High twist B meson LCDAs

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PQCD group meeting 2020

Definition of B meson LCDA

- two-particle B-meson LCDA:

$$\langle 0|\bar{q}^{\beta}(z)[z,0]h_v^{\alpha}(0)|\bar{B}(v)\rangle$$
$$= -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{\tilde{\Phi}^+(t,z^2)+\frac{\tilde{\Phi}^-(t,z^2)-\tilde{\Phi}^+(t,z^2)}{t}\not{z}\right\}\right]^{\alpha\beta}.$$

- light-cone: $z^2 \to 0; z = z^+ n, z^+ = \bar{n}\cdot z/2; t = v\cdot z = z^+$
- momentum space

$$\phi^{\pm}(\omega) = \int \frac{dt}{2\pi} e^{i\omega t}\tilde{\Phi}^{\pm}(t)\,. \quad (3)$$

- leading twist: $\phi^+(\omega)$

$$iF(\mu)\tilde{\Phi}_+(z,\mu) = \langle 0|\bar{q}(tn)\not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle,$$

- subleading twist $\phi^-(\omega)$

$$iF(\mu)\tilde{\Phi}_-(z,\mu) = \langle 0|\bar{q}(tn)\not{\bar{n}}\gamma_5 h_v(0)|\bar{B}(v)\rangle$$

Definition of twist

- definition of collinear twist:
$$t = d - s$$
- d: canonical dimension
$$d[\psi] = 3/2,\ d[A] = 1,\ d[G_{\mu\nu}] = 2$$
- s: projection of the spin along light-cone
- the spin operator
$$\Sigma_{\mu\nu}\phi = 0,\ \Sigma_{\mu\nu}\psi = \frac{i}{2}\sigma_{\mu\nu}\psi,\ \Sigma_{\mu\nu}G_{\alpha\beta} = g_{\nu\alpha}G_{\mu\beta} - g_{\mu\alpha}G_{\nu\beta} + g_{\mu\beta}G_{\nu\alpha} - g_{\nu\beta}G_{\mu\alpha}$$
- light-cone projection
$$\Sigma_{+-} = \frac{1}{2}\Sigma_{\mu\nu}n^{\mu}\bar{n}^{\nu}$$
$$\Sigma_{+-}\psi = -\frac{1}{8}(\not{n}\ \not{\bar{n}} - \not{\bar{n}}\ \not{n})\psi \equiv \frac{1}{2}(P_+ - P_-)\psi;$$
$$P_+^2 = P_+ = \frac{\not{\bar{n}}\ \not{n}}{4}, P_-^2 = P_- = \frac{\not{n}\ \not{\bar{n}}}{4}, P_+ + P_- = 1$$

twist of a field

- quark field:

$$\psi = P_+\psi + P_-\psi \equiv \psi_+ + \psi_-$$

$$\Sigma_{+-}\psi_+ = \frac{1}{2}\psi_+(s = \frac{1}{2}); \;\; \Sigma_{+-}\psi_- = -\frac{1}{2}\psi_-(s = -\frac{1}{2})$$

$$t[\psi_+] = \frac{3}{2} - \frac{1}{2} = 1; \;\; t[\psi_-] = \frac{3}{2} - (-\frac{1}{2}) = 2;$$

- gluon field

$$\Sigma_{+-}G_{+\perp} = g_{-+}G_{+\perp} - g_{++}G_{-\perp} + g_{-\perp}G_{++} - g_{+\perp}G_{-+} = G_{+\perp}$$

$$T_{+-} = \frac{1}{2}T_{\mu\nu}n^\mu \bar{n}^\nu, \;\; g_{-+} = g_{+-} = 1$$

$$\Sigma_{+-}G_{+-} = 0, \;\; \Sigma_{+-}G_{\perp\perp} = 0, \;\; \Sigma_{+-}G_{-\perp} = -G_{-\perp}$$

$$t[G_{+\perp}] = 2 - 1 = 1; t[G_{+-}] = t[G_{\perp\perp}] = 2 - 0 = 2; t[G_{-\perp}] = 2 - (-1) = 3$$

Example: Pion LCDAs

- leading twist: twist-2

$$if_\pi \bar{n}\cdot p\int du e^{\frac{i}{2}un\cdot z\bar{n}\cdot p}\phi_\pi(u) = \langle 0|\bar{q}_+(nz)\not{n}\gamma_5 q_+(0)|\pi(p)\rangle,$$

- twist-3

$$if_\pi r_\chi^\pi(\mu)\int du e^{\frac{i}{2}un\cdot z\bar{n}\cdot p}\phi_{+-}(u) = 2\langle 0|\bar{q}_+(nz)i\gamma_5 q_-(0)|\pi(p)\rangle,$$

$$if_\pi r_\chi^\pi(\mu)\int du e^{\frac{i}{2}un\cdot z\bar{n}\cdot p}\phi_{-+}(u) = 2\langle 0|\bar{q}_-(nz)i\gamma_5 q_+(0)|\pi(p)\rangle,$$

$$\phi_p(u) = \frac{1}{2}[\phi_{+-}(u)+\phi_{-+}(u)];\ \ \phi'_\sigma(u) = \frac{1}{6}[\phi_{+-}(u)-\phi_{-+}(u)]$$

$$\bar{q}(nz)i\gamma_5 q(0) = \bar{q}_+(nz)i\gamma_5 q_-(0)+\bar{q}_-(nz)i\gamma_5 q_+(0)$$

$$if_\pi r_\chi^\pi(\mu)\int du e^{\frac{i}{2}un\cdot z\bar{n}\cdot p}\phi_p(u) = \langle 0|\bar{q}(nz)i\gamma_5 q(0)|\pi(p)\rangle,$$

High twist B meson LCDA

- twist of heavy quark field:

$$\not{v} h_v = h_v, t[h_v] = 1$$

- leading twist: $\phi^+(\omega)$

  $iF(\mu)\tilde{\Phi}_+(z,\mu) = \langle 0|\bar{q}(tn)_+ \not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle,$

- next-to-leading twist $\phi^-(\omega)$

  $iF(\mu)\tilde{\Phi}_-(z,\mu) = \langle 0|\bar{q}(tn)_- \not{\bar{n}}\gamma_5 h_v(0)|\bar{B}(v)\rangle$

- obtain high twist LCDAs:
  - replace q_+ by q_-, or add more quark fields or gluon fields
  - expansion by z^2

High twist three-particle B meson LCDA

- covariant definition:

$$
\begin{aligned}
&\langle 0|\bar{q}(nz_1)gG_{\mu\nu}(nz_2)\Gamma h_v(0)|\bar{B}(v)\rangle \\
&= \frac{1}{2}F_B(\mu)Tr\Big\{\gamma_5\Gamma\frac{1+\not v}{2}\Big[(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\big[\Psi_A - \Psi_V\big] - i\sigma_{\mu\nu}\Psi_V \\
&\qquad - (n_\mu v_\nu - n_\nu v_\mu)X_A + (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\big[W + Y_A\big] \\
&\qquad - i\epsilon_{\mu\nu\alpha\beta}n^\alpha v^\beta\gamma_5\widetilde{X}_A + i\epsilon_{\mu\nu\alpha\beta}n^\alpha\gamma^\beta\gamma_5\widetilde{Y}_A \\
&\qquad - (n_\mu v_\nu - n_\nu v_\mu)\not n W + (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\not n Z\Big]\Big\}(z_1, z_2;\mu)\,.
\end{aligned}
$$

- The momentum space distributions are defined as

$$
\Psi_A(z_1, z_2) = \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \; e^{-i\omega_1 z_1 - i\omega_2 z_2}\,\psi_A(\omega_1, \omega_2) \qquad (15)
$$

- contracted with the light-like vector

$$
\begin{aligned}
&\langle 0|\bar{q}(z_1 n)gG_{\mu\nu}(z_2 n)n^\nu\Gamma h_v(0)|\bar{B}(v)\rangle \\
&= \frac{1}{2}F_B(\mu)Tr\Big\{\gamma_5\Gamma V_+\Big[(\not n v_\mu - \gamma_\mu)\big(\Psi_A - \Psi_V\big) - i\sigma_{\mu\nu}n^\nu\Psi_V - n_\mu X_A + n_\mu \not n Y_A\Big]\Big\},
\end{aligned}
$$

Three-particle B meson LCDA with definite twist

- twist three

$$2F_B(\mu)\Phi_3(z_1, z_2; \mu) = \langle 0|\bar{q}(z_1)gG_{\mu\nu}(z_2)n^\nu \; \not{n}\gamma_\perp^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle \,,$$

where

$$\Phi_3 = \Psi_A - \Psi_V \,,$$

- twist-four DAs

$$2F_B(\mu)\Phi_4(z_1, z_2; \mu) = \langle 0|\bar{q}(z_1)gG_{\mu\nu}(z_2)n^\nu \; \not{\bar{n}}\gamma_\perp^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle \,,$$

$$2F_B(\mu)\Psi_4(z_1, z_2; \mu) = \langle 0|\bar{q}(z_1)gG_{\mu\nu}(z_2)\bar{n}^\mu n^\nu \; \not{n}\gamma_5 h_v(0)|\bar{B}(v)\rangle \,,$$

$$2F_B(\mu)\widetilde{\Psi}_4(z_1, z_2; \mu) = \langle 0|\bar{q}(z_1)ig\widetilde{G}_{\mu\nu}(z_2)\bar{n}^\mu n^\nu \; \not{n} h_v(0)|\bar{B}(v)\rangle \,,$$

where

$$\Phi_4 = \Psi_A + \Psi_V \,,$$
$$\Psi_4 = \Psi_A + X_A \,,$$
$$\widetilde{\Psi}_4 = \Psi_V - \widetilde{X}_A \,,$$

Three-particle B meson LCDA with definite twist

- twist-five DAs

$$
\begin{aligned}
2F_B(\mu)\widetilde{\Phi}_5(z_1,z_2;\mu) &= \langle 0|\bar{q}(z_1)gG_{\mu\nu}(z_2)\bar{n}^\nu \not{n}\gamma_\perp^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle\,, \\
2F_B(\mu)\Psi_5(z_1,z_2;\mu) &= \langle 0|\bar{q}(z_1)gG_{\mu\nu}(z_2)\bar{n}^\mu n^\nu \not{\bar{n}}\gamma_5 h_v(0)|\bar{B}(v)\rangle\,, \\
2F_B(\mu)\widetilde{\Psi}_5(z_1,z_2;\mu) &= \langle 0|\bar{q}(z_1)ig\widetilde{G}_{\mu\nu}(z_2)\bar{n}^\mu n^\nu \not{\bar{n}} h_v(0)|\bar{B}(v)\rangle\,, \qquad (16)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\Phi}_5 &= \Psi_A + \Psi_V + 2Y_A - 2\widetilde{Y}_A + 2W\,, \\
\Psi_5 &= -\Psi_A + X_A - 2Y_A\,, \\
\widetilde{\Psi}_5 &= -\Psi_V - \widetilde{X}_A + 2\widetilde{Y}_A\,, \qquad (17)
\end{aligned}
$$

- twist-six DA

$$
2F_B(\mu)\widetilde{\Phi}_6(z_1,z_2;\mu) = \langle 0|\bar{q}(nz_1)gG_{\mu\nu}(nz_2)\bar{n}^\nu \not{\bar{n}}\gamma_\perp^\mu\gamma_5 h_v(0)|\bar{B}(v)\rangle \qquad (18)
$$

with

$$
\Phi_6 = \Psi_A - \Psi_V + 2Y_A + 2W + 2\widetilde{Y}_A - 4Z\,. \qquad (19)
$$

Two-particle higher twist B meson LCDA

- two-particle higher-twist DAs that arise as terms $\sim \mathcal{O}(x^2)$ in the expansion of the relevant nonlocal quark-antiquark operator close to the light cone.

$$\langle 0|\bar{q}(x)\Gamma[x,0]h_v(0)|\bar{B}(v)\rangle = -\frac{i}{2}F_B Tr\Big[\gamma_5\Gamma V_+\Big]\int_0^\infty d\omega\, e^{-i\omega(vx)}\Big\{\phi_+(\omega)+x^2 g_+(\omega)\Big\}$$

$$+\frac{i}{4}F_B Tr\Big[\gamma_5\Gamma V_+\,\not{x}\Big]\frac{1}{vx}\int_0^\infty d\omega\, e^{-i\omega(vx)}\Big\{[\phi_+-\phi_-](\omega)+x^2[g_+-g_-](\omega)\Big\}$$

- the constraints

$$\int_0^\infty d\omega\,\Big[\phi_+(\omega)-\phi_-(\omega)\Big]=0\,,\qquad \int_0^\infty d\omega\,\Big[g_+(\omega)-g_-(\omega)\Big]=0\,.$$

Relation from EOM

- the operator identity

$$\frac{\partial}{\partial x^\mu}\bar{q}(x)\gamma^\mu\Gamma[x,0]h_v(0) = -i\int_0^1 udu\,\bar{q}(x)[x,ux]x^\rho gG_{\rho\mu}(ux)[ux,0]\gamma^\mu\Gamma h_v(0)\,,$$

$$v^\mu\frac{\partial}{\partial x^\mu}\bar{q}(x)\Gamma[x,0]h_v(0) = \quad i\int_0^1 \bar{u}du\,\bar{q}(x)[x,ux]x^\rho gG_{\rho\mu}(ux)[ux,0]v^\mu\Gamma h_v(0) + (v\cdot\partial)\bar{q}(x)\Gamma[x,0]h_v(0)\,,$$

- the relations

$$\left[z\frac{d}{dz}+1\right]\Phi_-(z) = \Phi_+(z) + 2z^2\int_0^1 udu\,\Phi_3(z,uz)\,,$$

$$2z^2\mathrm{G}_+(z) = -\left[z\frac{d}{dz}-\frac{1}{2}+iz\bar{\Lambda}\right]\Phi_+(z) - \frac{1}{2}\Phi_-(z) - z^2\int_0^1 \bar{u}du\,\Psi_4(z,uz)\,,$$

$$2z^2\mathrm{G}_-(z) = -\left[z\frac{d}{dz}-\frac{1}{2}+iz\bar{\Lambda}\right]\Phi_-(z) - \frac{1}{2}\Phi_+(z) - z^2\int_0^1 \bar{u}du\,\Psi_5(z,uz)\,,$$

$$\Phi_-(z) = \left(z\frac{d}{dz}+1+2iz\bar{\Lambda}\right)\Phi_+(z) + 2z^2\int_0^1 du\left[u\Phi_4(z,uz)+\Psi_4(z,uz)\right],$$

- WW approximation(free parton)

$$\left[z\frac{d}{dz}+1\right]\Phi_-(z) = \Phi_+(z)$$

$$\Phi_-(z) = \left(z\frac{d}{dz}+1+2iz\bar{\Lambda}\right)\Phi_+(z)$$

WW approximation and beyond

- momentum space

$$\omega \frac{d}{d\omega}\phi_-(\omega) + \phi_+(\omega) = 0;$$
$$(\omega - 2\bar{\Lambda})\phi_+(\omega) + \omega\phi_-(\omega) = 0$$

- KKQT model(free parton model)

$$\phi_+^{WW}(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda} - \omega);$$
$$\phi_-^{WW}(\omega) = \frac{2\bar{\Lambda} - \omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda} - \omega)$$

- beyond WW approximation

$$\omega \frac{d}{d\omega}\phi_-(\omega) + \phi_+(\omega) = 2\omega \frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\frac{\partial \phi_3(\rho,\xi)}{\partial \xi};$$
$$(\omega - 2\bar{\Lambda})\phi_+(\omega) + \omega\phi_-(\omega) = -2\frac{d}{d\omega}\int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi}\left[2\frac{\partial \psi_V(\rho,\xi)}{\partial \xi} + \phi_4(\rho,\xi)\right]$$

Evolution of twist-3 LCDAs

- the leading twist LCDA satisfy the Lange-Nuebert evolution equation

$$\frac{d}{d\ln\mu}\phi^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega' \Gamma_+(\omega,\omega',\mu)\phi^+\left(\omega',\mu\right),$$

- position space

$$\frac{d}{d\ln\mu}\Phi^+(t,\mu) = -[\Gamma_{\rm cusp}(\alpha_s)\ln it\tilde{\mu} + \gamma_F(\alpha_s)]\Phi^+(t,\mu) + \int_0^1 dz K(z,\alpha_s)\Phi^+(zt,\mu),$$

- Mellin transform

$$\Phi_+(z,\mu) = -\frac{1}{z^2}\int_0^\infty ds\, s\, e^{is/z}\,\eta_+(s,\mu),$$

$$\Phi_-(z,\mu) = -\frac{i}{z}\int_0^\infty ds\, e^{is/z}\left[\eta_+(s,\mu) + \eta_3^{(0)}(s,\mu)\right],$$

- evolution equation in eigenspace

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \frac{\alpha_s}{2\pi}\mathcal{E}_+(s,\mu)\right]F(\mu)\,\eta_+(s,\mu) = 0,$$

$$\mathcal{E}_+(s,\mu) = 2C_F\left[\ln\left(\mu s\right) - \psi(1) - 5/4\right].$$

Evolution of twist-3 LCDAs

- The scale dependence of $\eta_+(s,\mu)$ and $\eta_3^{(0)}(s,\mu)$ is given by

$$\eta_+(s,\mu) = R(s;\mu,\mu_0)\eta_+(s,\mu_0)\,,$$
$$\eta_3^{(0)}(s,\mu) = L^{N_c/\beta_0} R(s;\mu,\mu_0)\eta_3^{(0)}(s,\mu_0)\,,$$

$$R(s;\mu,\mu_0) = L^{3C_F/(2\beta_0)} \exp\left[-\int_{\mu_0}^{\mu} \frac{d\tau}{\tau}\,\Gamma_{cusp}(\alpha_s(\tau))\,\ln(\tau s/s_0)\right]$$
$$= L^{3C_F/(2\beta_0)} \left(\frac{\mu}{\mu_0}\right)^{-\frac{2C_F}{\beta_0}} \left(\frac{\mu_0 s}{s_0}\right)^{\frac{2C_F}{\beta_0}\ln L} L^{-\frac{4C_F\pi}{\beta_0^2\,\alpha_s(\mu_0)}}\,.$$

- The DA $\Phi_3(z_1,z_2,\mu)$ can be expanded in terms the eigenfunctions of the large-N_c evolution kernel :

$$\Phi_3(\underline{z},\mu) = \int_0^\infty ds\Big[\eta_3^{(0)}(s,\mu)\,Y_3^{(0)}(s\,|\,\underline{z}) + \frac{1}{2}\int_{-\infty}^{\infty} dx\,\eta_3(s,x,\mu)\,Y_3(s,x\,|\,\underline{z})\Big],$$

$$Y_3(s,x\,|\,\underline{z}) = \frac{is^2}{z_1^2 z_2^3}\int_0^1 du\,u\bar{u}\,e^{is(u/z_1+\bar{u}/z_2)}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}-ix,\,-\frac{1}{2}+ix\\ 2\end{matrix}\,\Big|\,-\frac{u}{\bar{u}}\right),$$

$$Y_3^{(0)}(s\,|\,\underline{z}) = Y_3(s,x=i/2\,|\,\underline{z}) = \frac{is^2}{z_1^2 z_2^3}\int_0^1 du\,u\bar{u}\,e^{is(u/z_1+\bar{u}/z_2)}$$

$$\eta_3(s,x,\mu) = L^{\gamma_3(x)/\beta_0} R(s;\mu,\mu_0)\,\eta_3(s,x,\mu_0)\,,$$
$$\gamma_3(x) = N_c\big[\psi(3/2+ix)+\psi(3/2-ix)+2\gamma_E\big]\,,\gamma_3^{(0)} = \gamma_3(x=i/2) = N_c$$

Asymptotic form

- small momentum limit

$$\phi_+(\omega,\mu) = \int_0^\infty ds\,\sqrt{\omega s} J_1(2\sqrt{\omega s})\,\eta_+(s,\mu)\,,$$

$$\phi_-(\omega,\mu) = \int_0^\infty ds\,J_0(2\sqrt{\omega s})\,\left[\eta_+(s,\mu) + \eta_3^{(0)}(s,\mu)\right]$$

$$\phi_+(\omega) \sim \omega, \quad \phi_-(\omega) \sim 1$$

$$\Phi_3^{\rm as}(z_1,z_2,\mu) = \int_0^\infty ds\,\eta_0(s,\mu)\,Y_s^{(0)}(z_1,z_2).$$

The corresponding expression in momentum space reads

$$\phi_3^{\rm as}(\omega_1,\omega_2,\mu) = -\omega_2\sqrt{\omega_1}\int_0^\infty ds\sqrt{s}\eta^{(0)}(s,\mu)\int_0^1 du\sqrt{u}J_1\left(2\sqrt{s\omega_1 u}\right)J_2\left(2\sqrt{s\omega_2\bar{u}}\right)$$

$$\phi_3^{\rm as}(\omega_1,\omega_2,\mu) \sim \omega_1\omega_2^2$$

In general

$$\phi^{\rm as}(\omega_1,\omega_2) \sim \omega_1^{2j_1-1}\omega_2^{2j_2-1}$$

Conformal spin: $j = (d+s)/2$

$$\phi_3(\omega_1,\omega_2) \sim \omega_1\omega_2^2\,, \qquad \phi_4(\omega_1,\omega_2) \sim \omega_2^2\,, \qquad \psi_4(\omega_1,\omega_2) \sim \widetilde{\psi}_4(\omega_1,\omega_2) \sim \omega_1\omega_2\,.$$

- large momentum limit

Model 1: exponential model

- two-particle LCDA

$$\phi_+(\omega,\mu_0) = \frac{\omega}{\lambda_B^2}\, e^{-\omega/\lambda_B}\,, \qquad \eta_+(s,\mu) = e^{-s\lambda_B}\,,$$

WW approximation

$$\phi_-(\omega) = \frac{1}{\lambda_B}\, e^{-\omega/\lambda_B}$$

- three-particle LCDA

$$\phi_3(\omega_1,\omega_2,\mu_0) = \frac{\lambda_E^2-\lambda_H^2}{6\omega_0^5}\,\omega_1\omega_2^2\, e^{-(\omega_1+\omega_2)/\omega_0}\,,$$

$$\phi_4(\omega_1,\omega_2,\mu_0) = \frac{\lambda_E^2+\lambda_H^2}{6\omega_0^4}\,\omega_2^2\, e^{-(\omega_1+\omega_2)/\omega_0}\,,$$

$$\psi_4(\omega_1,\omega_2,\mu_0) = \frac{\lambda_E^2}{3\omega_0^4}\,\omega_1\omega_2\, e^{-(\omega_1+\omega_2)/\omega_0}\,,$$

$$\widetilde{\psi}_4(\omega_1,\omega_2,\mu_0) = \frac{\lambda_H^2}{3\omega_0^4}\,\omega_1\omega_2\, e^{-(\omega_1+\omega_2)/\omega_0}\,,$$

Model 2: Local duality

- two particle

$$
\begin{aligned}
& i\int d^4y\, e^{-i\omega(vy)}\langle 0|T\{\bar{q}(x)\Gamma_1[x,0]h_v(0)\bar{h}_v(y)\Gamma_2 q(y)|0\rangle \\
=& \langle 0|T\{\bar{q}(x)\Gamma_1[x,0]h_v(0)|\bar{B}(v)\rangle \frac{1}{\bar{\Lambda}-\omega}\langle \bar{B}(v)|\bar{h}_v(0)\Gamma_2 q(0)|0\rangle + \ldots \qquad (32)
\end{aligned}
$$

Evaluating the simple quark loop and expanding in powers of x^2 one obtains in this way

$$
\begin{aligned}
\phi_+^{\rm LD}(\omega) &= \frac{3}{4\omega_0^3}\omega(2\omega_0-\omega)\theta(2\omega_0-\omega)\,, \\
g_+^{\rm LD}(\omega) &= \frac{3}{16\omega_0^3}\omega^2(2\omega_0-\omega)^2\theta(2\omega_0-\omega)\,, \qquad (33)
\end{aligned}
$$

- Selfconsistent models of this type can be constructed by including three-particle contributions. It turns out that the constraints due to the EOM require using the same threshold and power behavior $\sim (2\omega_0-\omega)^p$ for all DAs except for ϕ_3 which has to be one power lower. $p=1, p=3$ is commonly used.

$B \to \gamma\nu\ell$

- hadronic tensor

$$T_{\mu\nu}^{(u)}(p,q) = -ie_u\sqrt{m_B}\int d^4x\, e^{ipx}\langle 0|T\{\bar{u}(x)\gamma_\mu u(x)\,\bar{u}(0)\gamma_\nu(1-\gamma_5)h_v(0)\}|B(v)\rangle$$

- quark propagator in background field

$$S(x,0) = \frac{i}{2\pi^2}\frac{\not{x}}{x^4} - \frac{1}{8\pi^2x^2}\int_0^1 du\Big\{ix^\rho g\widetilde{G}_{\rho\sigma}(ux)\gamma^\sigma\gamma_5 + (2u-1)x^\rho gG_{\rho\sigma}(ux)\gamma^\sigma\Big\} + \ldots$$

- the result

$$\begin{aligned}T_{\mu\nu}^{(u)} &= \frac{ie_uf_Bm_B}{2\pi^2}\int d^4x\,\frac{e^{ipx}}{x^4}\left[(vx)g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma}x^\rho v^\sigma\right] \\ &\quad\times\left\{\Phi_+(vx) + x^2G_+(vx) - \frac{x^2}{4}\int_0^1 du\left[(2u-1)\Psi_4 - \widetilde{\Psi}_4\right](vx,uvx)\right\} + \ldots \\ &= \frac{ie_uf_Bm_B}{2\pi^2}\int d^4x\,\frac{e^{ipx}}{x^4}\left[(vx)g_{\mu\nu} + i\epsilon_{\mu\nu\rho\sigma}x^\rho v^\sigma\right] \\ &\times\left\{\Phi_+(vx) + x^2G_+^{\mathrm{WW}}(vx) - \frac{x^2}{2(vx)^2}\Phi_-^{\mathrm{t3}}(vx) - \frac{x^2}{4}\int_0^1 du\left[\Psi_4 - \widetilde{\Psi}_4\right]^{\mathrm{t4}}(vx,uvx)\right\} \\ &\quad + \ldots\end{aligned}$$

Reference

1. arxiv: 1703.02446
2. arxiv: 1507.03445
3. arxiv: hep/ph-0309330