$\psi \to \pi^+\pi^-\eta'$过程的分波分析

M. Ablikim *et al*. (BESIII Collaboration), Phys. Rev. D **96**, 112012 (2017).

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衰变链:
$$\begin{tikzcd} \psi \to \pi^+\pi^- & \eta' \arrow[d] & \\ & \pi^+\pi^- & \eta \arrow[d] \\ & & \gamma\gamma \end{tikzcd}$$

分波分析

分波分析(PWA)的振幅是采用协变张量的形式 [1]: $\psi \to X\,\eta'$, $X \to \pi^+\pi^-$

最大似然值:

输入参数为： π^+, π^-, η' 的四动量

$$L = \prod_{i=1}^{N} P(\xi_i, \alpha) = \prod_{i=1}^{N} \frac{\omega(\xi_i, \alpha)\epsilon(\xi_i)}{\int d\xi_i \omega(\xi_i, \alpha)\epsilon(\xi_i)}$$

微分截面： $\omega(\xi_i, \alpha) \equiv \left(\frac{d\sigma}{d\Phi}\right)_i$

拟合参数为: 质量、宽度、系数、相对的相角

用MINUIT来最小化:

$$\mathcal{S} = -\ln L = -\sum_{i=1}^{N} \ln \frac{\omega(\xi_i, \alpha)}{\int d\xi_i \omega(\xi_i, \alpha)\epsilon(\xi_i)}$$

扣除本底的贡献之后:

不包括连续本底的贡献

$$\mathcal{S} = -(\ln L_{\text{data}} - \ln L_{\text{bkg}})$$

[1] B. S. Zou and D. V. Bugg, Eur. Phys. J. A **16**, 537 (2003).

中间共振态

PWA 基本解:

$J/\psi \to \pi^+\pi^-\eta'$: $\mathrm{non-Resonance}\,(1^{--})$, ρ, ω, $\rho(1450)$;

$\psi(3686) \to \pi^+\pi^-\eta'$: $\mathrm{non-Resonance}(1^{--})$, ρ;

Breit-Wigner 传播子:

对于 $\mathrm{non-Resonance}(1^{--})$ 和 ω :

$$BW = \frac{1}{m^2 - s - i\,m\,\Gamma}$$

对于ρ 和 $\rho(1450)$ [1,2]:

$$BW^{GS} = \frac{1 + d(m)\Gamma/\mathrm{m}}{m^2 - s + f(s,m,\Gamma) - im\Gamma(s,m,\Gamma)}$$

[1] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. **21**, 244 (1968).
[2] J. P. Lees et al., Phys. Rev. D **86**, 032013 (2012).

探测器分辨

用一个高斯函数$G_\sigma(x)$来描述

概率函数 $P(x)$ 卷积一个高斯函数$G_\sigma(x)$ 之后[1]变为:

$$u(x) = (P \otimes G_\sigma)(x) = \int G_\sigma(x-y)P(y)dy$$

↓

近似：卷积变为求和

$$u(x) = \sum_{m=-5\sigma}^{5\sigma} g_m P(x-m)\Delta m\,,$$

$$g_m = \frac{1}{T} G_\sigma(m),$$

$$T = \sum_{m=-5\sigma}^{5\sigma} G_\sigma(m)\Delta\, m\,,$$

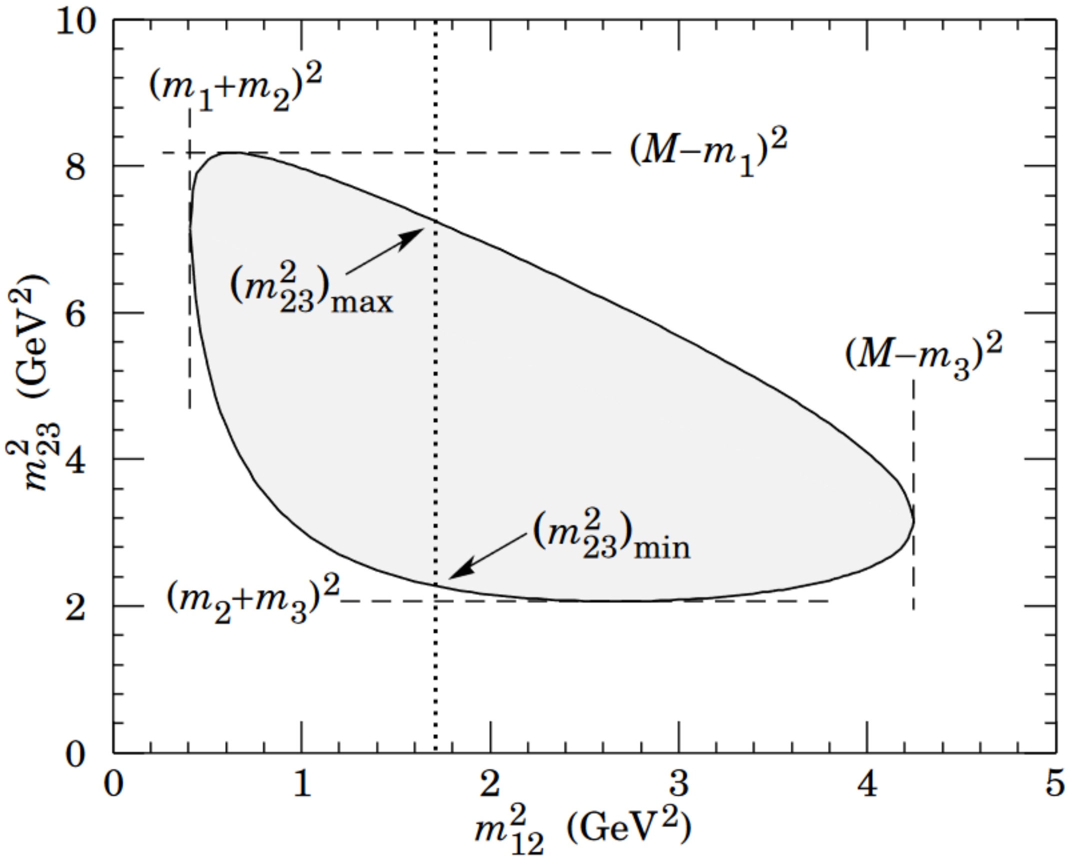
M$(\pi^+\pi^-)$ 的分辨率为: $\sigma = 3\ \mathrm{MeV}/c^2$

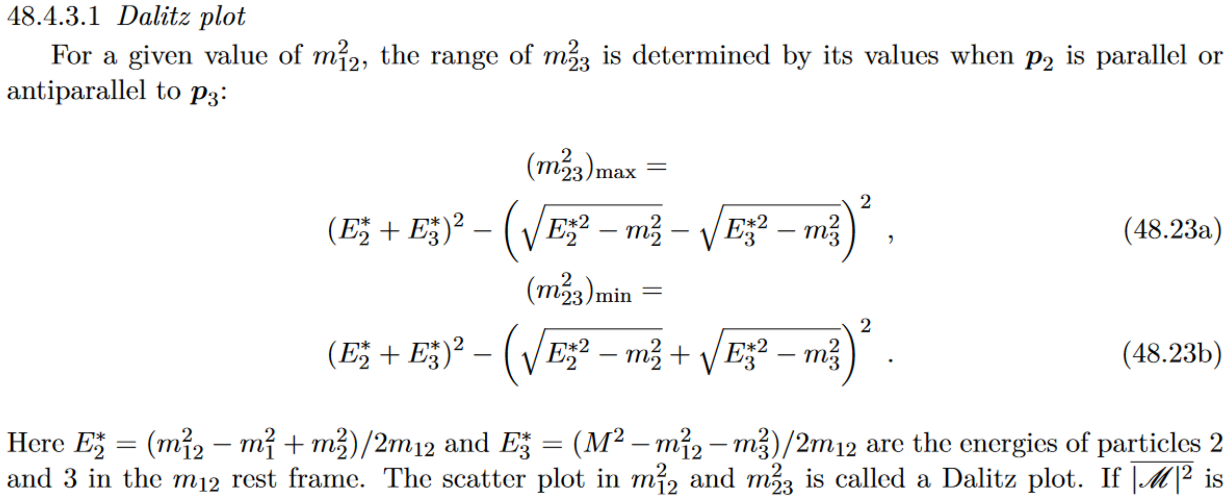
$\Delta m = \sigma.$

[1] Prof. Liu Beijiang, PWA meeting, 2014-12-24.

边界影响

Dalitz plot for a three-body final state. Four-momentum conservation restricts events to the shaded region [1].




48.4.3.1 *Dalitz plot*

For a given value of m^2_{12}, the range of m^2_{23} is determined by its values when $\boldsymbol{p}_2$ is parallel or antiparallel to $\boldsymbol{p}_3$:

$$(m^2_{23})_{\max} = (E^*_2 + E^*_3)^2 - \left(\sqrt{E^{*2}_2 - m^2_2} - \sqrt{E^{*2}_3 - m^2_3}\right)^2 , \tag{48.23a}$$

$$(m^2_{23})_{\min} = (E^*_2 + E^*_3)^2 - \left(\sqrt{E^{*2}_2 - m^2_2} + \sqrt{E^{*2}_3 - m^2_3}\right)^2 . \tag{48.23b}$$

Here $E^*_2 = (m^2_{12} - m^2_1 + m^2_2)/2m_{12}$ and $E^*_3 = (M^2 - m^2_{12} - m^2_3)/2m_{12}$ are the energies of particles 2 and 3 in the m_{12} rest frame. The scatter plot in m^2_{12} and m^2_{23} is called a Dalitz plot. If $\overline{|\mathscr{M}|^2}$ is

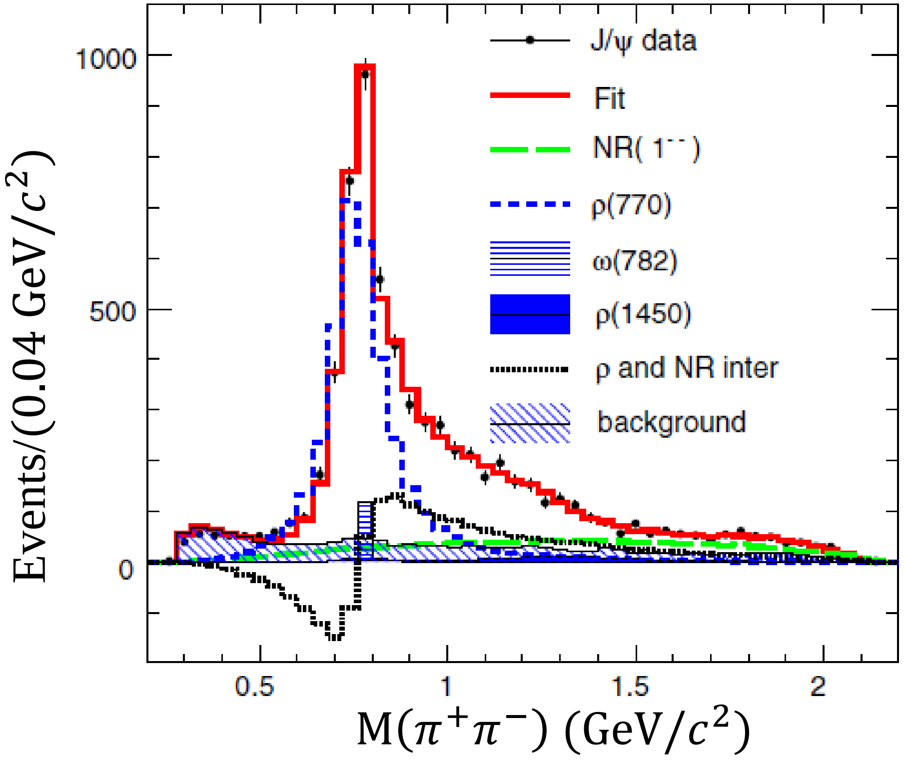
用近似的方法考虑探测器分辨影响时，需要检查一下是否有超出边界的点。如果有，就将其去除。

[1] PDG (2020), http://pdg.lbl.gov/2020/reviews/rpp2020-rev-kinematics.pdf

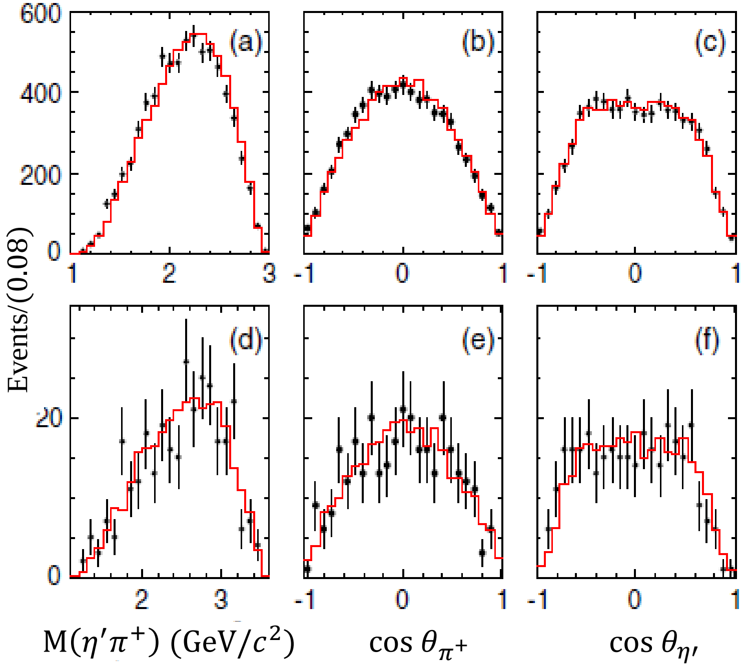
PWA 拟合结果的投影

$\psi(3686) \to \pi^+\pi^-\eta'$ 有两组解[1]：干涉相消和干涉相长.

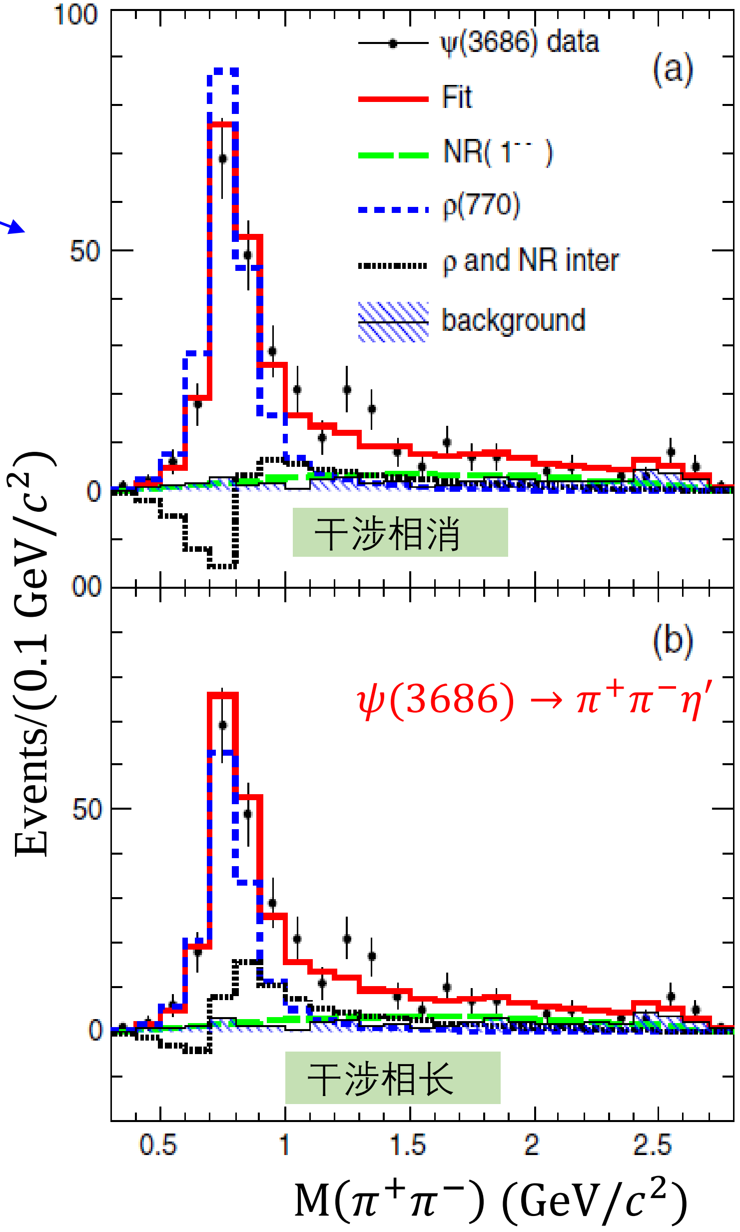
$J/\psi \to \pi^+\pi^-\eta'$



$J/\psi \to \pi^+\pi^-\eta'$



$\psi(3686) \to \pi^+\pi^-\eta'$



[1] K. Zhu, X. H. Mo, C. Z. Yuan, and P. Wang, Int. J. Mod. Phys. A **26**, 4511 (2011);

PWA 拟合参数的结果

没有误差的数值是固定在 PDG(2016).

组分	质量 (MeV/c^2)	宽度(MeV)	干涉角 (度)	显著性
ρ	766 ± 2	142 ± 5	0	$\geq 30\sigma$
ω	782.65	8.49	100.3 ± 5.3	10.7σ
$\rho(1450)$	1369 ± 38	386 ± 70	203.6 ± 11.9	12.5σ
PHSP(1^{--})	--	500 GeV	-269.7 ± 1.4	8.0σ

$J/\psi \to \pi^+\pi^-\eta'$

组分	干涉相消	干涉相长	质量(MeV/c^2)	宽度(MeV)	显著性
	干涉角 (度)				
PHSP(1^{--})	120.3 ±16.6	45.6 ±17.5	--	500(GeV)	15.1σ
ρ	0	0	775.26	149.1	20.0σ

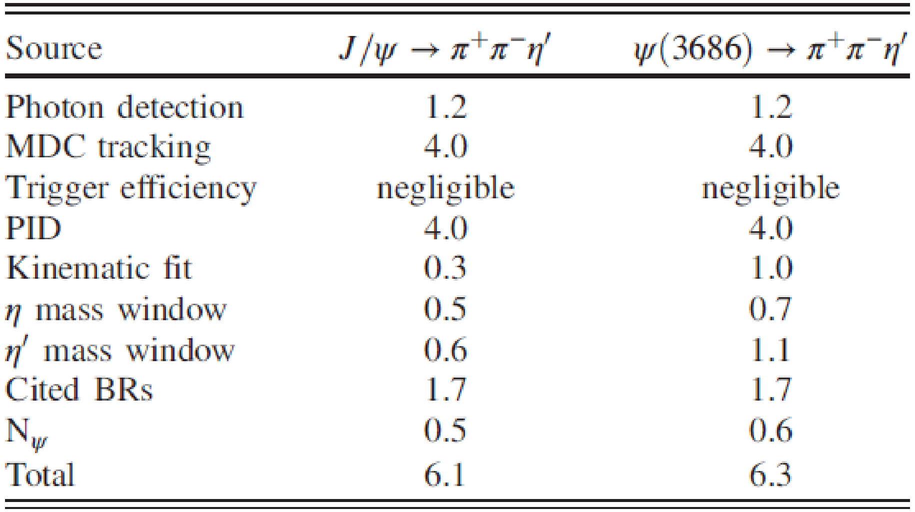
$\psi(3686) \to \pi^+\pi^-\eta'$

谢谢！

Back up

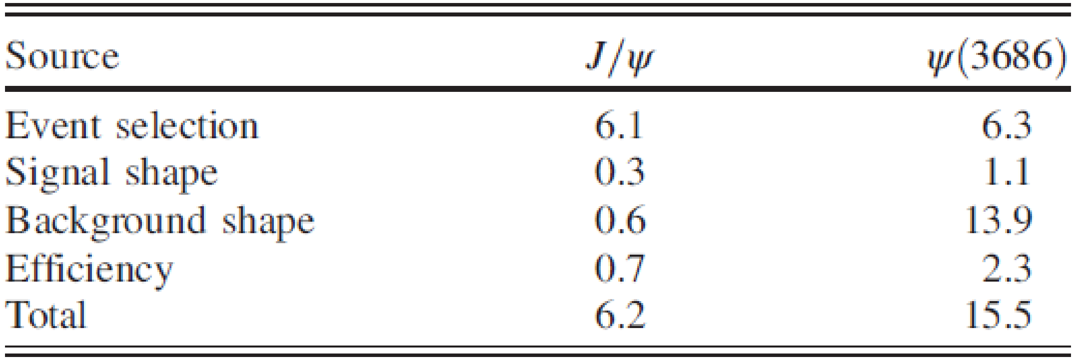
系统误差

来自于事例选择的相对系统误差(%).

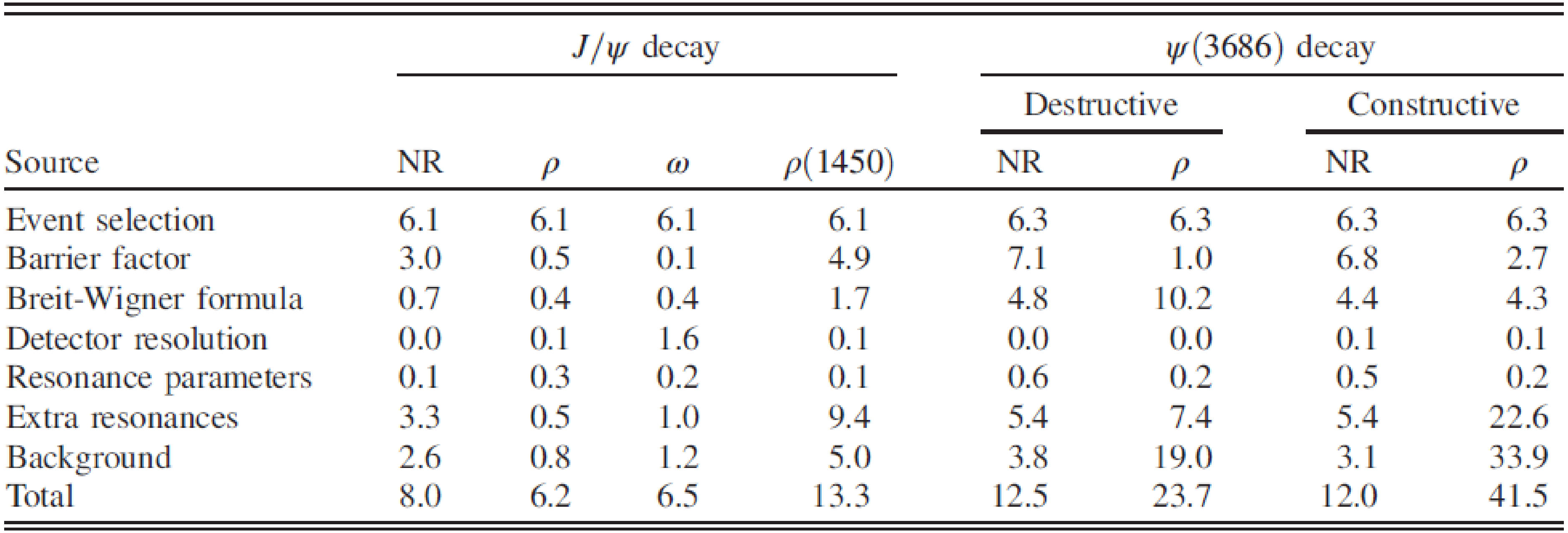


Source	$J/\psi \to \pi^+\pi^-\eta'$	$\psi(3686) \to \pi^+\pi^-\eta'$
Photon detection	1.2	1.2
MDC tracking	4.0	4.0
Trigger efficiency	negligible	negligible
PID	4.0	4.0
Kinematic fit	0.3	1.0
η mass window	0.5	0.7
η' mass window	0.6	1.1
Cited BRs	1.7	1.7
N_ψ	0.5	0.6
Total	6.1	6.3

影响单举衰变的相对系统误差(%).



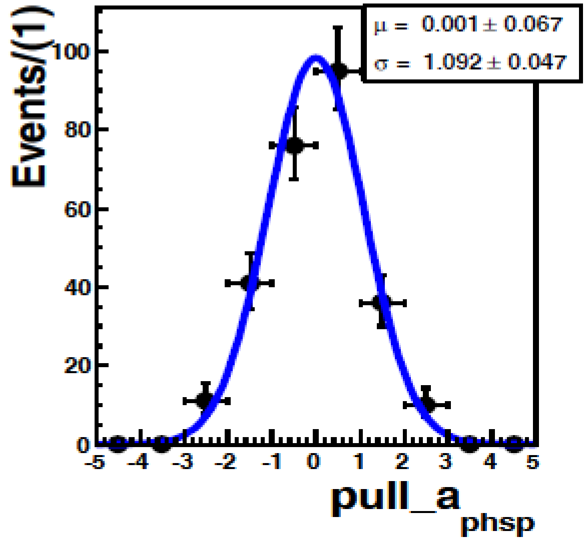
Source	J/ψ	$\psi(3686)$
Event selection	6.1	6.3
Signal shape	0.3	1.1
Background shape	0.6	13.9
Efficiency	0.7	2.3
Total	6.2	15.5

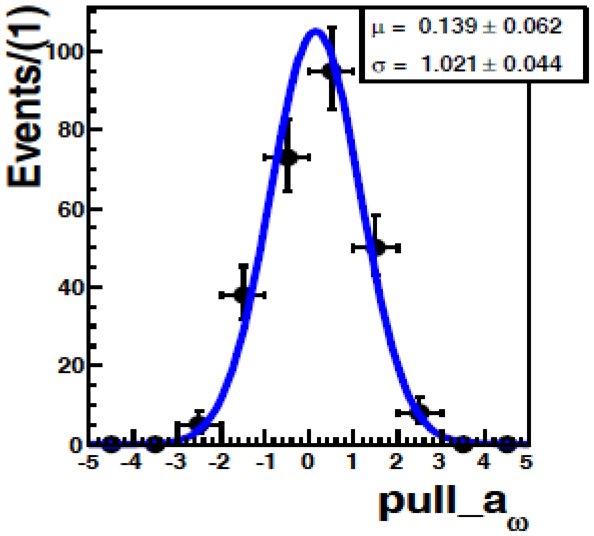


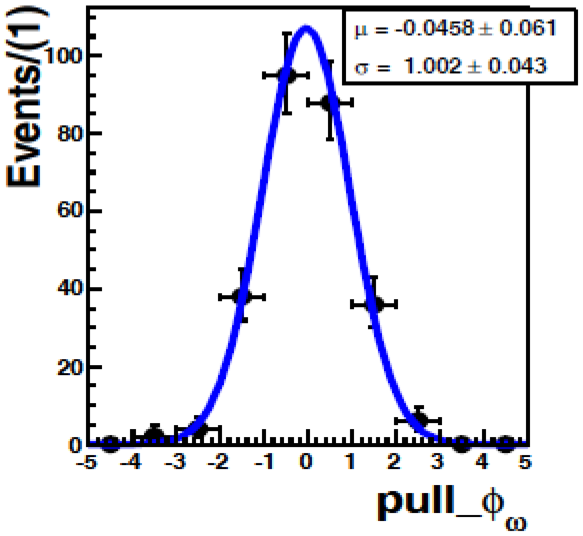
	J/ψ decay				$\psi(3686)$ decay			
					Destructive		Constructive	
Source	NR	ρ	ω	$\rho(1450)$	NR	ρ	NR	ρ
Event selection	6.1	6.1	6.1	6.1	6.3	6.3	6.3	6.3
Barrier factor	3.0	0.5	0.1	4.9	7.1	1.0	6.8	2.7
Breit-Wigner formula	0.7	0.4	0.4	1.7	4.8	10.2	4.4	4.3
Detector resolution	0.0	0.1	1.6	0.1	0.0	0.0	0.1	0.1
Resonance parameters	0.1	0.3	0.2	0.1	0.6	0.2	0.5	0.2
Extra resonances	3.3	0.5	1.0	9.4	5.4	7.4	5.4	22.6
Background	2.6	0.8	1.2	5.0	3.8	19.0	3.1	33.9
Total	8.0	6.2	6.5	13.3	12.5	23.7	12.0	41.5

影响各个组分的相对系统误差(%).

输入输出检查



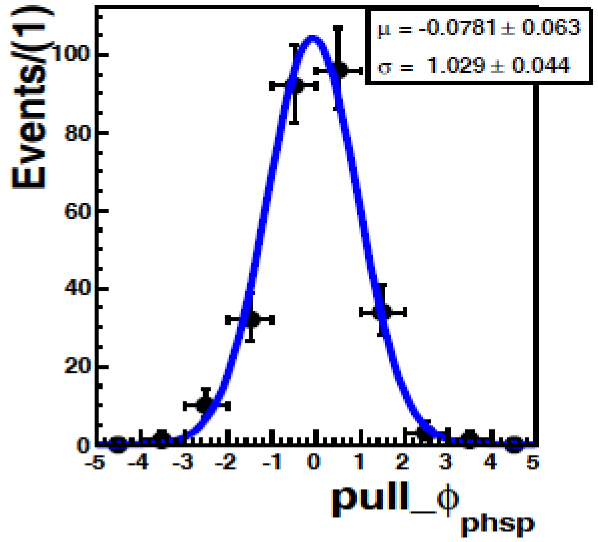


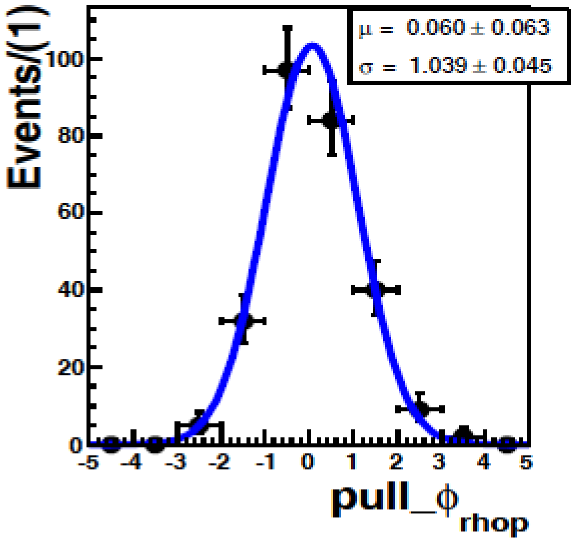


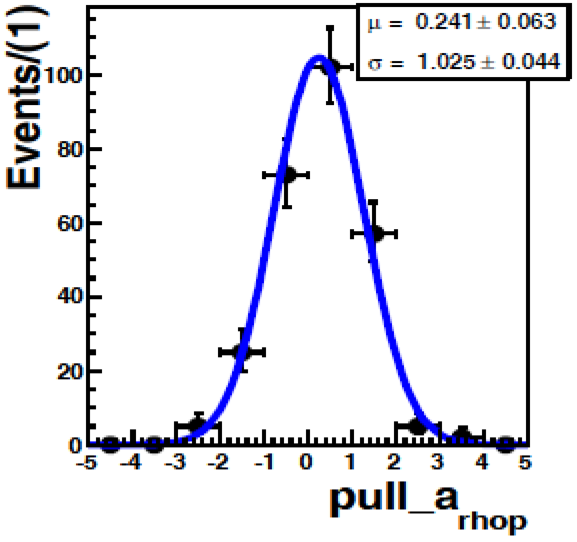
$$\text{pull} = \frac{\text{a}_{\text{meas}} - \text{a}_{\text{exp}}}{\sigma_{\text{meas}}}$$

每一个组分的系数和相对相角的 pull 分布情况.

这些 pull 分布是在误差范围内服从(0,1)的正态分布.



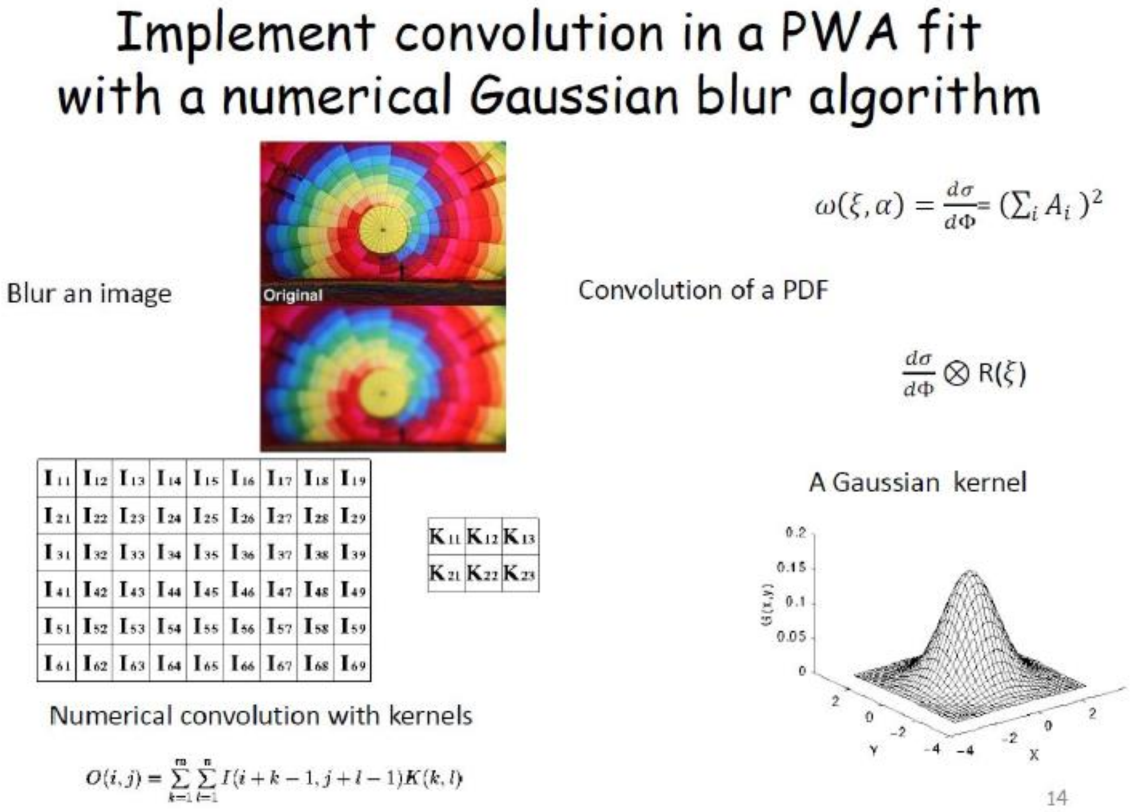




resolution

BES3 publication   Phys.Rev. D89 (2014) 052001,
convolution method adopted from CLEO

My talk @ coll. meeting



Implement convolution in a PWA fit with a numerical Gaussian blur algorithm

$$\omega(\xi,\alpha) = \frac{d\sigma}{d\Phi} = \left(\sum_i A_i\right)^2$$

Blur an image

Convolution of a PDF

$$\frac{d\sigma}{d\Phi} \otimes \mathrm{R}(\xi)$$

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline \mathbf{I}_{11} & \mathbf{I}_{12} & \mathbf{I}_{13} & \mathbf{I}_{14} & \mathbf{I}_{15} & \mathbf{I}_{16} & \mathbf{I}_{17} & \mathbf{I}_{18} & \mathbf{I}_{19} \\ \hline \mathbf{I}_{21} & \mathbf{I}_{22} & \mathbf{I}_{23} & \mathbf{I}_{24} & \mathbf{I}_{25} & \mathbf{I}_{26} & \mathbf{I}_{27} & \mathbf{I}_{28} & \mathbf{I}_{29} \\ \hline \mathbf{I}_{31} & \mathbf{I}_{32} & \mathbf{I}_{33} & \mathbf{I}_{34} & \mathbf{I}_{35} & \mathbf{I}_{36} & \mathbf{I}_{37} & \mathbf{I}_{38} & \mathbf{I}_{39} \\ \hline \mathbf{I}_{41} & \mathbf{I}_{42} & \mathbf{I}_{43} & \mathbf{I}_{44} & \mathbf{I}_{45} & \mathbf{I}_{46} & \mathbf{I}_{47} & \mathbf{I}_{48} & \mathbf{I}_{49} \\ \hline \mathbf{I}_{51} & \mathbf{I}_{52} & \mathbf{I}_{53} & \mathbf{I}_{54} & \mathbf{I}_{55} & \mathbf{I}_{56} & \mathbf{I}_{57} & \mathbf{I}_{58} & \mathbf{I}_{59} \\ \hline \mathbf{I}_{61} & \mathbf{I}_{62} & \mathbf{I}_{63} & \mathbf{I}_{64} & \mathbf{I}_{65} & \mathbf{I}_{66} & \mathbf{I}_{67} & \mathbf{I}_{68} & \mathbf{I}_{69} \\ \hline \end{array} \qquad \begin{array}{|c|c|c|} \hline \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} \\ \hline \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{K}_{23} \\ \hline \end{array}$$

A Gaussian kernel

Numerical convolution with kernels

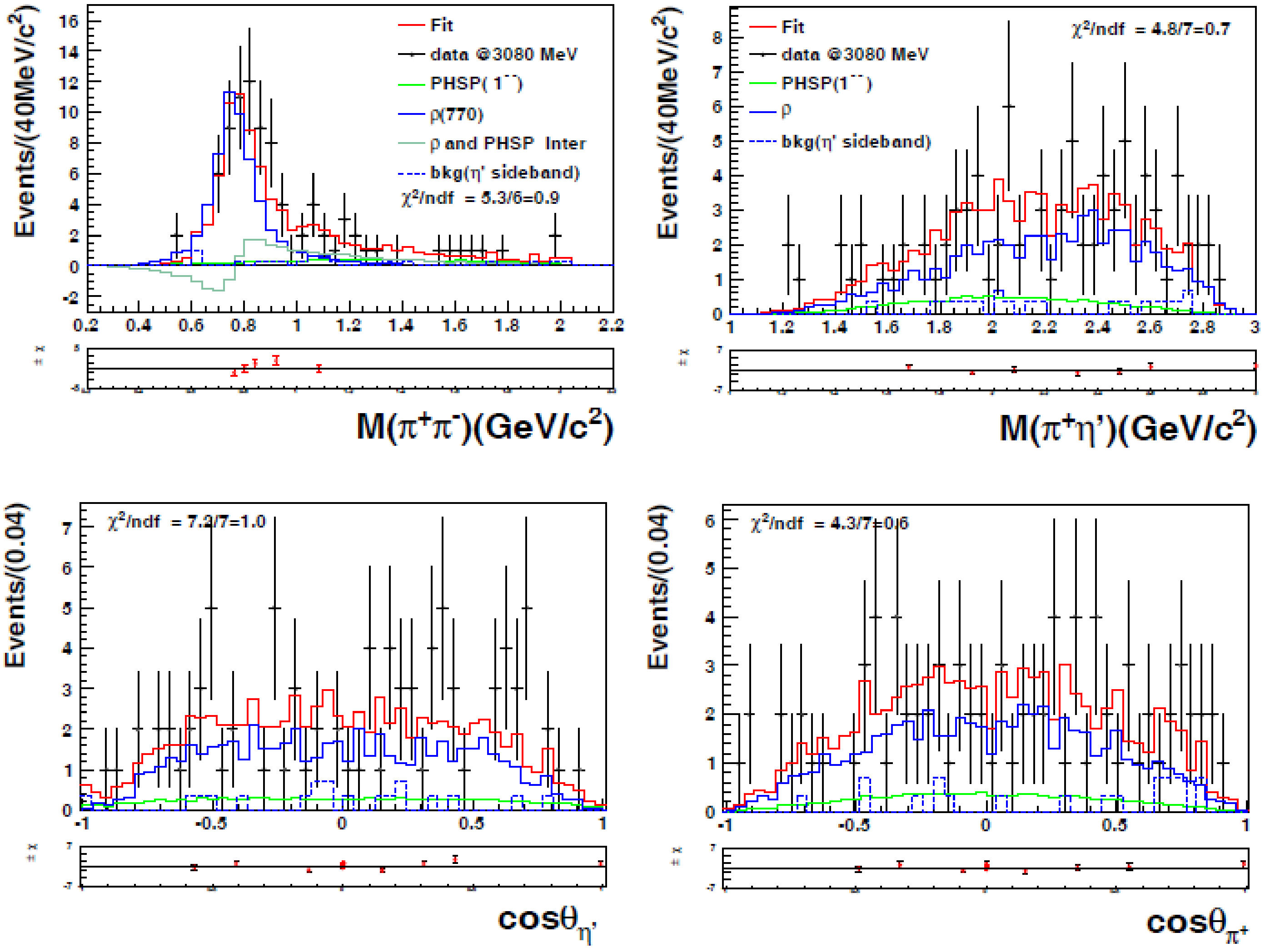
$$O(i,j) = \sum_{k=1}^{m}\sum_{l=1}^{n} I(i+k-1, j+l-1)K(k,l)$$

[1] Prof. Liu Beijiang, PWA meeting. 2014-12-24

$e^+e^- \to \gamma^* \to \pi^+\pi^-\eta'$

PWA拟合的投影结果

@3.08 GeV



由于统计量低，其共振参数是固定在 $J/\psi \to \pi^+\pi^-\eta'$的PWA拟合结果上。

组分	质量 (MeV/c^2)	宽度 (MeV)	相角 (度)	显著性
ρ	766	142	0	11.1σ
ω	移除			0.8σ
$\rho(1450)$	移除			1.5σ
PHSP(1^{--})	--	500 GeV	−273.9 ±37.1	6.6σ

由于QED连续本底的PWA 拟合结果与之前只拟合 $M(\pi^+\pi^-)$的得到的ρ 组分的事例数一致，所以在正则解中，我们用PWA拟合的结果来得到QED连续本底的贡献。