A new extraction of Gegenbauer moments

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a2pi-LCSRs

Overview

Applying pQCD theory on hadron involved observation,
hadron is presented by LCDA,

- DA: matrix element of nonlocal light-ray operator sandwiched between $\langle h|$ and $|0\rangle$.
- Physics: transparent in the infinite reference frame $P^3 \to \infty$ $(P_\perp = 0)$,
  hadron momentum $P = (P^+, P^-, P_\perp)$,
  x_i distribution of partons in a hadron at small b_i.
- Isochronous hadron Bethe-Salpeter wave function $\phi_{BS}(x_i, k_{\perp i}, \lambda_i)$,
  $\sum_{i=1}^n k_{\perp i} = 0, \quad \sum_{i=1}^n x_i = 1.$
- Light cone gauge $A^+ = A^0 + A^3 = 0$, physical partons,
  LCDA relates to the k_T integrals of ϕ_{BS},

$$\psi_n(x_i, \lambda_i) = \int^{|k_{\perp i}|<\mu} d^2 k_{\perp i}\, \phi_{BS}(x_i, k_{\perp i}, \lambda_i) \tag{1}$$

- ie., $|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$.
  for each component n, $\psi_n^\pi(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i|\pi\rangle$
- In large momentum transfer processes Q^2,
  $k_\perp$ can been neglected/integrated $\psi_n^\pi(x_i, \lambda_i)$,
  put out the spin and obtain the LCDA $\psi_n^\pi(x_i, Q)$.
- High order correction $\mathcal{O}(\alpha_s, k_\perp^2/Q^2, m^2/Q^2)$, scale dependence,
  RGE with the general solution in terms of Gegenbauer polynomials.
- asymptotic behaviour ($Q^2 \to \infty$): a_0^π determined by f_π;
  correlation: $a_{n\geqslant 2}^\pi(\mu)$ determined by non-perturbative theory.

- Phenomenologies in exclusive hard processes (factorisation methods):
  meson/baryon FFs, weak decay FFs, $B \to M_1 M_2$,
  radiative and semileptonic decays (FCNC) $\to$ NP.

  ie., the first few a_n^π is of utmost importance to an accuracy description of pion.

- In QCD,

$$a_n^\pi(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots |0\rangle \,, \quad (2)$$

- LQCD: $a_2^\pi(1GeV) =$

  0.334 ± 0.129 [R. Arthur et.al.(RBC and UKQCD), arXiv:1011.5906[hep-lat]]

  0.135 ± 0.032 [G. S. Bali et.al.(RQCD), arXiv:1903.03038[hep-lat]]

  a_4^π is not available $\leftarrow$ the growing number of derivatives in $q\bar{q}$ operator.
  new technique is being developed [G. S. Bali et al.(RQCD), arXiv:1709.04325,1807.06671[hep-lat]] .

- 2pSRs: $a_2^\pi(1GeV) =$

  0.19 ± 0.06, [V.L. Chernyak and A.R. Zhitnitsky, Phys.Rept. 112 (1984) 173]

  $0.26^{+0.21}_{-0.09}$, [A. Khodjamirian, T. Mannel and M. Melcher, hep-ph/0407226]

  0.28 ± 0.08, [P. Ball, V. Braun and A. Lenz, hep-ph/0603063]

  nonlocal vacuum condensate is introduced and modelled for $a_{n>2}^\pi$.
  [A.P. Bakulev, S.V Mikhailov and N.G Stefanis, hep-ph/0103119]

‡ QCD sum rules as an inverse problem, [H-n Li and H. Umeeda, arXiv:2006.16593[hep-ph]]
quark-hadron duality → Legendre expansion of spectral density

Background-a_n^π

- LCSRs: $a_2^\pi(1GeV) =$

$F_{B\to\pi}$: 0.19 ± 0.19, [P. Ball and R. Zwicky, hep-ph/0507076]
0.16, [A, Khodjamirian et al., 1103.2655 [hep-ph]] , uncertainty from B meson.

$F_{\pi\gamma\gamma^*}$: 0.14, BABAR+CLEO data , [S.S Agaev et.al., 1012.4671[hep-ph]]
0.10, Belle+CLEO data , [S.S Agaev et al., 1206.3968[hep-ph]]

∗ large uncertainty is obtained for the result of $a_{n>2}^\pi$.

∗ a mutual discrepancy between the results of different experiments, especially at large Q^2.

F_π: 0.24 ± 0.17, Wilson Lab+NA7 data, [C.J. Bebek, et al., Phys.Rev.D 17 (1978) 1693]
0.20 ± 0.03, Wilson Lab+JLab data [S.S Agaev, hep-ph/0509345]

∗ large uncertainty is obtained for the result of $a_{n>2}^\pi$.

- The valid region of LCSRs for F_π is $|q^2| \in [1, 10]$ GeV^2, the accurate measurement of F_π at JLab is limited to $|q^2| \leqslant 2.5$ GeV^2, more data can be expected to extract a_n^π precisely at JLab 12 GeV upgrade.
- The measurements of $F_\pi(q^2)$ at B factories provide independent an opportunity.

  BABAR: $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 , [J. Lees et al., 1205.2228 [hep-ex]]
  Belle: $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2, [M. Fujikawa et al., 0805.3773 [hep-ex]]
- BABAR measurement in timelike region, LCSR calculation in spacelike region, dispersion relation to relate these two FFs.
- dispersion relation is written in the integral of whole timelike region $[4m_\pi^2, \infty)$, the high energy tail should be considered, even though small.

Dispersion relation (DR)

The standard DR

$$F_\pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}F_\pi(s)}{s - q^2 - i\epsilon}\,, \quad q^2 < 0\,, \tag{3}$$

- integral over $\mathrm{Im}F_\pi(s)$ in $s > 4m_\pi^2$,
- the direct measurement at BABAR is $|F_\pi(s)|^2$,
- model dependence to parameterize $F_\pi(s)$ to reproduce $|F_\pi(s)|$, additional uncertainty.

The modulus representation of DR

$$\frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0 - q^2}} = \frac{1}{2\pi}\int_{s_0}^{\infty} \frac{ds\, \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}, \qquad q^2 < s_0, \tag{4}$$

$$\Leftrightarrow \qquad F_\pi(q^2) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} \frac{ds\, \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right], \quad q^2 < s_0. \tag{5}$$

The modulus representation of DR

- obtained by introducing an auxiliary function $g_\pi(q^2) \equiv \frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0 - q^2}}$.

  [B.V. Geshkenbein, arXiv: hep-ph/9806418]

- derived by Cauchy theorem and Schwartz reflection principle for g_π function.
- The only assumption is that the $F_\pi(q^2)$ is free from zeros in the complex q^2 plane. then $\ln F_\pi(q^2)$ does not diverge. [H. Leutwyler, [arXiv:hep-ph/0212324]

  [B. Ananthanarayan, I. Caprini and I. Imsong, arXiv:1102.3299 [hep-ph]]

- If $F_\pi(q^2)$ has zeros in the complex q^2 plane, deserves a separate analysis.

  [C.A. Dominguez, hep-ph/0102190], [B. Ananthanarayan, et al., hep-ph/0409222]

- $F_\pi(q^2)$ evaluated by the standard and modulus DR have a tiny difference, $\rightarrow$ the zeros of $F_\pi(q^2)$ are either absent or their influence is beyond our accuracy.

$F_\pi(|q^2|)$ from LCSRs

LCSRs: QCD $\leftrightarrow$ hadron spectral,
spacelike π transition, electromagnetic FFs; $B \to \pi$ FFs.

LCSRs on $F_\pi(|q^2|)$: energetic pion, [J. Bijnens and A. Khodjamirian, hep-ph/0206252] [V. Braun, A. Khodjamirian and M. Maul, hep-ph/9907495]

- soft dominated, one quark carries almost the whole momentum.
- asymptotic LCDA, asymptotic result of pQCD.
- $F_\pi(Q^2 = |q^2|)$ in terms of LCDA, Gegenbauer moments dependence.
- current accuracy:

$$F_\pi^{(\mathrm{LCSR})}(Q^2) = F_\pi^{(\mathrm{as})}(Q^2) + \sum_{n=2,4,..} a_n(\mu_0)\, f_n(Q^2, \mu, \mu_0)\,, \tag{6}$$

$$F_\pi^{(\mathrm{as})}(Q^2) = F_\pi^{(\mathrm{tw2,as})}(Q^2) + F_\pi^{(\mathrm{tw4,\,LO})}(Q^2) + F_\pi^{(\mathrm{tw6,\,fact})}(Q^2) \tag{7}$$

the coefficient functions $f_n(Q^2, \mu, \mu_0)$ are the integral of Gegenbauer polynomials with the Borel exponent.

$F_\pi(s)$ from the BABAR measurement, $[4m_\pi^2, 2.95^2]$ GeV^2

$$F_\pi^{data}(s) = \sum_{n=0,\cdots}^{N} \frac{c_n^\pi BW_n^{GS}(s)}{c_n^\pi}, \quad c_0^\pi \to \frac{1 + c_\omega^\pi \, BW_n^{KS}(s)}{1 + c_\omega^\pi}\,, \tag{8}$$

$$BW_{\rho_n}^{GS}(s) = \frac{m_n^2 + m_n \Gamma_n d(m_n)}{m_n^2 - s + f(s) - i\sqrt{s}\Gamma_n(s)}\,, \quad BW_\omega^{KS}(s) = \frac{m_\omega^2}{m_\omega^2 - s - i\, m_\omega \Gamma_\omega}\,. \tag{9}$$

- Vector dominate model (VDM), with the modified Breit-Winger formula, Gounaris-Sakuria (GS) and Kühn-Santamaria (KS) representations,
  [G.J. Gounaris and J.J Sakuria, Phys.Rev.Lett. 21(1968)244], [J. H. Kühn and A. Santamaria, Z.Phys.C48(1990)445]
- $N = 4$ & $\rho - \omega$ interaction, describes the BABAR data by 18 parameters.
  [J. Lees et al., 1205.2228 [hep-ex]]

Spacelike and timelike π electromagnetic FFs

High energy tail of $F_\pi(s)$, $[2.95^2, \infty)$ GeV^2

- The application of $F_\pi^{data}(s)$ at high energy tails is not physical: resonances above $N = 4$ are not included, $F_\pi^{data}(s) \nrightarrow 1/s$ at adequate large s.
- The dual-resonance models and $N_c = \infty$ limit of QCD.

[C.A. Dominguez, hep-ph/0102190], [C. Bruch, A. Khodjamirian and J H. Kuhn, hep-ph/0409080]

$$F_\pi^{(tail)}(s) = F_\pi^{(dQCD)}(s) = \sum_{n=0}^{\infty} c_n \, BW_n(s) \,, \quad BW_n(s) = BW_n^{KS}(s), \quad (10)$$

$$c_n = \frac{(-1)^n \Gamma(\beta - 1/2)}{\alpha' m_n^2 \sqrt{\pi} \, \Gamma(n+1)\Gamma(\beta - 1 - n)} \,, \quad m_n^2 = m_\rho^2(1 + 2n) \,. \quad (11)$$

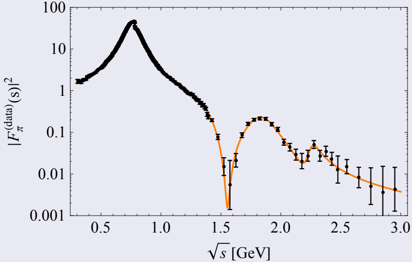
- $\alpha' = 1/2m_\rho^2$, $\Gamma_n = \gamma \, m_n$ and $\gamma = 0.193$ is adjusted to the total width of $\rho(770)$,
- The matching condition $|F_\pi^{(data)}(s_{max})| = |F_\pi^{(tail)}(s_{max})|$ indicates $\beta = 2.09 \pm 0.13$.

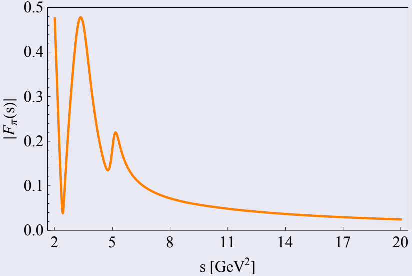
† the resonance width partially account for the coupling to the continuum of intermediate hadronic states.

† reproduce $F_\pi^{(\mathrm{dQCD})}(0) = 1$ and $\lim_{s \to -\infty} F_\pi^{(\mathrm{dQCD})}(s) \sim 1/s^{\beta - 1}$.

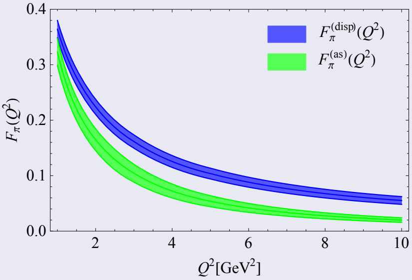
$F_\pi(s)$: $[4m_\pi^2, 2.95^2] + [2.95^2, \infty)$ GeV^2

$$|F_\pi(s)| = \Theta(s_{\rm max} - s)\,|F_\pi^{\rm (data)}(s)| + \Theta(s - s_{\rm max})\,|F_\pi^{\rm (tail)}(s)|\,. \qquad (12)$$





$F_\pi(|q^2|)$ obtained from asymptotic LCSRs and DR



- substitute Eq.(12) to Eq.(5) $\to F_\pi^{(disp)}(Q^2)$, Eq.(7) $\to F_\pi^{(asy)}(Q^2)$.
- the 2nd term in the r.h.s of Eq.(6) gives significant effect.
- do χ^2 fit to reveal the more inner structures of pion meson,

$$\chi^2 = \sum_{i=1}^{N_p} \frac{1}{\sigma_i^2} \left[\sum_{n=2,4,..}^{n_{\max}} a_n(\mu_0)\, f_n(Q_i^2, \mu_0) + F_\pi^{(\mathrm{as})}(Q_i^2) - F_\pi^{(\mathrm{disp})}(Q_i^2) \right]^2 . \qquad (13)$$

Result of a_n^π

$a_n^\pi(1\ GeV)$ obtained in our formalism

- Fitting points $Q^2 = 1.0, 1.5, 2.0, 3.0, 5.0, 7.0, 9.0,$
- The weighting coefficients

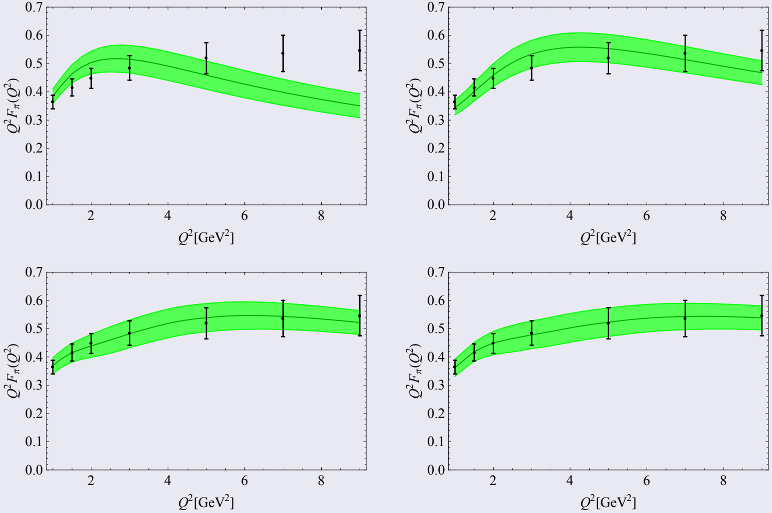
$$\sigma_i = \sqrt{[\Delta F_\pi^{(\mathrm{LCSR})}(Q_i^2, a_n(\mu_0) = 0)]^2 + [\Delta F_\pi^{(\mathrm{disp})}(Q_i^2)]^2}\,.$$

- $a_2^\pi(1\ GeV)$ obtained in our formalism,

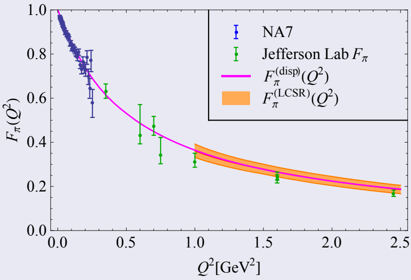
Model	$a_2(1\,\mathrm{GeV})$	$a_4(1\,\mathrm{GeV})$	$a_6(1\,\mathrm{GeV})$	$a_8(1\,\mathrm{GeV})$	$\chi^2_{\mathrm{min}}/\mathrm{ndf}$
$\{a_2\}$	0.302 ± 0.046				4.08
$\{a_2, a_4\}$	0.279 ± 0.047	0.189 ± 0.060			0.75
$\{a_2, a_4, a_6\}$	0.270 ± 0.047	0.179 ± 0.060	0.123 ± 0.086		0.073
$\{a_2, a_4, a_6, a_8\}$	0.269 ± 0.047	0.185 ± 0.062	0.141 ± 0.096	0.049 ± 0.116	0.013

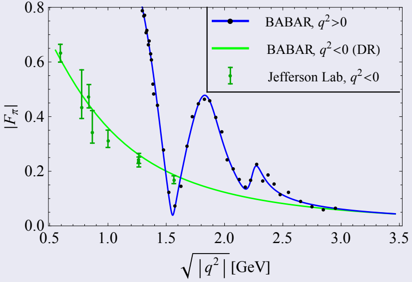
$\approx -15\%$ correlations are found between different moments.

$a_n^\pi(1\ GeV)$ obtained in our formalism



Performing with the direct measurement





- $|F_\pi(s)| \sim |F_\pi^{(disp)}(|q^2|)|$ at $|\sqrt{q^2}| \gtrsim 3\ GeV$,
- manifests analyticity of the modulus representation.

Conclusion

- Modulus representation of DR, LCSRs calculation + BABAR data,

  $a_2(1\,\mathrm{GeV}) = (0.22 - 0.33)\,, \quad a_4(1\,\mathrm{GeV}) = (0.12 - 0.25)\,.$

- Pion deviates from the purely asymptotic one,
  a_2^π is not enough, more inner structures.

† the role of form factor zeros.

† global fit with F_π with timelike and spacelike measurements, $F_{\pi\gamma\gamma^*}$.

† $\tau \to K\pi l\nu_l$ transition, K electromagnetic form factors $\to a_n^K$.

The End, Thanks.

Timelike and spacelike π electromagnetic FFs

Review of $F_\pi(Q^2)$ measurements, [K.K. Seth, arXiv:1401.7054[hep-ex]]

- Cyclotron Lab, $Q^2 = 0.176\mathrm{GeV}^2$ in electron production;
- NOVOSIBIRSK and ORSAY collaboration, $0.64 \leqslant Q^2 \leqslant 1.40\,\mathrm{GeV}^2$ & $1.35 \leqslant Q^2 \leqslant 2.38\,\mathrm{GeV}^2$, e^+e^- annihilation .
- CLEO collaboration, $Q^2 = 9.6, 13.48\,\mathrm{GeV}^2$.
- Belle collabaration, $4m_\pi^2 \leqslant Q^2 \leqslant 3.125\,\mathrm{GeV}^2$, $\tau^- \to \pi^-\pi^0\nu_\tau$ decay.
- BABAR collaboration, $4m_\pi^2 \leqslant Q^2 \leqslant 3.1^2\,\mathrm{GeV}^2$, $e^+e^- \to \pi^+\pi^-(\gamma)$.

Review of $F_\pi(q^2)$ measurements, [M.R. Whalley, et al., J.Phys. G 29 (2003) A1]

- Harvard & Cornell Collaboration, $0.15 \leqslant |q^2| \leqslant 10\mathrm{GeV}^2$;
- DESY collaboration, $|q^2| = 0.35, 0.70\mathrm{GeV}^2$, electron production;
- Jefferon Lab F_π collaboration, $0.6 \leqslant |q^2| \leqslant 1.6\mathrm{GeV}^2, |q^2| = 2.45\mathrm{GeV}^2$.