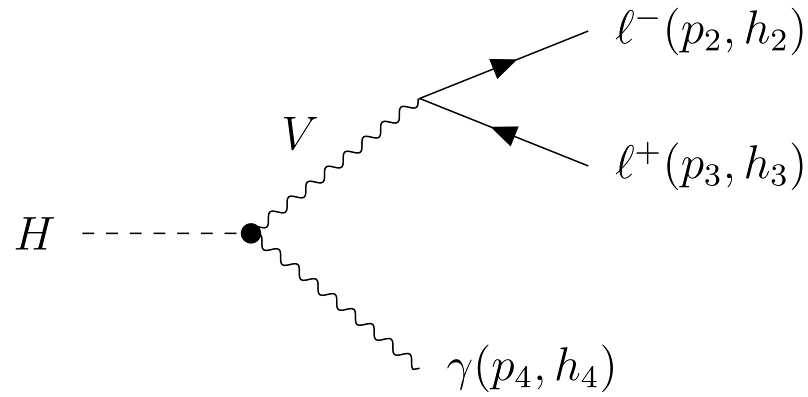
No fields, Lagrangian, Feynman rules.
Basic block is particle state.

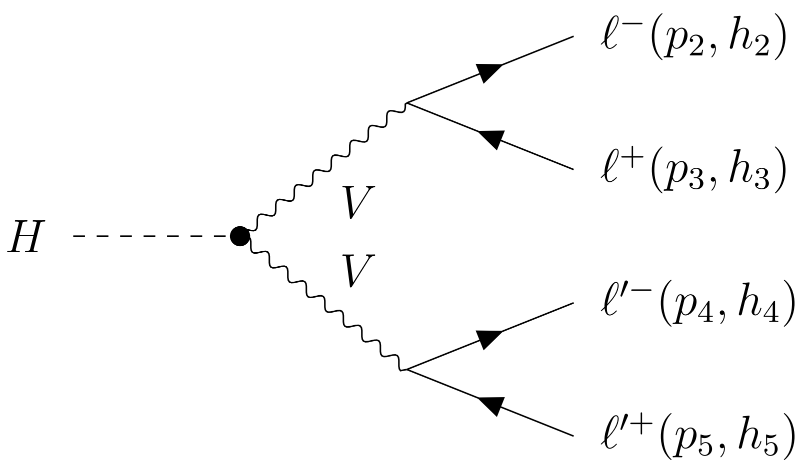
This talk focus on analytical structures of BSM amplitudes,
more relative research about CP-violating Higgs phenomenology
see *1705.00267, 1712.00267, 1902.04756*

SMEFT

LHC golden channels







$$\mathcal{L}^{int} = -\frac{c_{VV}}{v} H V^{\mu\nu} V_{\mu\nu} - \frac{\tilde{c}_{VV}}{v} H V^{\mu\nu} \tilde{V}_{\mu\nu}$$

$$\xi \equiv \tan^{-1}(\tilde{c}_{VV}/c_{VV})$$ **CP violation phase angle**

$$\mathcal{O}^6_{\Phi D} = (\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D^\mu \Phi),$$

$$\mathcal{O}^6_{\Phi W} = \Phi^\dagger \Phi W^I_{\mu\nu} W^{I\mu\nu}, \; \mathcal{O}^6_{\Phi B} = \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu}, \; \mathcal{O}^6_{\Phi WB} = \Phi^\dagger \tau^I \Phi W^I_{\mu\nu} B^{\mu\nu},$$

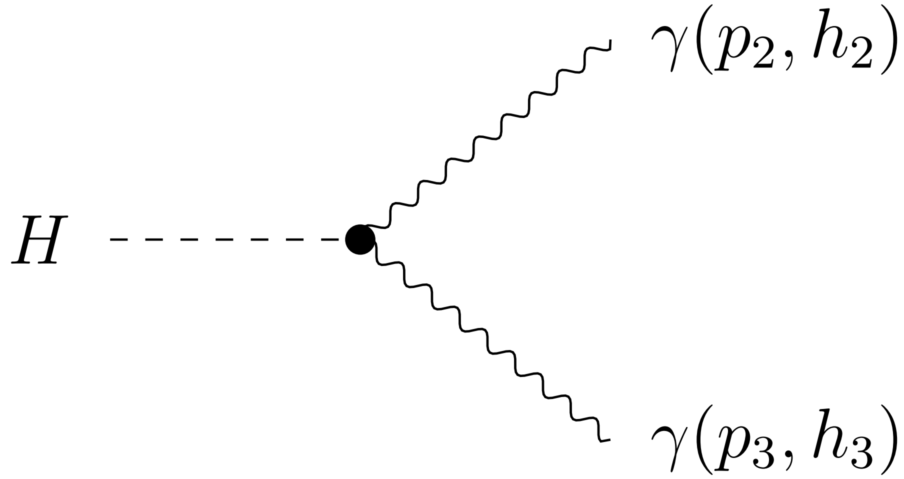
$$\mathcal{O}^6_{\Phi \tilde{W}} = \Phi^\dagger \Phi \tilde{W}^I_{\mu\nu} W^{I\mu\nu}, \; \mathcal{O}^6_{\Phi \tilde{B}} = \Phi^\dagger \Phi \tilde{B}_{\mu\nu} B^{\mu\nu}, \; \mathcal{O}^6_{\Phi \tilde{W} B} = \Phi^\dagger \tau^I \Phi \tilde{W}^I_{\mu\nu} B^{\mu\nu},$$

BSM amplitudes I

$$\langle ij\rangle = \overline{u_-(p_i)}u_+(p_j),\ \ [ij] = \overline{u_+(p_i)}u_-(p_j),$$

$$\langle ij\rangle[ji] = 2p_i\cdot p_j,\ \ s_{ij} = (p_i+p_j)^2,$$

$$\epsilon^{\pm}_{\mu}(p_i,q) = \pm\frac{\langle q^{\mp}|\gamma_\mu|p_i^{\mp}\rangle}{\sqrt{2}\langle q^{\mp}|p_i^{\pm}\rangle},$$



$$\mathcal{M}(2^+_\gamma,3^+_\gamma) = \frac{2c^S_{\gamma\gamma}}{v}e^{i\xi}[23]^2\ ,$$

$$\mathcal{M}(2^-_\gamma,3^-_\gamma) = \frac{2c^S_{\gamma\gamma}}{v}e^{-i\xi}\langle 23\rangle^2\ ,$$

$$\mathcal{M}(2^+_\gamma,3^-_\gamma) = 0\ ,$$

$$\mathcal{M}(2^-_\gamma,3^+_\gamma) = 0\ ,$$

BSM amplitudes II

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_\gamma) = f_V^-(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi}[23]\langle 24\rangle^2,$$

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^+_\gamma) = f_V^-(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{i\xi}\langle 23\rangle[34]^2,$$

$$\mathcal{M}(2^+_{\ell^-}, 3^-_{\ell^+}, 4^+_\gamma) = f_V^+(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{i\xi}\langle 23\rangle[24]^2,$$

$$\mathcal{M}(2^+_{\ell^-}, 3^-_{\ell^+}, 4^-_\gamma) = f_V^+(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi}[23]\langle 34\rangle^2,$$

$f_V^-(s) = \sqrt{2} e l_V P_V(s)$ and $f_V^+(s) = -\sqrt{2} e r_V P_V(s)$

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_{\ell'^-}, 5^+_{\ell'^+}) = f_V^-(s_{23}) f_V^-(s_{45}) \times \frac{2c^S_{VV}}{v}\left(e^{i\xi}\langle 23\rangle\langle 45\rangle[35]^2 + e^{-i\xi}[23][45]\langle 24\rangle^2\right)$$

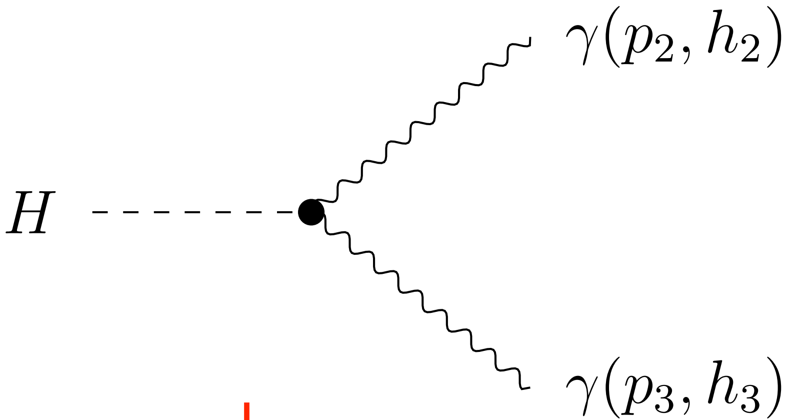
$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^+_{\ell'^-}, 5^-_{\ell'^+}) = f_V^-(s_{23}) f_V^+(s_{45}) \times \frac{2c^S_{VV}}{v}\left(e^{i\xi}\langle 23\rangle\langle 45\rangle[34]^2 + e^{-i\xi}[23][45]\langle 25\rangle^2\right)$$

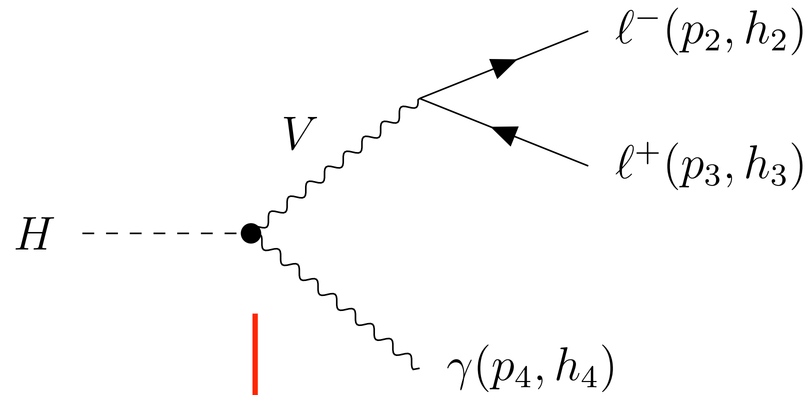
$$\mathcal{M}(2^+_{\ell^-}, 3^-_{\ell^+}, 4^-_{\ell'^-}, 5^+_{\ell'^+}) = f_V^+(s_{23}) f_V^-(s_{45}) \times \frac{2c^S_{VV}}{v}\left(e^{i\xi}\langle 23\rangle\langle 45\rangle[25]^2 + e^{-i\xi}[23][45]\langle 34\rangle^2\right)$$

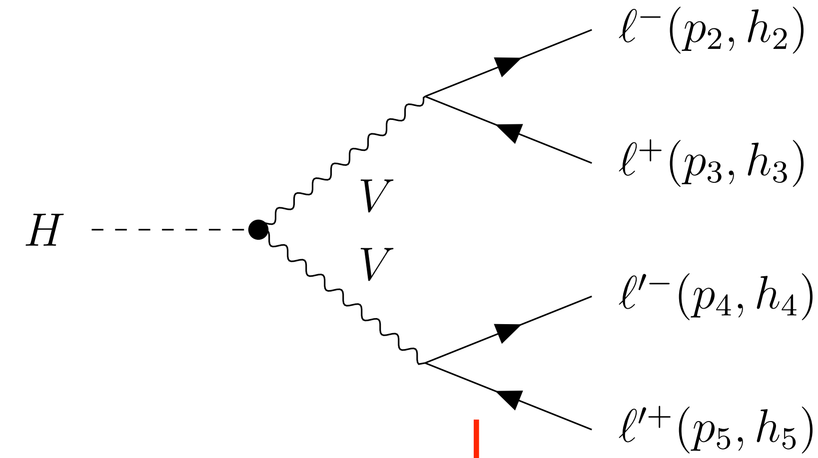
$$\mathcal{M}(2^+_{\ell^-}, 3^-_{\ell^+}, 4^+_{\ell'^-}, 5^-_{\ell'^+}) = f_V^+(s_{23}) f_V^+(s_{45}) \times \frac{2c^S_{VV}}{v}\left(e^{i\xi}\langle 23\rangle\langle 45\rangle[24]^2 + e^{-i\xi}[23][45]\langle 35\rangle^2\right)$$

CP-odd observables could be constructed when four external particles.

Analytical structures







$$\mathcal{M}(2_\gamma^+, 3_\gamma^+) = \frac{2c^S_{\gamma\gamma}}{v} e^{i\xi}[23]^2$$

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_\gamma) = f_V^-(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi}[23]\langle 24\rangle^2$$

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_{\ell'^-}, 5^+_{\ell'^+}) = f_V^-(s_{23}) f_V^-(s_{45}) \times \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle[35]^2 + e^{-i\xi}[23][45]\langle 24\rangle^2\right)$$

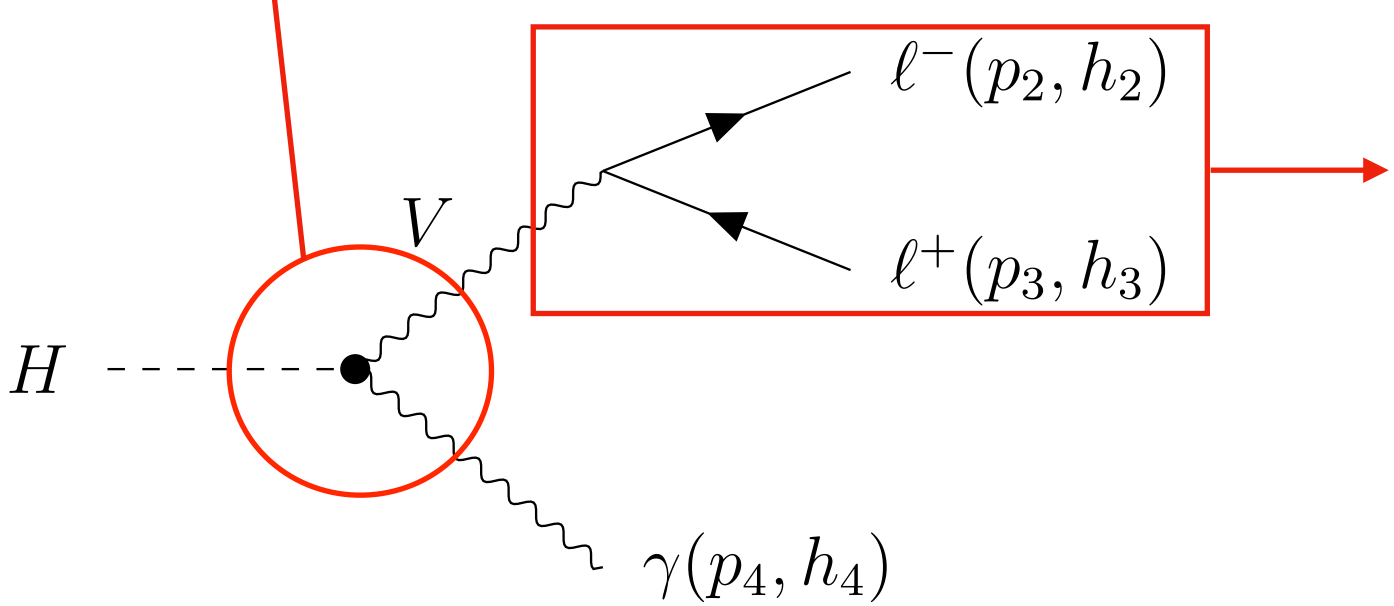
What is the principle?
What is the relations?

Decomposition relations I

$$\mathcal{L}^{int} = -\frac{c_{VV}}{v} H V^{\mu\nu} V_{\mu\nu} - \frac{\tilde{c}_{VV}}{v} H V^{\mu\nu} \tilde{V}_{\mu\nu}$$

$$\Gamma^{\mu\nu}(k,k') = -i\frac{4}{v}[c_{VV}\ (k^\nu k'^\mu - k\cdot k' g^{\mu\nu}) + \tilde{c}_{VV}\ \epsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma]$$

$$\begin{aligned}
&\Gamma^{\mu\nu}(k,k') \\
&= \Gamma^{\mu\nu}(p_2+p_3,k') = \Gamma^{\mu\nu}(p_2,k') + \Gamma^{\mu\nu}(p_3,k') \\
&= \Gamma^{\mu\nu}(p_2+p_3,p_4+p_5) = \Gamma^{\mu\nu}(p_2,p_4) + \Gamma^{\mu\nu}(p_2,p_5) + \Gamma^{\mu\nu}(p_3,p_4) + \Gamma^{\mu\nu}(p_3,p_5).
\end{aligned}$$

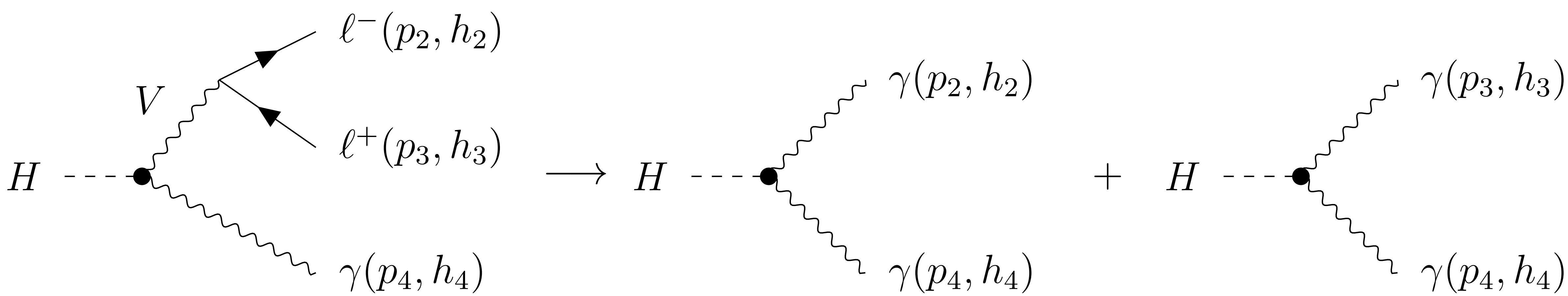


$$\begin{aligned}
J^{\pm}_\mu(s_{23}) &= \frac{f_V^{\mp}(s_{23})}{\sqrt{2}}\langle 2^{\mp}|\gamma_\mu|3^{\mp}\rangle \\
&= \pm f_V^{\mp}(s_{23})\langle 2^{\mp}|3^{\pm}\rangle \epsilon^{\pm}_\mu(3,2) \\
&= \pm f_V^{\mp}(s_{23})\langle 2^{\pm}|3^{\mp}\rangle \epsilon^{\mp}_\mu(2,3)
\end{aligned}$$

Decomposition relations II

$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_\gamma) = -i\Gamma^{\mu\nu}(p_2+p_3, p_4) J^+_\mu(s_{23}) \epsilon^-(4,q)$$
$$= -i\Gamma^{\mu\nu}(p_2, p_4) f^l_V(s_{23}) [23] \epsilon^-(2,3) \epsilon^-(4,q)$$
$$-i\Gamma^{\mu\nu}(p_3, p_4) f^l_V(s_{23}) \langle 23\rangle \epsilon^+(3,2) \epsilon^-(4,q)$$
$$= f^l_V(s_{23}) \times ([23]\mathcal{M}(2^-_\gamma, 4^-_\gamma) + \langle 23\rangle \mathcal{M}(3^+_\gamma, 4^-_\gamma)),$$

=0



$$\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_\gamma) = f^-_V(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi} [23]\langle 24\rangle^2$$

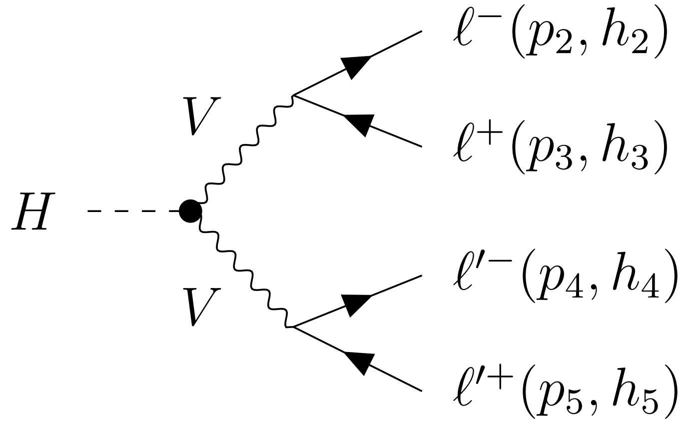
Decomposition relations III

$$
\begin{aligned}
&\mathcal{M}(2^-_{\ell^-},3^+_{\ell^+},4^-_{\ell'^-},5^+_{\ell'^+})\\
&= -i\Gamma^{\mu\nu}(p_2+p_3,p_4+p_5)J^+_\mu(s_{23})J^+_\mu(s_{45})\\
&= f^l_V(s_{23})f^l_V(s_{45})\times(\\
&- i\Gamma^{\mu\nu}(p_2,p_4)[23][45]\epsilon^-(2,3)\epsilon^-(4,5)\\
&- i\Gamma^{\mu\nu}(p_2,p_5)[23]\langle 45\rangle\epsilon^-(2,3)\epsilon^+(5,4)\\
&- i\Gamma^{\mu\nu}(p_3,p_4)\langle 23\rangle[45]\epsilon^+(3,2)\epsilon^-(4,5)\\
&- i\Gamma^{\mu\nu}(p_3,p_5)\langle 23\rangle\langle 45\rangle\epsilon^+(3,2)\epsilon^+(5,4)\ )
\end{aligned}
$$

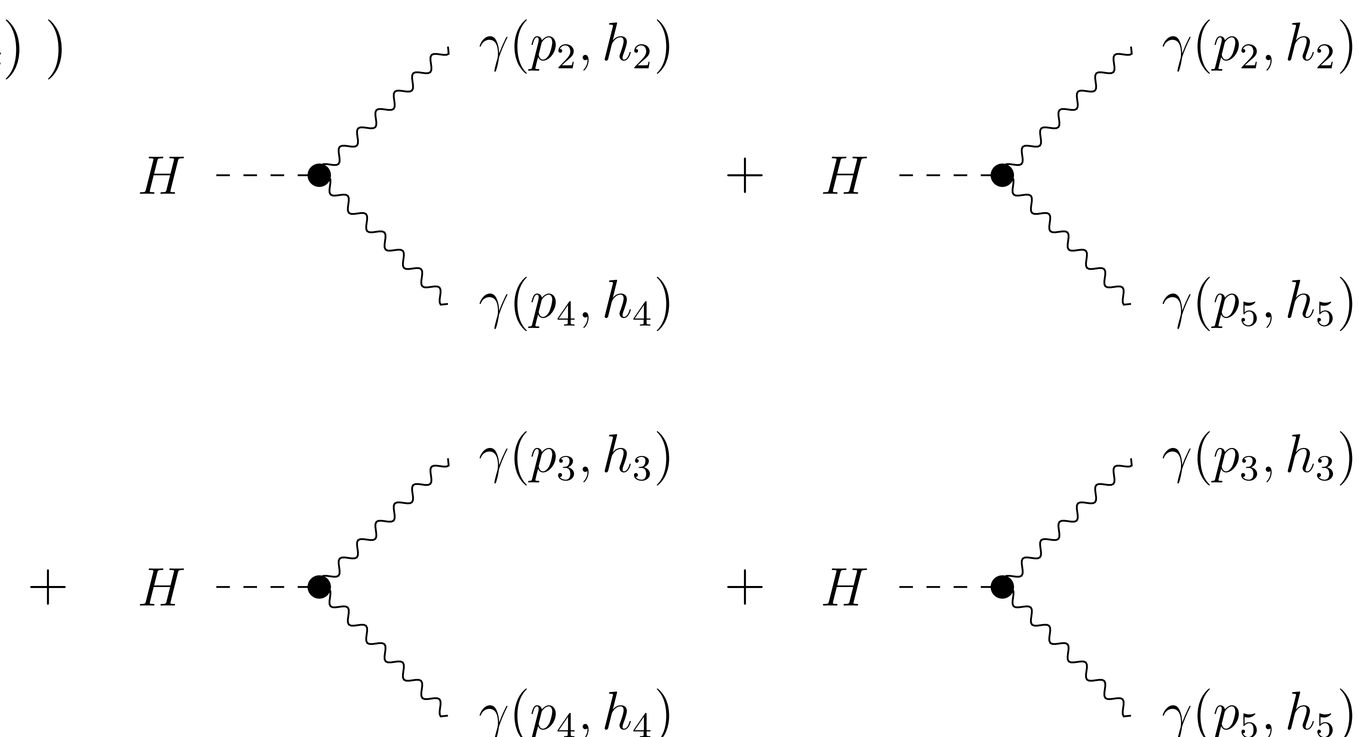
$$
\begin{aligned}
&= f^l_V(s_{23})f^l_V(s_{45})\times(\\
&\quad [23][45]\mathcal{M}(2^-_\gamma,4^-_\gamma)+[23]\langle 45\rangle\mathcal{M}(2^-_\gamma,5^+_\gamma)\\
&+\langle 23\rangle[45]\mathcal{M}(3^+_\gamma,4^-_\gamma)+\langle 23\rangle\langle 45\rangle\mathcal{M}(3^+_\gamma,5^+_\gamma)\ ),
\end{aligned}
$$

=0

=0



$\longrightarrow$



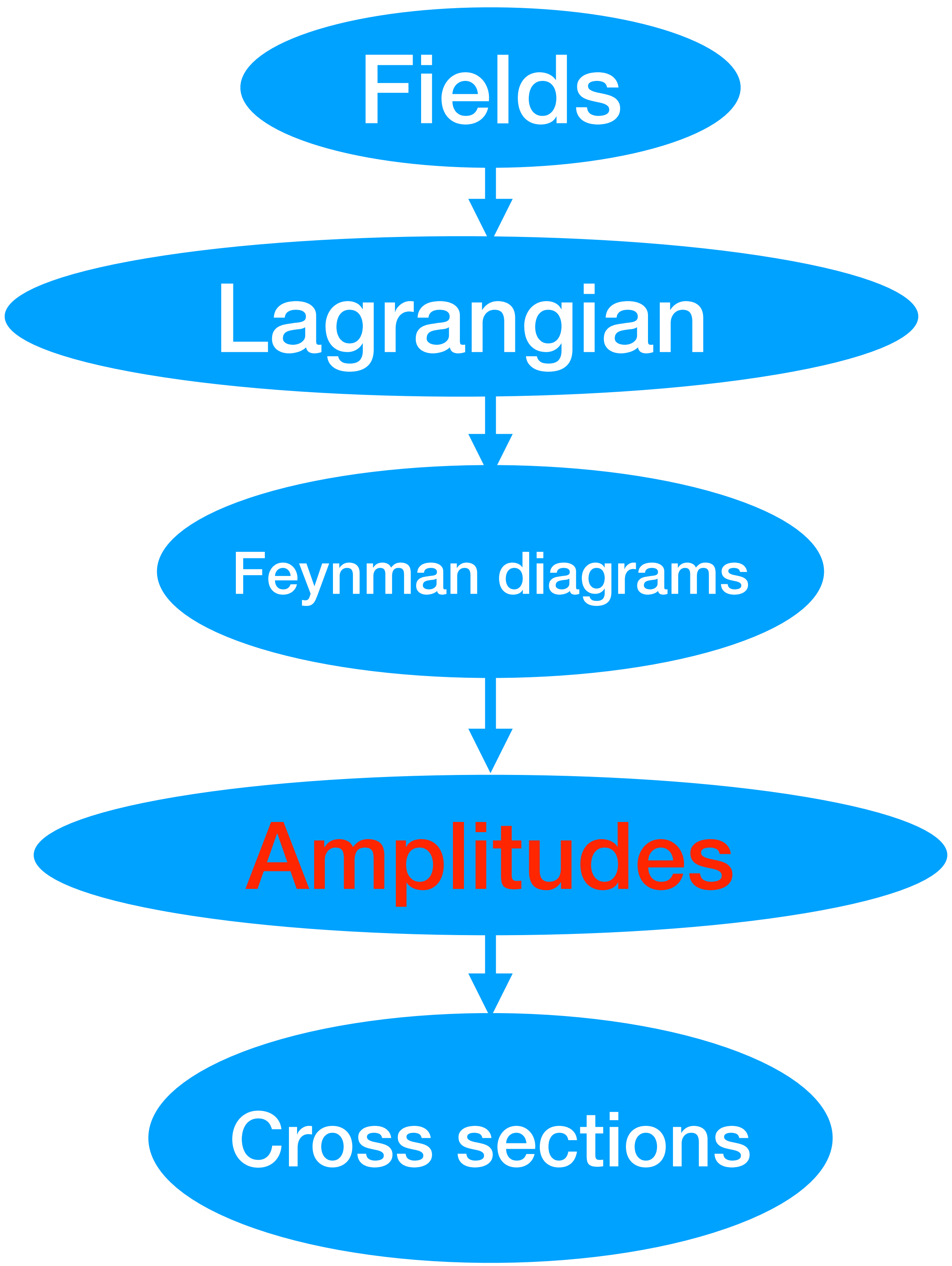
$$
\mathcal{M}(2^-_{\ell^-},3^+_{\ell^+},4^-_{\ell'^-},5^+_{\ell'^+}) = f^-_V(s_{23})f^-_V(s_{45})\times\frac{2c^S_{VV}}{v}\left(e^{i\xi}\langle 23\rangle\langle 45\rangle[35]^2+e^{-i\xi}[23][45]\langle 24\rangle^2\right)
$$

Summary of decomposition relations

- The HVV vertex are bilinear to the momenta of vector bosons. (SMEFT dimension-6 operators)

- Leptons are assumed to be massless. The propagators are gauge-independent.

On-shell scattering amplitudes

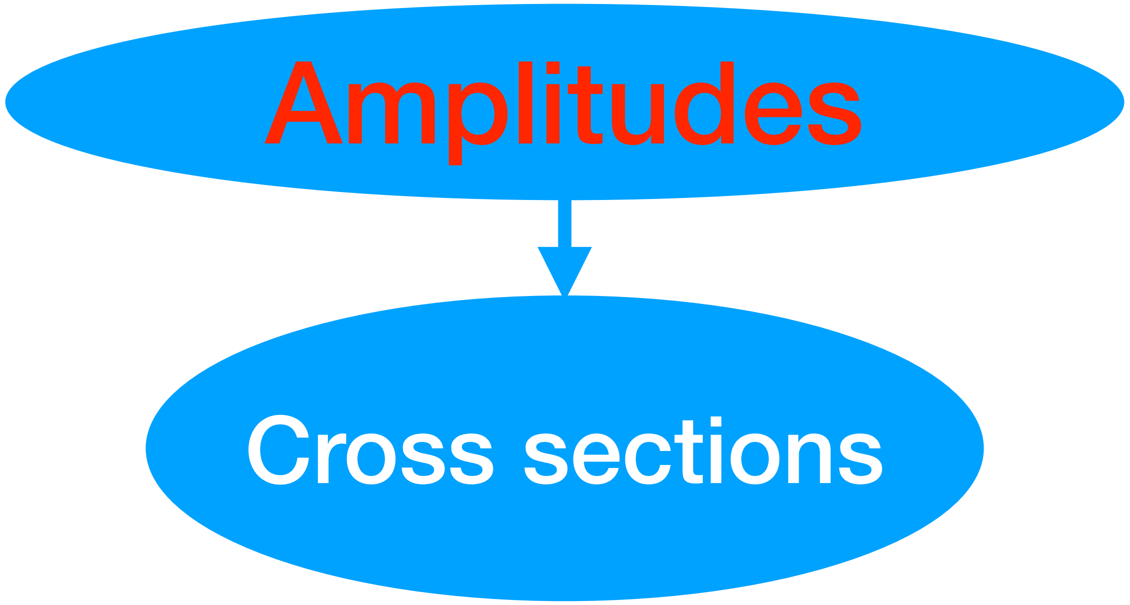


Drawbacks

- **Field definition**
- **Gauge redundancy**

How about constructing amplitude directly?

On-shell scattering amplitudes



How about constructing amplitude directly?

On-shell scattering amplitudes

$$|i_\alpha\rangle \equiv \lambda_{i\alpha} \equiv u_+(p_i) \equiv |i^+\rangle, \quad |i^{\dot\alpha}] \equiv \tilde\lambda_i^{\dot\alpha} \equiv u_-(p_i) \equiv |i^-\rangle,$$

$$\langle i^\alpha| \equiv \lambda_i^\alpha \equiv \overline{u_-(p_i)} \equiv \langle i^-|, \quad [i_{\dot\alpha}| \equiv \tilde\lambda_{i\dot\alpha} \equiv \overline{u_+(p_i)} \equiv \langle i^+|,$$

Weyl Spinors

$$\langle ij\rangle \equiv \lambda_i^\alpha \lambda_{j\alpha}, \quad [ij] \equiv \tilde\lambda_{i\dot\alpha}\tilde\lambda_j^{\dot\alpha}$$

$$p_{\alpha\dot\alpha} \equiv p_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}$$

Little Group Scaling: $\lambda_\alpha \to t\lambda_\alpha, \tilde\lambda_{\dot\alpha} \to t^{-1}\tilde\lambda_{\dot\alpha}$

$$A_n\big(\{|1\rangle, |1], h_1\}, \ldots, \{t_i|i\rangle, t_i^{-1}|i], h_i\}, \ldots\big) = t_i^{-2h_i} A_n\big(\ldots\{|i\rangle, |i], h_i\}\ldots\big)$$

Dimensional analysis

1308.1697, H. Elvang, Y.T. Huang

1708.03872.,C. Cheung

Massless three-particle amplitude

1708.03872.,C. Cheung

$$p_1 + p_2 + p_3 = 0 \quad \Rightarrow \quad \begin{array}{l} (p_1+p_2)^2 = \langle 12\rangle[12] = p_3^2 = 0 \\ (p_2+p_3)^2 = \langle 23\rangle[23] = p_1^2 = 0 \\ (p_3+p_1)^2 = \langle 31\rangle[31] = p_2^2 = 0 \end{array}$$

One solution:

$$[12] = [23] = [31] = 0 \quad \Rightarrow \quad \tilde{\lambda}_1 \propto \tilde{\lambda}_2 \propto \tilde{\lambda}_3,$$

$$A(1^{h_1}2^{h_2}3^{h_3}) = \langle 12\rangle^{n_3}\langle 23\rangle^{n_1}\langle 31\rangle^{n_2}$$

Little Group Scaling:

$$\begin{array}{l} -2h_1 = n_2 + n_3 \\ -2h_2 = n_3 + n_1 \\ -2h_3 = n_1 + n_2 \end{array} \quad \Rightarrow \quad \begin{array}{l} n_1 = h_1 - h_2 - h_3 \\ n_2 = h_2 - h_3 - h_1 \\ n_3 = h_3 - h_1 - h_2 \end{array}$$

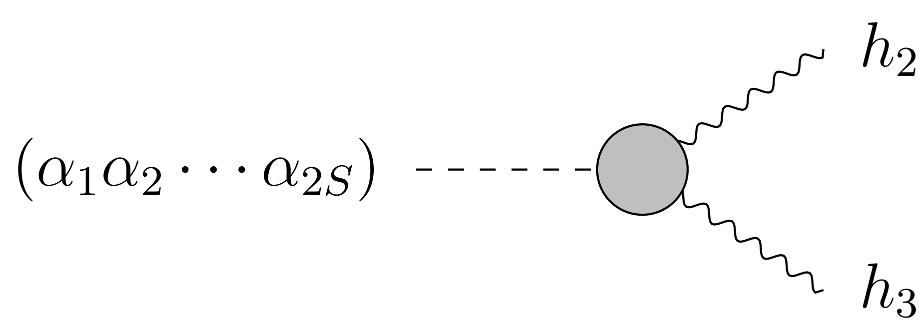
Dimensional analysis

$$A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, & h \leq 0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, & h \geq 0 \end{cases}$$

$H \to \gamma\gamma$

One massive two massless three-particle amplitude

1709.04891 N. Arkani-Hamed, T.C. Huang, Y.T. Huang



$\mathcal{M}^{h_2h_3}\{\alpha_1\alpha_2\cdots\alpha_{2S}\}$

$$\mathcal{M}(2_\gamma^{h_2}, 3_\gamma^{h_3}) = e^{i\xi^{h_2,h_3}} \frac{g}{m^{h_2+h_3-1}} [23]^{h_2+h_3}$$

$$\mathcal{M}(2_\gamma^+, 3_\gamma^+) = e^{i\xi'} \frac{g}{m} [23]^2 \ ,$$

$$\mathcal{M}(2_\gamma^-, 3_\gamma^-) = e^{-i\xi'} \frac{g}{m} \langle 23\rangle^2$$

$$\frac{g}{m} = \frac{2c_{\gamma\gamma}^S}{v} \text{ and } \xi' = \xi$$

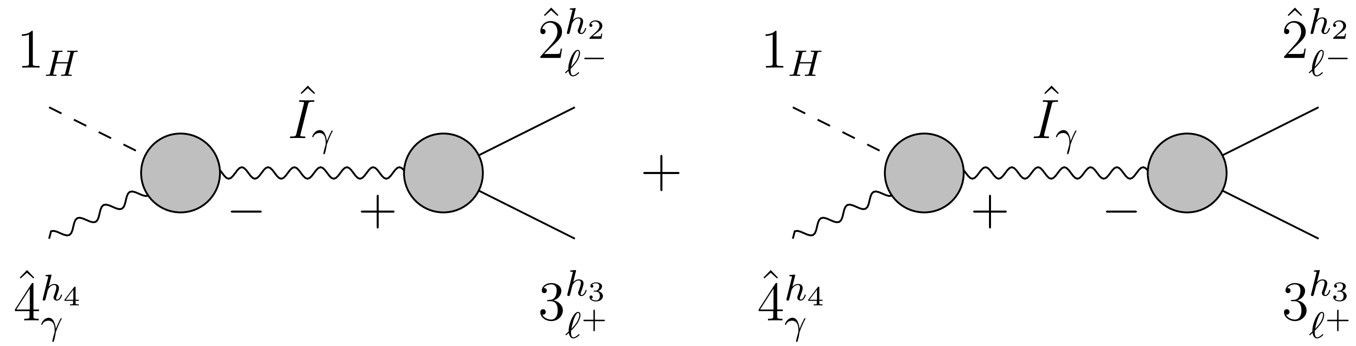
$$\mathcal{M}(2_\gamma^+, 3_\gamma^+) = \frac{2c_{\gamma\gamma}^S}{v} e^{i\xi} [23]^2 \ ,$$

$$\mathcal{M}(2_\gamma^-, 3_\gamma^-) = \frac{2c_{\gamma\gamma}^S}{v} e^{-i\xi} \langle 23\rangle^2 \ ,$$

BCFW recursion relation I

hep-th/0412308, R.Britto, F.Cachazo, B. Feng
hep-th/0501052, R.Britto, F.Cachazo, B. Feng, E. Witten

$$\mathcal{M}_4(1_H, 2_{\ell-}^{h_2}, 3_{\ell+}^{h_3}, 4_\gamma^{h_4}) =$$



$$|\hat{2}] = |2], \quad |\hat{4}] = |4] + z|2], \quad |\hat{4}\rangle = |4\rangle, \quad |\hat{2}\rangle = |2\rangle - z|4\rangle$$

$$\mathcal{M}(1_\gamma^-, 2_{\ell-}^-, 3_{\ell+}^+) = \tilde{e}\frac{\langle 12\rangle^2}{\langle 23\rangle}$$

$$\mathcal{M}(1_\gamma^-, 2_{\ell-}^+, 3_{\ell+}^-) = \tilde{e}\frac{\langle 13\rangle^2}{\langle 23\rangle}$$

$$\mathcal{M}(1_\gamma^+, 2_{\ell-}^-, 3_{\ell+}^+) = \tilde{e}\frac{[13]^2}{[23]}$$

$$\mathcal{M}(1_\gamma^+, 2_{\ell-}^+, 3_{\ell+}^-) = \tilde{e}\frac{[12]^2}{[23]}$$

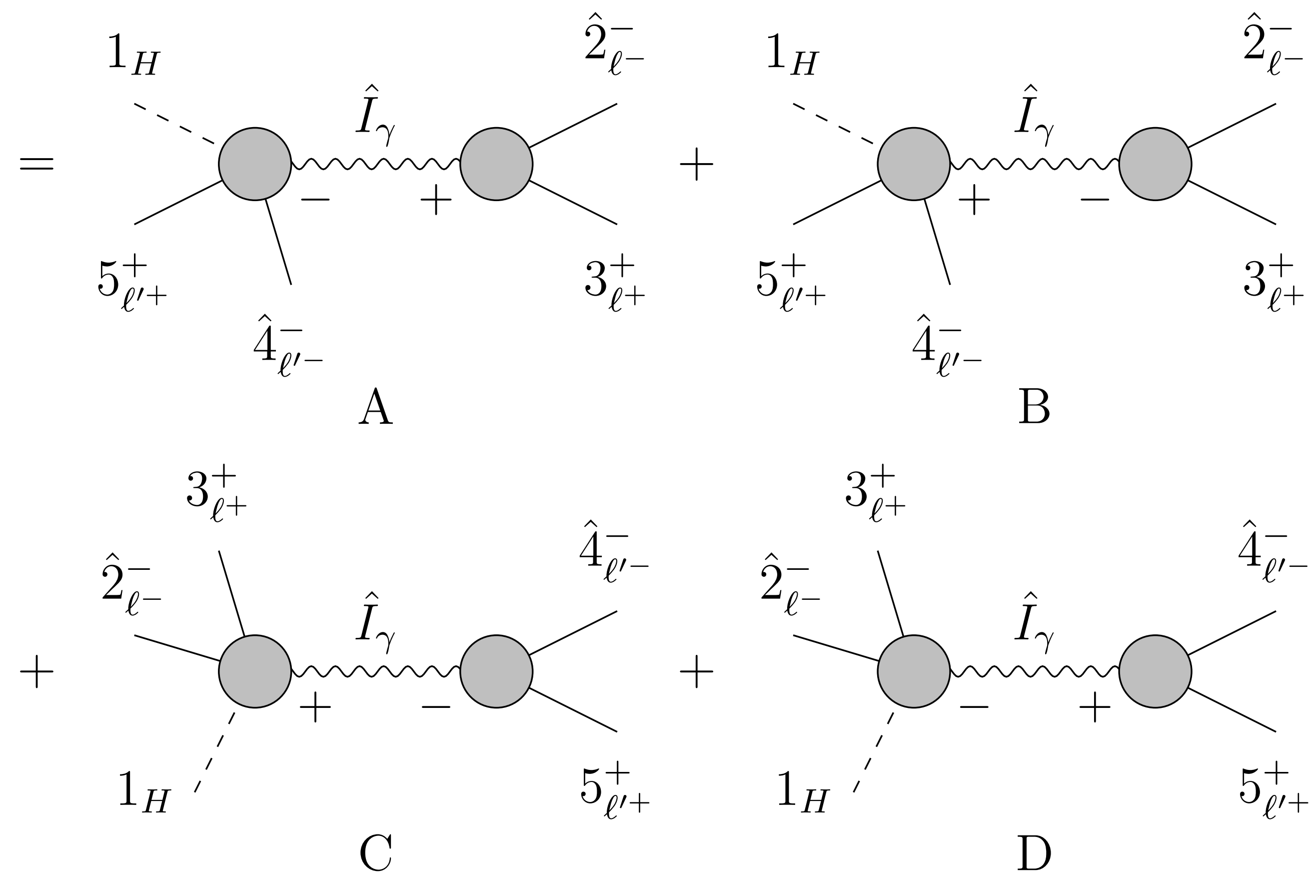
$$\mathcal{M}(1_H, 2_{\ell-}^-, 3_{\ell+}^+, 4_\gamma^-) = \tilde{e}P_\gamma(s_{23}) \times \frac{2c_{\gamma V}^S}{v} e^{-i\xi} \frac{\langle \hat{I}\hat{4}\rangle^2 [\hat{I}3]^2}{[\hat{2}3]},$$

$$= \tilde{e}P_\gamma(s_{23}) \times \frac{2c_{\gamma V}^S}{v} e^{-i\xi} [23]\langle 24\rangle^2,$$

$$\boxed{\mathcal{M}(2_{\ell-}^-, 3_{\ell+}^+, 4_\gamma^-) = f_V^-(s_{23}) \times \frac{2c_{\gamma V}^S}{v} e^{-i\xi} [23]\langle 24\rangle^2}$$

BCFW recursion relation II

$$\mathcal{M}_5(1_H, 2^-_{\ell-}, 3^+_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) =$$



A $P_\gamma(s_{23})\mathcal{M}(1_H, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}, \hat{I}^-_\gamma)\mathcal{M}(\hat{I}^+_\gamma, \hat{2}^-_{\ell-}, 3^+_{\ell+})$

$$= \frac{2c^S_{\gamma\gamma}}{v} e^{-i\xi} P_\gamma(s_{23}) P_\gamma(s_{\hat{4}5}) [\hat{4}5] \langle \hat{4}\hat{I}\rangle^2 \frac{[\hat{I}3]^2}{[\hat{2}3]}$$

$$= \frac{2c^S_{\gamma\gamma}}{v} e^{-i\xi} P_\gamma(s_{23}) P_\gamma(s_{45}) [45][23]\langle 24\rangle^2 \,,$$

B

$P_\gamma(s_{23})\mathcal{M}(1_H, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}, \hat{I}^+_\gamma)\mathcal{M}(\hat{I}^-_\gamma, \hat{2}^-_{\ell-}, 3^+_{\ell+}) = 0$

C $P_\gamma(s_{45})\mathcal{M}(1_H, \hat{2}^-_{\ell-}, 3^+_{\ell+}, \hat{I}^+_\gamma)\mathcal{M}(\hat{I}^-_\gamma, \hat{4}^-_{\ell'-}, 5^+_{\ell'+})$

$$= \frac{2c^S_{\gamma\gamma}}{v} e^{i\xi} P_\gamma(s_{45}) P_\gamma(s_{\hat{2}3}) \langle \hat{2}3\rangle [3\hat{I}]^2 \times \frac{\langle \hat{I}\hat{4}\rangle^2}{\langle \hat{4}5\rangle}$$

$$= \frac{2c^S_{\gamma\gamma}}{v} e^{i\xi} P_\gamma(s_{45}) P_\gamma(s_{23}) \langle 23\rangle\langle 45\rangle [35]^2$$

D

$P_\gamma(s_{45})\mathcal{M}(1_H, \hat{2}^-_{\ell-}, 3^+_{\ell+}, \hat{I}^-_\gamma)\mathcal{M}(\hat{I}^+_\gamma, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}) = 0$

$$\mathcal{M}(2^-_{\ell-}, 3^+_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) = f^-_V(s_{23}) f^-_V(s_{45}) \times \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle [35]^2 + e^{-i\xi}[23][45]\langle 24\rangle^2 \right)$$

Summary of on-shell recursion relations

- No assumption for vertex, especially vertex bilinear to momenta of vector bosons.
- Only massless propagator is considered. Massless propagator, massless leptons.
- Do not consider boundary conditions.

Summary

- CP violation in new physics is needed, they have close relation with Higgs.

- BSM amplitudes of $H \to \gamma\gamma, H \to \gamma 2\ell, H \to 4\ell$ are given in SMEFT.

- Decomposition relations are derived, which explains the behavior of CP violation phase.

- Recursion relations are derived through on-shell scattering amplitude approach, consistent results are obtained. Probably it is a first time using on-shell approach to calculate a realistic massive process.

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