

Exclusive weak decays of D_s^* meson

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Overview

1 Motivation

- The desirability
- The significances

2 $D_s^* \rightarrow \phi$ helicity form factors

- OPE evaluation
- hadron interpolation
- duality
- result

3 Exclusive D_s^* weak decays

4 Conclusion

Motivation-desirability

- $\alpha_s : \alpha : G_F \sim \mathcal{O}(1) : \mathcal{O}(\frac{1}{137}) : \mathcal{O}(10^{-5})$
- very hard to measure weak decay from strong and e.m interactions

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- very hard to measure weak decay from strong and e.m interactions
- so the total widths of heavy-light vector mesons are still in lack
 - ★ $\Gamma_{D^{*+}} = 84.3 \pm 1.8 \text{ keV} \rightarrow D^0\pi^+, D^+\pi^0, D^+\gamma$
 - ★ $\Gamma_{D^{*0}} < 2.1 \text{ MeV} \quad \Gamma_{D_s^{*+}} < 1.9 \text{ MeV} \quad [\text{PDG 2020}]$
 $\rightarrow D^0\pi^0, D^0\gamma \quad \rightarrow D_s^+\gamma, D_s^+\pi^0, D_s^+e^+e^-$
 - ★ $\Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement

D_s^* weak decays

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★ $\Gamma_{B^*}, \Gamma_{B_s^*}$ no measurement

- but they are very important properties, structures, $g_{D_s^* D_s \gamma}$, non-perturbative approaches, et.al.,
- impressive lattice QCD evaluation

$$\Gamma_{D_s^{*+}} = 0.070(28) \text{ keV} \quad [\text{HPQCD 2013}]$$

- encourage us to study the exclusive D_s^* weak decay

D_s^* weak decays

Motivation-significances

$\Gamma_{D_s^*} \rightarrow g_{D_s^* D_s \gamma}$ et.al \rightarrow nonpert. approaches

fruitful significances in phenomenology

† leptonic decays, helicity enhanced $D_s^* \rightarrow l\nu$, **decay constant**

$$\Gamma_{D_s^* \rightarrow l\nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 \left(1 - \frac{m_l^2}{m_{D_s^*}^2}\right) \left(1 + \frac{m_l^2}{m_{D_s^*}^2}\right) = 2.44 \times 10^{-12} \text{ GeV}. \quad (1)$$

† semileptonic decays, $D_s^* \rightarrow \phi l\nu$, **| V_{cs} |** and **helicity form factors**,
QCD-based approaches

† **lepton flavour university** (LFU) in vector charm sector

† hadronic decays, $D_s^* \rightarrow \phi\rho, \phi\pi$, factorisation theorem or topological analysis

† inclusive decays, $D_s^* \rightarrow X_s l\nu$, HQET and reliability of power expansion

D_s^* weak decays

D_s^* $\rightarrow \phi$ helicity form factors

- heavy-to-light form factors (FFs) play the key role in weak decays
- both pert. and **nonpert. physics** enter into the game
- the measurement would reveal the **inner structures of hadrons**
- QCD-based approaches to calculate FFs, **LCSR**s, **PQCD**, **LQCD**
- implement of LCSR in charm sector, $D \rightarrow \pi, K, \eta^{(\prime)}, \phi$ et.al
[Khodjamirian 2000, Ball 2006, Offen 2013, Du 2003, Wu 2006]

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[Khodjamirian 2000, Ball 2006, Offen 2013, Du 2003, Wu 2006]
- $D_s^* \rightarrow \phi$ FFs in this work
 - † first LCSR prediction of $V \rightarrow V'$ type FFs
 - † helicity decomposition with four FFs, saying **L**, **LT**, **TL**, **T**
 - † LCSR prediction is reliable in large recoiled region $[0, 0.4] \text{ GeV}^2$
 - † parameterisations to the full kinematical region $[0, 1.2] \text{ GeV}^2$
- experiment potential of D_s^* weak decays

$D_s^* \rightarrow \phi$ helicity form factors

- start with the correlation function

$$F_{\mu a}(p_1, q) = i \int d^4x e^{iq \cdot x} \langle \phi(p_2, \epsilon_2^*) | T\{J_\mu^W(x), J_a^V(0)\} | 0 \rangle, \quad (2)$$

- heavy-light weak current $J_\mu^W = \bar{s}\gamma_\mu(1 - \gamma_5)c$ and vector current $J_a^V = \bar{c}\gamma_a s$
- modify the correlation function by multiplying $\bar{\epsilon}^\mu$ to obtain the **helicity correlator**

$$\bar{\epsilon}^\mu F_{\mu a}(p_1, q) = \epsilon_{1a}^*(0) F_{\epsilon_1}^{(i=L,T)}(q^2, p_1^2) + \epsilon_{1a}^*(\pm) F_{\epsilon_1}^{(i=LT,TL)}(q^2, p_1^2), \quad (3)$$

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- Eq. (4) is considered in twofold ways

- † at quark-gluon level in $q^2 < 0$ by **OPE**, $\sim \sum_i H_i(u, \mu) \otimes \phi_i(u)$
- † at hadron level in $q^2 > 0$ & $\rightarrow m_R^2$, **sum over intermediate states**
- † QCD asymptotic behaviour, quark-hadron duality to equal, s_0
- † to improve the accuracy of duality, Borel transformation, M^2

- OPE is valid for the large energies of the final state meson $E_\phi \gg \Lambda_{QCD}$,
 $0 \leq |q^2| \leq m_{D_s^*}^2 - 2m_{D_s^*}E_\phi \sim m_c^2 - 2m_c\chi$, $\chi \sim 500 \text{ MeV} \Leftarrow q \cdot x \sim 0, x^2 \sim 0$
- $|q^2| \in [0, 0.4] \text{ GeV}^2$, the lower part of $0 < |q^2| < (m_{D_s^*}^2 - m_\phi^2)^2 \approx 1.2 \text{ GeV}^2$

- † $|q^2| \rightarrow \mathcal{O}(m_c^2)$, the virtuality of c -quark decreases to a soft scale, OPE fails
 † $|q^2|, |(p_2 + q)^2| \ll m_c^2$, the intermediate c -quark field has large virtuality,

$$\text{LO}, \quad S(x, 0) = -i\langle 0 | T\{c(x), \bar{c}(0)\} | 0 \rangle = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} \frac{|p + m_c|}{p^2 - m_c^2} \quad (6)$$

NLO, $\mathcal{O}(\alpha_s)$ correction with gluon interactions ...

- only ϕ meson is on shell, $p_2^2 = m_\phi^2$, dispersion integral in $(p_2 + q)^2$

$$\epsilon^\mu F_{\mu a}(q, p_1) = \epsilon_{1a}^* \sum_i F^{\text{OPE},(i)}(q^2, (p_2 + q)^2) \quad (7)$$

$$\begin{aligned} F^{\text{OPE},(i)}(q^2, (p_2 + q)^2) &= \sum \int_0^1 du T^{(n)}(u, q^2, p_1^2) \phi^{(n)}(u) \\ &= \frac{1}{\pi} \int_0^1 du \sum_n \frac{\text{Im} F_n^{\text{OPE},(i)}(q^2, u)}{[-u(p_2 + q)^2 - \bar{u}q^2 + u\bar{u}m_\phi^2 + m_c^2]^n}. \end{aligned} \quad (8)$$

D_s^* weak decays

$D_s^* \rightarrow \phi$ helicity form factors hadron interpolation

- the hadron dispersion relation in $p_1^2 > 0$

$$\bar{\epsilon}^\mu F_{\mu a}^{(i)}(q^2, p_1^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} ds \frac{\epsilon_{1a}^* \text{Im } F^{(i)}(q^2, s)}{s - p_1^2} = \frac{\rho^{0,(i)}}{m_{D_s^*}^2 - p_1^2} + \int_{s_0}^{\infty} \frac{\rho^{h,(i)}(q^2, s)}{s - p_1^2} \quad (9)$$

$$2 \text{Im } \bar{\epsilon}^\mu F_{\mu a}(q, p_1) = \bar{\epsilon}^\mu \langle \phi(p_2, \epsilon_2^*) | J_\mu^W(x) | D_s^*(\epsilon_1, p_1) \rangle \langle D_s^*(\epsilon_1^*, p_1) | J_a^V(0) | 0 \rangle + \dots \quad (10)$$

- the matrix elements

$$\langle D_s^{*+}(p_1, \epsilon_1^*) | \bar{s} \gamma_a c | 0 \rangle = \epsilon_{1a}^* m_{D_s^*} f_{D_s^*}, \quad (11)$$

$$\langle \phi(p_2, \epsilon_2^*) | \bar{s} J_\mu^W c | D_s^*(\epsilon_1, p_1) \rangle = (\epsilon_1 \cdot \epsilon_2^*) [p_{1\mu} \mathcal{V}_1(q^2) - p_{2\mu} \mathcal{V}_2(q^2)] + \dots \quad (12)$$

- introduce the helicity form factors

$$\mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2 > 0) \equiv \bar{\epsilon}^\mu \langle \phi(p_2, \epsilon_2^*) | J_\mu^W | D_s^*(\epsilon_1, p_1) \rangle \quad (13)$$

- isolate the ground state contribution

$$\bar{\epsilon}^\mu F_{\mu a}(q, p_1) = \sum_i \left[\frac{\epsilon_{1a}^{*(i)} m_{D_s^*} f_{D_s^*} \mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2 > 0)}{m_{D_s^*}^2 - p_1^2} + \int_{s_0}^{\infty} ds \frac{\epsilon_{1a}^{*(i)} \rho^{h,(i)}(q^2 > 0, s)}{s - p_1^2} \right]. \quad (14)$$

D_s^* weak decays

$D_s^* \rightarrow \phi$ helicity form factors duality

- the same correlator in OPE calculation Eq.(7) and hadron interpolation Eq.(14)
- QCD property, like $F_\pi(q^2)$ and $G_\pi(s)$ have the similar asymptotic behaviour
- semi-local duality $s \equiv s(q^2, u) = \bar{u}m_\phi^2 + (m_c^2 - \bar{u}q^2)/u$

$$\frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{u^2(s)}{[u^2(s)m_\phi^2 - q^2 + m_c^2]} \sum_n \frac{\epsilon_{1a}^* \text{Im}F_n^{\text{OPE},(i)}(q^2, s)}{u^n(s)[s - (p_2 + q)^2]^n} \Big|_{q^2, (p_2 + q)^2 < 0} = \int_{s_0}^{\infty} ds \frac{\rho^{h,(i)}(q^2, s)}{s - p_1^2} \quad (15)$$

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† Borel trans. to suppress the pollution introduced by duality

$$\hat{B} \left[\int_{u_0}^1 du \frac{F(u)}{\Delta} \right] = \int_{u_0}^1 du \frac{F(u)}{u} e^{-s(u)/M^2}, \dots \quad (17)$$

† $\mu_f^2 = m_{D_s^*}^2 - m_c^2 = 1.66^2 \text{ GeV}^2$, $M^2 \sim \mathcal{O}(u m_{D_s^*}^2 + \bar{u} Q^2 - u \bar{u} m_\phi^2) < s_0 < \mu_f^2$, $s_0 \approx (m_{D_s^*} + \chi)^2$

† compromise between the overwhelming ground state and the convergent OPE,

$$\frac{d}{d(1/M^2)} \ln \mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2) = 0. \quad (18)$$

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$$\dagger \quad \mu_f^2 = m_{D_s^*}^2 - m_c^2 = 1.66^2 \text{ GeV}^2, \quad M^2 \sim \mathcal{O}(u m_{D_s^*}^2 + \bar{u} Q^2 - u \bar{u} m_\phi^2) < s_0 < \mu_f^2, \quad s_0 \approx (m_{D_s^*} + \chi)^2$$

† compromise between the overwhelming ground state and the convergent OPE,

$$\frac{d}{d(1/M^2)} \ln \mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2) = 0 \quad (21)$$

- the sum rule with leading power approximation

$$\frac{1}{\pi} \int_{u_0}^1 du \frac{\text{Im} F_1^{\text{OPE},(i)}(q^2 < 0, u)}{u} e^{-s(u)/M^2} = m_{D_s^*} f_{D_s^*} \mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2 > 0) e^{-m_{D_s^*}^2/M^2} \quad (22)$$

D_s^* weak decays

$D_s^* \rightarrow \phi$ helicity form factors small recoiling

- the helicity form factors with small recoil $q^2 \in [0.4, \approx 1.2 \text{ GeV}^2]$
- consider two parameterisations
- two-pole (BK) mode [Becirevic 1999]

$$F^{(i)}(q^2 > 0) \rightarrow \frac{c_{D_s^* \phi}^{(i)} \left(1 - \alpha_{D_s^* \phi}^{(i)} \right)}{\left(1 - q^2/m_{D1}^2 \right) \left(1 - \alpha_{D_s^* \phi}^{(i)} q^2/m_{D1}^2 \right)} \quad (23)$$

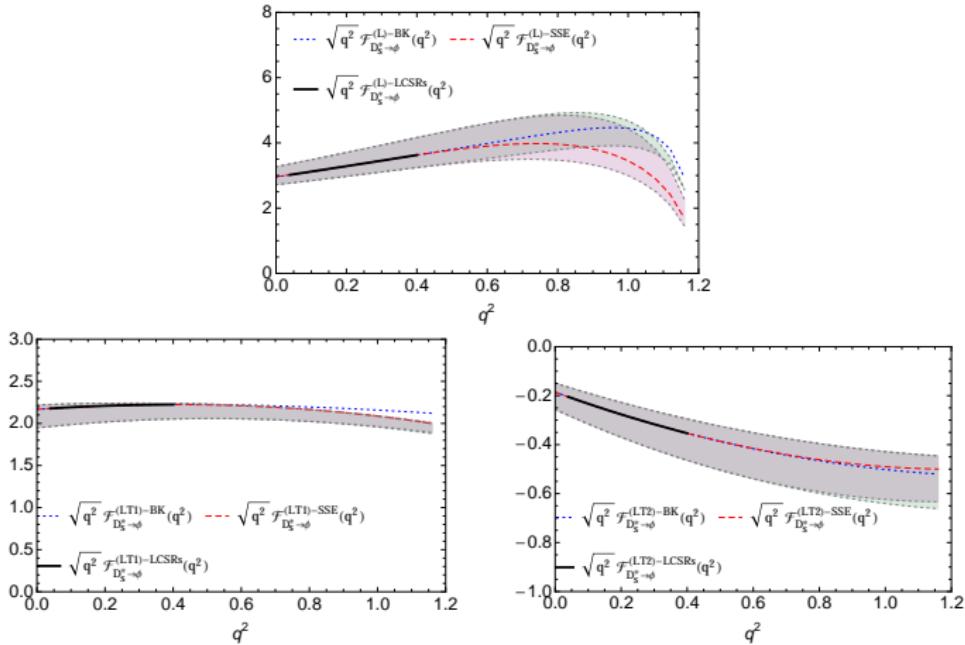
- z -series expansion [Bourrely 2008]

$$F^{(i)}(q^2 > 0) = \frac{a_{F(i)}(q^2)}{1 - q^2/m_{D1}^2} \left\{ 1 + b_{F(i)} [z(q^2) - z(0)] \right\}, \quad z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad (24)$$

D_s^* weak decays

$D_s^* \rightarrow \phi$ helicity form factors Preliminary result

- $s_0 = 6.8 \pm 0.5 \text{ GeV}^2$ and $M^2 = 2.0 \pm 0.2 \text{ GeV}^2$
- power hierarchy of the form factors: $\mathcal{O}(1) : \mathcal{O}\left(\frac{m_\phi}{m_{D_s^*}}\right) : \mathcal{O}\left(\frac{\sqrt{Q^2}}{m_{D_s^*}}\right) : \mathcal{O}\left(\frac{\sqrt{Q^2} m_\phi}{m_{D_s^*}^2}\right)$



D_s^* weak decays

- leptonic decays

$$\Gamma_{D_s^* \rightarrow l\nu} = 2.44 \times 10^{-12} \text{ GeV} \quad (25)$$

- semileptonic decays

$$\frac{d\Gamma^{(i)}(D_s^* \rightarrow \phi l\nu_l)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s^*}^3} \lambda^{1/2}(m_{D_s^*}^2, m_\phi^2, q^2) q^2 |\mathcal{F}_{D_s^* \rightarrow \phi}^{(i)}(q^2 > 0)|^2,$$

$$\Gamma_{D_s^* \rightarrow \phi l\nu_l} = \int_0^{q_0^2} \sum_i d\Gamma^{(i)}(D_s^* \rightarrow \phi l\nu_l) = \begin{cases} (2.10^{+0.46}_{-0.23}) \times 10^{-13} \text{ GeV}, & \text{BK} \\ (2.00^{+0.42}_{-0.25}) \times 10^{-13} \text{ GeV}. & \text{SSE} \end{cases} \quad (26)$$

- hadronic decays (naive factorisation)

$$\mathcal{A}(D_s^{*+} \rightarrow \phi \rho^+) = \frac{G_F}{\sqrt{2}} V_{cs} a_1 m_\rho f_\rho^{\parallel(\perp)} \sum_i \mathcal{F}^{(i)}(m_\rho^2). \quad (27)$$

$$\Gamma_{D_s^{*+} \rightarrow \phi \pi^+} = (1.29^{+0.11}_{-0.11}) \times 10^{-13} \text{ GeV}, \quad (28)$$

$$\Gamma_{D_s^{*+} \rightarrow \phi \rho^+} = \begin{cases} (6.75^{+2.49}_{-1.47}) \times 10^{-13} \text{ GeV}, & \text{BK} \\ (6.65^{+2.58}_{-1.45}) \times 10^{-13} \text{ GeV}. & \text{SSE} \end{cases} \quad (29)$$

- † 2023, 400 fb^{-1} data, reconstruct 2×10^5 data samples of D_s and D_s^* at Belle II
- † phase 3 running (2024-2027), 10 ab^{-1} , 5×10^6 data sample of D_s^*
- † clear background
- with phase 3 running, the measurement ability of $\Gamma_{D_s^*}$ down to $\{12.2, 1.05, 0.65, 3.38\} \times 10^{-6} \text{ GeV}$ in the leptonic, semileptonic and hadronic channels, respectively
- † lattice result $7 \times 10^{-8} \text{ GeV}$ [HPQCD 2013] 50 ab^{-1} at Belle II is hottest expected
- † LHCb ? [talk from Liang Sun]
- BESIII ? [talks from Cong Geng and Bai-qian, Poster Presentation from Ziyi Wang]

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- with the lattice evaluation of $\Gamma_{D_s^*} = (0.70 \pm 0.28) \times 10^{-8} \text{ GeV}$

$$\mathcal{B}(D_s^* \rightarrow l\nu) = (3.48 \pm 1.39) \times 10^{-5}, \quad (30)$$

$$\mathcal{B}(D_s^* \rightarrow \phi l\nu) = \begin{cases} (2.29^{+0.80}_{-0.43} \pm 0.92) \times 10^{-6}, & \text{BK} \\ (2.17^{+0.75}_{-0.37} \pm 0.87) \times 10^{-6}, & \text{SSE} \end{cases} \quad (31)$$

$$\mathcal{B}(D_s^{*+} \rightarrow \phi \pi^+) = (2.74^{+0.76}_{-0.50} \pm 1.10) \times 10^{-6}, \quad (32)$$

$$\mathcal{B}(D_s^{*+} \rightarrow \phi \rho^+) = \begin{cases} (6.98^{+2.38}_{-1.43} \pm 2.79) \times 10^{-6}, & \text{BK} \\ (6.91^{+2.34}_{-1.43} \pm 2.76) \times 10^{-6}. & \text{SSE} \end{cases} \quad (33)$$

D_s^* weak decays

Conclusion

- we study the D_s^* weak decay
 - we discuss the experiment potentials
- † first direct measurement of weak decays of vector meson
- † shine light on the study of $\Gamma_{D_s^*}$ and $g_{D_s^* D_s \gamma} \dots$
- † new playground to examine SM

D_s^* weak decays

The End, Thanks.

- $D_s^* \rightarrow \phi$ transition form factors

$$\begin{aligned}
& \langle \phi(p_2, \epsilon_2^*) | \bar{s} \gamma_\mu (1 - \gamma_5) c | D_s^*(\epsilon_1, p_1) \rangle \\
= & (\epsilon_1 \cdot \epsilon_2^*) \left[p_{1\mu} \mathcal{V}_1(q^2) - p_{2\mu} \mathcal{V}_2(q^2) \right] + \frac{(\epsilon_1 \cdot q)(\epsilon_2^* \cdot q)}{m_{D_s^*}^2 - m_\phi^2} \left[p_{1\mu} \mathcal{V}_3(q^2) + p_{2\mu} \mathcal{V}_4(q^2) \right] \\
- & (\epsilon_1 \cdot q) \epsilon_{2\mu}^* \mathcal{V}_5(q^2) + (\epsilon_2 \cdot q) \epsilon_{1\mu}^* \mathcal{V}_6(q^2) - i \varepsilon_{\mu\nu\rho\sigma} \epsilon_1^\rho \epsilon_2^{*\sigma} \left[p_1^\nu \mathcal{A}_1(q^2) + p_2^\nu \mathcal{A}_2(q^2) \right] \\
+ & i \varepsilon_{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma \frac{1}{m_{D_s^*}^2 - m_\phi^2} \left[\epsilon_1^\nu (\epsilon_2^* \cdot q) \mathcal{A}_3(q^2) - \epsilon_2^\nu (\epsilon_1^* \cdot q) \mathcal{A}_4(q^2) \right]
\end{aligned} \tag{34}$$