PQCD Group Meeting

Aug. 14 – 15, *2020*

Semileptonic decays $B_s \to \phi\ell^+\ell^-$ in the perturbative QCD factorization approach with lattice QCD input

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Outline

Introduction: P_5' anomaly

Update of the $B^0 \to K^{*0}\mu^+\mu^-$ angular analysis at LHCb[1]

- Tension is confirmed using data collected during Run I and 2016

$4.0 < q^2 < 6.0\,\mathrm{GeV}^2/c^4$

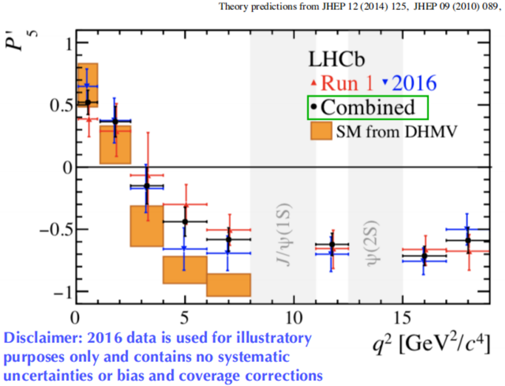
Run1+2016: 2.5σ

Run1 only: 2.8σ

$6.0 < q^2 < 8.0\,\mathrm{GeV}^2/c^4$

Run1+2016: 2.9σ

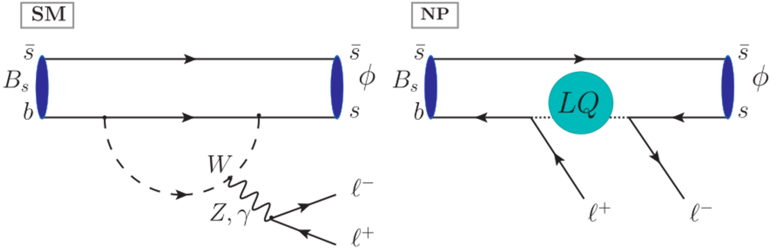
Run1 only: 3.0σ



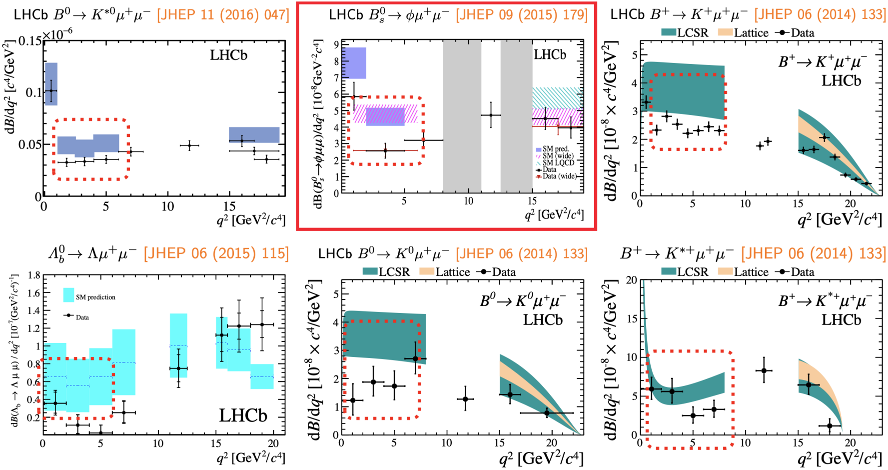
[1] R. Aaij *et al.* [LHCb Collaboration],PRL**125**,011802(2020)

Rare Decays of B mesons: $b \to s \ell^+ \ell^-$

1. Proceed via a **flavour-changing neutral current**(FCNC)
   - Forbidden at tree level in the Standard Model(SM)
   - Occur at loop level only via electroweak penguin and box
2. **As suppressed in the SM** more sensitive to New Physics(NP)
3. As NP particles appear virtually can probe heavier NP scales than those accessible via direct searches



Motivation (experiment)

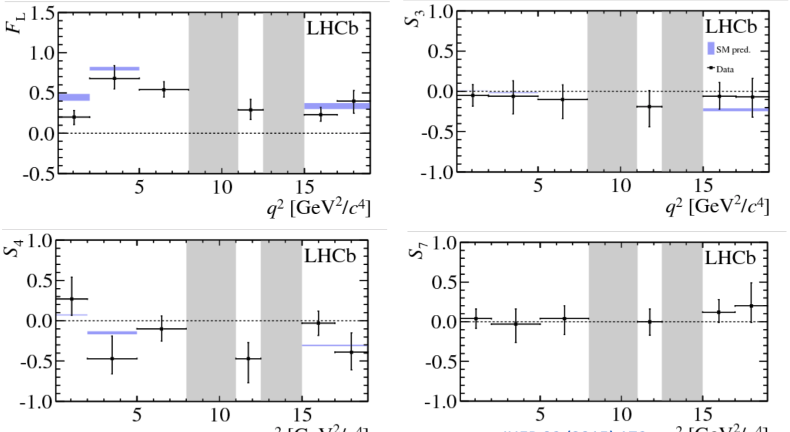


- Pattern: Data consistently below SM predictions
- But sizeable hadronic theory uncertainties
- Tension at $1-3\sigma$ level

Motivation (experiment)

$$B_s^0 \to \phi(\to K^+K^-)\mu^+\mu^-$$

- Equivalent process of $B^0 \to K^{*0}\mu^+\mu^-$ for B_s^0 mesons
- Not as powerful a channel because the process is not self tagging
- Angular observables are consistent with the Standard Model



JHEP 09 (2015) 179

Motivation (theory)

PHYSICAL REVIEW D **102**, 013001 (2020)

Study of $B_s \to K^{(*)}\ell^+\ell^-$ decays in the PQCD factorization approach with lattice QCD input

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(Received 31 March 2020; accepted 22 June 2020; published 7 July 2020)

1. Calculation of **Form Factor**(FF):
   - **PQCD**: H.N Li, C.D Lü, Z.J Xiao *et al.*
   - **PQCD**+"**Lattice**":Combined with lattice data[1]
2. **Extraplation** and **Fit**: Bourrely-Caprini-Lellouch(BCL) parametrization method[2]
3. **Angular Analysis**: J. Matias, W. Altmannshofer, B. Kindra *et al.*

[1]R. R. Horgan, Z. Liu and S. Meinel, Phys. Rev. D**89**,094501(2014)

[2]C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D**79**,013008(2009)

Theoretical Framework

$b \to s\ell^+\ell^-$ transitions in effective theory: Effective Hamiltonian[1]

- Write Hamiltonian as combination of these two:

  1. **Wilson coefficients**, C_i, [short distance]
  2. **Operators**, $\mathcal{O}_i$, [long distance, low energy] $\Rightarrow$ **Form Factor**

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* (\mathcal{H}_{\text{eff}}^{(t)} + \lambda_u \mathcal{H}_{\text{eff}}^{(u)}) + h.c., \tag{1}$$

with the CKM ratio $\lambda_u \equiv V_{ub} V_{us}^* / (V_{tb} V_{ts}^*)$ and

$$\mathcal{H}_{\text{eff}}^{(t)} = C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{10} C_i \mathcal{O}_i,$$

$$\mathcal{H}_{\text{eff}}^{(u)} = C_1 [\mathcal{O}_1^c - \mathcal{O}_1^u] + C_2 [\mathcal{O}_2^c - \mathcal{O}_2^u].$$

[1]W. Altmannshofer, P. Ball, A. Bharucha *et al.* ,JHEP **0901**, 019 (2009)

A traditional basis of the operators $\{\mathcal{O}_i\}$[1]:

- current-current operators $\mathcal{O}_{1,2}^{u,c}$:

$$\mathcal{O}_1^c = (\bar{s}_\alpha \gamma_\mu P_L c_\beta)(\bar{c}_\beta \gamma_\mu P_L b_\alpha), \quad \mathcal{O}_2^c = (\bar{s}_\alpha \gamma_\mu P_L c_\alpha)(\bar{c}_\beta \gamma_\mu P_L b_\beta),$$

$$\mathcal{O}_1^u = (\bar{s}_\alpha \gamma_\mu P_L u_\beta)(\bar{u}_\beta \gamma_\mu P_L b_\alpha), \quad \mathcal{O}_2^u = (\bar{s}_\alpha \gamma_\mu P_L u_\alpha)(\bar{u}_\beta \gamma_\mu P_L b_\beta),$$

- QCD penguin operators $\mathcal{O}_{3-6}$:

$$\mathcal{O}_3 = (\bar{s}_\alpha \gamma_\mu P_L b_\alpha)\sum_q (\bar{q}_\beta \gamma_\mu P_L q_\beta), \quad \mathcal{O}_4 = (\bar{s}_\alpha \gamma_\mu P_L b_\beta)\sum_q (\bar{q}_\beta \gamma_\mu P_L q_\alpha),$$

$$\mathcal{O}_5 = (\bar{s}_\alpha \gamma_\mu P_L b_\alpha)\sum_q (\bar{q}_\beta \gamma_\mu P_L q_\beta), \quad \mathcal{O}_6 = (\bar{s}_\alpha \gamma_\mu P_L b_\beta)\sum_q (\bar{q}_\beta \gamma_\mu P_L q_\alpha),$$

- electromagnetic and chromomagnetic penguin operators $\mathcal{O}_{7,8}$

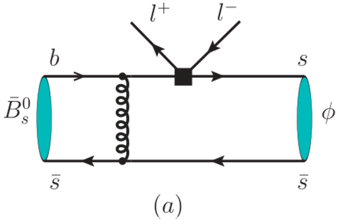
$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} P_R b_\alpha F_{\mu\nu}, \quad \mathcal{O}_8 = \frac{g}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} T^a_{\alpha\beta} P_R b_\beta G^a_{\mu\nu},$$

- semileptonic operators $\mathcal{O}_{9,10}$

$$\mathcal{O}_9 = \frac{\alpha_{em}}{4\pi} (\bar{s}_\alpha \gamma_\mu P_L b_\alpha)\sum_\ell (\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}_{10} = \frac{\alpha_{em}}{4\pi} (\bar{s}_\alpha \gamma_\mu P_L b_\alpha)\sum_\ell (\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

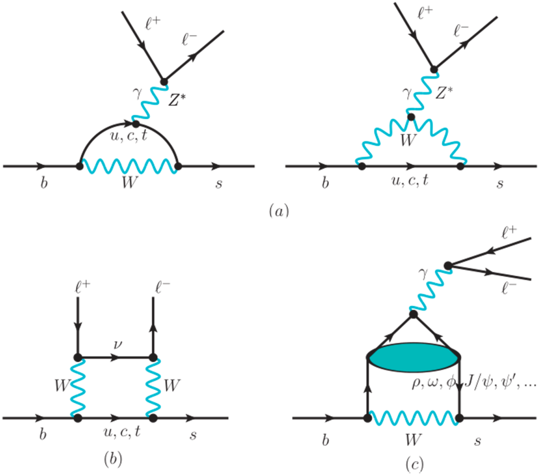
[1]G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev.Mod.Phys.**68**,1125 (1996)

Typical Feynman diagrams in PQCD



- Typical Feynman diagrams in PQCD for semileptonic decays $\bar{B}_s^0 \to \phi(1020)\ell^+\ell^-$ with the flavor-changing neutral current contributions due to the operators $\mathcal{O}_i$ denoted as black squares

Typical Feynman loop diagrams:



γ-penguin(a) $\leftrightharpoons$ $\mathcal{O}_7$, loops(c) $\leftrightharpoons$ $\mathcal{O}_{1,2}^{u,c}$,

Z/γ-penguin(a) & W-box(b) $\leftrightharpoons$ $\mathcal{O}_{9,10}$.

Theoretical Framework

SM Wilson coefficients in the NLL approximation.[1]

$\mu \backslash C_i(\mu)$	C_1	C_2	$C_3(\%)$	$C_4(\%)$	$C_5(\%)$	$C_6(\%)$	C_7	C_8	C_9	C_{10}
$m_b/2$	−0.276	1.131	2.005	−4.845	1.375	−5.841	−0.329	−0.165	4.450	−4.410
m_b	−0.175	1.076	1.258	−3.279	1.112	−3.634	−0.302	−0.148	4.232	−4.410
$3m_b/2$	−0.129	1.053	0.966	−2.608	0.964	−2.786	−0.287	−0.139	4.029	−4.410

Decay Amplitude for $b \to sl^+l^-$ loop transition [2]

$$\mathcal{A}(b \to sl^+l^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} V_{tb} V_{ts}^* \times$$

$$\left\{ C_9^{\rm eff}(q^2)[\bar{s}\gamma_\mu P_L b][\bar{l}\gamma^\mu l] + C_{10}[\bar{s}\gamma_\mu P_L b][\bar{l}\gamma^\mu \gamma_5 l] - 2m_b C_7^{\rm eff}\left[\bar{s} i\sigma_{\mu\nu}\frac{q^\nu}{q^2} P_R b\right][\bar{l}\gamma^\mu l] \right\}$$

[1] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev.Mod.Phys.**68**,1125 (1996)

[2] B. Kindra and N. Mahajan, Phys. Rev. D **98**, 094012 (2018)

The effective Wilson coefficient C_9^{eff}[1]

$$C_9^{\text{eff}}(q^2) = C_9(\mu) + Y_{\text{pert}}(\hat{s}) + Y_{\text{res}}(q^2).$$

1. Y_{pert}: short distance, pertubative
2. Y_{res}: non-pertubative, resonances [Breit-Wigner form]

$$\begin{aligned} Y_{\text{pert}}(\hat{s}) &= 0.124\,\omega(\hat{s}) + g(\hat{m}_c,\hat{s})C_0 + \lambda_u\left[g(\hat{m}_c,\hat{s}) - g(\hat{m}_u,\hat{s})\right](3C_1 + C_2) \\ &\quad -\frac{1}{2}g(\hat{m}_d,\hat{s})(C_3 + 3C_4) - \frac{1}{2}g(\hat{m}_b,\hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) \\ &\quad +\frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned}$$

where, $C_0 = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$

Note: $\omega(\hat{s}) \Rightarrow$ soft-gluon correction, $g(\hat{m}_q,\hat{s}) \Rightarrow$ loop coefficient function

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[1] P. Nayek, P. Maji and S. Sahoo, Phys.Rev.D**99**,013005(2019)

Theoretical Framework

The effective Wilson coefficient $C_9^{\rm eff}$[1]

$$C_9^{\rm eff}(q^2) = C_9(\mu) + Y_{\rm pert}(\hat{s}) + Y_{\rm res}(q^2).$$

1. $Y_{\rm pert}$: short distance, pertubative
2. $Y_{\rm res}$: non-pertubative, resonances [Breit-Wigner form]

$$Y_{\rm res}(q^2) = -\frac{3\pi}{\alpha_{\rm em}^2}\Bigg[C_0 \times \sum_{V=J/\Psi,\Psi'\ldots} \frac{m_V \mathcal{B}(V \to l^+ l^-)\Gamma_{\rm tot}^V}{q^2 - m_V^2 + i m_V \Gamma_{\rm tot}^V}$$
$$-\lambda_u g(\hat{m}_u, \hat{s})(3C_1 + C_2) \times \sum_{V=\rho,\omega,\phi} \frac{m_V \mathcal{B}(V \to l^+ l^-)\Gamma_{\rm tot}^V}{q^2 - m_V^2 + i m_V \Gamma_{\rm tot}^V}\Bigg]$$

[1]P. Nayek, P. Maji and S. Sahoo, Phys.Rev.D**99**,013005(2019)

Long-distance resonances contributions[1]

$$B_s \to \phi V^* \to \phi(V^* \to \ell^+\ell^-)$$

V	Mass[GeV]	$\Gamma^V_{\rm tot}$[MeV]	$\mathcal{BR}(V \to \ell^+\ell^-)$ with $\ell = e, \mu$
$\rho(770)$	0.775	149.1	4.63×10^{-5}
$\omega(782)$	0.782	8.490	7.38×10^{-5}
$\phi(1020)$	1.019	4.249	2.92×10^{-4}
$J/\psi(1S)$	3.096	0.093	5.96×10^{-2}
$\psi(2S)$	3.686	0.294	7.96×10^{-3}
$\psi(3770)$	3.773	27.2	9.60×10^{-6}
$\psi(4040)$	4.039	80	1.07×10^{-5}
$\psi(4160)$	4.191	70	6.90×10^{-6}
$\psi(4415)$	4.421	62	9.40×10^{-6}

[1] M. Tanabashi *et al.* [Particle Data Group], Phys.Rev.D **98**, 030001 (2018)

$B_s \to \phi$ transition **Form Factors**

Form Factor Definition

- $B_s \to \phi$ form factors read:$[T^\mu = \bar{d}\sigma^{\mu\nu} b\ , T_5^\mu = \bar{d}\sigma^{\mu\nu}\gamma_5 b]$

$$\langle\phi(p_2)|V^\mu|\bar{B}_s(p_1)\rangle = \epsilon^{\mu\nu\alpha\beta}\epsilon_{\nu *}p_{1\alpha}p_{2\beta}\frac{2V(q^2)}{m_{B_s}+m_\phi}\ ,$$

$$\langle\phi(p_2)|A^\mu|\bar{B}_s(p_1)\rangle = 2i\frac{m_\phi(\epsilon^*\cdot q)}{q^2}q^\mu A_0(q^2) + i\left[\epsilon^{\mu *} - \frac{\epsilon^*\cdot q}{q^2}q^\mu\right]\cdot$$

$$(m_{B_s}+m_\phi)A_1(q^2) - i\left[(p_1+p_2)^\mu - \frac{m_{B_s}^2 - m_\phi^2}{q^2}q^\mu\right]\frac{(\epsilon^*\cdot q)A_2(q^2)}{m_{B_s}+m_\phi},$$

$$\langle\phi(p_2)|T^{\mu\nu}q_\nu|B_s(p_1)\rangle = 2i\epsilon^{\mu\nu\alpha\beta}\epsilon_{*\nu}p_{1\alpha}p_{2\beta}T_1(q^2),$$

$$\langle\phi(p_2)|T_5^{\mu\nu}q_\nu|B_s(p_1)\rangle = \left[\epsilon^{*\mu}(m_{B_s}^2 - m_\phi^2) - (\epsilon^*\cdot q)(p_1+p_2)^\mu\right]T_2(q^2)$$

$$+(\epsilon^*\cdot q)\left[q^\mu - \frac{q^2}{m_{B_s}^2 - m_\phi^2}(p_1+p_2)^\mu\right]T_3(q^2).$$

Form Factors Calculation Expressions in **PQCD** : $V\ A_{0,1,2}\ T_{1,2,3}$

$$V(q^2) = 8\pi m_{B_s}^2 C_F (1+r) \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1) \times$$

$$\Big\{ \Big[-x_2 r \phi_\phi^v(x_2) + \phi_\phi^T(x_2) - \frac{1+x_2 r\eta}{\sqrt{\eta^2-1}} \phi_\phi^a(x_2) \Big] \cdot H_1(t_1)$$

$$+ \Big[(r + \frac{x_1}{2\sqrt{\eta^2-1}}) \phi_\phi^v(x_2) + \frac{x_1 - 2r\eta}{2\sqrt{\eta^2-1}} \phi_\phi^a(x_2) \Big] \cdot H_2(t_2) \Big\},$$

$$A_0(q^2) = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1) \times$$

$$\Big\{ \Big[(1+x_2 r(2\eta - r)) \phi_\phi(x_2) + (1-2x_2) r \phi_\phi^t(x_2) - \frac{(1-r\eta) - 2x_2 r(\eta - r)}{\sqrt{\eta^2-1}} \phi_\phi^s(x_2) \Big] \cdot H_1(t_1)$$

$$+ \Big[[\frac{x_1}{\sqrt{\eta^2-1}} (\frac{\eta+r}{2} - r\eta^2) + (\frac{x_1}{2} - x_1 r\eta + r^2)] \phi_\phi(x_2) + [\frac{x_1(1-r\eta) + 2r(r-\eta)}{\sqrt{\eta^2-1}} - x_1 r] \phi_\phi^s(x_2) \Big] \cdot H_2(t_2) \Big\},$$

$$T_1(q^2) = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1) \times$$

$$\Big\{ \Big[(1-2x_2) r \phi_\phi^v(x_2) + (1+2x_2 r\eta - x_2 r^2) \phi_\phi^T(x_2) - \frac{1+2x_2 r^2 - (1+2x_2) r\eta}{\sqrt{\eta^2-1}} \phi_\phi^a(x_2) \Big] \cdot H_1(t_1)$$

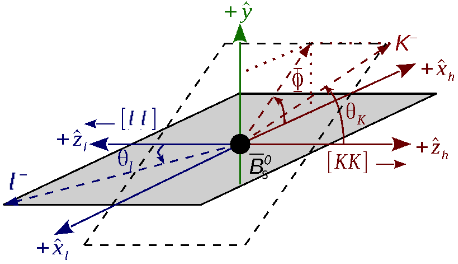
$$+ [(1 - \frac{x_1}{2}) r - \frac{x_1(r\eta - 1)}{2\sqrt{\eta^2-1}}] \phi_\phi^v(x_2) - [\frac{r(\eta - r)}{\sqrt{\eta^2-1}} + \frac{x_1}{2} (r + \frac{r\eta - 1}{\sqrt{\eta^2-1}})] \phi_\phi^a(x_2) \Big] \cdot H_2(t_2) \Big\},$$

...

Angular analysis for $B_s \to \phi \ell^+ \ell^-$

Why angular analysis?

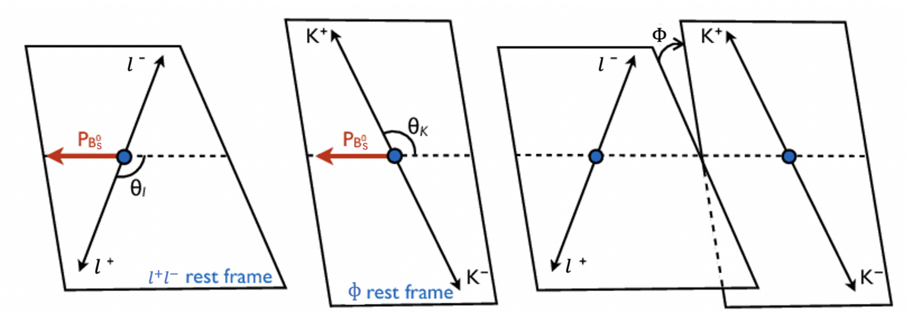
1. **Angular analysis**: measure the **rate of a decay process** as a **function of the angles** of the final decay products
2. Compared to measuring the decay rate(i.e. branching fractions)alone, angular analysis can give access to a large ranges of observables with reduced theory uncertainties



Angular analysis for $B_s \to \phi \ell^+ \ell^-$

$\bar{B}_s \to \phi[\to K^+K^-]\ell^+\ell^-$ angular description[1]

$$\frac{d^4\Gamma(\bar{B}_s \to \phi \ell^+ \ell^-)}{dq^2\, d\vec{\Omega}} = \frac{9}{32\pi} \sum_i \mathcal{I}_i f_i(\vec{\Omega}), \quad d\vec{\Omega} = d\cos\theta_K\, d\cos\theta_\ell\, d\Phi,$$



[1]W. Altmannshofer, P. Ball, A. Bharucha *et al.* ,JHEP **0901**, 019 (2009)

What are angular coefficients?

Angular coefficients $\mathcal{I}_i$ are combinations of different amplitudes $\mathcal{A}$ [1]:

i	$\mathcal{I}_i$	f_i
1s	$(\frac{3}{4}-\hat{m}_\ell^2)\left[\vert\mathcal{A}_\parallel^L\vert^2+\vert\mathcal{A}_\perp^L\vert^2+\vert\mathcal{A}_\parallel^R\vert^2+\vert\mathcal{A}_\perp^R\vert^2\right]+4\hat{m}_\ell^2\mathrm{Re}\left[\mathcal{A}_\perp^L\mathcal{A}_\perp^{R*}+\mathcal{A}_\parallel^L\mathcal{A}_\parallel^{R*}\right]$	$\sin^2\theta_V$
1c	$\vert\mathcal{A}_0^L\vert^2+\vert\mathcal{A}_0^R\vert^2+4\hat{m}_\ell^2\left[\vert\mathcal{A}_t\vert^2+2\mathrm{Re}[\mathcal{A}_0^L\mathcal{A}_0^{R*}]\right]$	$\cos^2\theta_V$
2s	$\frac{1}{4}\beta_\ell^2\left[\vert\mathcal{A}_\parallel^L\vert^2+\vert\mathcal{A}_\perp^L\vert^2+\vert\mathcal{A}_\parallel^R\vert^2+\vert\mathcal{A}_\perp^R\vert^2\right]$	$\sin^2\theta_V\cos 2\theta_\ell$
2c	$-\beta_\ell^2\left[\vert\mathcal{A}_0^L\vert^2+\vert\mathcal{A}_0^R\vert^2\right]$	$\cos^2\theta_V\cos 2\theta_\ell$
3	$\frac{1}{2}\beta_\ell^2\left[\vert\mathcal{A}_\perp^L\vert^2-\vert\mathcal{A}_\parallel^L\vert^2+\vert\mathcal{A}_\perp^R\vert^2-\vert\mathcal{A}_\parallel^R\vert^2\right]$	$\sin^2\theta_V\sin^2\theta_\ell\cos 2\Phi$
4	$\sqrt{\frac{1}{2}}\beta_\ell^2\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}+\mathcal{A}_0^R\mathcal{A}_\parallel^{R*})$	$\sin 2\theta_V\sin 2\theta_\ell\cos\Phi$
5	$\sqrt{2}\beta_\ell\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}-\mathcal{A}_0^R\mathcal{A}_\perp^{R*})$	$\sin 2\theta_V\sin\theta_\ell\cos\Phi$
6s	$2\beta_\ell\mathrm{Re}(\mathcal{A}_\parallel^L\mathcal{A}_\perp^{L*}-\mathcal{A}_\parallel^R\mathcal{A}_\perp^{R*})$	$\sin^2\theta_V\cos\theta_\ell$
7	$\sqrt{2}\beta_\ell\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L*}-\mathcal{A}_0^R\mathcal{A}_\parallel^{R*})$	$\sin 2\theta_V\sin\theta_\ell\sin\Phi$
8	$\sqrt{\frac{1}{2}}\beta_\ell^2\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\perp^{L*}+\mathcal{A}_0^R\mathcal{A}_\perp^{R*})$	$\sin 2\theta_V\sin 2\theta_\ell\sin\Phi$
9	$\beta_\ell^2\mathrm{Im}(\mathcal{A}_\parallel^{L*}\mathcal{A}_\perp^L+\mathcal{A}_\parallel^{R*}\mathcal{A}_\perp^R)$	$\sin^2\theta_V\sin^2\theta_\ell\sin 2\Phi$

[1] R. Aaij *et al.* [LHCb Collaboration], JHEP **1602**, 104(2016)

How to describe amplitudes $\mathcal{A}$?

$\mathcal{A}_i^{L/R}$[1] depend on **Wilson Coefficients** and **Form Factors**

$$\mathcal{A}_\perp^{L,R} = -N_\ell \sqrt{2N_\phi}\sqrt{\lambda}\left[(C_9^{\rm eff} \mp C_{10})\frac{V(q^2)}{m_{B_s}+m_\phi} + 2\hat{m}_b C_7^{\rm eff} T_1(q^2)\right]$$

$$\mathcal{A}_\parallel^{L,R} = N_\ell \sqrt{2N_\phi}[(C_9^{\rm eff} \mp C_{10})(m_{B_s}+m_\phi)A_1(q^2) + 2\hat{m}_b C_7^{\rm eff}(m_{B_s}^2 - m_\phi^2)T_2(q^2)]$$

$$\mathcal{A}_0^{L,R} = \frac{N_\ell\sqrt{N_\phi}}{2m_\phi\sqrt{q^2}}\Big\{(C_9^{\rm eff} \mp C_{10})[(m_{B_s}^2 - m_\phi^2 - q^2)(m_{B_s}+m_\phi)A_1(q^2) - \frac{\lambda}{m_{B_s}+m_\phi}A_2(q^2)]$$
$$+ 2m_b C_7^{\rm eff}[(m_{B_s}^2 + 3m_\phi^2 - q^2)T_2(q^2) - \frac{\lambda}{m_{B_s}^2 - m_\phi^2}T_3(q^2)]\Big\}$$

$$\mathcal{A}_t = 2N_\ell\sqrt{N_\phi}\frac{\sqrt{\lambda}}{\sqrt{q^2}}C_{10}A_0(q^2)$$

where, $N_\ell = \frac{i\alpha_{em}G_F}{4\sqrt{2}\pi}V_{tb}V_{ts}^*$, $N_\phi = \frac{8\beta_\ell\sqrt{\lambda}q^2}{3\times 256\pi^3 m_{B_s}^3}\mathcal{B}(\phi\to K^+K^-)$,

$$\lambda \equiv (m_{B_s}^2 - m_\phi^2 - q^2)^2 - 4m_\phi^2 q^2, \beta_\ell = \sqrt{1-4\hat{m}_\ell}, \hat{m}_\ell = m_\ell/q^2, \hat{m}_b = m_b/q^2.$$

Note : $\mathcal{A}_i^{L/R}(i=\parallel,\perp) \Rightarrow$ *transverse*, $(i=0) \Rightarrow$ *longitudinal*, $(i=t) \Rightarrow$ *timelike*

Why? : $\mathcal{B}(\phi\to K^+K^-) = 0.492, \mathcal{B}(K^*\to K\pi) \approx 1$(*omitted*)[*PDG*2018],

What about the CP-conjugated condition?

$B_s \to \phi[\to K^+K^-]\ell^+\ell^-$ angular description[1]

$$\frac{d^4\bar{\Gamma}(B_s \to \phi\ell^+\ell^-)}{dq^2\, d\vec{\Omega}} = \frac{9}{32\pi}\sum_i \bar{\mathcal{I}}_i f_i(\vec{\Omega}), \quad d\vec{\Omega} = d\cos\theta_K\, d\cos\theta_\ell\, d\Phi,$$

1. CP transformation : $\ell \rightleftarrows \bar{\ell}$
2. Modification: $\theta_\ell \to \theta_\ell - \pi, \quad \Phi \to -\Phi$
3. Substitution: $\mathcal{I}_{1,2,3,4,7} \to \bar{\mathcal{I}}_{1,2,3,4,7}, \quad \mathcal{I}_{5,6,8,9} \to -\bar{\mathcal{I}}_{5,6,8,9}$
4. $\bar{\mathcal{I}}_i \equiv$ making the complex conjugation for all weak phases in $\mathcal{I}_i$.

[1]W. Altmannshofer, P. Ball, A. Bharucha *et al.* ,JHEP **0901**, 019 (2009)

How to construct observables ?

The CP averaged(asymmetry) angular coefficients $S_i(A_i)$[1]

:

$$S_i = \frac{\mathcal{I}_i + \bar{\mathcal{I}}_i}{d(\Gamma + \bar{\Gamma})/dq^2}, \quad A_i = \frac{\mathcal{I}_i - \bar{\mathcal{I}}_i}{d(\Gamma + \bar{\Gamma})/dq^2},$$

- the differential decay rate: $d\Gamma/dq^2 = \frac{1}{4}\left(3\mathcal{I}_1^c + 6\mathcal{I}_1^s - \mathcal{I}_2^c - 2\mathcal{I}_2^s\right)$
- the CP asymmetry : $\mathcal{A}_{CP} = \frac{1}{4}(3A_1^c + 6A_1^s - A_2^c - 2A_2^s)$
- lepton forward-backward asymmetry: $\mathcal{A}_{\mathrm{FB}} = \frac{3}{4}S_6^s, \quad \mathcal{A}_{\mathrm{FB}}^{CP} = \frac{3}{4}A_6^s$
- ϕ polarization fractions(massless limit): $F_L = S_1^c, \quad F_T = 4S_2^s$

[1]W. Altmannshofer, P. Ball, A. Bharucha *et al.* ,JHEP **0901**, 019 (2009)

Observables for $B_s \to \phi[\to K^+K^-]\ell^+\ell^-$

Clean(no S-wave pollution)observables $P_{1,2,3}$ and $P'_{4,5,6}$[1]

- Corresponding Relationship:

$$\frac{S_{3,6,9}}{F_T} \to P_{1,2,3} \quad , \quad \frac{S_{4,5,6,7}}{\sqrt{F_L(1-F_L)}} \to P'_{4,5,6,8}$$

- Comparison with the LHCb definition[2]:

$$P_1 = \frac{2S_3}{F_T} = A_T^{(2)} = P_1{}^{\mathrm{LHCb}}, \quad P_2 = \frac{S_6^s}{2F_T} = \frac{1}{2}A_T^{(\mathrm{re})} = P_2{}^{\mathrm{LHCb}},$$

$$P_3 = \frac{-S_9}{F_T} = -\frac{1}{2}A_T^{(\mathrm{im})} = P_3{}^{\mathrm{LHCb}}, \quad P'_4 = \frac{2S_4}{\sqrt{F_L(1-F_L)}} = 2P'_4{}^{\mathrm{LHCb}},$$

$$P'_5 = \frac{S_5}{\sqrt{F_L(1-F_L)}} = P'_5{}^{\mathrm{LHCb}}, \quad P'_6 = -\frac{S_7}{\sqrt{F_L(1-F_L)}} = -P'_6{}^{\mathrm{LHCb}},$$

$$P'_8 = -\frac{2S_8}{\sqrt{F_L(1-F_L)}} = -2P'_8{}^{\mathrm{LHCb}}.$$

[1] J. Matias, F. Mescia, M. Ramon and J. Virto,JHEP**1204**, 104 (2012)

[2] R. Aaij *et al.* [LHCb Collaboration],JHEP **1602**, 104 (2016)

Form Factors **Extrapolation** and **Fit**

1. The z-series expansion based on the BCL[1] parametrization method:$|z(q^2)| \leq 1$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \ ,$$

where,$t_\pm \equiv (m_B \pm m_\phi)^2$ and $t_0 \equiv t_+(1 - \sqrt{1 - t_-/t_+})$.

2. Form Factors are parameterized as a series expansion(SE)[2],

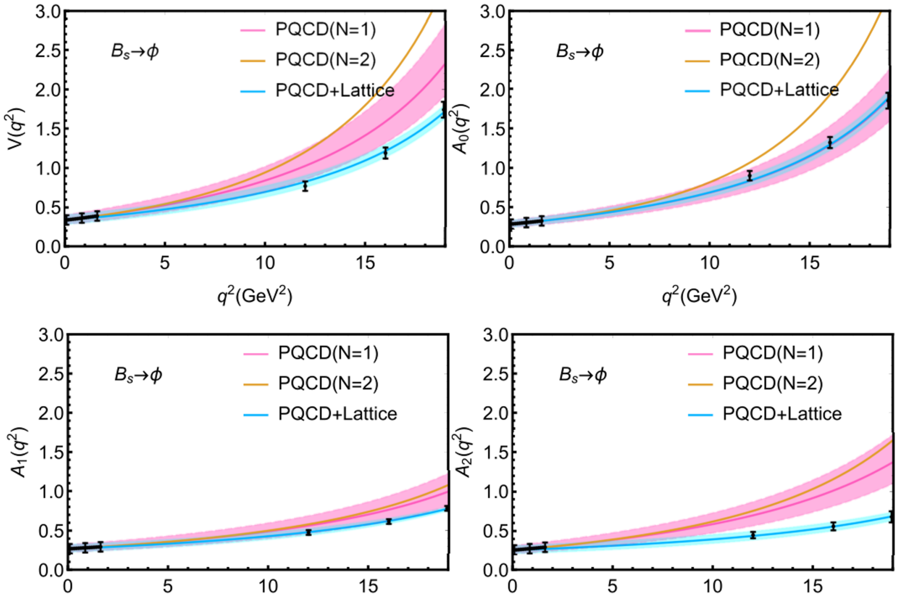
$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/m^2_{i,\mathrm{pole}}} \left\{ 1 + \sum_{k=1}^{N} b_k^i \left[z(q^2, t_0)^k - z(0, t_0)^k \right] \right\} .$$

Note: SE can be truncated after the first two terms, i.e.$N = 1$.

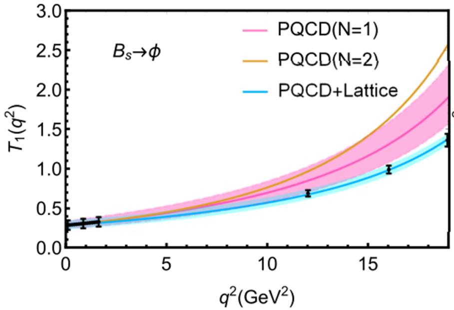
[1]C. Bourrely, I. Caprini and L. Lellouch, Phys. Rev. D**79**,013008(2009)

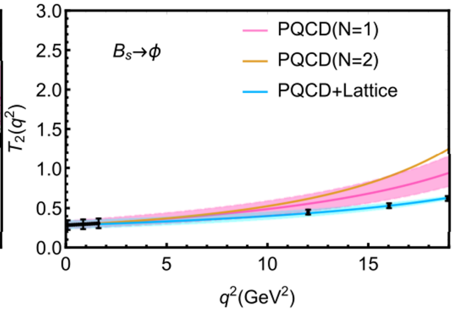
[2]A. Bharucha, T. Feldmann and M. Wick, JHEP **1009** (2010)090

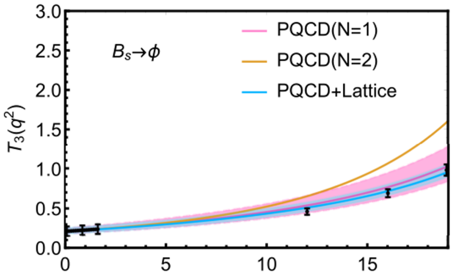
Numerical Results



Numerical Results







Theoretical predictions for the **Form Factors** at $q^2 = 0$

TABLE V. The theoretical predictions for the central values of form factors of the $B_s \to \phi$ transitions at $q^2 = 0$ obtained by using rather different theories or models.

	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$	$T_{1,2}(0)$	$T_3(0)$
This paper	0.337	0.284	0.267	0.259	0.286	0.212
PQCD[73]	0.26	0.31	0.18	0.12	0.23	0.19
PQCD[41]	0.25	0.30	0.19	–	–	
LCSR[23]	0.434	0.474	0.311	0.234	0.349	0.175
LCSR[74]	0.433	0.382	0.296	0.255	0.348	0.254
LCSR[26]	0.387	0.389	0.296	–	0.309	–
QCDSR[75]	0.45	0.30	0.32	0.30	0.33	0.22
RDA[76]	0.44	0.42	0.34	0.31	0.38	0.26
RQM[77]	0.406	0.322	0.320	0.318	0.275	0.133
SCET[78]	0.329	0.279	0.232	0.210	0.276	0.170
HQEFT[24]	0.339	0.269	0.271	0.212	0.299	0.191
SQEH[79]	0.259	0.311	0.194	–	–	–
CQM[21]	0.31	0.28	0.27	0.27	0.27	0.18

Numerical Results

- $\omega_{B_s} = 0.50 \pm 0.05, f_\phi = 0.231 \pm 0.04, f_\phi^T = 0.20 \pm 0.01$,(GeV) $a_{2\phi}^{||} = 0.18 \pm 0.08, a_{2\phi}^{\perp} = 0.14 \pm 0.07$

TABLE VII. The PQCD and "PQCD+Lattice" predictions for the form factors of $B_s \to \phi$ transitions.

Form Factors	Parameter	Central value	ω_{B_s}	$a_{2\phi}^{\vert\vert}$	$a_{2\phi}^{\perp}$	f_ϕ	f_ϕ^T
$V^{B_s\to\phi}$	$V(0)$	0.337	$^{+0.068}_{-0.055}$	$^{+0.0}_{-0.0}$	$^{+0.008}_{-0.008}$	$^{+0.004}_{-0.004}$	$^{+0.005}_{-0.005}$
PQCD	b_1^V	-9.181	$^{+0.178}_{-0.229}$	$^{+0.0}_{-0.0}$	$^{+0.087}_{-0.113}$	$^{+0.010}_{-0.006}$	$^{+0.034}_{-0.037}$
"PQCD+Lattice"		-5.144	$^{+1.463}_{-1.531}$	$^{+0.0}_{-0.0}$	$^{+0.289}_{-0.302}$	$^{+0.154}_{-0.156}$	$^{+0.169}_{-0.172}$
$A_0^{B_s\to\phi}$	$A_0(0)$	0.284	$^{+0.050}_{-0.042}$	$^{+0.010}_{-0.009}$	$^{+0.0}_{-0.0}$	$^{+0.002}_{-0.002}$	$^{+0.008}_{-0.008}$
	$b_1^{A_0}$	-8.123	$^{+0.218}_{-0.233}$	$^{+0.089}_{-0.060}$	$^{+0.0}_{-0.0}$	$^{+0.015}_{-0.00}$	$^{+0.00}_{-0.019}$
		-8.238	$^{+1.581}_{-1.671}$	$^{+0.494}_{-0.529}$	$^{+0.0}_{-0.0}$	$^{+0.116}_{-0.120}$	$^{+0.419}_{-0.439}$
$A_1^{B_s\to\phi}$	$A_1(0)$	0.267	$^{+0.057}_{-0.046}$	$^{+0.0}_{-0.0}$	$^{+0.006}_{-0.007}$	$^{+0.003}_{-0.004}$	$^{+0.003}_{-0.004}$
	$b_1^{A_1}$	-4.112	$^{+0.278}_{-0.235}$	$^{+0.0}_{-0.0}$	$^{+0.103}_{-0.062}$	$^{+0.031}_{-0.018}$	$^{+0.069}_{-0.044}$
		-1.832	$^{+1.235}_{-1.306}$	$^{+0.0}_{-0.0}$	$^{+0.198}_{-0.205}$	$^{+0.117}_{-0.119}$	$^{+0.114}_{-0.115}$
$A_2^{B_s\to\phi}$	$A_2(0)$	0.259	$^{+0.058}_{-0.045}$	$^{+0.004}_{-0.004}$	$^{+0.009}_{-0.008}$	$^{+0.004}_{-0.004}$	$^{+0.002}_{-0.001}$
	$b_1^{A_2}$	-8.556	$^{+0.402}_{-0.370}$	$^{+0.048}_{-0.098}$	$^{+0.067}_{-0.002}$	$^{+0.060}_{-0.081}$	$^{+0.202}_{-0.193}$
		-1.115	$^{+0.909}_{-0.860}$	$^{+0.139}_{-0.146}$	$^{+0.273}_{-0.086}$	$^{+0.129}_{-0.134}$	$^{+0.041}_{-0.039}$

$F_i^{B_s\to\phi}(q^2)$	$B(J^P)$	$m_{i,\text{pole}}^{b\to s}/\text{GeV}$
$A_0(q^2)$	0^-	5.366
$V(q^2),\ T_1(q^2)$	1^-	5.415
$A_{1,2}(q^2),\ T_{2,3}(q^2)$	1^+	5.829

TABLE VIII. The PQCD and "PQCD+Lattice" predictions for the form factors of $B_s \to \phi$ transitions.

Form Factors	Parameter	Central value	ω_{B_s}	$a_{2\phi}^{\parallel}$	$a_{2\phi}^{\perp}$	f_ϕ	f_ϕ^T
$T_1^{B_s\to\phi}$	$T_1(0)$	0.286	$^{+0.056}_{-0.046}$	$^{+0.0}_{-0.0}$	$^{+0.007}_{-0.007}$	$^{+0.004}_{-0.003}$	$^{+0.005}_{-0.005}$
PQCD	$b_1^{T_1}$	-8.659	$^{+0.317}_{-0.252}$	$^{+0.0}_{-0.0}$	$^{+0.102}_{-0.074}$	$^{+0.033}_{-0.016}$	$^{+0.073}_{-0.055}$
"PQCD+Lattice"		-4.520	$^{+1.388}_{-1.453}$	$^{+0.0}_{-0.0}$	$^{+0.276}_{-0.287}$	$^{+0.134}_{-0.136}$	$^{+0.193}_{-0.198}$
$T_2^{B_s\to\phi}$	$T_2(0)$	0.286	$^{+0.057}_{-0.046}$	$^{+0.0}_{-0.0}$	$^{+0.007}_{-0.007}$	$^{+0.004}_{-0.003}$	$^{+0.005}_{-0.004}$
	$b_1^{T_2}$	-2.874	$^{+0.226}_{-0.238}$	$^{+0.0}_{-0.0}$	$^{+0.060}_{-0.091}$	$^{+0.018}_{-0.003}$	$^{+0.051}_{-0.029}$
		-0.181	$^{+0.860}_{-0.550}$	$^{+0.0}_{-0.0}$	$^{+0.185}_{-0.008}$	$^{+0.278}_{-0.0}$	$^{+0.242}_{-0.0}$
$T_3^{B_s\to\phi}$	$T_3(0)$	0.212	$^{+0.045}_{-0.035}$	$^{+0.004}_{-0.004}$	$^{+0.008}_{-0.008}$	$^{+0.004}_{-0.003}$	$^{+0.002}_{-0.001}$
	$b_1^{T_3}$	-7.293	$^{+0.363}_{-0.376}$	$^{+0.120}_{-0.118}$	$^{+0.034}_{-0.009}$	$^{+0.072}_{-0.046}$	$^{+0.195}_{-0.168}$
		-6.307	$^{+1.076}_{-1.571}$	$^{+0.223}_{-0.231}$	$^{+0.485}_{-0.528}$	$^{+0.202}_{-0.207}$	$^{+0.092}_{-0.092}$

Numerical Results

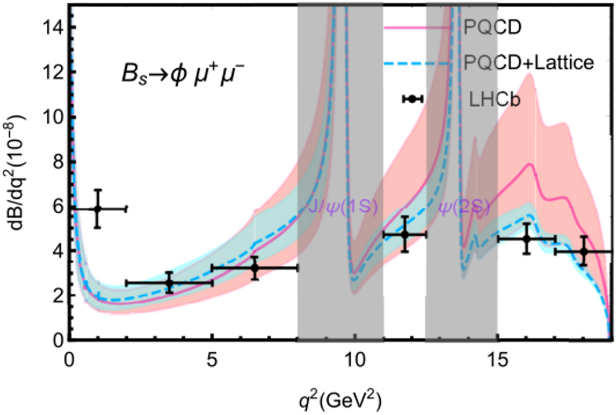


Figure: The theoretical prediction for the differential q^2 distributions of the semileptonic $B_s \to \phi\, \mu^+\mu^-$ decays compared with the LHCb data[1]

- $\mathcal{B}(B_s \to \phi\mu^+\mu^-)$ (in unit of 10^{-7})

1. **PQCD**:
   $\mathcal{B} = 8.41^{+2.00}_{-2.66}$
2. **PQCD+Lattice**:
   $\mathcal{B} = 7.44^{+1.65}_{-1.28}$
3. **LHCb**:
   $\mathcal{B} = 7.97^{+0.82}_{-0.80}$

Why f_{veto}?

The total branching fraction of the signal decay is given by the integral over the six q^2 bins. To account for the fraction of signal events in the vetoed q^2 regions, a correction factor $f_{\text{veto}} = 1.520 \pm 0.003 \pm 0.043$ is applied, which is determined using the calculation in

q^2 bin [GeV^2/c^4]	$N_{\phi\mu\mu}$	$\frac{\text{d}\mathcal{B}(B_s^0\to\phi\mu\mu)}{\mathcal{B}(B_s^0\to J/\psi\,\phi)\text{d}q^2}$ [$10^{-5}\,\text{GeV}^{-2}c^4$]	$\frac{\text{d}\mathcal{B}(B_s^0\to\phi\mu^+\mu^-)}{\text{d}q^2}$ [$10^{-8}\,\text{GeV}^{-2}c^4$]
$0.1 < q^2 < 2.0$	85^{+11}_{-10}	$5.44^{+0.68}_{-0.64} \pm 0.13$	$5.85^{+0.73}_{-0.69} \pm 0.14 \pm 0.44$
$2.0 < q^2 < 5.0$	60^{+10}_{-9}	$2.38^{+0.39}_{-0.37} \pm 0.06$	$2.56^{+0.42}_{-0.39} \pm 0.06 \pm 0.19$
$5.0 < q^2 < 8.0$	83^{+12}_{-11}	$2.98^{+0.41}_{-0.39} \pm 0.07$	$3.21^{+0.44}_{-0.42} \pm 0.08 \pm 0.24$
$11.0 < q^2 < 12.5$	70^{+10}_{-10}	$4.37^{+0.64}_{-0.61} \pm 0.14$	$4.71^{+0.69}_{-0.65} \pm 0.15 \pm 0.36$
$15.0 < q^2 < 17.0$	83^{+10}_{-10}	$4.20^{+0.53}_{-0.50} \pm 0.11$	$4.52^{+0.57}_{-0.54} \pm 0.12 \pm 0.34$
$17.0 < q^2 < 19.0$	54^{+8}_{-7}	$3.68^{+0.53}_{-0.50} \pm 0.13$	$3.96^{+0.57}_{-0.54} \pm 0.14 \pm 0.30$
$1.0 < q^2 < 6.0$	101^{+13}_{-12}	$2.40^{+0.30}_{-0.29} \pm 0.07$	$2.58^{+0.33}_{-0.31} \pm 0.08 \pm 0.19$
$15.0 < q^2 < 19.0$	136^{+13}_{-13}	$3.75^{+0.37}_{-0.35} \pm 0.12$	$4.04^{+0.39}_{-0.38} \pm 0.13 \pm 0.30$

Table 1. The signal yields for $B_s^0 \to \phi\mu^+\mu^-$ decays, as well as the differential branching fraction relative to the normalisation mode and the absolute differential branching fraction, in bins of q^2. The given uncertainties are (from left to right) statistical, systematic, and the uncertainty on the branching fraction of the normalisation mode.

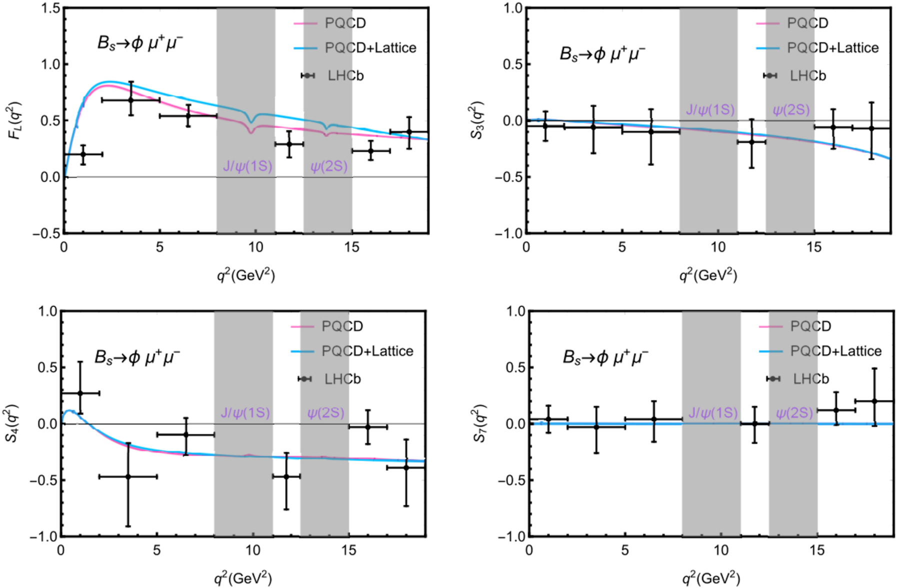
ref. [36] with updated form factors from ref. [37]. The first given uncertainty is statistical, the second is systematic.

The resulting relative and total branching fractions are

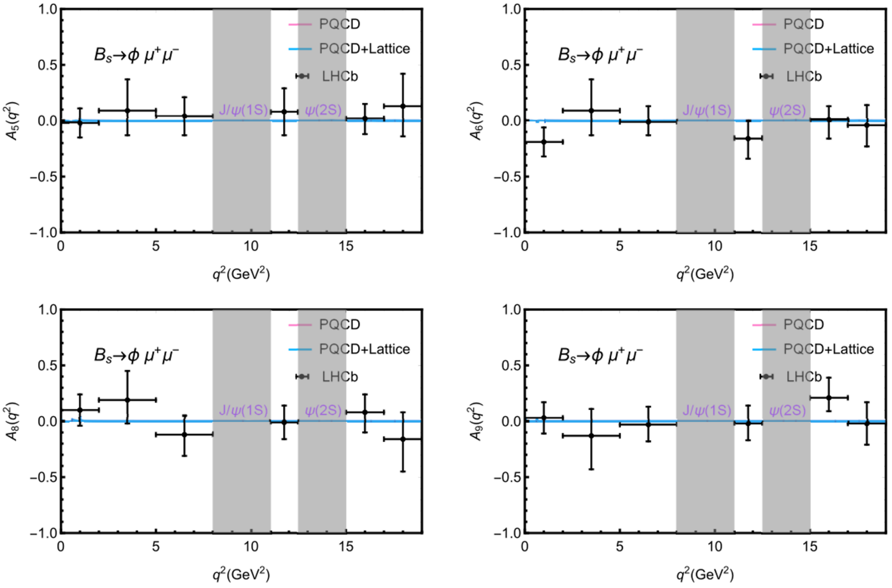
$$\frac{\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-)}{\mathcal{B}(B_s^0 \to J/\psi\,\phi)} = (7.41^{+0.42}_{-0.40} \pm 0.20 \pm 0.21) \times 10^{-4},$$

$$\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-) = (7.97^{+0.45}_{-0.43} \pm 0.22 \pm 0.23 \pm 0.60) \times 10^{-7},$$

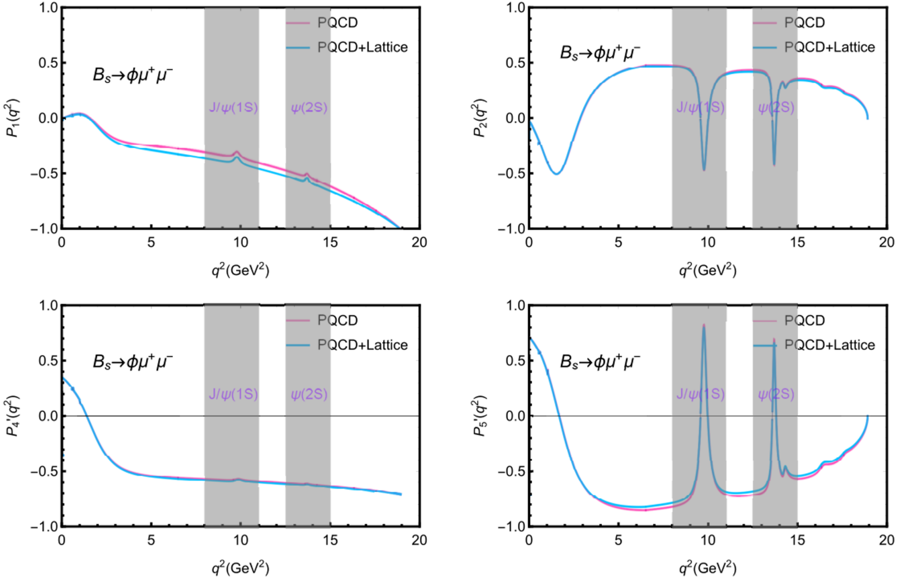
Numerical Results



Numerical Results



Numerical Results



Summary

Semileptonic decays $B_s \to \phi\ell^+\ell^-$

- A previous result of branching ratio $\mathcal{B}(B_s^0 \to \phi\mu^+\mu^-)$ from LHCb is consistent with our prediction both PQCD and "PQCD+Lattice" approaches within the errors.
- A decrease by about $(10-30)\%$ will be produced when the lattice QCD input for the form factors are taken into account in the extrapolation of the relevant form factors to higher q^2 region.
- The value of physical observables calculated in this work have no obvious deviations with other theories in SM frame.
- We are looking forward to improved measurements of various physical observables which will be useful to understand the pattern of deviations.

Thank You!