Heavy baryon two body decays from light-cone sum rules

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- Two-point light-cone sum rules
- Results and discussion
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- **Motivation and framework**
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Matter-antimatter asymmetry

- One of the most important mission is to understand the matter- antimatter asymmetry.
- A baryon-generating interaction must satisfy to produce matter and antimatter at different rates.
- Sakharov three conditions:

  Baryon number violation.

  *C and **CP-symmetry violation**.*

  Interactions out of thermal equilibrium.

CP-violation in heavy baryon decays

- The first evidence of baryon CP-violation appeared from four-body decays $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$. [LHCb 2017]
- The CP-violation in heavy baryon two-body decays

Channel	$\Lambda_b \to p\pi$	$\Lambda_b \to pK$
$\mathrm{BR(PDG)}(10^{-6})$	4.5 ± 0.8	5.4 ± 1.0
CPV(PDG)	-0.025 ± 0.029	-0.025 ± 0.022
$\Delta A_{CP}(pK^-/\pi^-)$	0.014 ± 0.024	

- The only QCD-based study from [C.-D. Lu, Y.-M. Wang, H. Zou, A. Ali, G. Kramer, (2009)] in pQCD approach, our understanding is still very limited.
- So we consider to study baryon decays from LCSRs.

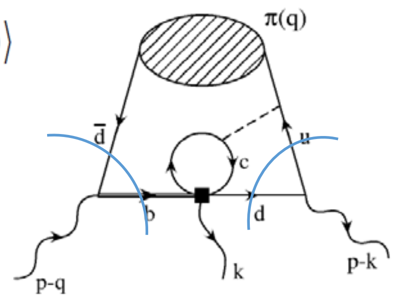
Framework of LCSRs

- LCSRs is a useful method for the calculation of different kinds of form factors, such as for $\Lambda_b \to p$.

  [A. Khodjamirian, Ch. Klein, Th. Mannel, Y.-M. Wang (2011)]

- It also has been used to study heavy meson two-body decays, such as $B \to \pi\pi$ and $D^0 \to \pi^+\pi^-, K^+K^-$

$$F_\alpha^{(\mathcal{O}_1^c)} = i^2 \int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{i(p-k)y} \langle 0|T\{j_{\alpha 5}^{(\pi)}(y)\mathcal{O}_1^c(0) j_5^{(B)}(x)\}|\pi^-(q)\rangle$$



[A. Khodjamirian, (2001);

A. Khodjamirian, T. Mannel and P. Urban (2003);

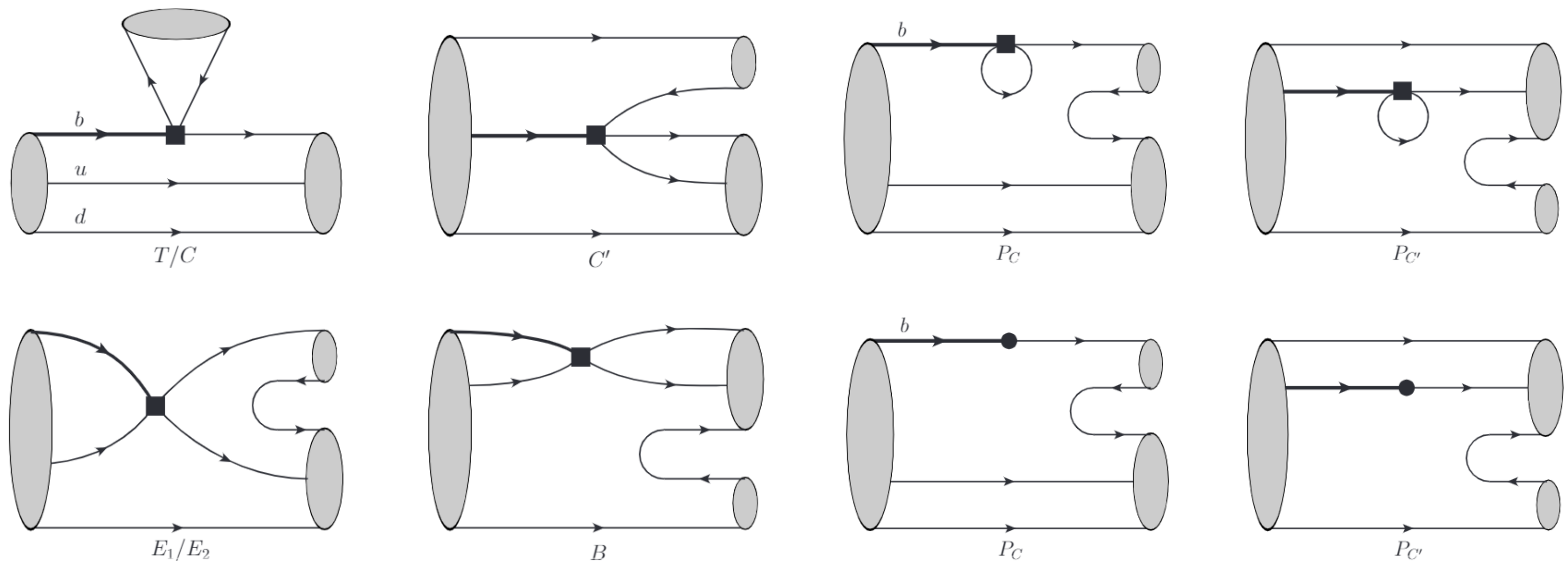
A. Khodjamirian, T. Mannel, B. Melic (2003);

A. Khodjamirian, T. Mannel, M. Melcher and B. Melic (2005);

A. Khodjamirian, A. A. Petrov (2017)]

The topology diagram

- The topology after the insertion of effective operators



$$\langle p\pi|O_i|\Lambda_b\rangle = \bar{u}_N(q)(\mathcal{A}_i + \mathcal{B}_i\gamma_5)u_{\Lambda_b}(p)$$

Channel	$\Lambda_b \to p\pi$	$\Lambda_b \to pK$
topology	T, C', E_2, B $P_C, P_{C'}$	T, E_2 P_C

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- **Three-point light-cone sum rules**
- Two-point light-cone sum rules
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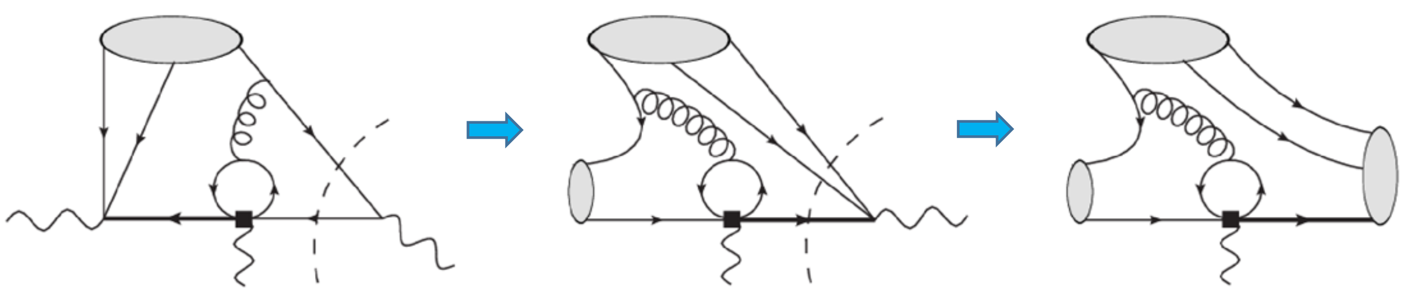
The three-point correlation function

- The three-point correlation function

$$\Pi^{\mathcal{O}}_{3\text{pt},\alpha}((p-k)^2,(p-q)^2,P^2,\lambda)$$
$$= i^2\int d^4x e^{-i(p-q)\cdot z}\int d^4y e^{i(p-k)\cdot y}\langle 0|T\left\{j^{(\pi)\dagger}_{\alpha 5}(y)\eta_{\Lambda_b}(z)\mathcal{O}(0)\right\}|\mathbf{p}(q,\lambda)\rangle$$
$$= (p-k)_\alpha\Pi^{\mathcal{O}}_{3\text{pt}} + q_\alpha\Pi^{\mathcal{O}}_{3\text{pt},1} + k_\alpha\Pi^{\mathcal{O}}_{3\text{pt},2} + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho\Pi^{\mathcal{O}}_{3\text{pt},3} + \gamma_\alpha\Pi^{\mathcal{O}}_{3\text{pt},4}$$

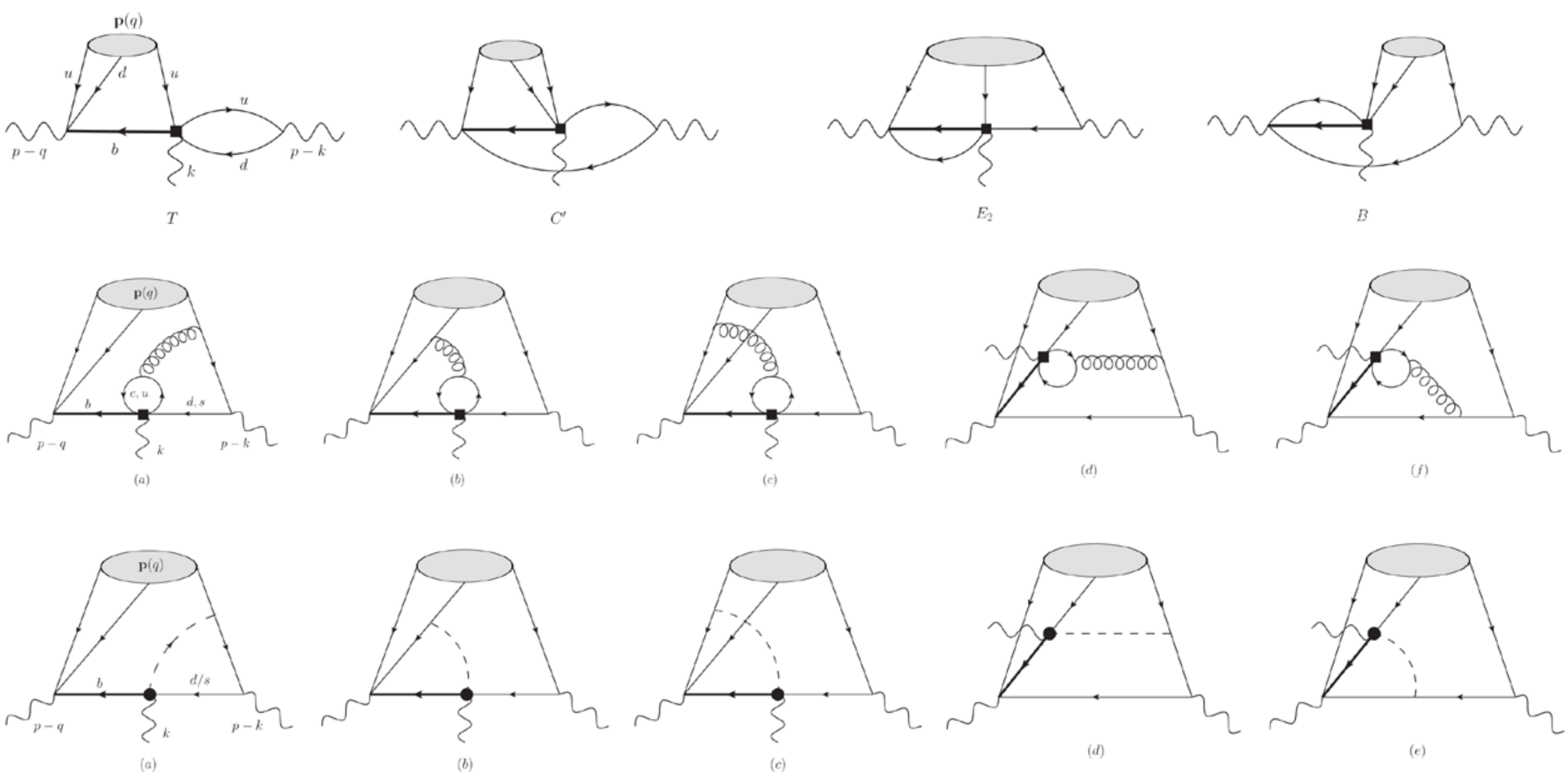
where $j^{(\pi)}_{\alpha 5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $\eta^{(\mathcal{P})}_{\Lambda_b} = \epsilon^{ijk}(u^{iT}C\gamma_5 d^j)b^k$

- The procedure for the extraction of matrix elements



The light-cone OPE calculation

- The topology in the three-point correlator scheme



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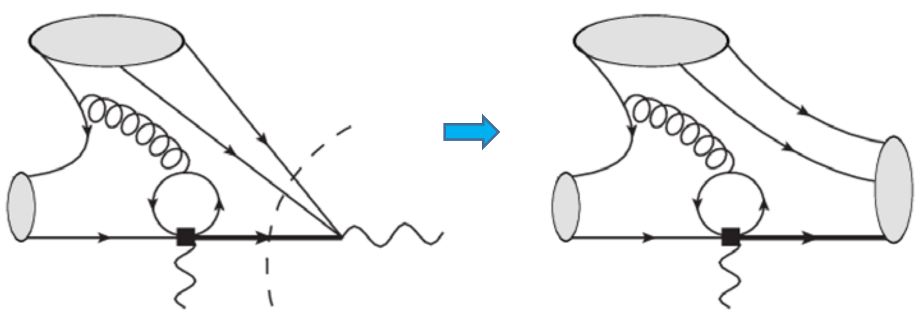
The two-point correlator scheme

- In this case, the starting point will be the two-point correlation function

$$\Pi^{\mathcal{O}}_{2\text{pt}}((p-q)^2, P^2, \lambda) = i \int d^4 z e^{-i(p-q)\cdot z} \langle \pi^+(p-k)|T\{\eta_{\Lambda_b}(z)\mathcal{O}(0)\}|\mathbf{p}(q,\lambda)\rangle$$

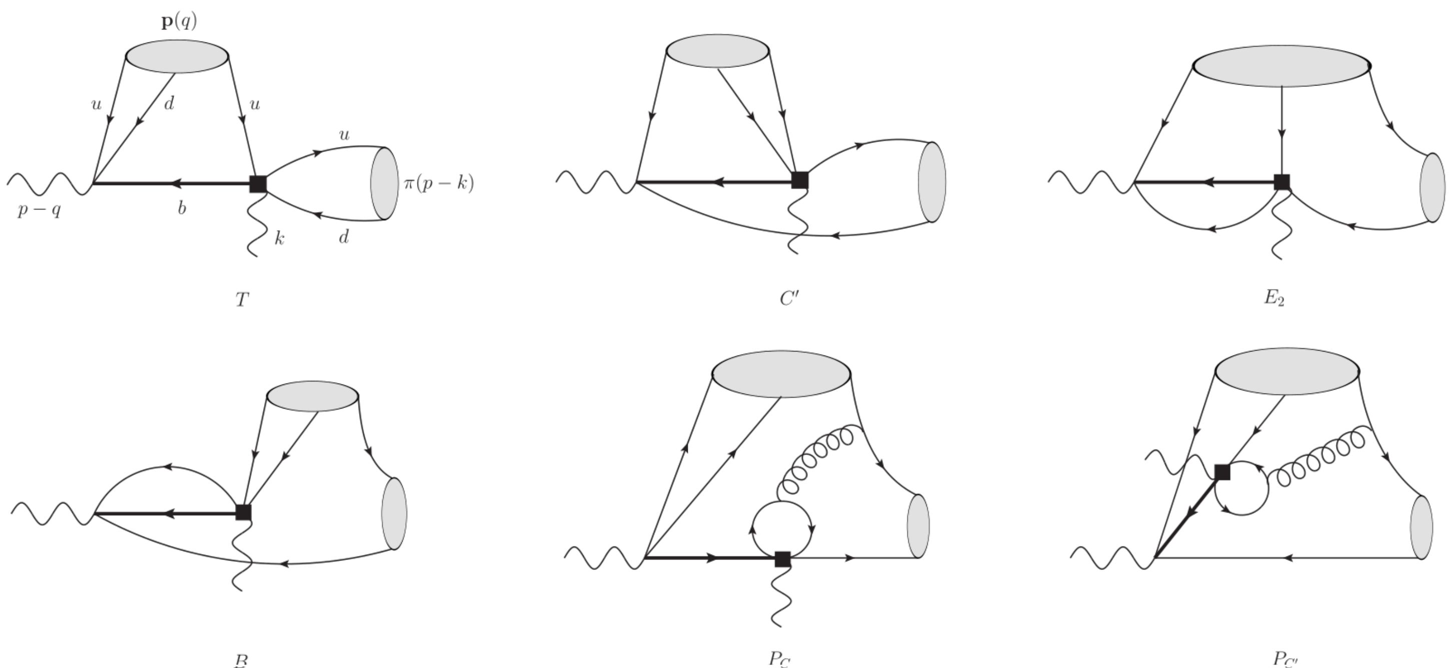


- We get the following invariant amplitudes

$$\mathcal{A}^{\mathcal{O}} = \frac{(-1)(m_{\Lambda_b^*} - m_N)}{\pi m_{\Lambda_b} \lambda_{\Lambda_b} (m_{\Lambda_b} + m_{\Lambda_b^*})} \int_{m_b^2}^{s_0^{\Lambda_b}} ds e^{-s/M^2 + m_{\Lambda_b}^2/M^2} \text{Im}_s \Big(F_2^{\mathcal{O}}(s) + \frac{F_1^{\mathcal{O}}(s)}{m_N - m_{\Lambda_b^*}} \Big)$$

The topology in the 2pt LCSRs

- The topology in the 2pt scheme



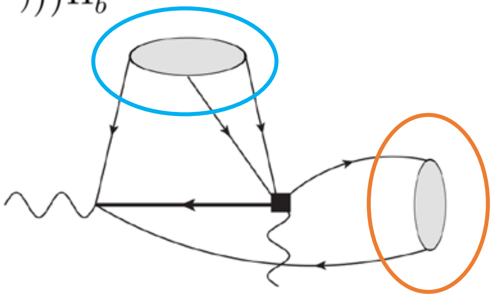
The contribution from C'

- The topology C' in the two-point correlator scheme

$$\Pi^{\mathcal{O}}_{2\text{pt},C'} = i\int d^4z e^{-i(p-q)\cdot z}\langle\pi(p-k)|T\left\{\eta_{\Lambda_b}(z)\mathcal{O}(0)\right\}|\mathbf{p}(q,\lambda)\rangle$$
$$= \int d^4z\int\frac{d^4p_2}{(2\pi)^4}e^{-i(p-q+p_2)\cdot z}\frac{[\Gamma_2(\not{p}_2+m_b)\Gamma^\mu]^{\tau\eta}}{p_2^2-m_b^2}(C\Gamma_1)^{\sigma\gamma}(\Gamma_\mu)^{\delta\rho}$$
$$\times\langle\pi(p-k)|\bar{u}_\delta(0)d_\gamma(z)|0\rangle\langle 0|\epsilon^{lhi}u^l_\eta(0)u^h_\sigma(z)d^i_\rho(0)|\mathbf{p}(q,\lambda)\rangle$$

- The invariant functions

$$F_1^c(s_2) = 2f_\pi\int_0^1\varphi_\pi(u)du\int_0^1 dx_2 m_N(m_N^2-m_{\Lambda_b}^2)\Big[(-\bar{\Phi}^c_{21}-2\bar{x}_2\bar{\Phi}^c_{11})\Pi_b$$
$$+\bar{x}_2\big(s_2\bar{\Phi}^c_{21}-m^2_{\Lambda_b}u\bar{\Phi}^c_{21}-m_N^2\big(\bar{u}\bar{\Phi}^c_{21}+2(\bar{\Phi}^c_{31}+\bar{x}_2\bar{\Phi}^c_{22}+9\tilde{T}_1^M)\big)\big)\Pi_b^2$$
$$+2\bar{x}_2^2m_N^2\Big(s_2(\bar{\Phi}^c_{32}+6\tilde{T}_1^M)-um^2_{\Lambda_b}(\bar{\Phi}^c_{32}+6\tilde{T}_1^M)$$
$$+m_N^2\big(6\bar{x}_2(\tilde{T}_1^M+\tilde{\tilde{\tilde{T}}}_{125678})-\bar{u}(\bar{\Phi}^c_{32}+6\tilde{T}_1^M)\big)\Big)\Pi_b^3\Big]$$
$$\equiv f_\pi\int_0^1\varphi_\pi(u)du\otimes\mathbb{F}_1^c(s_2,u)$$



The contribution from E_2 and B

- The topology E_2 and B in the 2pt scheme

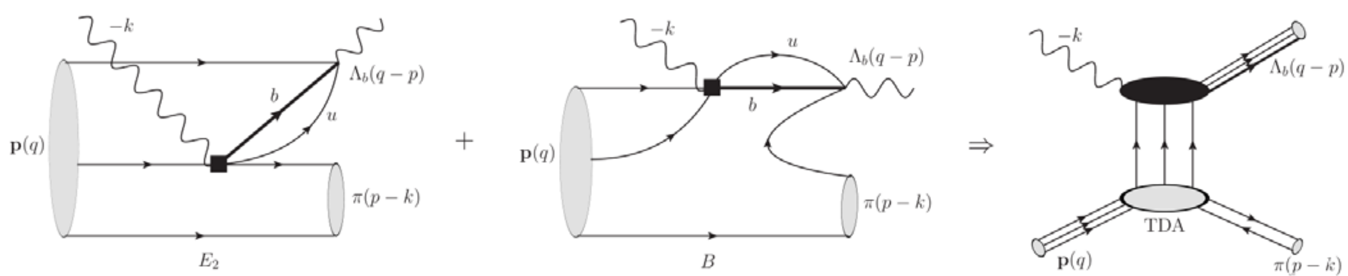
$$\Pi_{2\text{pt}}^{E_2+B} = i\int d^4z e^{-i(p-q)\cdot z}\langle\pi(p-k)|T\left\{\eta_{\Lambda_b}(z)\mathcal{O}(0)\right\}|\mathbf{p}(q,\lambda)\rangle$$

$$= i^3\int d^4z\int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{[\Gamma_\mu^T \not{p}_1^T C\Gamma_1]^{\rho\gamma}[\Gamma_2(\not{p}_2+m_b)\Gamma^\mu]^{\sigma\tau}}{p_1^2(p_2^2-m_b^2)}e^{-i[(p-q)+p_1+p_2]\cdot z}$$

$$\times\left\langle\pi(p-k)\middle|\epsilon^{khl}d_\gamma^k(z)d_\rho^h(0)u_\tau^l(0)\middle|\mathbf{p}(q,\lambda)\right\rangle$$

Transition distribution amplitude

[B. Pire, K. Semenov-Tian-Shansky, L. Szymanowski (2021)]



- We can't calculate such kind of diagram so far.

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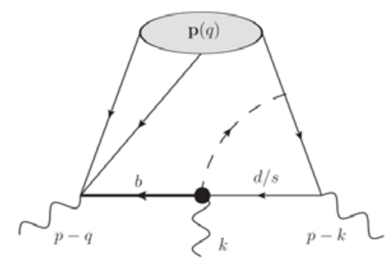
Branching ratio and CPV

- The branching fraction and CP violation from 3pt

channel	$\Lambda_b \to p\pi$	$\Lambda_b \to pK$
topology	T,C', E_2, B P_C, $P_{C'}$	T,E_2 P_C
BR (10^{-6})	5.94	6.50
BR (PDG) (10^{-6})	4.5 ± 0.8	5.4 ± 1.0
A_{CP}	-0.018	-0.001
A_{CP} (PDG)	-0.025 ± 0.029	-0.025 ± 0.022

Without gluon penguin contribution

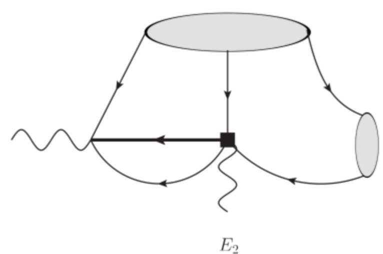


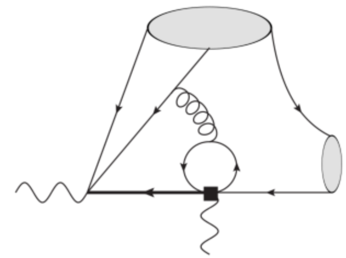
- The contributions to $(\mathcal{A}, \mathcal{B})$ from different topologies

channel	$T(10^{-9})$	$C'(10^{-9})$	$E_2(10^{-9})$	Penguin (10^{-9})
$\Lambda_b \to p\pi$	$(-1.57i, -1.51i)$	$(0.20i, 0.20i)$	$(-1.37i, -2.55i)$	$(0.94e^{-1.51i}, 0.89e^{1.62i})$
$\Lambda_b \to pK$	$(-0.59i, -0.58i)$	—	$(-0.26i, -0.48i)$	$(2.25e^{-1.57i}, 5.57e^{1.58i})$

The light-cone OPE of 2pt

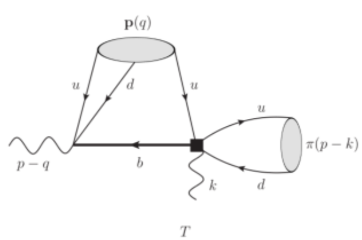
- The general form

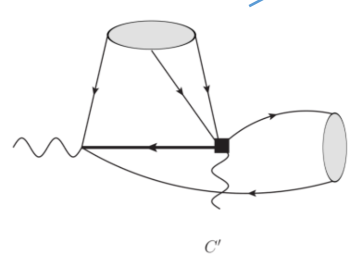


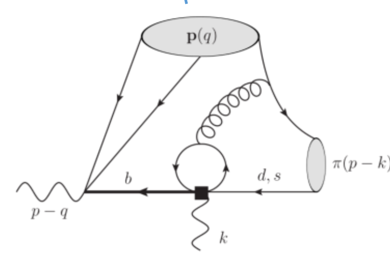


$$\Pi^{\mathcal{O}}_{2\mathrm{pt}}((p-q)^2, P^2, \lambda) = i\int d^4z e^{-i(p-q)\cdot z}\langle \pi^+(p-k)|T\{\eta_{\Lambda_b}(z)\mathcal{O}(0)\}|\mathbf{p}(q,\lambda)\rangle$$

$$= i\int d^4z e^{-i(p-q)\cdot z}\Big\{\mathbf{C}_1\langle\pi^+|udd|\mathbf{p}\rangle + \mathbf{C}_2\langle\pi^+|\bar{u}d|0\rangle\langle 0|uud|\mathbf{p}\rangle + \cdots\Big\}$$

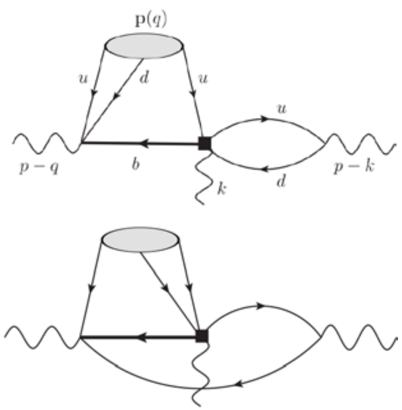




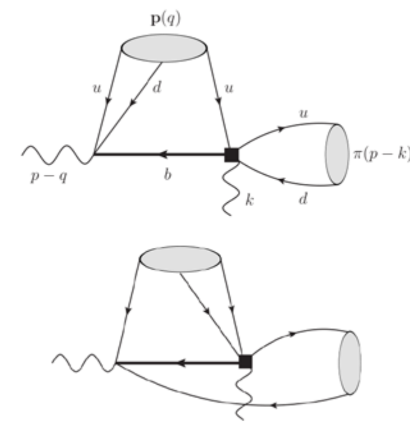


3pt scheme vs 2pt scheme

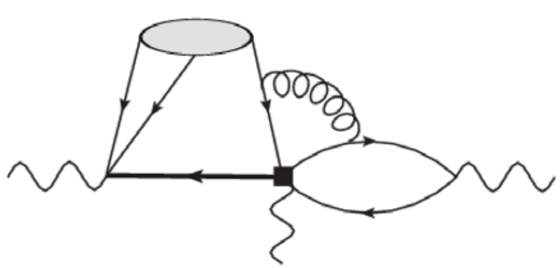
- The topology T and C' in the two schemes

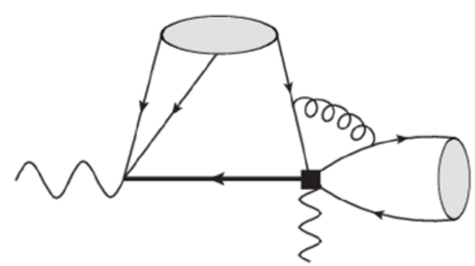


Topology	3pt scheme	2pt scheme
$T(10^{-9})$	$(-1.57i, -1.51i)$	$(-1.79i, -1.80i)$
$C'(10^{-9})$	$(0.20i, 0.20i)$	$(0.26i, 0.25i)$



- The NLO correction is achievable from 2pt





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Summary

- As a first try, we found the heavy baryon two-body decays can be well studied within the framework of LCSRs .
- The branching ratio and CP-violation are estimated and consistent with the experimental data.
- Our LCSRs scheme can deal fairly well with W-exchange diagram.
- The proposed two-point correlator scheme has special advantages for simplifying the calculation procedure and estimating the higher order correction of tree topology.

Thanks for your attention!

Back up

- The fundamental constituents of effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_u \left(c_1 O_1^u + c_2 O_2^u \right) + \lambda_c \left(c_1 O_1^c + c_2 O_2^c \right) + \left(\lambda_u + \lambda_c \right) \left[\sum_{i=3}^{10} c_i O_i + c_{7\gamma} O_{7\gamma} + c_{8g} O_{8g} \right] \right\} + h.c.$$

$$O_1^p = (\bar{q}\Gamma^l_\mu p)(\bar{p}\Gamma^{l\mu} b) = \frac{1}{3} O_2^p + 2\widetilde{O}_2^p$$

$$O_2^p = (\bar{p}\Gamma^l_\mu p)(\bar{q}\Gamma^{l\mu} b) = \frac{1}{3} O_1^p + 2\widetilde{O}_1^p$$

$$O_3 = \sum_f \left(\bar{f}\Gamma^l_\mu f\right)\left(\bar{q}\Gamma^{l\mu} b\right) = \frac{1}{3} O_4 + 2\widetilde{O}_4,$$

$$O_4 = \sum_f \left(\bar{q}\Gamma^l_\mu f\right)\left(\bar{f}\Gamma^{l\mu} b\right) = \frac{1}{3} O_3 + 2\widetilde{O}_3,$$

$$O_5 = \sum_f \left(\bar{f}\Gamma^r_\mu f\right)\left(\bar{q}\Gamma^{l\mu} b\right) = \frac{1}{3} O_6 + 2\widetilde{O}_6$$

$$O_6 = -2\sum_f \left(\bar{q}\left(1+\gamma_5\right) f\right)\left(\bar{f}\left(1-\gamma_5\right) b\right) = \frac{1}{3} O_5 + 2\widetilde{O}_5,$$

$$O_7 = \frac{3}{2}\sum_f e_f \left(\bar{f}\Gamma^r_\mu f\right)\left(\bar{q}\Gamma^{l\mu} b\right) = \frac{1}{3} O_8 + 2\widetilde{O}_8$$

$$O_8 = -3\sum_f e_f \left(\bar{q}\left(1+\gamma_5\right) f\right)\left(\bar{f}\left(1-\gamma_5\right) b\right) = \frac{1}{3} O_7 + 2\widetilde{O}_7,$$

$$O_9 = \frac{3}{2}\sum_f e_f \left(\bar{f}\Gamma^l_\mu f\right)\left(\bar{q}\Gamma^{l\mu} b\right) = \frac{1}{3} O_{10} + 2\widetilde{O}_{10},$$

$$O_{10} = \frac{3}{2}\sum_f e_f \left(\bar{q}\Gamma^l_\mu f\right)\left(\bar{f}\Gamma^{l\mu} b\right) = \frac{1}{3} O_9 + 2\widetilde{O}_9,$$

$$O_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}\sigma^{\mu\nu}\left(1+\gamma_5\right) F_{\mu\nu} b$$

$$O_{8g} = \frac{g_s}{8\pi^2} m_b \bar{q}\sigma^{\mu\nu}\left(1+\gamma_5\right) t^a G^a_{\mu\nu} b$$

$$\widetilde{O}_1^p = (\bar{q}\Gamma^l_\mu t^a p)(\bar{p}\Gamma^{l\mu} t^a b) \quad \widetilde{O}_2^p = (\bar{p}\Gamma^l_\mu t^a p)(\bar{q}\Gamma^{l\mu} t^a b)$$

Proton light-cone distribution amplitude

In the light-cone limit, $y_1 = a_1 z, y_2 = a_2 z, y_3 = a_3 z, \quad z^2 = 0$

$$
\begin{aligned}
&4 \langle 0| \varepsilon^{ijk} u_\alpha^i(a_1 z) u_\beta^j(a_2 z) d_\gamma^k(a_3 z) |P\rangle = \\
&= \mathcal{S}_1 M C_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{S}_2 M^2 C_{\alpha\beta} (\not{z}\gamma_5 N)_\gamma + \mathcal{P}_1 M (\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{P}_2 M^2 (\gamma_5 C)_{\alpha\beta} (\not{z} N)_\gamma \\
&\quad + \mathcal{V}_1 (\not{P} C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{V}_2 M (\not{P} C)_{\alpha\beta} (\not{z}\gamma_5 N)_\gamma + \mathcal{V}_3 M (\gamma_\mu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma \\
&\quad + \mathcal{V}_4 M^2 (\not{z} C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{V}_5 M^2 (\gamma_\mu C)_{\alpha\beta} (i\sigma^{\mu\nu} z_\nu \gamma_5 N)_\gamma + \mathcal{V}_6 M^3 (\not{z} C)_{\alpha\beta} (\not{z}\gamma_5 N)_\gamma \\
&\quad + \mathcal{A}_1 (\not{P}\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_2 M (\not{P}\gamma_5 C)_{\alpha\beta} (\not{z} N)_\gamma + \mathcal{A}_3 M (\gamma_\mu \gamma_5 C)_{\alpha\beta} (\gamma^\mu N)_\gamma \\
&\quad + \mathcal{A}_4 M^2 (\not{z}\gamma_5 C)_{\alpha\beta} N_\gamma + \mathcal{A}_5 M^2 (\gamma_\mu \gamma_5 C)_{\alpha\beta} (i\sigma^{\mu\nu} z_\nu N)_\gamma + \mathcal{A}_6 M^3 (\not{z}\gamma_5 C)_{\alpha\beta} (\not{z} N)_\gamma \\
&\quad + \mathcal{T}_1 (P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma + \mathcal{T}_2 M (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma_5 N)_\gamma + \mathcal{T}_3 M (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \gamma_5 N)_\gamma \\
&\quad + \mathcal{T}_4 M (P^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 N)_\gamma + \mathcal{T}_5 M^2 (z^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma \\
&\quad + \mathcal{T}_6 M^2 (z^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta} (\not{z}\gamma_5 N)_\gamma + \mathcal{T}_7 M^2 (\sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\nu} \not{z}\gamma_5 N)_\gamma \\
&\quad + \mathcal{T}_8 M^3 (z^\nu \sigma_{\mu\nu} C)_{\alpha\beta} (\sigma^{\mu\varrho} z_\varrho \gamma_5 N)_\gamma \,, \qquad (2.3)
\end{aligned}
$$

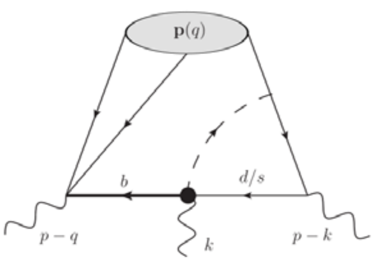
[V. Braun , R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. B 589 (2000) 381-409]

The invariant amplitudes

- Summary of the invariant amplitudes

$$\mathcal{A}_{\rm tot} = \frac{G_F}{\sqrt{2}}\Big[\lambda_u(a_1\mathcal{A}^{O_1^u}_{\rm tree} + 2c_1\mathcal{A}^{\tilde{O}_2^u}_{\rm peng}) + 2\lambda_c c_1\mathcal{A}^{\tilde{O}_2^c}_{\rm peng} + (\lambda_u+\lambda_c)\big[a_4\mathcal{A}^{O_4^u}_{\rm tree} + 2c_4(\mathcal{A}^{\tilde{O}_3^u}_{\rm peng} + \mathcal{A}^{\tilde{O}_3^c}_{\rm peng})$$
$$+ a_6\mathcal{A}^{O_6^u}_{\rm tree} + 2c_6(\mathcal{A}^{\tilde{O}_5^u}_{\rm peng} + \mathcal{A}^{\tilde{O}_5^c}_{\rm peng}) + a_8\mathcal{A}^{O_8^u}_{\rm tree} + 2c_8(\mathcal{A}^{\tilde{O}_7^u}_{\rm peng} + \mathcal{A}^{\tilde{O}_7^c}_{\rm peng})$$
$$+ a_{10}\mathcal{A}^{O_{10}^u}_{\rm tree} + 2c_{10}(\mathcal{A}^{\tilde{O}_9^u}_{\rm peng} + \mathcal{A}^{\tilde{O}_9^c}_{\rm peng}) + \underline{c_{7\gamma}\mathcal{A}^{O_{7\gamma}}_{\rm peng} + c_{8g}\mathcal{A}^{O_{8g}}_{\rm peng}}\big]\Big]$$
$$= \mathbf{A}_{\rm tree}e^{i\delta_T} + \mathbf{A}_{\rm peng}e^{i\delta_P}$$

$$\mathcal{B}_{\rm tot} = \frac{G_F}{\sqrt{2}}\Big[\lambda_u(a_1\mathcal{B}^{O_1^u}_{\rm tree} + 2c_1\mathcal{B}^{\tilde{O}_2^u}_{\rm peng}) + 2\lambda_c c_1\mathcal{B}^{\tilde{O}_2^c}_{\rm peng} + (\lambda_u+\lambda_c)\big[a_4\mathcal{B}^{O_4^u}_{\rm tree} + 2c_4(\mathcal{B}^{\tilde{O}_3^u}_{\rm peng} + \mathcal{B}^{\tilde{O}_3^c}_{\rm peng})$$
$$+ a_6\mathcal{B}^{O_6^u}_{\rm tree} + 2c_6(\mathcal{B}^{\tilde{O}_5^u}_{\rm peng} + \mathcal{B}^{\tilde{O}_5^c}_{\rm peng}) + a_8\mathcal{B}^{O_8^u}_{\rm tree} + 2c_8(\mathcal{B}^{\tilde{O}_7^u}_{\rm peng} + \mathcal{B}^{\tilde{O}_7^c}_{\rm peng})$$
$$+ a_{10}\mathcal{B}^{O_{10}^u}_{\rm tree} + 2c_{10}(\mathcal{B}^{\tilde{O}_9^u}_{\rm peng} + \mathcal{B}^{\tilde{O}_9^c}_{\rm peng}) + \underline{c_{7\gamma}\mathcal{B}^{O_{7\gamma}}_{\rm peng} + c_{8g}\mathcal{B}^{O_{8g}}_{\rm peng}}\big]\Big]$$
$$= \mathbf{B}_{\rm tree}e^{i\delta'_T} + \mathbf{B}_{\rm peng}e^{i\delta'_P}$$



$$\Gamma(\Lambda_b \to p\pi) = \frac{p_{cm}}{8\pi}\Big[\frac{(m_{\Lambda_b}+m_N)^2 - m_\pi^2}{m_{\Lambda_b}}|\mathcal{A}_{\rm tot}|^2 + \frac{(m_{\Lambda_b}-m_N)^2 - m_\pi^2}{m_{\Lambda_b}}|\mathcal{B}_{\rm tot}|^2\Big]$$

The light-cone distribution amplitudes

- For nucleon, the non-perturbative parameters

Model	$f_N \cdot 10^3$	$\lambda_1 \cdot 10^3$	$\lambda_2 \cdot 10^3$	A_1^u	V_1^d	f_1^u	f_1^d	f_2^d
BLW (2006)	5.0(5)	−27(9)	54(19)	0.13	0.30	0.33	0.09	0.25
CZ (2002)	5.0(5)	−27(9)	54(19)	0.38(15)	0.23(3)	0.40(5)	0.07(5)	0.22(5)
LAT (2019)	3.54(6)	−44.9(4.2)	93.4(4.8)	0.300(32)	0.191(22)	–	–	–

- The RG running of part parameters

$$f_N(\mu) = f_N(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{2/(3\beta_0)}$$

$$\varphi_{nk}(\mu) = \varphi_{nk}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{nk}/\beta_0}$$

$$\lambda_{1,2}^N(\mu) = \lambda_{1,2}^N(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-2/\beta_0}$$

$$\beta_0 = 11 - \tfrac{2}{3}N_f$$

$$\gamma_{00} = 0, \quad \gamma_{10} = \frac{20}{9}, \quad \gamma_{11} = \frac{8}{3},$$
$$\gamma_{20} = \frac{32}{9}, \quad \gamma_{21} = \frac{40}{9}, \quad \gamma_{22} = \frac{14}{3}$$

[I. V. Anikin, V. M. Braun and N. Offen, (2013)]

- So far we temporarily use the leading twist asymptotic function for light meson $\varphi_\pi^{(as)}(u) = 6u(1-u)$