Study on pure annihilation type B $\to V\gamma$ decay

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Overview

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1. Motivation

The exclusive $B \to V\gamma$ are very improtant processes of heavy flavor physics, since they provide an excellent platform to constrain standard model parameters, to understand QCD factorization of the decay amplitudes and to test new physics models. Most $B \to V\gamma$ decays occur via the flavour-changing neutral-current (FCNC) transitions $b \to d\gamma$, like $B_d^0 \to \rho^0\gamma$ and $B_d^0 \to \omega\gamma$. For rare annihilation decay of $B_d^0 \to \phi\gamma$, there are some new physics modles like RPV to enhanced branching fraction. It is more profound to calculate this exclusive modes more precisely in standard modle, especially mixing enhancement.

pQCD of $B \to V\gamma$ Wei Wang, Run-Hui Li, Cai-Dian Lü hep-ph/0711.0432

SCET of $B \to V\gamma$ T. Becher, R. J. Hill and M. Neubert hep-ph/0503263

SCET of $B \to V\gamma$ Ahmed Ali, Ben D. Pecjak hep-ph/0709.4422

We estimate the relative size of $B \to \phi\gamma$ decays of different contributions. In the following table, we assumes the size of $B \to \rho/\omega\gamma$ decay amplitude to be unit.

Contributions	Suppression	Enhancement	total
Direct annihilation	$(\alpha_3 - 1/2\alpha_{3_{EW}})/C_{7\gamma} \times m_V/m_b$	-	0.004
EM penguin operator	$\alpha_{\rm em} \times a_\phi$	m_b/m_V	0.011
Mixing	0.061	-	0.061

Table 1: estimation of the relative size.

pQCD of $B_{Anni} \to \phi\gamma$ Ying Li, Cai-Dian Lü hep-ph/0605220

QCDF of $B_{Anni} \to \phi\gamma$ Xinqiang Li, Gongru Lu, Rumin Wang, Y.D. Yang hep-hp/0308303

pQCD of $B_{EM} \to \phi\gamma$ Cai-Dian Lü, Yue-Long, Shen and Wei Wang hep-ph/0606092

2. $B \to V\gamma$ decay in SCET

The effective weak Hamiltonian of the type $b \to d\gamma$ has the form :

$$\mathcal{H}_w = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{ps}^* V_{pb} [C_1 O_1^p + C_2 O_2^P + \sum_{i=3}^{8} C_i O_i]$$

$$O_1^p = \bar{s}\gamma^\mu(1-\gamma_5)p\bar{p}\gamma_\mu(1-\gamma_5)b \quad O_2^p = \bar{s}^i\gamma^\mu(1-\gamma_5)p^j\bar{p}^j\gamma_\mu(1-\gamma_5)b^i$$

$$O_3 = \bar{s}\gamma^\mu(1-\gamma_5)b\sum_q \bar{q}\gamma_\mu(1-\gamma_5)q \quad O_4 = \bar{s}^i\gamma^\mu(1-\gamma_5)b^j\sum_q \bar{q}^j\gamma_\mu(1-\gamma_5)q^i$$

$$O_5 = \bar{s}\gamma^\mu(1-\gamma_5)b\sum_q \bar{q}\gamma_\mu(1+\gamma_5)q \quad O_6 = \bar{s}^i\gamma^\mu(1-\gamma_5)b^j\sum_q \bar{q}^j\gamma_\mu(1+\gamma_5)q^i$$

$$O_{7\gamma} = -\frac{e}{8\pi^2}m_b\bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu} \quad O_{8g} = -\frac{g}{8\pi^2}m_b\bar{s}\sigma^{\mu\nu}(1+\gamma_5)T^a bG^a_{\mu\nu}$$

(1). Soft collinear effective theory

For $B \to V\gamma$ decays, it is convenient to work in the light-cone coordinate system

Collinear vector meson momentum

$$p = (n \cdot p, \bar{n} \cdot p, p_\perp) \sim (\lambda^2, 1, \lambda) m_b$$

Anti-collinear photon momentum

$$q = (\bar{n} \cdot q, n \cdot q, q_\perp) \sim (1, \lambda^2, \lambda) m_b$$

We consider the collinear part of the fermion field and we further spilt it into two components as follows

$$\psi_c \equiv \xi + \eta$$

$$\xi = \frac{\not{n}\not{\bar{n}}}{4}\psi_c \sim \lambda, \quad \eta = \frac{\not{\bar{n}}\not{n}}{4}\psi_c \sim \lambda^2$$

QCD $\to$ SCET$_{\mathrm{I}}$

In SCET, the matrix element of $\langle V\gamma|\mathcal{H}_{eff}|B\rangle$ can be evaluated by a two-step matching process QCD$\to$SCET$_{\mathrm{I}}$$\to$SCET$_{\mathrm{II}}$.

$$\begin{aligned}
\mathcal{H}^{eff}_{current} &= \left[\bar{\psi}(x)\Gamma Q(x)\right]_{\mathrm{QCD}} \\
&= \int ds \int da \tilde{C}^A(s,a) J^A(s,a) \\
&+ \sum_{j=1,2} \int ds \int dr \int da \tilde{C}^B_j J^B_j(s,r,a) \\
&+ \int ds \int dr \int da \tilde{C}^C(s,r,a) J^C(s,r,a) + \ldots
\end{aligned}$$

For $B \to V\gamma$, step-one matching result takes the form:

$$\mathcal{H}^{eff}_{current} \to \Delta C^A J^A + \Delta C^{B1} \otimes J^{B1} + \Delta C^{B2} \otimes J^{B2} + \Delta C^C \otimes J^C$$

T. Becher, R. J. Hill and M. Neubert hep-ph/0503263

In the first step, we integrate the hard mode and reach to $\mathrm{SCET_I}$.

$$J^A = (\bar{\xi}W)_{hc} \not{\mathcal{A}}_{\perp}^{em} (1-\gamma_5) h_v$$

$$J^{B1} = (\bar{\xi}W)_{hc} \not{\mathcal{A}}_{\perp}^{em} \not{\mathcal{A}}_{\perp hc} (1+\gamma_5) h_v$$

$$J^{B2} = (\bar{\xi}W)_{hc} \not{\mathcal{A}}_{\perp hc} \not{\mathcal{A}}_{\perp}^{em} (1+\gamma_5) h_v$$

$$J^C = (\bar{\xi}W)_{hc} (1+\gamma_5) \frac{\not{\bar{n}}}{2} (\xi W^{\dagger})_{hc} (\bar{\xi}W)_{hc} (1+\gamma_5) \frac{\not{\bar{n}}}{2} h_v$$

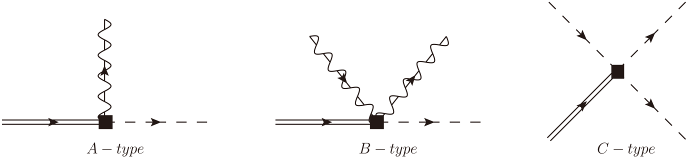

Figure 1: $\mathrm{SCET_I}$ current

A-type current

There are some singularities of A-type operator. Soft-form factor $\zeta_{V_\perp}$ will be defined to absored end-point singularities.

$$\langle V\gamma| C^A J^A |B\rangle = \langle V\gamma| C^A (\bar{\xi} W)_{hc} \not{A}_\perp^{em} (1-\gamma_5) h_v |B\rangle \equiv C^A \zeta_{V_\perp}$$

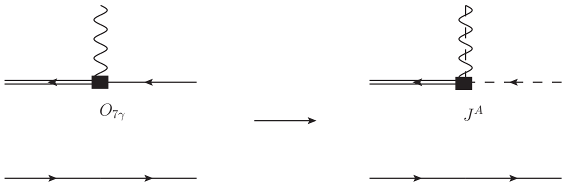

Figure 2: $\mathrm{SCET_I}$ current

Jing Gao, Cai-Dian Lü, Yue-Long Shen, Yu-Ming Wang, Yan-Bing Wei hep-hp/1907.11092

$\text{SCET}_{\text{I}} \to \text{SCET}_{\text{II}}$
B-type current

In the second step, the hard-collinear modes are integrated out, and only soft and collinear fields left.

$$\int dx \left\langle \mathcal{L}_{\xi q}^{(1)}(x),\, J^{B1}(0) \right\rangle = \int dt \tilde{J}_\perp(s,t) O^{B1}(s,t)$$

where

$$O^{B1}(s,t) = \left[\bar{\chi}(s\bar{n})(1+\gamma_5)\not{\mathcal{A}}_{\bar{c}}^{em}\frac{\not{\bar{n}}}{2}\chi_c(0)\right]\left[\bar{\mathcal{Q}}_s(tn)(1+\gamma_5)\frac{\not{n}}{2}\mathcal{H}_s(0)\right]$$

At tree level, jet function $J_\perp$ can be written as:

$$J_\perp(u,v) = -\frac{4\pi C_F \alpha_s}{N}\frac{1}{2E\bar{u}}\delta(u-v)$$

The matrix elements of O_1^B can then be written convergen convolution interals over the mesons LCDAS:

$$\langle V(p)|\,\bar{\chi}_c(s\bar{n})\Gamma\frac{\not{\bar{n}}}{2}\chi_c(0)\,|0\rangle = \frac{if_V(\mu)}{4}\bar{n}\cdot p\ \mathrm{tr}\left[\frac{\not{n}\not{\bar{n}}}{2}\not{\epsilon}^*\Gamma\frac{\not{\bar{n}}}{2}\right]\int_0^1 d\omega e^{ius\bar{n}\cdot p}\phi_V(u,\mu)$$

$$\langle 0|\,\bar{\mathcal{Q}}_s(tn)\frac{\not{n}}{2}\Gamma\mathcal{H}_s(0)\,|B_v\rangle = \frac{F(\mu)}{2}\sqrt{m_B}\ \mathrm{tr}\left[\frac{\not{n}}{2}\Gamma\frac{1+\not{v}}{2}\gamma_5\right]\int_0^\infty d\omega e^{-i\omega tn\cdot v}\phi_B(\omega,\mu)$$

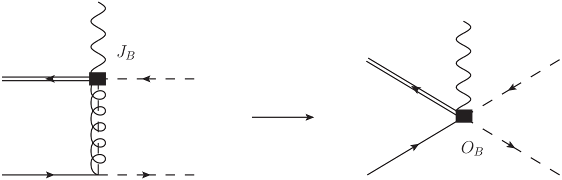

Figure 3: $\mathrm{SCET_I}$ to $\mathrm{SCET_{II}}$ matching

M. Beneke and D.Yang hep-hp/0508250

(2). Factorization of $B \to V\gamma$

At leading power, the decay amplitude can be factorized to the form:

$$\langle V\gamma|\mathcal{H}_W|B_v\rangle = 2m_B C^A(\mu)\zeta_{V_\perp}(\frac{m_B}{2},\mu) + \frac{m_B^{3/2}F(\mu)}{2}\int_0^\infty \frac{d\omega}{\omega}\phi_B(\omega,\mu) \times \int_0^1 du f_{V_\perp}(\mu)\phi_{V_\perp}(u,\mu)\int_0^1 dv J_\perp(u,v,\ln\frac{m_B\omega}{2},\mu)C_1^B(v,\mu)$$

$$\langle V\gamma|\mathcal{H}_W|B_v\rangle|_{\mathrm{LP}} \equiv 2m_B\left[C^A\zeta_{V_\perp} + \frac{\sqrt{m_B F}}{4}\phi_B \otimes f_{V_\perp}\phi_{V_\perp} \otimes J_\perp^B \otimes C^{B1}\right]$$

Using these results, we can calculate the branching fraction of $B \to \rho\gamma$ and $B \to \omega\gamma$ decays at leading power in SCET.

3. $B^0 \to \phi\gamma$
(1). Weak Annihilation processes

Weak annihilation process of $B \to \phi\gamma$ is a pure annihilation process. QCD$\to$SCET$_\text{I}$ matching process can be decuded as follow:

$$\begin{aligned} H_{\text{Anni}} &= C_{\text{Anni}}[\bar{\xi}\gamma_\mu(1-\gamma_5)h_v][\bar{\xi}(1+\gamma_5)\gamma^\mu\eta + h.c.] \\ &\equiv C_{\text{Anni}} \cdot J_{\text{Anni}} \end{aligned}$$

where C_{Anni} is the hard cofficient function.

$$\begin{aligned} C_{\text{Anni}} = 1 - \frac{\alpha_s C_F}{4\pi}\bigg[& 2\ln^2\frac{\mu}{n\cdot p} + 5\ln\frac{\mu}{m_b} - 2\text{Li}_2\Big(1-\frac{1}{r}\Big) \\ & - \ln^2 r + \frac{3r-2}{1-r}\ln r + \frac{\pi^2}{12} + 6\bigg] \end{aligned}$$

Through the definition of the LCDA of tansversely polized vector meson and the B$\to \gamma$ form factor:

$$\langle \gamma(p,\varepsilon)| \, C_{Anni}\bar{\xi}\gamma^{\mu}(1-\gamma_5)h_v \, |B_v\rangle = \quad v\cdot p\,\varepsilon^{*}_{\nu}(g_{\perp}^{\mu\nu}F_A + i\epsilon_{\perp}^{\mu\nu}F_V)$$

where

$$F_{V,LP}(n\cdot p) = F_{A,LP}(n\cdot p)$$
$$= \frac{Q_u m_B}{n\cdot p}\tilde{f}_B(\mu) C_{Anni}\int_0^{\infty} d\omega \frac{\phi_B^{+}(\omega,\mu)}{\omega} J_{\perp}(n\cdot p,\omega,\mu)$$

and vector meson LCDA

$$\langle V(p,\varepsilon)| \, [\bar{\chi}_c\gamma_{\perp}^{\mu}\frac{\not{\bar{n}}}{2}\eta_c + h.c.] \, |0\rangle = f_V m_V \varepsilon_{\perp}^{*\mu}\int du\, e^{ius\bar{n}\cdot p} g_{\perp}^{(v)}(u)$$

$$\langle V(p,\varepsilon)| \, [\bar{\chi}_c\gamma_{\perp}^{\mu}\gamma_5\frac{\not{\bar{n}}}{2}\eta_c + h.c.] \, |0\rangle = \frac{i}{4} f_V m_V \epsilon_{\perp}^{\mu\nu}\varepsilon^{*}_{\perp\nu}\int du\, e^{ius\bar{n}\cdot p} g_{\perp}^{(a)}(u)$$

Yu-Ming Wang hep-hp/1606.03086

The transition amplitude $B \to \phi\gamma$ is then written by

$$\langle \phi(\epsilon_1)\gamma(\epsilon_2)| \mathcal{H}_{\text{eff}} |B_v\rangle_{\text{Anni}}$$
$$= -\frac{G_F}{\sqrt{2}} V_{td} V_{td}^* \left(\alpha_3 - \frac{1}{2}\alpha_{3_{\text{EW}}}\right) f_\phi m_\phi F_V (g_\perp^{\mu\nu} + i\epsilon_\perp^{\mu\nu})\varepsilon_{\mu 1}^* \varepsilon_{\nu 2}^*$$

where

$$\alpha_3 = C_3 + \frac{C_4}{N_c} + C_5 + \frac{C_6}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\frac{f_{\phi_\perp}}{f_\phi}(C_4 V_1 + C_6 V_2)$$
$$\alpha_{3_{EW}} = C_7 + \frac{C_8}{N_c} + C_9 + \frac{C_{10}}{N_C} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_C}\frac{f_{\phi_\perp}}{f_\phi}(C_8 V_1 + C_{10} V_2)$$

(2). Electro-magnetic penguin operator

An important enhanced contribution of the $B \to \phi\gamma$ decay is from Electro-magnetic penguin operator. In a more intutive word, the transverse polarized vector meson can be generated by one of the photons in the $B \to \gamma\gamma$ decay.

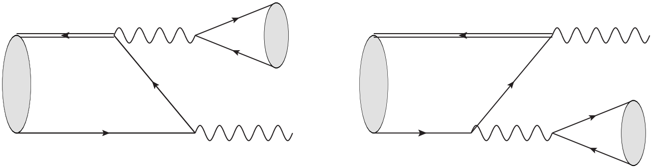

Figure 4: Production of a vector meson via electromagnetic penguin operator

When the photon field is sandwiched by the vector meson state and the vacuum, the matrix element reads

$$\left\langle V(p,\varepsilon)\left|eA^{em}_{\perp\mu}\right|0\right\rangle = \left\langle V(p,\varepsilon)\left|eW^{+}iD_{\mu\perp}W\right|0\right\rangle = -\frac{2}{3}ia_V\frac{e^2}{m_V}\varepsilon^*_{\perp\mu}$$

with $a_\rho = 3/2, a_\omega = 2/3, a_\phi = -1/3$. The amplitude of $B \to \gamma\gamma$ decay is required.

$$\langle\gamma(k1,\varepsilon_1^*)\gamma(k2,\varepsilon_2^*)\,|H_{eff}|\,B\rangle = \left(4\frac{G_F}{\sqrt{2}}\right)\left(-i\frac{\alpha_{em}}{\pi}m_B^2 f_B\right)\varepsilon_1^{*\mu}\varepsilon_2^{*\nu}(g_{\perp\mu\nu}G_A - i\epsilon_{\perp\mu\nu}G_V)$$

Finally, we can get the factorized result.

$$\left\langle\phi(k_1,\varepsilon^*_{\phi1})\gamma(k_2,\varepsilon_2^*)\,|H_{eff}|\,B\right\rangle = -\left(4\frac{G_F}{\sqrt{2}}\right)\left(-i\frac{\alpha_{em}}{\pi}m_B^2 f_B\right)\left(\frac{2a_\phi e^2 f_\phi}{3m_\phi}\right) \times \varepsilon^{*\mu}_{\gamma1}\varepsilon^{*\nu}_{\phi2}(g_{\perp\mu\nu}G_A - i\epsilon_{\perp\mu\nu}G_V)$$

M. Beneke, J. Rohrer and D. Yang hep-ph/0512258

Numerical result

Compared with previous work and PDG, we can list the branching fraction as follow.

	SCET	Previous work\PDG
$B^0 \to \rho^0\gamma$	9.9×10^{-7}	$8.6 \pm 1.5 \times 10^{-7}$
$B^0 \to \omega\gamma$	9.8×10^{-7}	$4.4^{+1.8}_{-1.6} \times 10^{-7}$
$B^0_{Em} \to \phi\gamma$	1.3×10^{-11}	1×10^{-11}
$B^0_{Anni} \to \phi\gamma$	1.8×10^{-12}	3.6×10^{-12}

Table 2: Branching fraction of $B \to V\gamma$ decay

4. Mixing of neutral vector meson

Under SU(3) symmetry, vector mesones octect are express as follow:

$$M = \begin{pmatrix} \frac{2\mathrm{u}\bar{\mathrm{u}} - \mathrm{d}\bar{\mathrm{d}} - \mathrm{s}\bar{\mathrm{s}}}{3} & \mathrm{u}\bar{\mathrm{d}} & \mathrm{u}\bar{\mathrm{s}} \\ \mathrm{d}\bar{\mathrm{u}} & \frac{2\mathrm{d}\bar{\mathrm{d}} - \mathrm{u}\bar{\mathrm{u}} - \mathrm{s}\bar{\mathrm{s}}}{3} & \mathrm{d}\bar{\mathrm{s}} \\ \mathrm{s}\bar{\mathrm{u}} & \mathrm{s}\bar{\mathrm{d}} & \frac{2\mathrm{s}\bar{\mathrm{s}} - \mathrm{u}\bar{\mathrm{u}} - \mathrm{d}\bar{\mathrm{d}}}{3} \end{pmatrix}$$

$$M = \begin{pmatrix} \frac{\omega_8}{\sqrt{6}} - \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ -\rho^- & \frac{\omega_8}{\sqrt{6}} + \frac{\rho^0}{\sqrt{2}} & K^{*0} \\ -K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} \end{pmatrix} \qquad \omega_1 = \frac{\mathrm{u}\bar{\mathrm{u}} + \mathrm{d}\bar{\mathrm{d}} + \mathrm{s}\bar{\mathrm{s}}}{\sqrt{3}}$$

SU(3) symmetry is violated slightly, which will induce mixing effect.

$$\phi = \omega_8 \cos\theta - \omega_1 \sin\theta$$
$$\omega = \omega_8 \sin\theta + \omega_1 \cos\theta$$

For ideal mixing, mixing angle is chosed $\theta = 35^\circ$, which means

$$\phi = \frac{\mathsf{u\bar{u} + dd - 2s\bar{s}}}{\sqrt{6}} \cdot \sqrt{\frac{2}{3}} - \frac{\mathsf{u\bar{u} + d\bar{d} + s\bar{s}}}{\sqrt{3}} \cdot \sqrt{\frac{1}{3}} = -\mathsf{s\bar{s}}$$
$$\omega = \frac{\mathsf{u\bar{u} + dd - 2s\bar{s}}}{\sqrt{6}} \cdot \sqrt{\frac{1}{3}} + \frac{\mathsf{u\bar{u} + d\bar{d} + s\bar{s}}}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}} = \frac{\mathsf{u\bar{u} + d\bar{d}}}{\sqrt{2}}$$

From the experiment results of PDG, $\theta = 38.7^\circ$ or $\theta = 36.0^\circ$.

The states of definite isospin are given by

$$|\rho_I^0\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle - |\bar{d}d\rangle) \qquad |\omega_I\rangle = \frac{1}{\sqrt{2}}(|\bar{u}u\rangle + |\bar{d}d\rangle) \qquad |\phi_I\rangle = |\bar{s}s\rangle$$

where ρ has isospin 1 and ω and ϕ have isospin 0.The physical states are different from the ideal state by the following mixing matrix.

$$\begin{pmatrix} |\omega\rangle \\ |\rho\rangle \\ |\phi\rangle \end{pmatrix} = \begin{pmatrix} 1 & -0.0054 + 0.029i & -0.061 \\ 0.0054 - 0.029i & 1 & 0.00057 + 0.00116i \\ 0.061 & -0.00057 - 0.00116i & 1 \end{pmatrix} \begin{pmatrix} |\omega_I\rangle \\ |\rho_I\rangle \\ |\phi_I\rangle \end{pmatrix}$$

The mixing of $|\phi\rangle$ state

$$|\phi\rangle = |\phi_I\rangle + 0.061|\omega_I\rangle - (0.00057 + 0.00116i)|\rho_I\rangle$$

M. Benayoun and H.B. O'Connell hep-ph/0107047

$\rho - \omega - \phi$ mixing amplitude can be written as

$$\mathcal{A}(B \to \phi\gamma)_{mixing} = \mathcal{A}(B \to \phi_I\gamma) + 0.061\mathcal{A}(B \to \omega_I\gamma) - (0.00057 + 0.00116i)\mathcal{A}(B \to \rho_I\gamma)$$

After the mixing effect, the $B \to \phi\gamma$ decays can be expressed in terms of the decay amplitude with the ideal mixing meson final state.

$$\mathrm{Br}(B \to \phi\gamma)_{\mathrm{mixing}} = \tau_B \frac{m_B}{4\pi}\left(1 - \frac{m_\phi^2}{m_B^2}\right)|\mathcal{A}(B \to \phi\gamma)_{\mathrm{mixing}}|^2$$

Finally, we can get the branching fraction of $B^0 \to \phi\gamma$ decay, which is enforced by ρ-ω-ϕ mixing:

$$\mathrm{Br}(B \to \phi\gamma)_{\mathrm{mixing}} = 3.8 \times 10^{-9}$$

5. Conclusion

- In SCET, we have calculated radiative decay $B \to \rho(\omega)\gamma$ and pure penguin radiative annihilation processes $B \to \phi\gamma$.
- We estimate $\text{Br}(B \to \phi\gamma) = 1.5 \times 10^{-11}$ in standard modle.
- We also find that the decay of $B \to \rho(\omega)\gamma$ is very sensitive of $\rho - \omega - \phi$ mixing. After $\rho - \omega - \phi$ mixing, it is found that $\text{Br}(B \to \phi\gamma)$ could be enhanced to order of 10^{-9}.
- If $B \to \phi\gamma$ was found in Belle II, it maybe a mixing enhanced rather than new physics.

Thank You!