B-meson DA. Introduction

Grozin Neubert hep-ph/9607366 ⊖ 95

KKQT hep-ph/0109181

Lange Neubert hep-ph/0303082

Cheng & Li PRD 72, 014003 (2005)

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<u>Evolution equation</u>

1 Composite 人 renormalization

2. B-meson LCDA evolution equation

3. Solution

① $S^{(0)} = \bar\psi^{(0)}\psi^{(0)}(x)$ $\psi^{(0)} = Z_\psi^{1/2}\psi$

$$\underline{S} = \frac{S^{(0)}}{Z_S} = \frac{\bar\psi^{(0)}\psi^{(0)}}{Z_S} = \frac{Z_\psi}{Z_S}\bar\psi\psi \qquad \boxed{S \neq \bar\psi\psi}$$

$\langle \underline{S} \rangle$ 有限值

$$= \left\langle \frac{Z_\psi}{Z_S}\bar\psi\psi \right\rangle = \frac{Z_\psi}{Z_S}\langle\bar\psi\psi\rangle = \langle\bar\psi\psi\rangle + \underbrace{\overbrace{\left(\frac{Z_\psi}{Z_S}-1\right)}^{\alpha_s}}_{1+\alpha_s}\langle\bar\psi\psi\rangle$$

$\langle\bar\psi\psi\rangle = \langle q|\bar\psi\psi|q\rangle$

 +  + $\left(\frac{Z_\psi}{Z_S}-1\right)$  ⇒ 有限值

$\bar u u \quad + \quad \bar u \delta\Gamma u \quad + \quad \left(\frac{Z_\psi}{Z_S}-1\right)\bar u u \Rightarrow$ 有限

$\alpha_s^0 \qquad \alpha_s^1 \qquad \alpha_s^1$

 $\Rightarrow \mu^{2\varepsilon}(ig_s)^2 C_F \int \frac{d^dq}{(2\pi)^d}\gamma^\alpha \frac{i\not q}{q^2}\frac{i\not q}{q^2}\gamma^\beta \frac{(-i)}{q^2} g_{\alpha\beta} = -4ig_s^2 C_F \int \frac{d^dq}{(2\pi)^d}\frac{1}{(q^2)^2}$

$$= \frac{8 g_s^2 C_F}{32\pi^2 \varepsilon}$$

$$Z_s = 1 + 6\,\frac{g_s^2 C_F}{32\pi^2 \varepsilon}$$

$$\frac{dO}{d\ln\mu} = \gamma_0\, O \Rightarrow \quad \gamma_0 = \mu\,\frac{\ln Z_s}{d\mu} = -\frac{6 g_s^2 C_F}{32\pi^2}$$

② . evolution equation: (BL)

π介子:

$$\frac{d\,\Phi_\pi(x)}{d\ln\mu} = \frac{\alpha_s}{\pi} C_F \int_0^1 dy\; V_0(x,y)\, \Phi(y,\mu)$$

$$V_0(x,y) = \left[\frac{1-x}{1+y}\left(1+\frac{1}{x-y}\right)\theta(x-y) + \frac{x}{y}\left(1+\frac{1}{y-x}\right)\theta(y-x)\right]_+$$

$$\frac{d}{d\ln\mu}\,\phi_B^+(\omega) = -\int_0^\infty d\omega'\; \gamma_+(\omega,\omega',\mu)\, \phi_+^B(\omega',\mu)$$

$$\to \gamma_+(\omega,\omega',\mu) = \left[\Gamma_{cusp} \ln\frac{\mu}{\omega} + \gamma\right]\delta(\omega-\omega') + \omega\,\Gamma(\omega,\omega',\mu)$$

$$\Gamma_{cusp}^{(1)} = 4. \qquad \gamma^{(1)} = -2$$

$$\Gamma^{(1)}(\omega,\omega') = -\Gamma_{cusp}^{(1)}\left[\frac{\theta(\omega-\omega')}{\omega'(\omega'-\omega)} + \frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+$$

重夸克场 h_v

$$\frac{1}{v\cdot q}$$

$$\frac{1}{n_+ q}$$

$$\langle\Phi\rangle^{(a)} = -i g_s^2 C_F\, \delta(\omega-\omega_0)\, \mu^{2\varepsilon} \int \frac{d^d q}{(2\pi)^d}\; \frac{n_+ v}{n_+ q}\, \frac{1}{v\cdot q}\, \frac{1}{q^2}\; \frac{\bar{v}(\omega_0)\, \not{n}_+ \gamma_5\, u_b(p_b)}{B}$$

$$\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}$$

$\langle \bar{q}_{2\pi}^{(a)} |$

$$\int \frac{d^dq}{(2\pi)^d} \frac{(\not p+\not q)}{\underline{(p+q)^2}} \frac{1}{n_+ q} \frac{1}{q^2} \left(- - - \right) \leftarrow \pi$$

$$\rightarrow \left(\frac{1}{\varepsilon}\right)$$

$$\langle 0 | \underline{\bar q^{\beta}(z)} [z,0] \underline{h_v^{\alpha}(0)} | \bar B(v)\rangle = -\frac{i \tilde f_B m_B}{4}\left[\frac{1+\not v}{2}\left\{2\tilde\Phi_B^+(t,z^2) - \frac{\tilde\Phi_B^- - \tilde\Phi_B^+}{t}\not z\right\}\gamma_5\right]^{\alpha\beta}$$

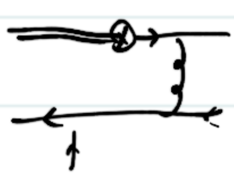
$t = v\cdot z$ ↓

light-like $\underline{z^2 \to 0}$

$\underline{n_+ . n_-}$

$$\doteq \left\{ \underline{\Phi_B^+} \not n_+ + \tilde\Phi_B^- \not n_- + \cdots \right\}^{\alpha\beta}$$

↑ 大分量



$\not p\, u(p) = m\, u(p) = 0$

$$\boxed{\not n_+ u(p) = 0}$$

$v^2 = 1$. $v^\mu = (1, 0, 0, 0)$

③ solution:

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{cusp} \ln\frac{\mu}{\omega} + \gamma\right]\phi_B^+(\omega,\mu) - \omega\int_0^\infty d\eta\, \Gamma_+(\omega,\eta,\alpha_s)\, \phi_B^+(\eta,\mu)$$

Neubert:

$$\varphi_B^+(\theta,\mu) = \int_0^\infty \frac{d\omega}{\omega}\left(\frac{\omega}{\mu}\right)^{-i\theta} \phi_B^+(\omega,\mu)$$

↓

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$$\underline{\phi_B^+(\omega)} = \int_0^\infty \frac{d\omega'}{\omega'}\sqrt{\frac{\omega}{\omega'}}\; \underline{J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)}\; \rho_B^+(\omega')$$

$$\underline{\rho_B^+(\omega')} = \int_0^\infty \frac{d\omega}{\omega}\sqrt{\frac{\omega}{\omega'}}\; J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \phi_B^+(\omega)$$

$$\frac{d\rho_B^+(\omega')}{d\ln\mu} = -\left[\underline{\Gamma_{cusp} \ln\frac{\mu}{\omega} + \gamma}\right] \rho_B^+(\omega') \Rightarrow$$

$$\varphi_B^+(\omega',\mu) = U_{\omega'}(\mu,\mu_0)\;\varphi_B^+(\omega',\mu_0)$$