B-meson quasi-DA in LaMET

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PQCD group meeting 2020
15.08.2020 HUST, Wuhan

In collaboration with Wei Wang, Yu-Ming Wang, Shuai Zhao

Phys. Rev. D 102, 011502(R)

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Outline

- **Introduction to LaMET**
  1. Deep inelastic scattering (DIS) and parton distribution function (PDF)
  2. Larger Momentum Effective Theory (LaMET)
  3. Universality to calculate parton structure
- **B-meson quasi-DA in LaMET**
  1. Introduction to B-meson LCDA
  2. Matching between B-meson quasi-DA and LCDA
  3. Calculation of One-loop matching coefficients
  4. Perspectives for lattice calculations
- **Summary**

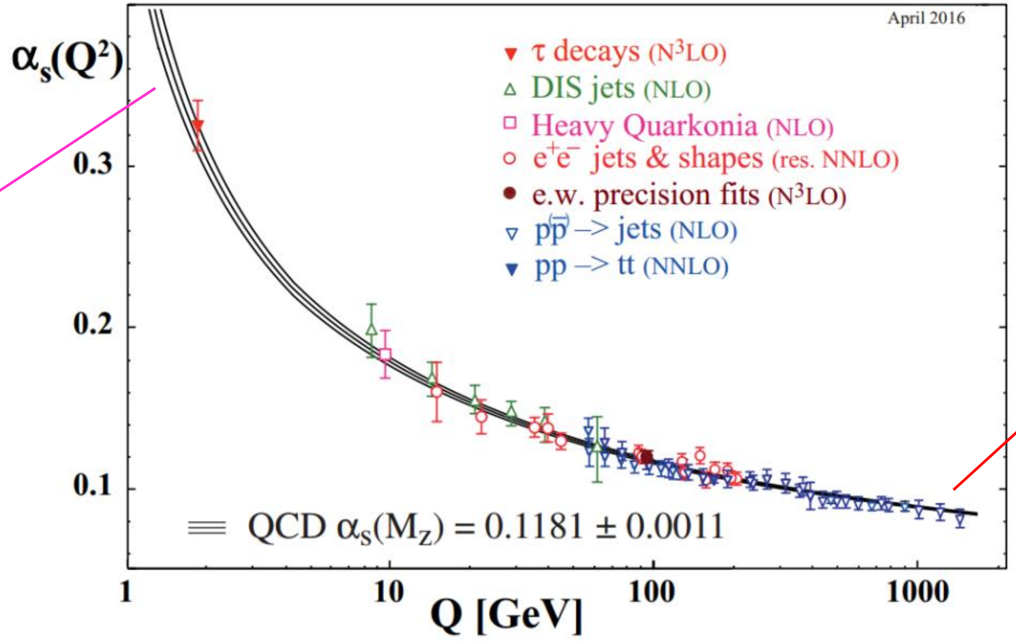
DIS and PDF

The proton and neutron, collectively called the nucleon, are the basic building blocks of visible matter in the universe today. The most revealing discovery of them came from the electron deep inelastic scattering (DIS) on the proton and nuclei at Stanford Linear Accelerator Center (SLAC) in the late 1960s.

Soon after, quantum chromodynamics (QCD) was established as the fundamental theory of strong interactions, and thus for the internal structure of the nucleon as well.

QCD has two sides:

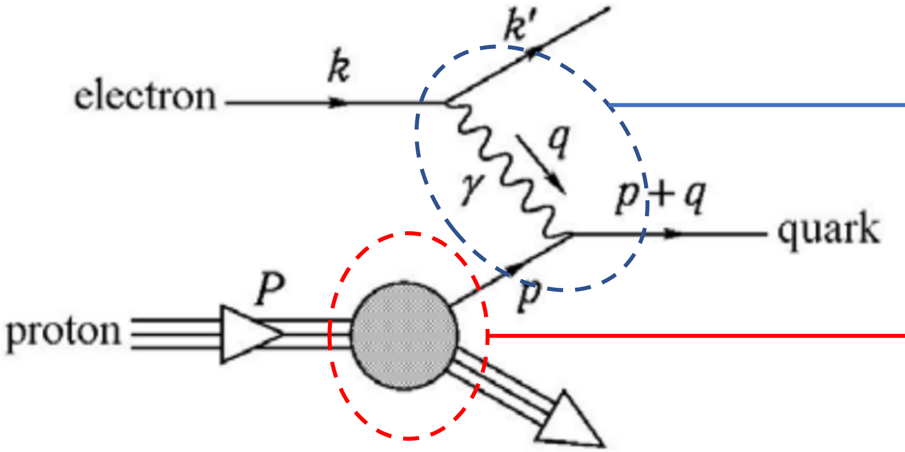
Color confinement



Asymptotic free

DIS and PDF

Based on the QCD factorization theorems, derived from perturbative QCD analyses beyond the Feynman's parton model, the parton (quark and gluon) distribution functions (PDFs) have been obtained.



Perturbatively calculable coefficient

Parton distribution function, non-perturbative

$$F_2\left(x, Q^2\right) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n\left(\mu_R^2\right)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}\left(z, Q^2, \mu_R^2, \mu_F^2\right) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

DIS and PDF

PDFs can be obtained from global fits to experimental data. A recent result of the phenomenological proton PDFs is shown below. *T.-J. Hou et al., arXiv:1912.10053 [hep-ph]*

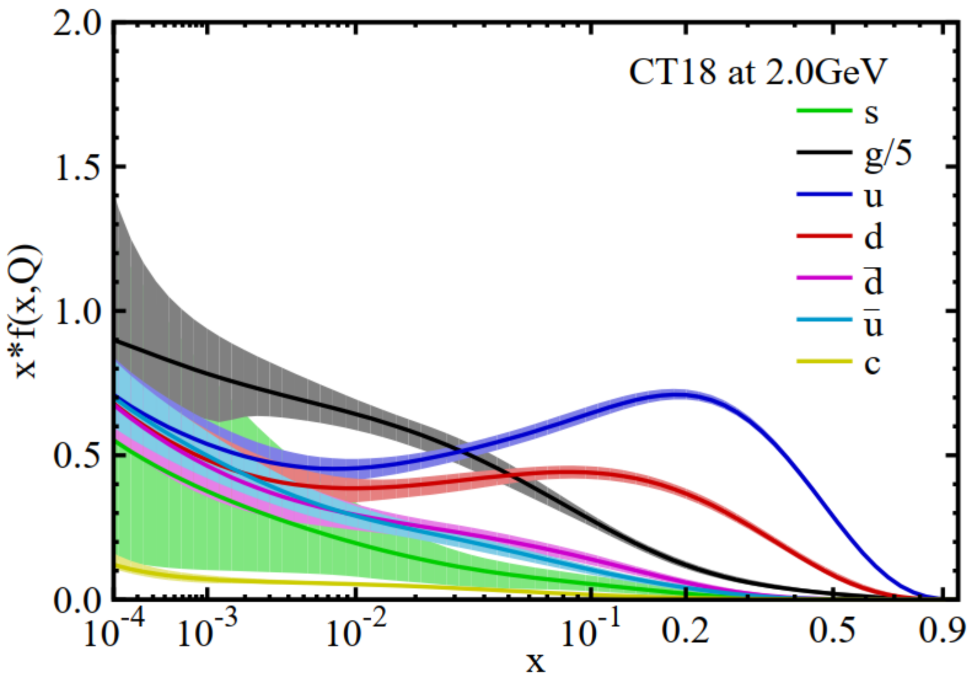


FIG. 1: CT18 phenomenological parton distributions obtained from fits to global high-energy scattering data [13].

The PDFs describe the probability distributions of quarks and gluon inside nucleon.

The PDFs provide a comprehensive description of the quark and gluon content of the nucleon.

Despite these impressive achievements, we have not been able to explain the phenomenological partonic structure of the proton from the first principles.

DIS and PDF

LQCD privides a systematic *ab initio* calculations of the non-perturbative strong interactions.

- The standard formulation of parton physics is accomplished through the dynamical correlators of quantum fields on the light-front (LF), which has the important feature of being independent of the proton's momentum.

$$q\left(x,\mu^2\right)=\int\frac{d\xi^-}{4\pi}e^{-ix\xi^-P^+}\langle P|\bar{\psi}\left(\xi^-\right)\gamma^+\exp\left(-ig\int_0^{\xi^-}d\eta^-A^+\left(\eta^-\right)\right)\psi(0)|P\rangle$$

- On the other hand, lattice QCD is intrinsically a Euclidean approach and cannot be used to directly calculate the dynamical correlations.

Although significant advances have been made over the years, a systematic approximation to calculate the nucleon PDFs is still missing.

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LaMET

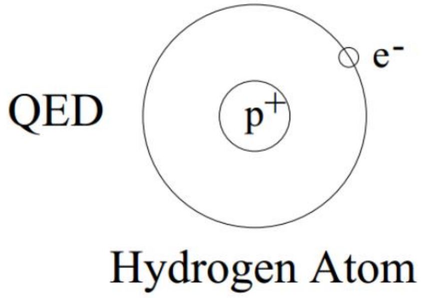
Large Momentum Effective Theory

X. Ji, Phys. Rev. Lett. 110 (2013) 262002 X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412

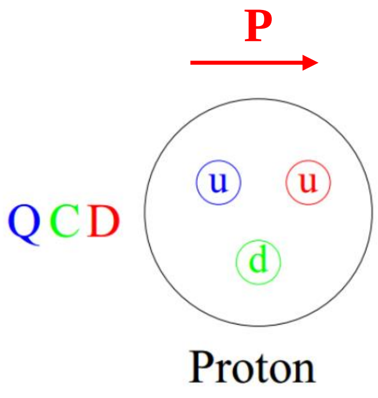
With this approach, parton physics can be extracted using effective field theory (EFT) methods from the physical properties of the proton at a moderately-large momentum, e.g., with a Lorentz boost factor γ = 2-5.
Thus, the theory has been named as *large-momentum effective theory* (LaMET).

LaMET is not merely a theoretical trick, but is based on an important physical insight by Feynman.
Naive parton model: The structure of the proton shall be approximately independent of its momentum so long as it is much larger than a typical strong-interaction scale Λ. For example, the quark momentum distribution in the proton at $P = |\vec{P}|$= 5 GeV shall not be very different from that at P = 50 GeV or P = 5 TeV.

One might call this phenomenon Large-momentum symmetry.



$M_e = 0.5$ MeV
$M_p = 938$ MeV
$E_{binding} = 13.6$ eV
(EM force)



$M_u \sim 3$ MeV
$M_d \sim 6$ MeV
$M_p = 938$ MeV
(Strong force)

LaMET

Assuming this symmetry, Feynman replaced the protons probed at different large momenta in high energy scattering with the one at the infinite momentum P = ∞, therefore the idealized concepts of a proton in the infinite-momentum frame (IMF) and its constituents—partons—were born.

In QFTs, whether a large-momentum symmetry exists depends on their ultraviolet (UV) behavior.

Physical limit shall be $\Lambda_{UV} \gg P \to \infty$
Parton model and subsequent QCD factorization theorems use $P \gg \Lambda_{UV} \to \infty$

Fortunately, because of asymptotic freedom, the above differences can all be calculated in perturbative QCD.

LaMET is a theory to systematically compute effects of large-momentum symmetry breaking and non-commuting $P \to \infty$ limits through EFT matching and running.

LaMET

Go back to PDF

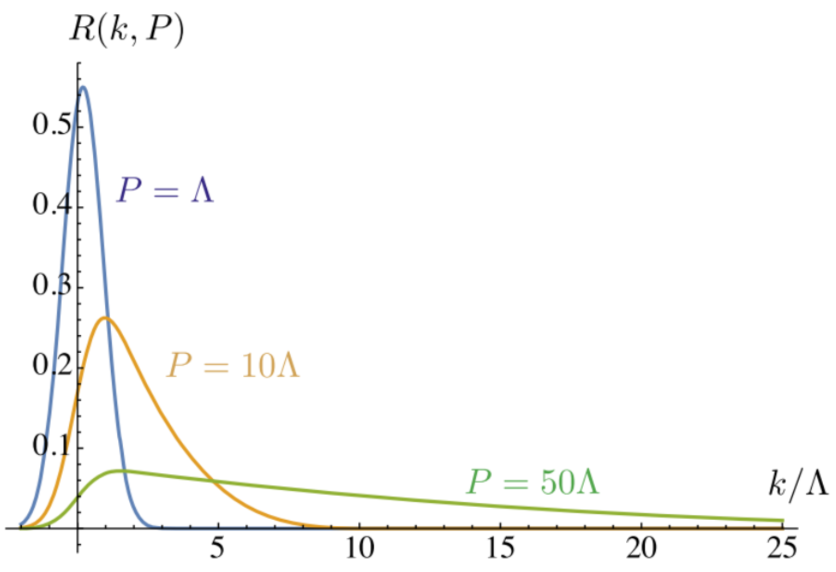
- Feynman introduced the *parton model* to describe deep inelastic scattering (DIS) on the proton.
- The assumption that all large-momentum protons have a similar structure appears intuitive and natural.
  1. The internal structure of non-relativistic systems is center-of-mass (COM) momentum independent.
  2. The situation is different for intrinsically relativistic systems. In general though, the internal structures are inexplicably mixed with the COM motion, and their dependence on the external momentum is a dynamical problem.

$$R\left(k_3, P\right) = \int_{-1}^{1} dx \mathcal{R}\left(x, k_3 - xP\right) .$$

$$\mathcal{R}\left(x, k_3 - xP\right) = f(x) r\left(k_3 - xP\right) ,$$

A. V. Radyushkin, Phys. Rev. D 96, 034025 (2017)

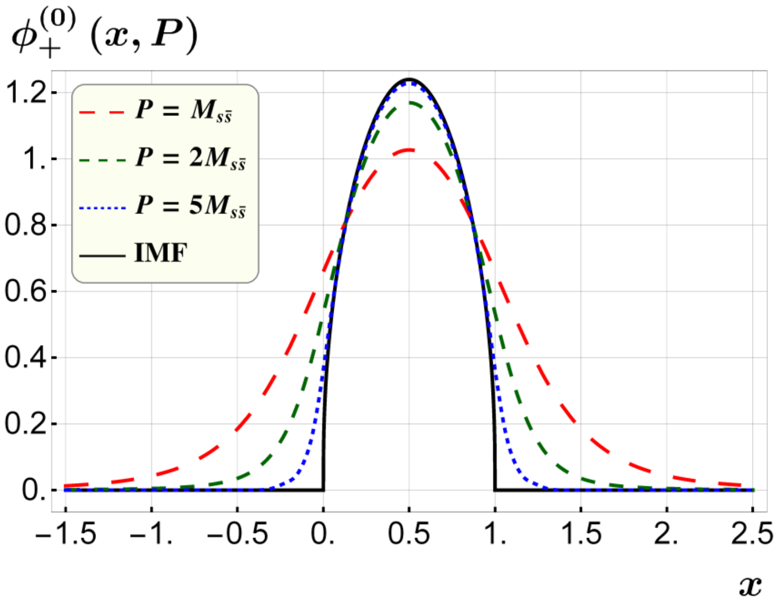
The space-time structure of a bound state is frame-dependent. This dependence is not as simple as kinematic transformations.



LaMET

Nonetheless, it is generally expected that the large momentum limit of the proton state exists and is smooth, and some small parameters such as Λ/P^z control the limiting process.

This is true in certain simple QFT models such as ’t Hooft model. X. Ji et al., arXiv:2004.03543v1



Wave function amplitudes of a meson in the ’t Hooft model at different external momenta.

This is the type of examples that Feynman’s intuition applies.

However, in general this is not the case for 3+1 dimensional QFTs. A singularity (cut) at $P \to \infty$ can exist in field theories, making $P \to \infty$ limit ill-defined and the large momentum expansion impossible.

LaMET

Fortunately, in QCD, these dangerous symmetry breaking effects can be resummed using the RGE method. Therefore, large-momentum symmetry is still a useful concept, subject to corrections from perturbative large logarithms.

In QFTs, UV divergences bring in complications.

- The physically relevant one is clearly $\Lambda_{UV} \gg P \to \infty$, as discussed in the previous subsection.
- Historically, It was found that taking $P \to \infty$ by ignoring the UV divergences considerably simplifies the perturbation theory rules.

It is the "naive" limit, $P \gg \Lambda_{UV} \to \infty$, that corresponds to Feynman's parton model, and hence we name the resulting theory as effective field theory for partons.

In asymptotically free theories such as QCD, differences (or discontinuities) in taking the limits of $P \gg \Lambda_{UV}$ and $\Lambda_{UV} \gg P \to \infty$ are perturbatively calculable, as only the high-momentum modes matter. The differences are called **matching coefficients.**

LaMET

Compare parton model with HQET

- Taking $P \to \infty$ first.

- Limits of taking $\Lambda_{UV} \to \infty$ and infinite P are not changeable, due to the presence of the large logarithms $\ln P$.

- One can systematically calculate power-suppressed terms $O(1/P^2)$.

- Taking $m_Q \to \infty$ first.

- Limits of taking $\Lambda_{UV} \to \infty$ and infinite m_Q are not changeable, due to the presence of the large logarithms $\ln m_Q$.

  In an EFT approach, one takes the $m_Q \to \infty$ limit first, this will result in a new theory with different UV behavior, but without the heavy-quark mass to worry about.
  A. G. Grozin and M. Neubert, Phys. Rev. D 55 (1997) 272: A new kind of ultraviolet divergence appears.

- One can systematically calculate power-suppressed terms $O(1/m_Q)$.

LaMET

As we have explained above, Feynman's partons in the IMF correspond to the EFT description of the proton structure on the light-cone or LF.

However, directly solving the structure on the LF has been proven challenging. LaMET provides a new possibility to access parton physics in which the direct LFQ problems can be avoided.

Its strategy can be concisely stated as follow: Use whatever theoretical approaches to calculate the structural properties of a proton travelling at a moderately-large momentum P, and match them to the standard partonic quantities on the LF using EFT methods.

- Large momentum symmetry
- Asymptotic freedom of QCD

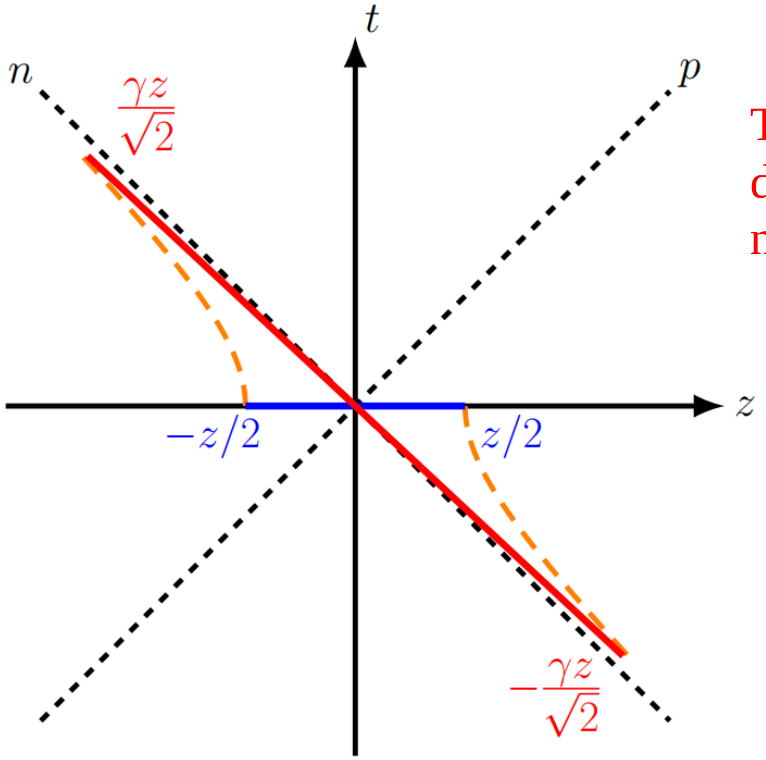
While LFQ may provide a physical picture for the proton, the Euclidean equal-time formulation is more practical for carrying out the calculations, and LaMET serves to build a bridge between them.

LaMET

$$\tilde{q}\left(x,\mu^2,P^z\right)=\int\frac{dz}{4\pi}e^{izk^z}\langle P|\bar{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z\left(z'\right)\right)\psi(0)|P\rangle$$

$$q\left(x,\mu^2\right)=\int\frac{d\xi^-}{4\pi}e^{-ix\xi^-P^+}\langle P|\bar{\psi}\left(\xi^-\right)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+\left(\eta^-\right)\right)\psi(0)|P\rangle$$

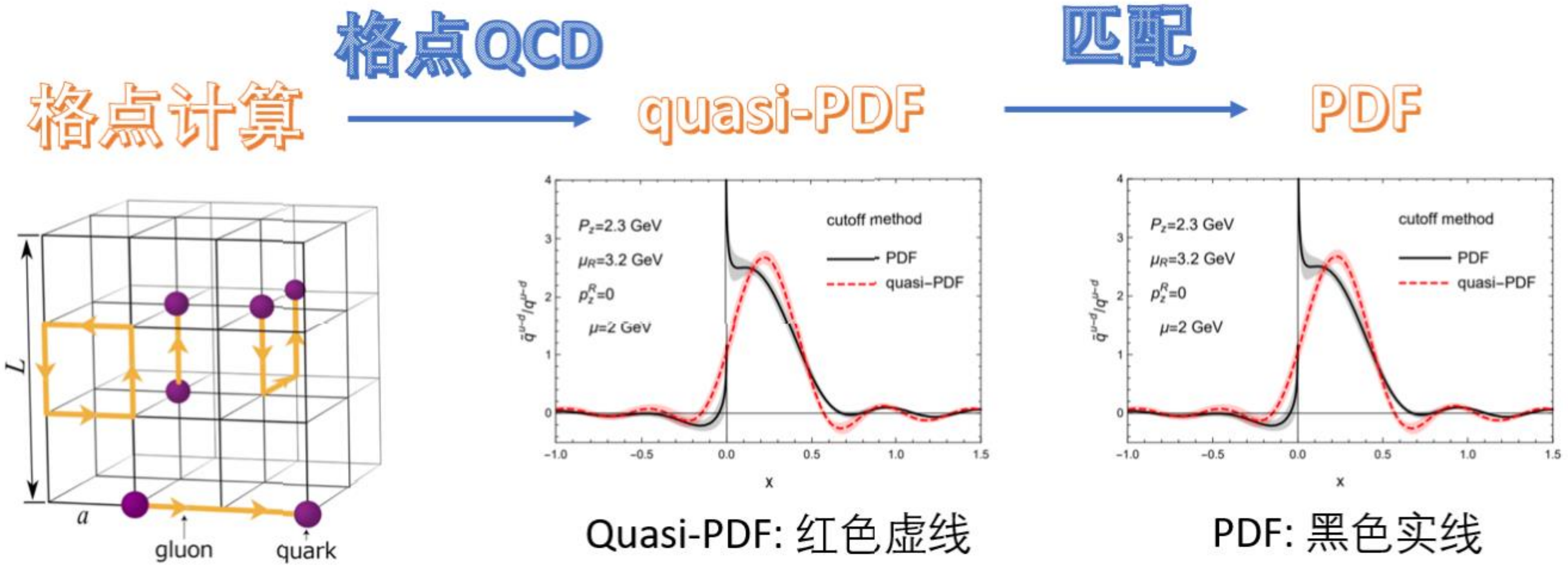
The momentum distribution defined above has been called *quasi-PDF*, but in reality it is a physical momentum distribution in a proton of momentum *P* .



The PDFs describe the probability distributions of quarks and gluon inside nucleon.

LaMET

- **Setp1：** Constructing lattice operators and evaluate the ME
- **Step2：** Lattice calculations
- **Step3：** Extracting the light-cone physics from the lattice ME



LaMET

Factorization formula

$$\tilde{q}\left(x, \mu^2, P^z\right) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\bar{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z\left(z'\right)\right)\psi(0)|P\rangle$$

$$q\left(x, \mu^2\right) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\bar{\psi}\left(\xi^-\right)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+\left(\eta^-\right)\right)\psi(0)|P\rangle$$

$$\tilde{q}\left(x, \mu^2, P^z\right) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q\left(y, \mu^2\right) + \mathcal{O}\left(\Lambda^2/\left(P^z\right)^2, M^2/\left(P^z\right)^2\right)$$

Matching coefficent (for $Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right)$)

$$Z\left(x, \mu/P^z\right) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}\left(x, \mu/P^z\right) + \ldots$$

LaMET

What have we learned above?

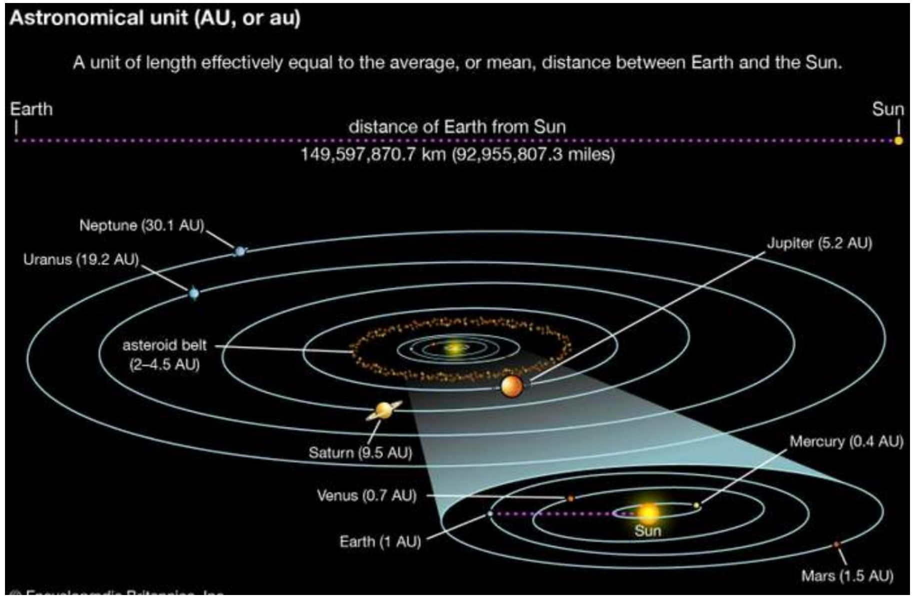
This is just Taylor expansion. X. Ji, arXiv:2007.06613v1

Imagine some physics quantity f(x, ϵ, δ ...) depending on the variable x and many other parameters , ϵ, δ,... etc, one can simplify the problem by expanding around the ideal limit, $\epsilon = \delta = 0$,

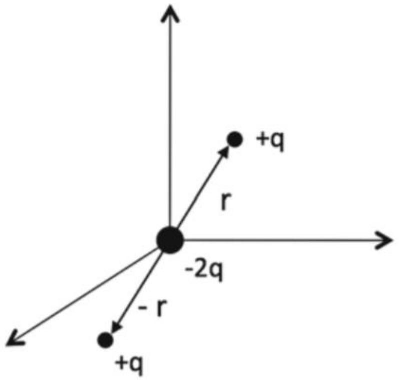
$$f\,(\mathrm{x}, \epsilon, \delta\ \ldots) = f(\mathrm{x}, 0{,}0, \ldots) + \epsilon\, f_\epsilon(x, 0{,}0\ \ldots) + \delta f_\delta(x, 0{,}0\ \ldots) + \cdots$$

This way of doing physics may be called an effective approach.

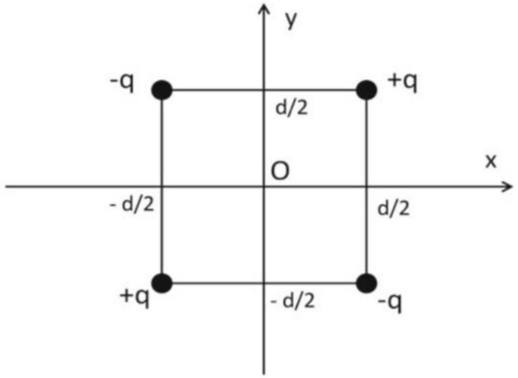
the radius of the Earth when studying its rotation around the Sun



Multipole Expansion of the Electrostatic Potential



Electric Dipole Moment



The bidimensional electric quadrupole

LaMET

What have we learned above?

This is just Taylor expansion, but need to consider the ultraviolet (UV) divergences carefully!

In some sense, all QFTs are EFTs!

UV divergences sometimes make Taylor expansions not so straightforward.

There is a difference because taking $\epsilon \to 0$ does not commute with $\Lambda \to \infty$. Thus, an EFT often deals with a Taylor expansion around a singular point of the relevant parameters, which is a bit tricky.

An effective Lagrangian is constructed to evaluate $\mathrm{f}(\mathrm{x}, \epsilon = 0, \ldots, \Lambda)$, and this calculation is presumably simpler. However this does not give the right answer $\mathrm{f}(\mathrm{x}, \epsilon \to 0, \ldots, \Lambda)$, One needs to figure out what is their difference, and this is very important!

Once an effective theory calculation is done, one can get the right Taylor series by adding up the difference. This is called EFT matching!

LaMET

What have we learned above?
LaMET is an effective field theory.

Assuming $P^z \to \infty$ is analytic, one can Taylor-expand the momentum distribution there, and the famous parton distribution is just the first term of the expansion,

$$f(k^z, P^z) = f(x) + f_2(x)\left(\frac{M}{P^z}\right)^2 + \cdots$$

where $x = k^z/P^z$ and M is a hadron mass scale.

The quark and gluon momentum distributions $f(k^z, P^z)$ are quantities that can routinely be simulated in lattice QCD for a moderately-large momentum P^z.

- One would naively expect that since M is on the order of $1GeV$, $P^z \sim 2GeV$ already gives $\epsilon = (M/P^z)^2 = 0.25$.
- With incoming exascale computing, one can simulate in lattice QCD a proton at $P^z \sim (3 \sim 5)GeV$, making ϵ as small as 0.03.

LaMET

What have we learned above?

LaMET is an effective field theory.

One can invert the expansion in page 19 through

$$f\ (\mathrm{x},0,0,\ldots) = f(\mathrm{x},\epsilon,\delta\ \ldots) - \epsilon\ f_\epsilon(x,0,0\ \ldots) - \delta f_\delta(x,0,0\ \ldots) + \cdots$$

Now one can regard $f(\mathrm{x},\epsilon,\delta\ \ldots)$ as an effective description of $f\ (\mathrm{x},0,0,\ldots)$. Therefore, the magic word "effective" is not absolute, but mutual, analogous to a mirror symmetry.

- Partons provide a powerful language which is used everyday by thousands of physicists to describe high-energy collisions. Although conceptually simple, the mathematics of describing parton interactions forming the high energy proton is very challenging. Therefore, it is important that any approach to calculating PDFs must have systematic controls of errors.

LaMET provide a method to do this.

LaMET

We can generalize the discussions in the previous subsection to any type of physical observables for the large momentum proton, which will be generally called quasi-parton observables.

- Quark-PDF

  X.Xiong, X. Ji, J.-H. Zhang, Y. Zhao Phys.Rev.D 2014; Y.-Q. Ma and J.-W. Qiu 2014 ......

- TMD

  X. Ji, P.Sun, X. Xiong and F. Yuan, Pys. Rev. D 2015; X.Ji,L.-C. Jin, F.Yuan, J.-H. Zhang, Y. Zhao Phys.Rev.D 2015; Markus A. Ebert, Iain W. Stewart, Yong Zhao .JHEP 1909 (2019) 037......

- GPD

  X. Ji, A. Schafer, X. Xiong and J.-H. Zhang Phys. Rev. D 2015; X. Xiong and J.-H. Zhang Phys. Rev. D 2015 ......

- LCDA

  J.-H. Zhang, J.-W. C, X. Ji, Lu. J, H.-W. L Phys.Rev. D 2017; Hiroyuki Kawamura and Kazuhiro Tanaka, Proceedings of Science 2017; J. Xu, Q.-A. Zhang and S. Zhao, Phys. Rev. D 2018 ......

- Gluon-PDFs

  W. Wang, S. Zhao and R.Zhu, Eur.Phys.J. C 2018; W. Wang and S. Zhao, JHEP 2018; J. H. Zhang, X. Ji, A. Schafer, W. Wang and S. Zhao, Phys. Rev.Lett 2019......

LaMET

Lattice simulation in LaMET

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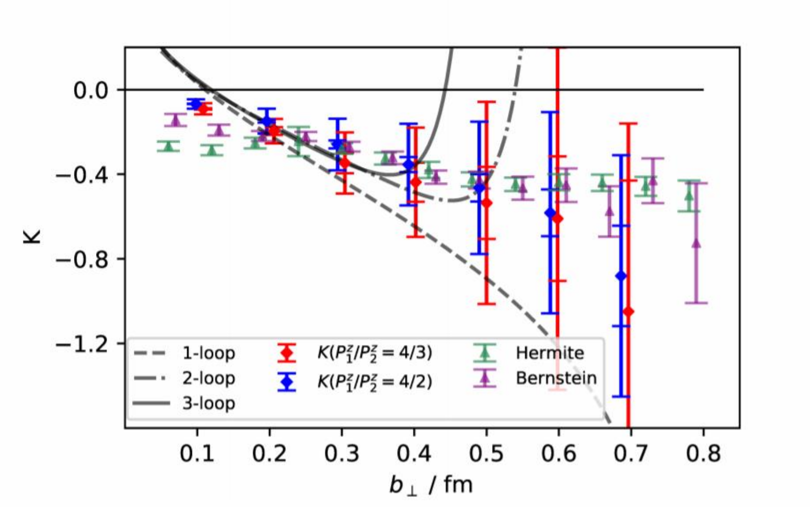
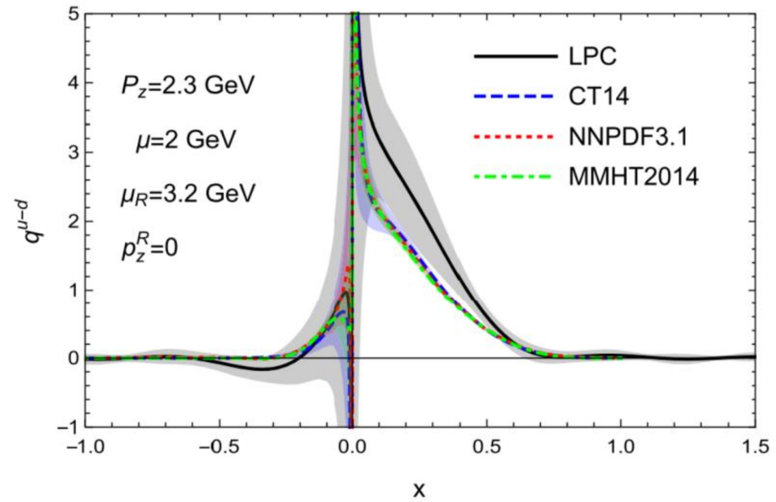
  J. H. Zhang, J. W. Chen, X. Ji, L. Jin and H. W. Lin, Phys. Rev. D 2017; J. H. Zhang et al. [LP3 Collaboration], Nucl. Phys. B 2019……



Lattice parton Collaboration, arXiv: 2005.14572



Y. S. Liu et al. [Lattice Parton Collaboration], Phys. Rev. D 2020

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Universality

LaMET provides a framework connecting the properties of a large-momentum proton with its partonic observables at LF. However, the relationship is not one-to one.

In the case of unpolarized PDFs, the initial proposal in LaMET starts from the matrix element of the following operator

$$O_1(z) = \bar{\psi}(0)\gamma^z W(0,z)\psi(z)$$

However, one can equally start from

$$O_2(z) = \bar{\psi}(0)\gamma^t W(0,z)\psi(z)$$

Another example of Euclidean operators for PDFs is the current-current correlators in a pure space separation,

$$O(z) = J^\mu(0)J^\mu(z)$$

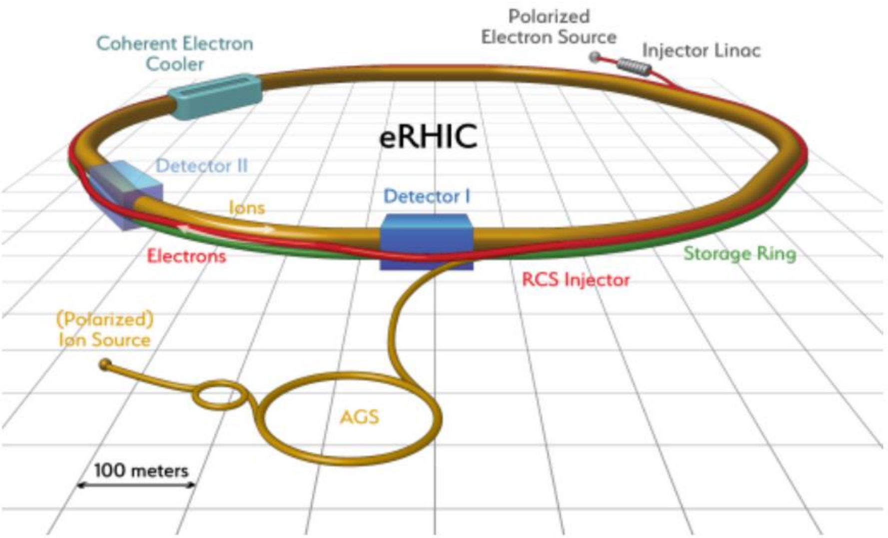
One can also start with a coordinate-space factorization approach known as pseudo-PDF approach.

Dr. Shuai's talk in PQCD group meeting 2020

Universality

Universality in the large-*P* limit provides rich possibilities in calculating the partonic structure of nucleons and other hadrons.

At a practical level, it is very useful to find which operator has the fastest convergence in the LaMET expansion.



A.Aprahamian et al., (2015)

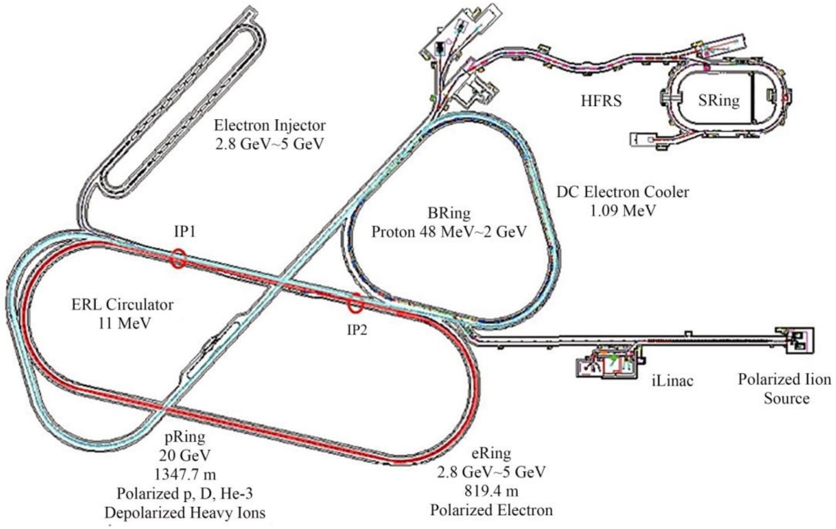


图31 EicC装置总体布局
Fig.31 The layout of EicC

《中国极化电子离子对撞机计划》

- Sub-threshold production of heavy flavor in e A collisions, including J/ψ and Υ production.

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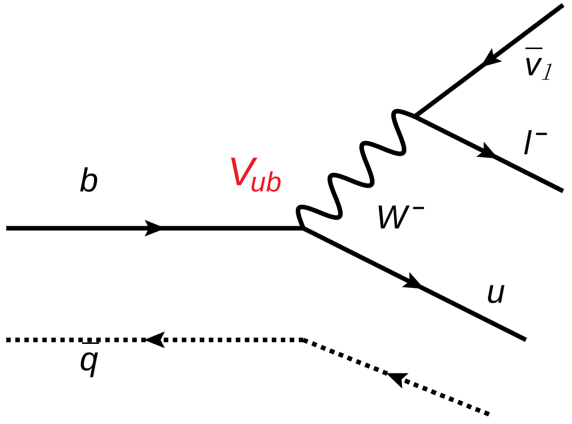
B-meson LCDA

Importance of B physics

- Accurate measurement of standard model parameters: V_{ub}, V_{cb}

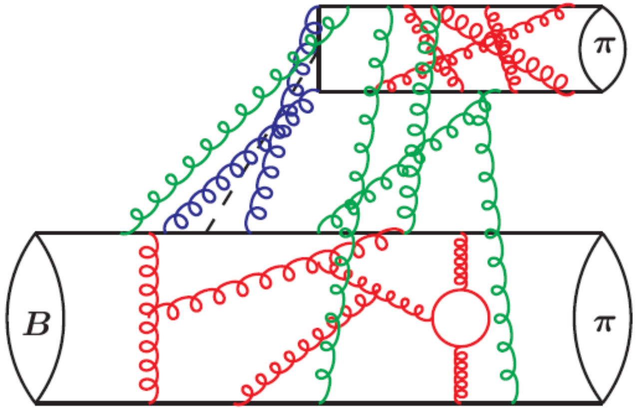
  K. A. Olive et al., Chin. Phys. C 38, 090001 (2014)



- Factorization theory in B-meson decay

  Phys. Rev. Lett.83, 1914 (1999)

$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$



$B\to\pi$ form factor

Hard kernel

B-meson LCDA

The significance of the factorization formula is that all leading-power nonperturbative effects in the $B\to\pi\pi$ amplitudes can be absorbed into the form factor and the light-cone wave functions.

B-meson LCDA

B-meson light-cone distribution amplitude (LCDA) in heavy-quark effective theory (HQET) serves as an indispensable ingredient for

- Establishing QCD factorization theorems of exclusive B-meson decay amplitudes
- Accurate measurement of observable quantities in B-meson decay process
- Constructing light cone sum rules of numerous hadronic matrix elements

It's definition in HQET was proposed in 1997 A. G. Grozin and M. Neubert, Phys. Rev. D 55 (1997) 272

The meson matrix element of a heavy-light bilocal operator operator can be expressed as

$$\langle 0|\tilde{O}(\tau)|B\rangle = f\left(\tilde{\phi}_B^+(\tau) + \frac{1}{2\tau}\left[\tilde{\phi}_B^-(\tau) - \tilde{\phi}_B^+(\tau)\right] \not{z}\right) u\,,$$

LCDAs of B-meson

B-meson LCDA

Evolution equations

- DGLAP for PDFs
- ERBL for light meson LCDAs
- Lange-Neubert for B-meson LCDAs

Defined as the light-ray matrix elements of the composite HQET quark-gluon operators, they encode information of the non-perturbative strong interaction dynamics from the soft-scale fluctuation of the *B*-meson system.

Propose to extract the leading-twist B-meson LCDA $\phi_B^+(\omega, \mu)$ in the frame of LaMET

W. Wang et al. Phys. Rev. D 102, 011502(R)

B-meson LCDA

The leading-twist LCDA $\phi_B^+(\eta,\mu)$ in coordinate space is defined by the renormalized HQET matrix element of a light-ray operator.

$$\langle 0|\,(\bar{q}_s\,Y_s)\,(\eta\,\bar{n})\;\;\bar{\not n}\,\gamma_5\,\left(Y_s^\dagger\,h_v\right)\,(0)|\bar{B}(v)\rangle = i\tilde{f}_B(\mu)\,m_B\,\tilde{\phi}_B^+(\eta,\mu)$$

Applying the Fourier transformation for $\phi_B^+(\eta,\mu)$ leads to the momentum-space distribution function

$$\phi_B^+(\omega,\mu) = \frac{1}{2\,\pi}\int_{-\infty}^{+\infty} d\eta\, e^{i\,\bar{n}\cdot v\,\omega\,\eta}\,\tilde{\phi}_B^+(\eta,\mu)$$

Following the construction presented above, we will employ the following B-meson quasi-distribution amplitude

$$i\tilde{f}_B(\mu)\,m_B\,\varphi_B^+(\xi,\mu) = \int_{-\infty}^{+\infty}\frac{d\tau}{2\,\pi}\,e^{i\,n_z\cdot v\,\xi\,\tau}\langle 0|\,(\bar{q}_s\,Y_s)\,(\tau\,n_z)\;\;\not n_z\,\gamma_5\,\left(Y_s^\dagger\,h_v\right)\,(0)|\bar{B}(v)\rangle$$

Here $n_z = (1,0,0,0)$.

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  4. Perspectives for lattice calculations

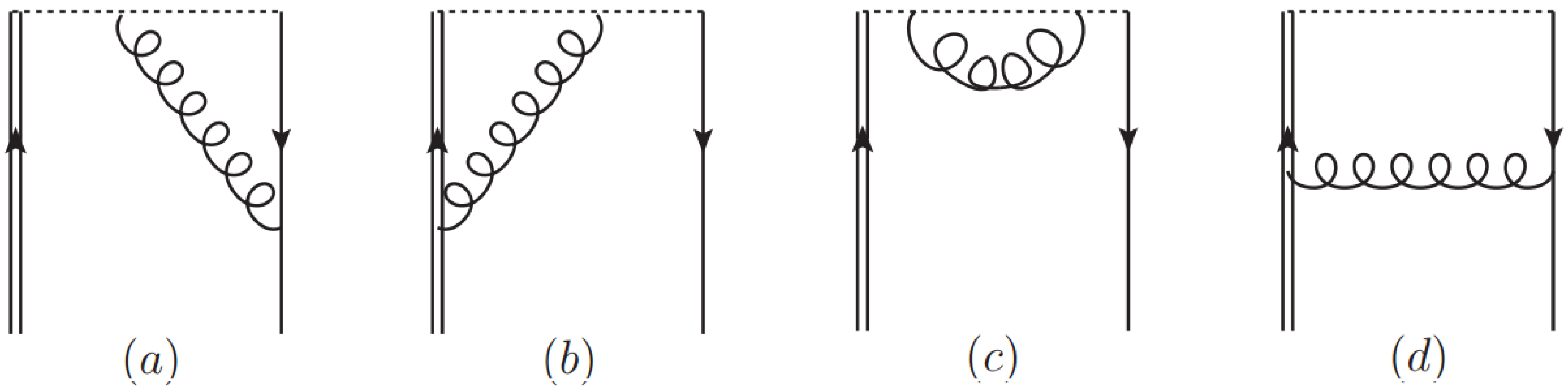
- **Summary**

Matching

We now proceed to determine the perturbative matching coefficient function entering the hard-collinear factorization formula

$$\varphi_B^+(\xi,\mu) = \int_0^\infty d\omega\, H(\xi,\omega,n_z\cdot v,\mu)\,\phi_B^+(\omega,\mu) + O\left(\frac{\Lambda_{\rm QCD}}{n_z\cdot v\,\xi}\right)$$

Matching coefficient got by calculating these diagrams below



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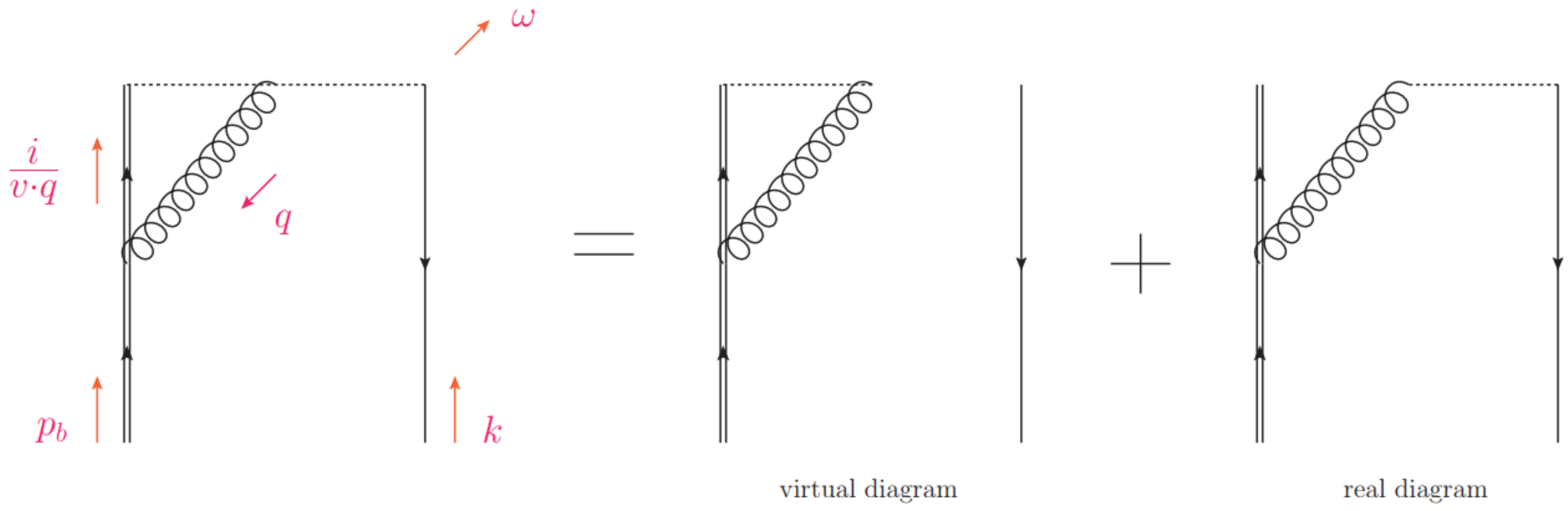
- Summary

Matching

Example 1,



Here we use off-shell ness of light quark to regularize the collinear divergence, $\rho = -k^2/k_z^2$. The amplitude for virtual diagram is

$$M_{Quasi}^{(1,b,vir.)} = (-ig_s^2 C_F n_z.v)\bar{v}(k)\gamma^z\gamma_5 u_v(p_b)\delta(\omega - k^z)\int \frac{d^d q}{(2\pi)^d}\frac{1}{q^2}\frac{1}{v.q}\frac{1}{q^z}$$

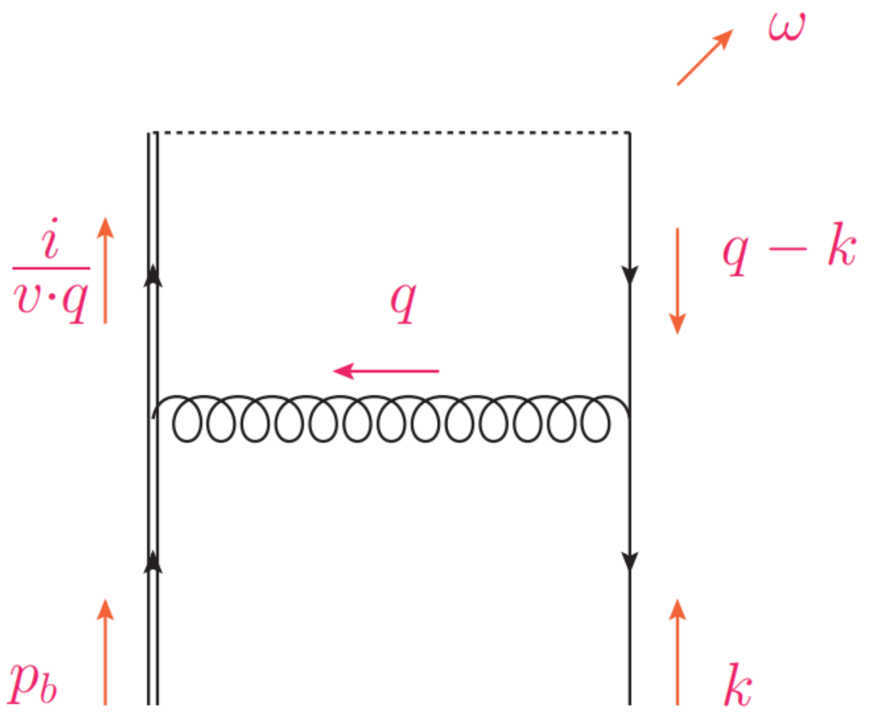
$$M_{Quasi}^{(1,b,real)} = \ldots$$

$$M_{Quasi}^{(1,b)} = M_{Quasi}^{(1,b,vir.)} + M_{Quasi}^{(1,b,real)}$$

Matching

Example 2,

Applying the default power counting scheme one can readily identify that the hard correction from the one-loop box diagram is power suppressed.



Here we take

$k^\mu \sim (k^+, k^-, \vec{k}_\perp) \sim (1, \lambda^2, \lambda)$

$v^\mu \sim (v^+, v^-, \vec{v}_\perp) \sim (\lambda^{-1}, \lambda, 0)$

$$\mathcal{M}^{(1,d)}_{\rm LCDA} = -ig_s^2 C_F \times \tilde{\mu}^{2\epsilon} \int \frac{d^d q}{(2\pi)^d} \frac{1}{(q-k)^2} \frac{1}{v\cdot q} \frac{1}{q^2} \bar{v}(k) \not{v} (\not{q} - \not{k}) \not{n}_+ \gamma^5 u_v(p_b) \delta(\omega v^+ - k^+ + q^+) ,$$

$$\mathcal{M}^{(1,d)}_{\rm Quasi} = -ig_s^2 C_F \times \tilde{\mu}^{2\epsilon} \int \frac{d^d q}{(2\pi)^d} \frac{1}{(q-k)^2} \frac{1}{v\cdot q} \frac{1}{q^2} \bar{v}(k) \not{v} (\not{q} - \not{k}) \gamma^z \gamma^5 u_v(p_b) \delta(\omega v^z - k^z + q^z) .$$

硬区域： $(q^+, q^-, \vec{q}_\perp) \sim (1, 1, 1)$,

共线区域： $(q^+, q^-, \vec{q}_\perp) \sim (1, \lambda^2, \lambda)$,

软区域： $(q^+, q^-, \vec{q}_\perp) \sim (\lambda^2, \lambda^2, \lambda^2)$.

Matching

Example 2,

In hard region,

$$\mathcal{M}_{\text{Quasi}}^{(1,d,\text{hard})} = -ig_s^2 C_F \tilde{\mu}^{2\epsilon} \int \frac{d^d q}{(2\pi)^d} \frac{1}{2(q^+ - k^+)q^- - q_\perp^2} \frac{1}{v^+ q^-} \frac{1}{q^2} v^- \times \bar{v}(k) \not{n}_+ (\not{q} - \not{n}_- k^+) \gamma^z \gamma^5 u_v(p_b) \delta(\omega v^z - k^z + q^z) \, .$$

Here $\int d^d q \sim 1, \quad 2(q^+ - k^+)q^- - q_\perp^2 \sim 1, \quad v^+ q^- \sim \lambda^{-1}, \quad q^2 \sim 1, \quad v^- \sim \lambda.$

So $M_{Quasi}^{(1,d,hard)} \sim \lambda^2$, it is $O(\lambda^2)$ suppressed.

Matching

$$H(\xi,\,\omega,\,n_z\cdot v,\,\mu)=\delta(\xi-\omega)$$
$$+\frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left\{\frac{1}{\omega-\xi}\,\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\,\theta(-\xi)\,\,\theta(\omega)\right.$$
$$+\left\{\frac{1}{\omega-\xi}\,\left[3-2\,\left(1+\frac{2\,\xi}{\omega}\right)\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\omega-\xi)}\right)-\frac{2\,\xi}{\omega}\,\left(\ln\left(\frac{\omega-\xi}{\xi}\right)+1\right)\right]\right\}_{\oplus}$$
$$\times\theta(\xi)\,\,\theta(\omega-\xi)$$
$$+\left\{\frac{1}{\xi-\omega}\,\left[3-2\,\ln\left(\frac{\mu}{2\,n_z\cdot v\,(\xi-\omega)}\right)-\frac{2\,\xi}{\omega}\,\ln\left(\frac{\xi}{\xi-\omega}\right)\right]\right\}_{\oplus}\,\theta(\omega)\,\,\theta(\xi-\omega)$$
$$\left.+2\,\left[\ln^2\frac{\mu}{n_z\cdot v\,\xi}-3\,\ln\frac{\mu}{n_z\cdot v\,\xi}+f(a)\right]\,\delta(\xi-\omega)\right\}$$

$$\begin{aligned}f(a)\quad=\quad&\ln\frac{a^2}{4(a-1)^3}\,\ln\frac{\mu}{n_z\cdot v\,\xi}+\ln(a-1)\,\ln\frac{8\,(a-1)}{a}\\&+\mathrm{Li}_2(1-a)+\ln a\,\ln\left(\frac{a}{4}\right)-\frac{1}{2}\ln(a-1)\\&+\ln(8\,a)+\ln^2 2+\frac{\pi^2}{8}-3\end{aligned}$$

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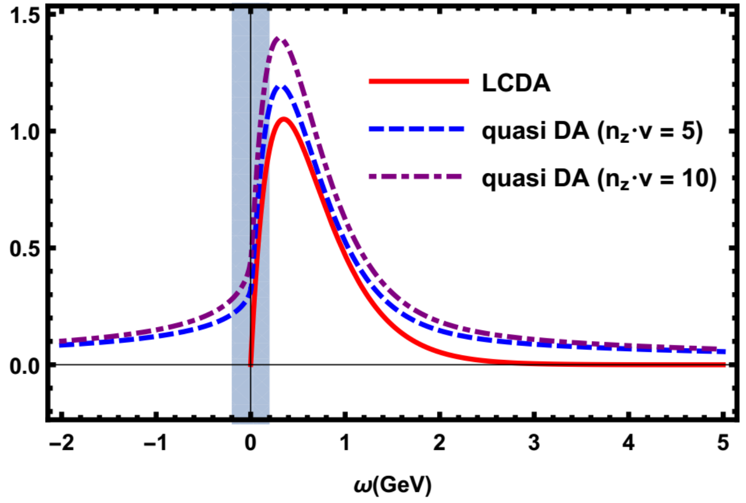
- **Summary**

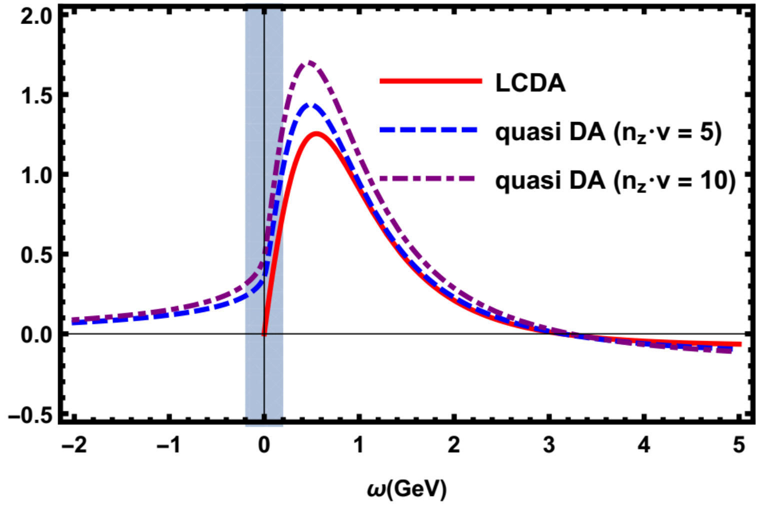
Perspectives for lattice calculations

- It will be instructive to understand the characteristic feature of $\varphi_B^+(\xi, \mu)$ with distinct nonperturbative models of $\phi_B^+(\omega, \mu)$.
- Taking advantage of the two phenomenological models motivated by the HQET sum rule calculation at leading order and at next-to-leading order

$$\phi_{B,\,\mathrm{I}}^+(\omega,\,\mu = 1.5\,\mathrm{GeV}) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0},$$

$$\phi_{B,\,\mathrm{II}}^+(\omega,\,\mu = 1.5\,\mathrm{GeV}) = \left[\frac{1}{k^2+1} - \frac{2\,(\sigma_B^{(1)}-1)}{\pi^2}\,\ln k\right] \times \frac{4}{\pi\,\omega_0}\,\frac{k}{k^2+1},\quad k = \frac{\omega}{1\,\mathrm{GeV}},$$





Perspectives for lattice calculations

- Our major objective is to explore the opportunity of accessing the light-cone dynamics of $\phi_B^+(\omega,\mu)$ starting from the Euclidean space.

- Fairly encouraging results from the state-of-art computations of the nucleon PDFs and the light-meson distribution amplitudes vidently demonstrate that the newly constructed LaMET formalism allows for a promising future.

- The infamous signal-to-noise problem indeed brought progress in the lattice HQET to a stop in the beginning of nineties. JHEP 08 (2005) 051

- Major challenges for the advanced lattice QCD lie in constructing a fully non-perturbative renormalization program for the (non-local) heavy-light currents.

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Summary

- It is highly desirable to predict parton observables from *ab initio* calculations such as lattice QCD.

- LaMET provides a systematically improvable method to calculate parton physics from first principles.

- Improvements of LaMET

  - ➢ Large hadron momentum.
  - ➢ Renormalization. X. Ji et al., 2008.03886
  - ➢ Higher-order perturbative matching. L.-B. Chen, W.Wei and R.Zhu, arXiv: 2006.14825 Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, arXiv: 2006.12370
  - ➢ Power corrections.

Thanks！