The semi-leptonic form factors of $\Xi_b \to \Xi_c$ and $\Lambda_b \to \Lambda_c$ in QCD sum rules

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preliminary

Outline

- Motivation
- Two-point correlation function: pole residue
- Three-point correlation function: form factors
- Summary and outlook

Motivation

Eur. Phys. J. C (2020) 80:568
https://doi.org/10.1140/epjc/s10052-020-8096-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

QCD Sum Rules Analysis of Weak Decays of Doubly-Heavy Baryons

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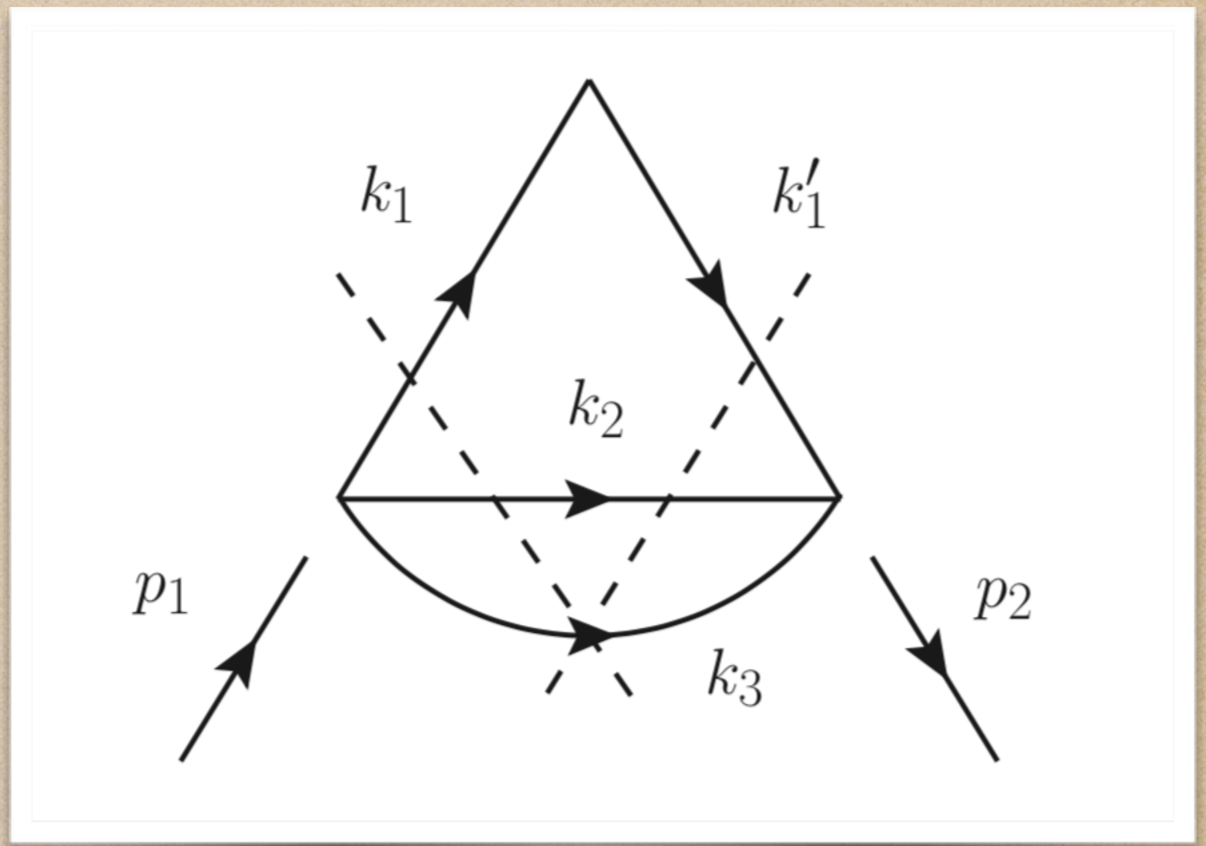
Received: 18 January 2020 / Accepted: 29 May 2020

- Eur.Phys.J.C 80 (2020) 6, 568

Two key techniques

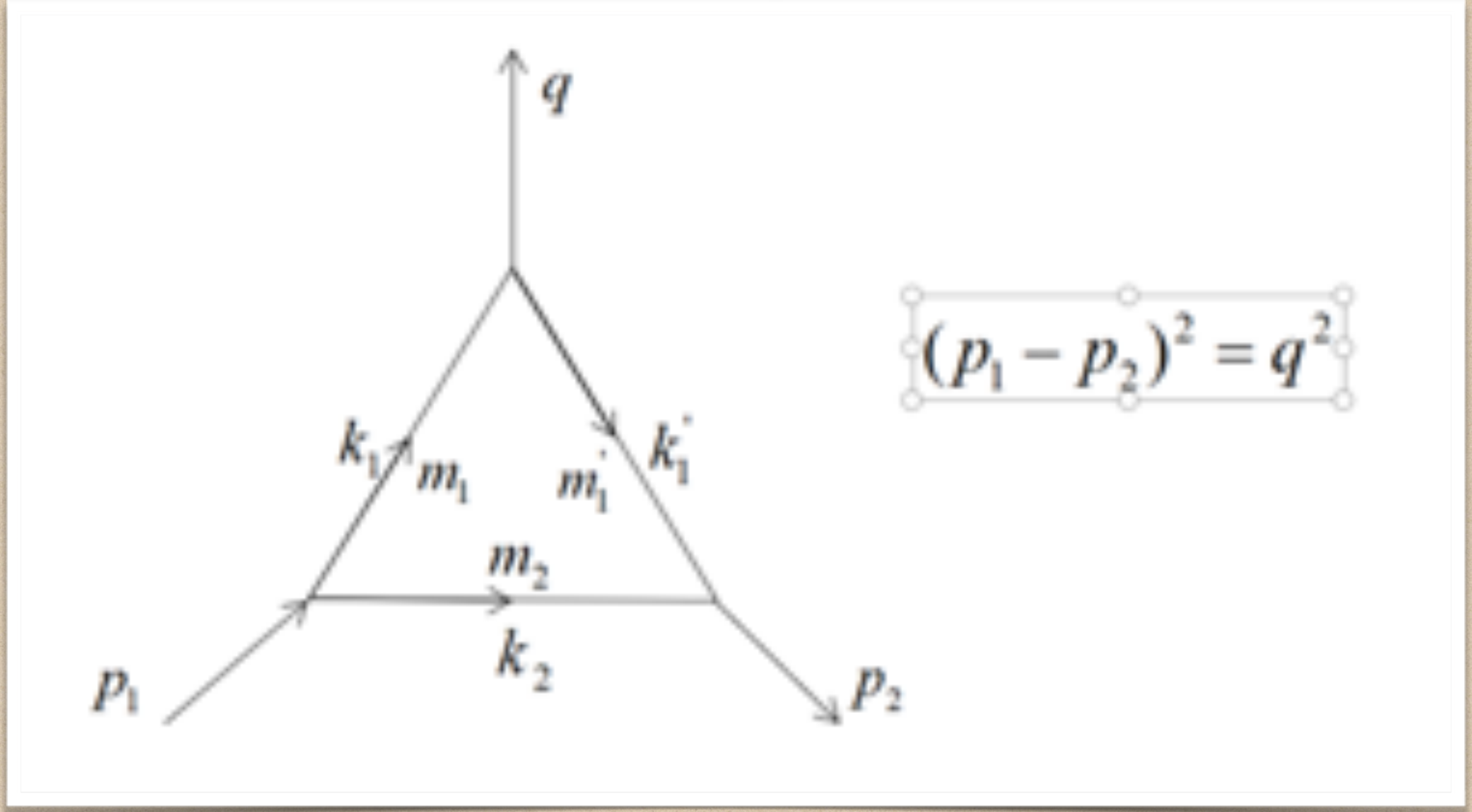
- Cutkosky cutting rules — Double Dispersion Relation (DDR)
- Dirac structures — Consider negative parity baryons' contributions



Why $\Xi_b \to \Xi_c$ and $\Lambda_b \to \Lambda_c$?

- Widely investigated: Quark model, HQET, Lattice, Exp.
- $\mathscr{B}(\Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e) = (6.2^{+1.4}_{-1.3}) \times 10^{-2}$ (PDG2016)

Verify the DDR



way 1

$$\Pi = \Pi(p_1, p_2, q, k_1, k_2, k_1', m_1, m_2, m_1') = \int \frac{d^4 k_2}{(2\pi)^4} \frac{1}{A_1 A_2 A_3}$$

Finite!

Verify the DDR

way 2

为了检验双重色散关系，我们还需要得到上述关联函数如下的色散积分：

$$\Pi\ (p_1^2, p_2^2, q^2) = \int_{m_1^2}^{\infty} ds_1 \int_{m_1'^2}^{\infty} dx_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} \tag{4-2-1}$$

DDR

其中，我们认为 m_1 和 m_1' 是重夸克。

在上式中 $\rho(s_1, s_2, q^2)$ 项就是谱密度：

$$\rho(p_1^2, p_2^2, q^2) = \frac{(-2\pi i)^3}{(2\pi i)^2} \frac{1}{(2\pi)^4} I_\Delta \tag{4-2-2}$$

其中：

$$I_\Delta = \int d^4k\,\delta(k_1^2 - m_1^2)\,\delta(k_2^2 - m_2^2)\,\delta(k_1'^2 - m_1'^2) \tag{4-2-3}$$

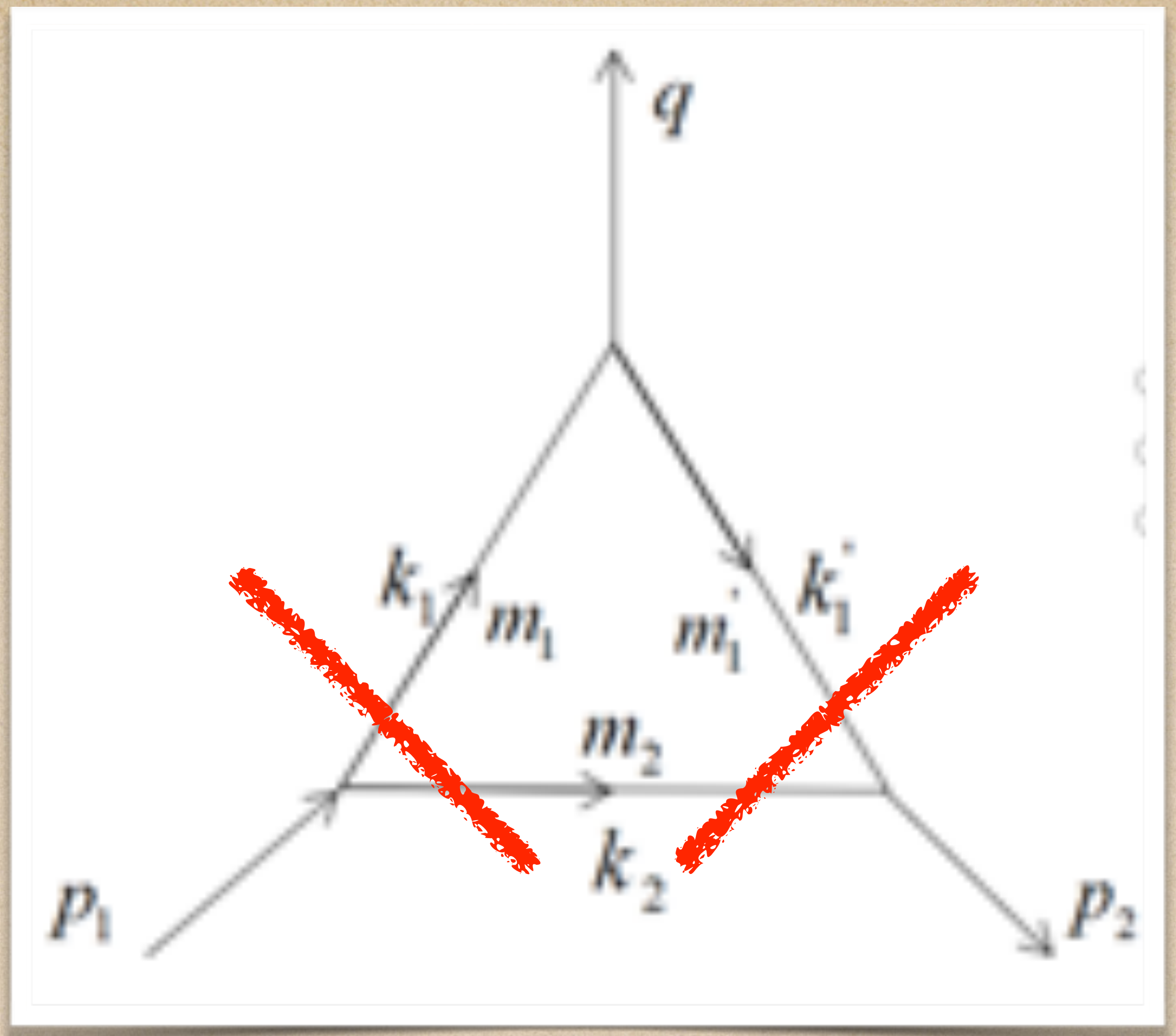
Verify the DDR

$p_1^{\ 2} = -100GeV^2$， $p_2^{\ 2} = -100GeV^2, q^2 = 0.0GeV^2,$

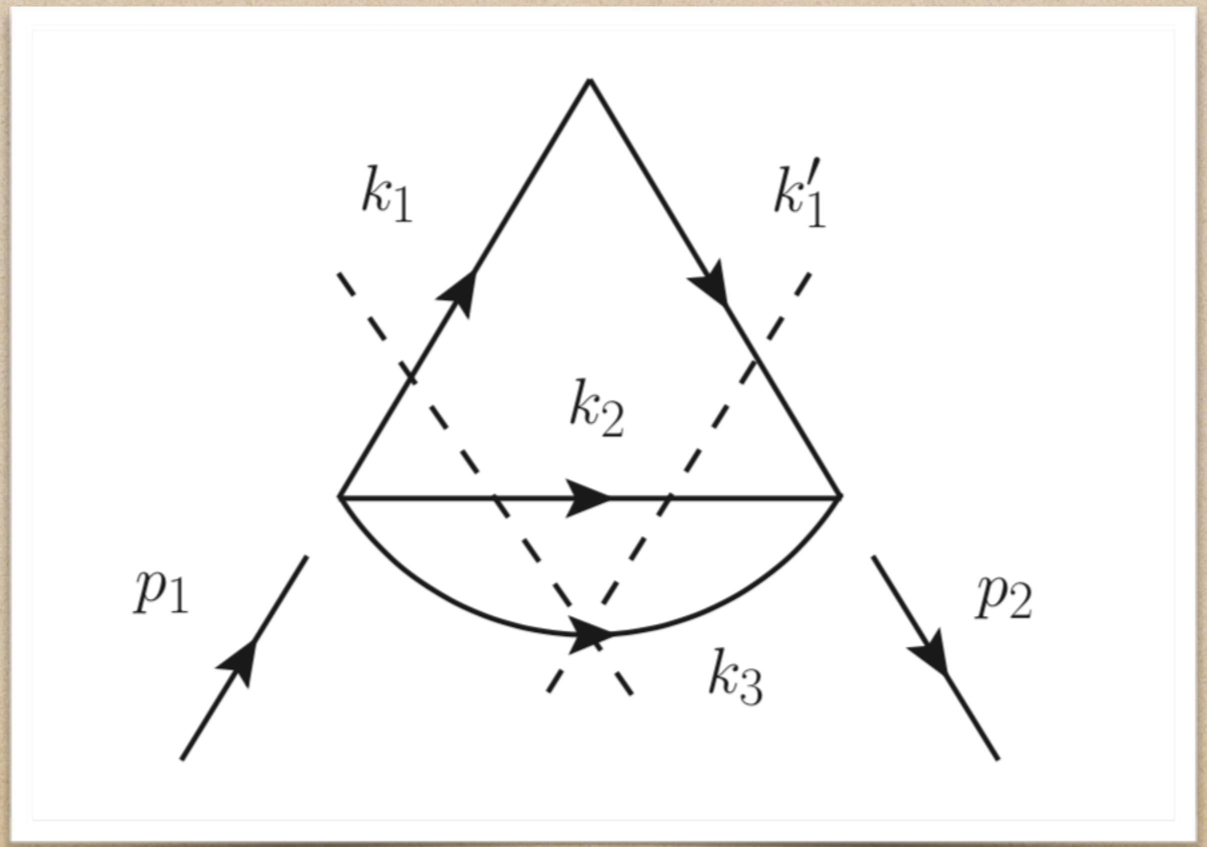
$m_1 = 4.7GeV, m_1^{'} = 1.35GeV, m_2 = 0.0GeV$

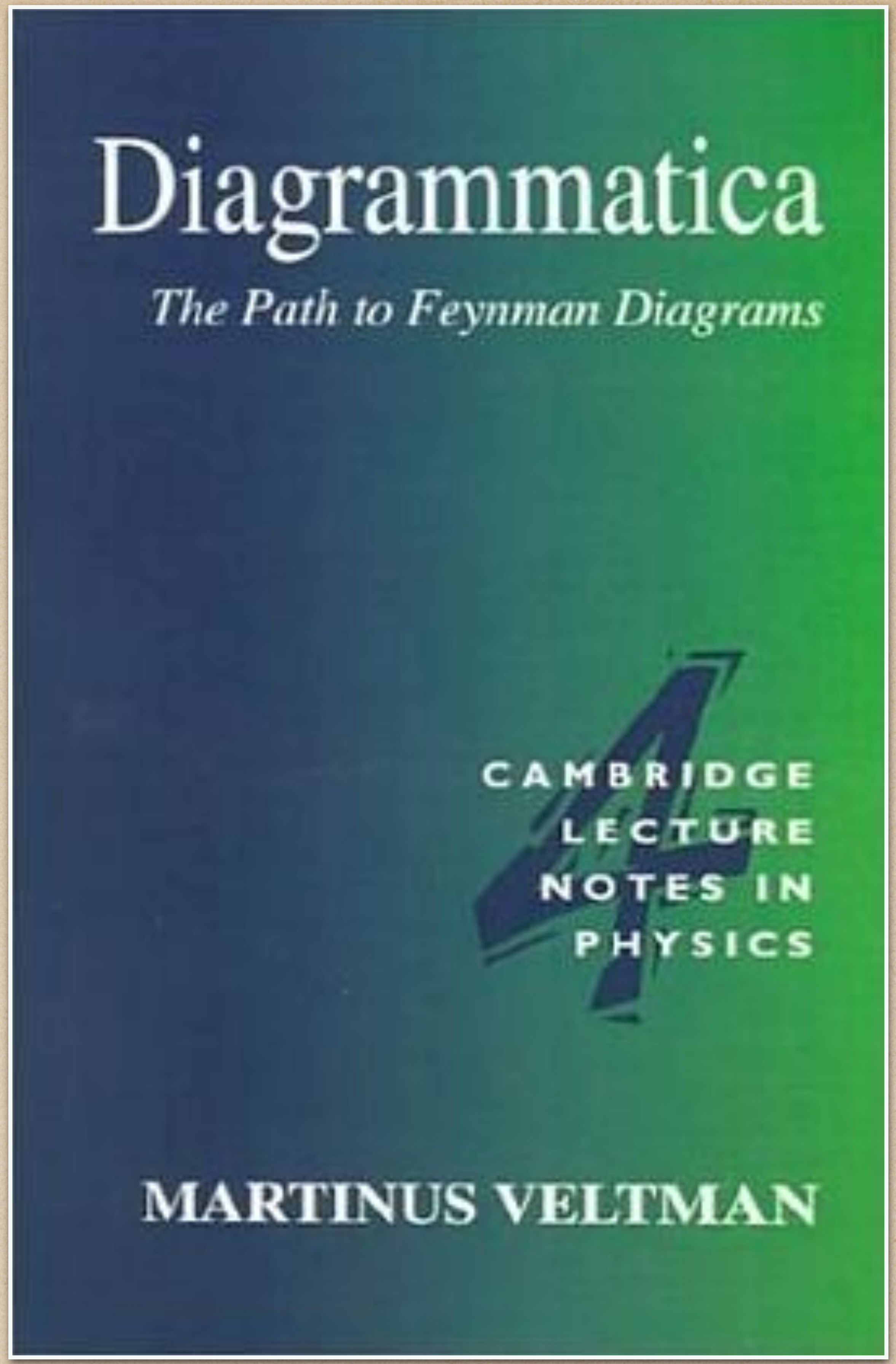
直接计算和色散关系形式的最终结果都约为 0-0.000151808i 左右，于是我们便从数值上验证了上述双重色散关系。

ffs
for
meson



ffs
for
baryon





Veltman:
Largest Time Equation

Too many Dirac structures

- Universal when using QCDSR method to deal with baryons
- The prescription of considering negative parity baryons' contribution may be a universal solution
- Can be seen in the following slides…

Two-point correlation function

$$J_{\Lambda_Q} = \epsilon_{abc}(u_a^T C\gamma_5 d_b)Q_c,$$
$$J_{\Xi_Q} = \epsilon_{abc}(q_a^T C\gamma_5 s_b)Q_c,$$

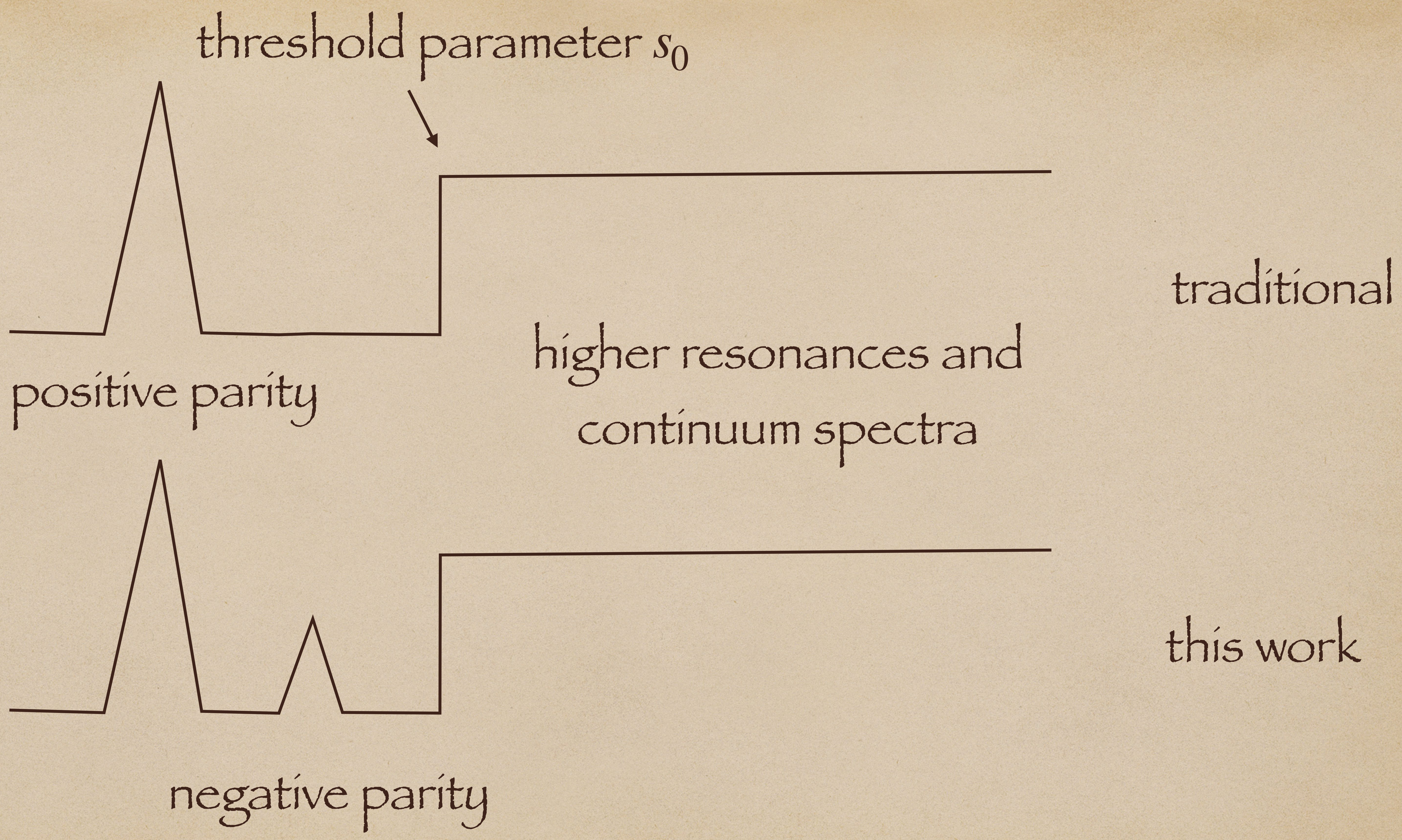
$$\Pi(p) = i\int d^4x\; e^{ip\cdot x}\langle 0|T\{J(x)\bar{J}(0)\}|0\rangle.$$

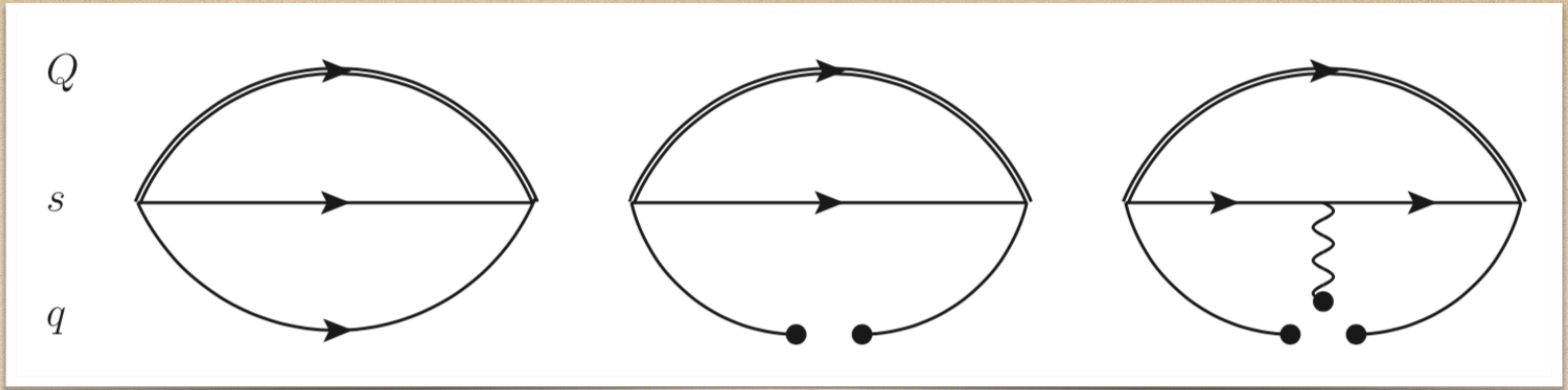
$$\Pi^{\rm QCD}(p) = A(p^2)\not{p} + B(p^2).$$

$$\Pi^{\rm had}(p) = \lambda_+^2 \frac{\not{p} + M_+}{M_+^2 - p^2} + \lambda_-^2 \frac{\not{p} - M_-}{M_-^2 - p^2} + \cdots,$$

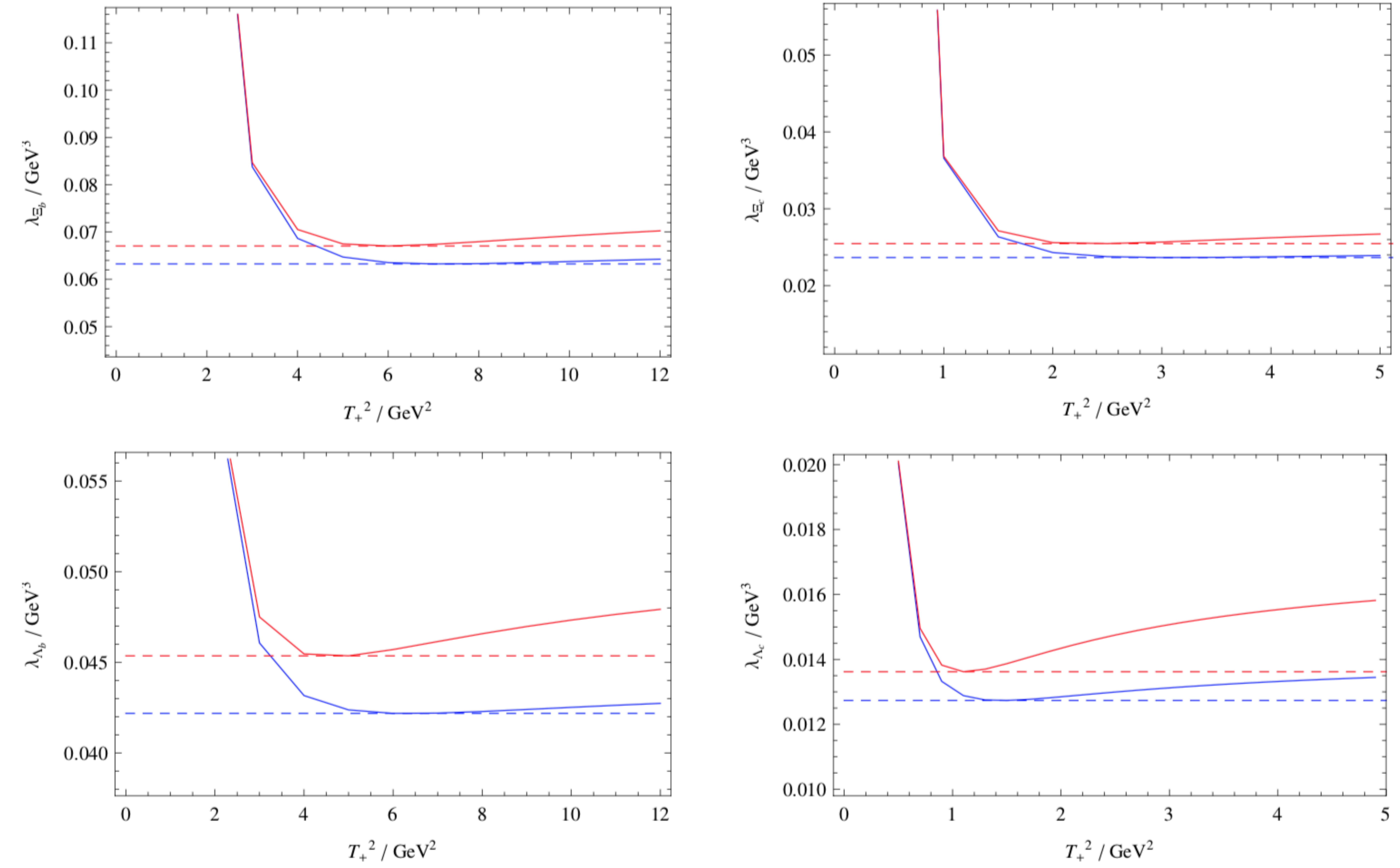
$$(M_+ + M_-)\lambda_+^2 \exp(-M_+^2/T_+^2) = \int_{m_Q^2}^{s_+} ds\; (M_-\rho^A + \rho^B)\exp(-s/T_+^2),$$

Weighted average


threshold parameter s_0
traditional
higher resonances and
continuum spectra
positive parity
this work
negative parity



- dim-0,3,5 for Ξ_Q, 1 for each
- dim-3,5 ~ m_s ===> Λ_Q only dim-0 survive
- dim-4 neglected


0.11
0.10
0.09
0.08
0.07
0.06
0.05
λ_{Ξ_b} / GeV^3
0 2 4 6 8 10 12
$T_+{}^2$ / GeV^2
0.05
0.04
0.03
0.02
λ_{Ξ_c} / GeV^3
0 1 2 3 4 5
$T_+{}^2$ / GeV^2
0.055
0.050
0.045
0.040
λ_{Λ_b} / GeV^3
0 2 4 6 8 10 12
$T_+{}^2$ / GeV^2
0.020
0.018
0.016
0.014
0.012
0.010
λ_{Λ_c} / GeV^3
0 1 2 3 4 5
$T_+{}^2$ / GeV^2

	$(s_+/\mathrm{GeV}^2,\ T_+^2/\mathrm{GeV}^2)$	λ_+/GeV^3	M_+/GeV	$M_+^{\exp}/\mathrm{GeV}$
Ξ_b	$(6.3^2, 7),\ (6.4^2, 6)$	0.0633 ± 0.0038	5.786 ± 0.016	5.793
Ξ_c	$(2.9^2, 3),\ (3.0^2, 2.5)$	0.0237 ± 0.0018	2.461 ± 0.029	2.468
Λ_b	$(6.0^2, 6)\ (6.1^2, 5)$	0.0422 ± 0.0032	5.612 ± 0.030	5.620
Λ_c	$(2.6^2, 1.5),\ (2.7^2, 1.1)$	0.0127 ± 0.0009	2.292 ± 0.008	2.286

optimal suboptimal

- The accuracy is very high
- For Λ_Q, only dim-0
- $\overline{\mathrm{MS}}$ mass for quark — not pole mass
- Results of pole residues are different in the two schemes

Three-point correlation function

$$\Pi_\mu^{V,A}(p_1^2,p_2^2,q^2) = i^2\int d^4x d^4y\ e^{-ip_1\cdot x+ip_2\cdot y}\langle 0|T\{J_{\Xi_c}(y)(V_\mu,A_\mu)(0)\bar{J}_{\Xi_b}(x)\}|0\rangle,$$

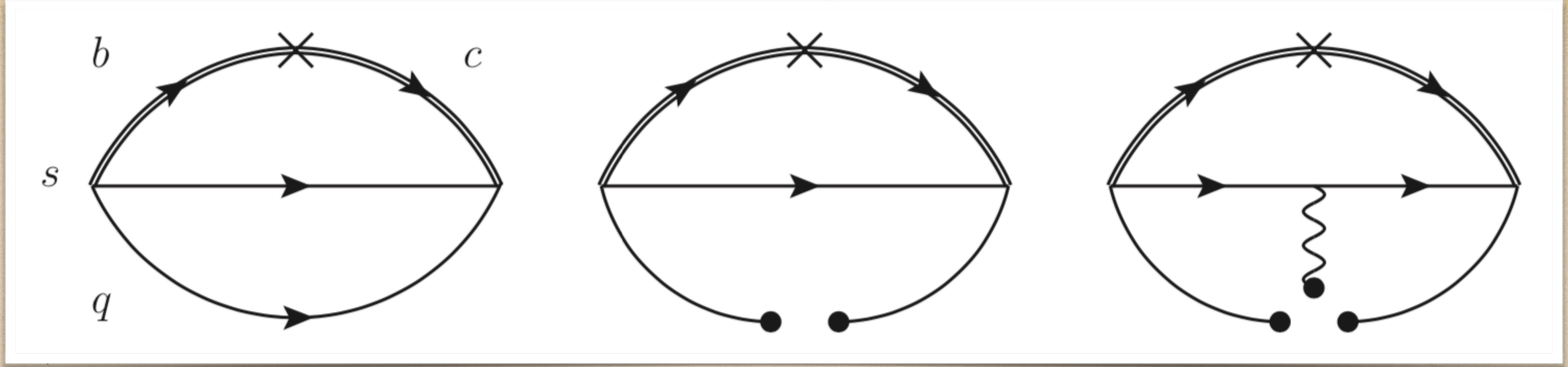
$$\begin{aligned}\Pi_\mu^{V,\text{had}}(p_1^2,p_2^2,q^2) &= \lambda_f^+\lambda_i^+\frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^+}F_1^{++}+\frac{p_{2\mu}}{M_2^+}F_2^{++}+\gamma_\mu F_3^{++})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{+2})}\\ &+ \lambda_f^+\lambda_i^-\frac{(\not{p}_2+M_2^+)(\frac{p_{1\mu}}{M_1^-}F_1^{+-}+\frac{p_{2\mu}}{M_2^+}F_2^{+-}+\gamma_\mu F_3^{+-})(\not{p}_1-M_1^-)}{(p_2^2-M_2^{+2})(p_1^2-M_1^{-2})}\\ &+ \lambda_f^-\lambda_i^+\frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^+}F_1^{-+}+\frac{p_{2\mu}}{M_2^-}F_2^{-+}+\gamma_\mu F_3^{-+})(\not{p}_1+M_1^+)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{+2})}\\ &+ \lambda_f^-\lambda_i^-\frac{(\not{p}_2-M_2^-)(\frac{p_{1\mu}}{M_1^-}F_1^{--}+\frac{p_{2\mu}}{M_2^-}F_2^{--}+\gamma_\mu F_3^{--})(\not{p}_1-M_1^-)}{(p_2^2-M_2^{-2})(p_1^2-M_1^{-2})}\\ &+ \cdots .\end{aligned}$$

$$\frac{\lambda_i^+ \lambda_f^+ (F_1^{++}/M_1^+)}{(p_1^2 - M_1^{+2})(p_2^2 - M_2^{+2})} = \frac{\{M_1^- M_2^-, M_2^-, M_1^-, 1\}.\{A_1, A_2, A_3, A_4\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)},$$

$$\frac{\lambda_i^+ \lambda_f^+ (F_2^{++}/M_2^+)}{(p_1^2 - M_1^{+2})(p_2^2 - M_2^{+2})} = \frac{\{M_1^- M_2^-, M_2^-, M_1^-, 1\}.\{A_5, A_6, A_7, A_8\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)},$$

$$\frac{\lambda_i^+ \lambda_f^+ F_3^{++}}{(p_1^2 - M_1^{+2})(p_2^2 - M_2^{+2})} = \frac{\{M_1^- M_2^-, M_2^-, M_1^-, 1\}.\{A_9, A_{10}, A_{11}, A_{12}\}}{(M_1^+ + M_1^-)(M_2^+ + M_2^-)}.$$

Weighted average



- dim-0,3,5 for $\Xi_b \to \Xi_c$, 1 for each
- dim-3,5 ~ m_s ===> $\Lambda_b \to \Lambda_c$ only dim-0 survives
- dim-4 neglected

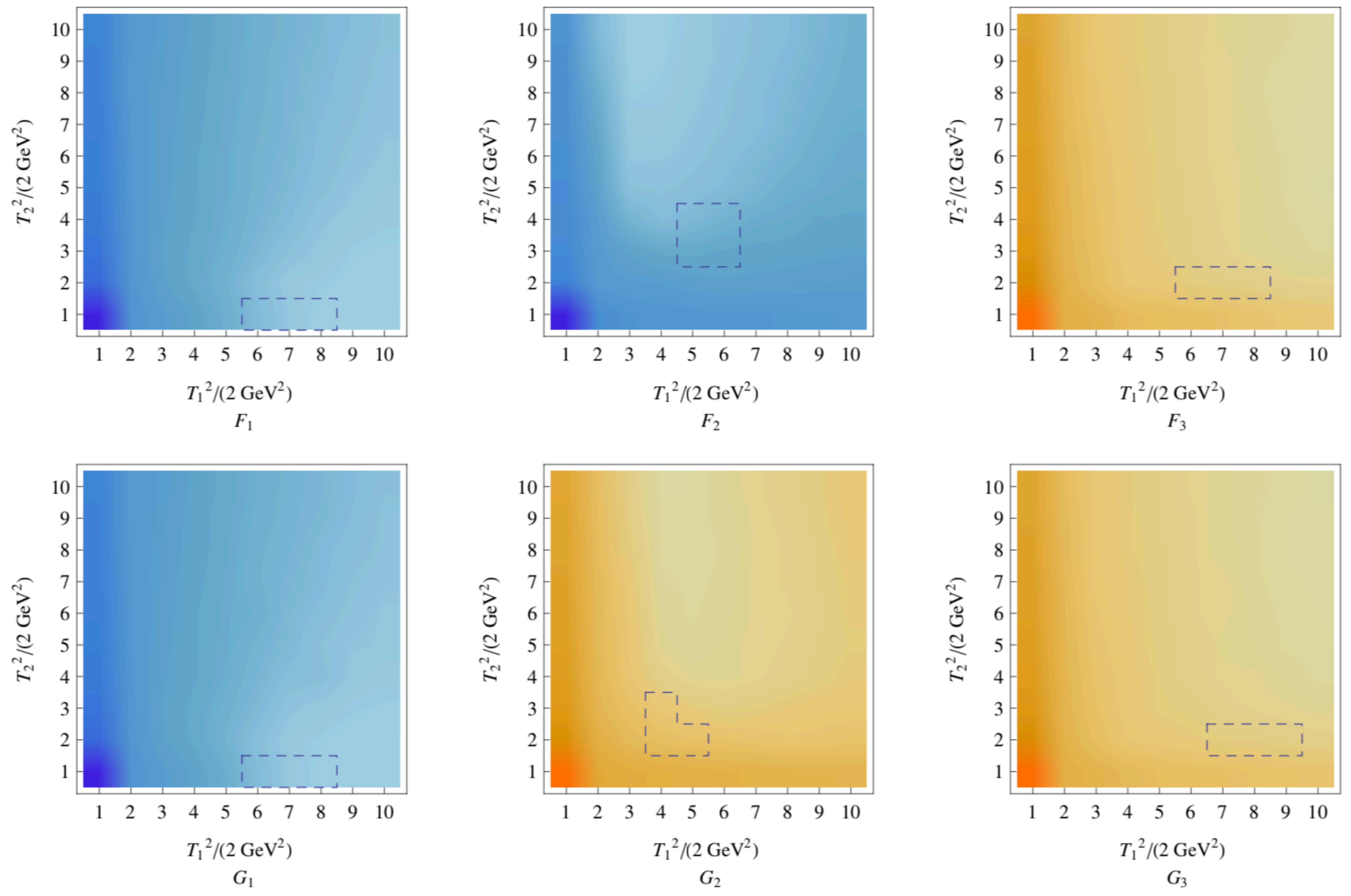
- Pole dominance. It can be described quantitatively by

$$r \equiv \frac{\int^{s_1^0} ds_1 \int^{s_2^0} ds_2 \rho^{\rm QCD}(s_1, s_2, q^2) \exp\left(-s_1/T_1^2 - s_2/T_2^2\right)}{\int^{\infty} ds_1 \int^{\infty} ds_2 \rho^{\rm QCD}(s_1, s_2, q^2) \exp\left(-s_1/T_1^2 - s_2/T_2^2\right)} \gtrsim 0.5 \tag{26}$$

- OPE convergence. It can be achieved by demanding that dim-5/Total should be small enough.

- Weak dependence on the Borel parameters.

The constraint is too strong



$T_2{}^2/(2\ \mathrm{GeV}^2)$
$T_1{}^2/(2\ \mathrm{GeV}^2)$
F_1
F_2
F_3
G_1
G_2
G_3

Transition	F	This work	LQCD [4]	HQET [12]
$\Xi_b \to \Xi_c$	F_1	$(-0.176, -0.490)$	- -	-0.321
	F_2	$(-0.062, -0.159)$	- -	-0.094
	F_3	$(0.784, 1.928)$	- -	1.415
	G_1	$(-0.218, -0.589)$	- -	-0.321
	G_2	$(0.107, 0.305)$	- -	0.094
	G_3	$(0.545, 1.047)$	- -	1
$\Lambda_b \to \Lambda_c$	F_1	$(-0.163, -0.452)$	$(-0.174, -0.419)$	-0.321
	F_2	$(-0.060, -0.173)$	$(-0.010, -0.086)$	-0.094
	F_3	$(0.751, 1.891)$	$(0.558, 1.492)$	1.415
	G_1	$(-0.198, -0.576)$	$(-0.210, -0.493)$	-0.321
	G_2	$(0.097, 0.304)$	$(0.082, 0.196)$	0.094
	G_3	$(0.525, 1.244)$	$(0.388, 0.907)$	1

$(F(0), F(q^2_{max}))$ $(F(0), F(q^2_{max}))$ $F(q^2_{max})$ at $1/m_Q$

Channel	This work	Lattice QCD [4]	Experimental data [5]
$\Xi_b \to \Xi_c e^- \bar{\nu}_e$	9.31 ± 1.58	$--$	$--$
$\Lambda_b \to \Lambda_c e^- \bar{\nu}_e$	9.41 ± 2.71	5.32 ± 0.35	$6.2^{+1.4}_{-1.3}$

- $m_c(m_b) = 0.906$ GeV at 2-loop ===>
  $m_c(m_b) = 0.997$ GeV at 1-loop
- Consider the scale dependence of interpolating currents
- The definition of pole dominance should be relaxed.

Summary and outlook

- $T\{Q_1(x)Q_2(0)\} = \sum_i C_i(x)O_i(0)$, current masses for quarks
- Full LO results in QCD for the Wilson coefficients are obtained
- Heavy quark limit can be achieved from the full QCD results ===> doubly heavy baryons
- The two key techniques are correct
- Can be safely applied to evaluate the four-quark operator matrix elements for lifetimes of baryons

Thank you for your attention!

Backup

$$
\begin{aligned}
f_1 &= \left[1 + \left(\frac{\bar{\Lambda}_\Lambda}{2m_c} + \frac{\bar{\Lambda}_\Lambda}{2m_b}\right)\right]\zeta(w), \\
f_2 &= -\frac{\bar{\Lambda}_\Lambda}{m_c}\left(\frac{1}{1+w}\right)\zeta(w), \\
f_3 &= -\frac{\bar{\Lambda}_\Lambda}{m_b}\left(\frac{1}{1+w}\right)\zeta(w), \\
g_1 &= \left[1 - \left(\frac{\bar{\Lambda}_\Lambda}{2m_c} + \frac{\bar{\Lambda}_\Lambda}{2m_b}\right)\left(\frac{1-w}{1+w}\right)\right]\zeta(w), \\
g_2 &= -\frac{\bar{\Lambda}_\Lambda}{m_c}\left(\frac{1}{1+w}\right)\zeta(w), \\
g_3 &= \frac{\bar{\Lambda}_\Lambda}{m_b}\left(\frac{1}{1+w}\right)\zeta(w).
\end{aligned}
\tag{4.47}
$$

$$\bar{\Lambda}_\Lambda = 0.9 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}$$