Mass Resolution in GPUPWA

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Outline

- Introduction
- Mass resolution in GPUPWA
- Summary

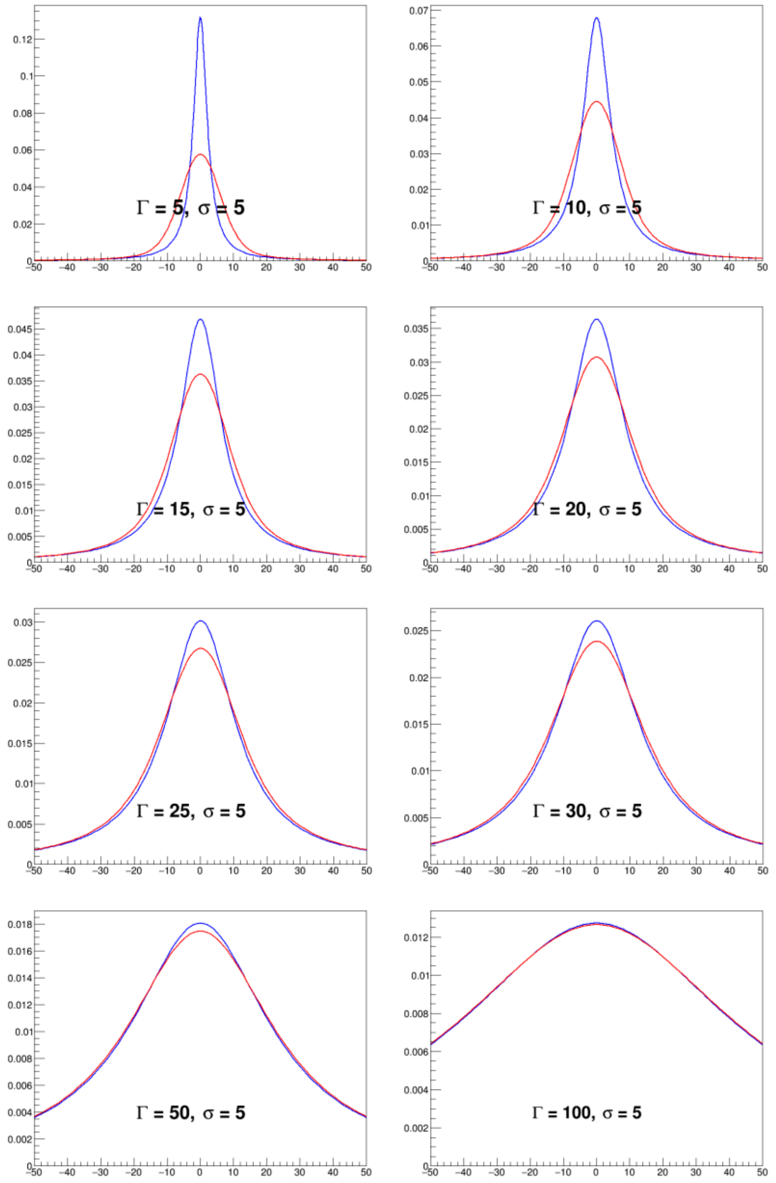
Introduction

- Mass resolution: essential for narrow resonances
  - Width measurement
  - Correct treatment of interference term not only on the peak but also in the tail

In Breit-Wigner⊗Gaussian case, mass resolution should be considered when $\Gamma \lesssim 6\sigma$

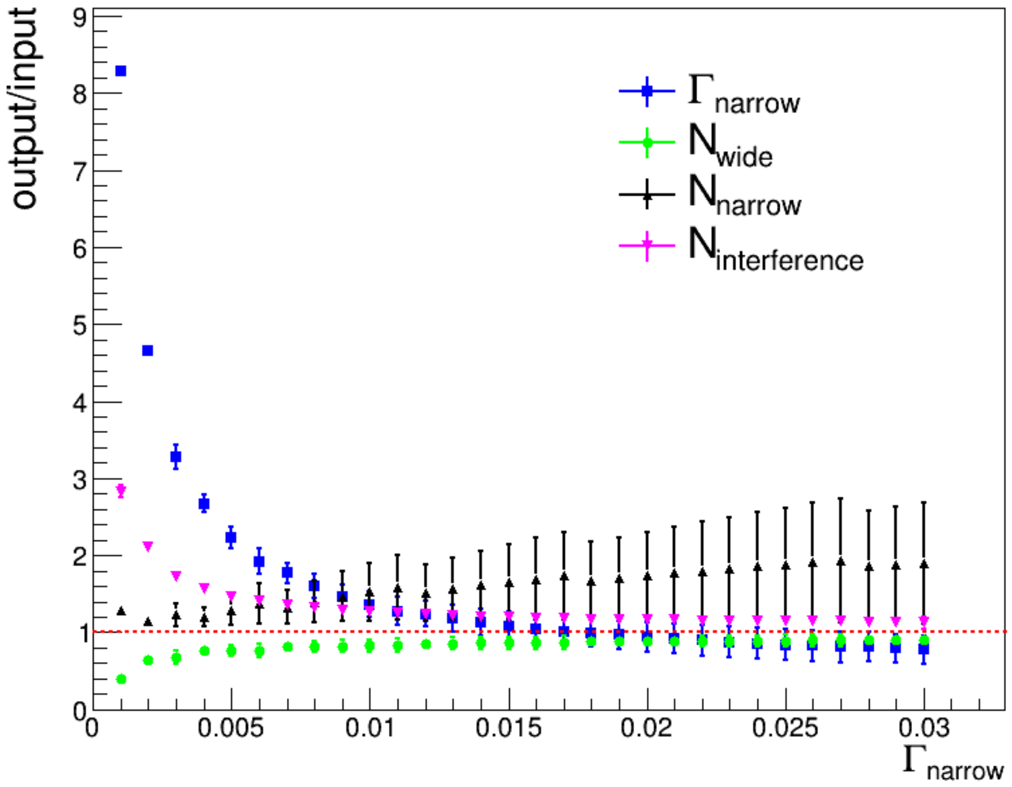
✓ Typical mass resolution at BESIII is 5 MeV/c^2

✓ Mass resolution should be considered for states such as ***ω, φ, $f_1(1285)$, η_c, Z_c, …***



Introduction

- Mass resolution affects both width and branching ratio measurements



✓ Model: X(wide) + X(narrow) + interference
  - Generate toy MC with resolution (5 MeV/c^2)
  - Fit to toy MC without resolution (5 MeV/c^2)

✓ **Width and branching ratio measurements are affected when *$\Gamma \lesssim 6\sigma$***

ω, φ, $f_1(1285)$, η_c, Z_c, …

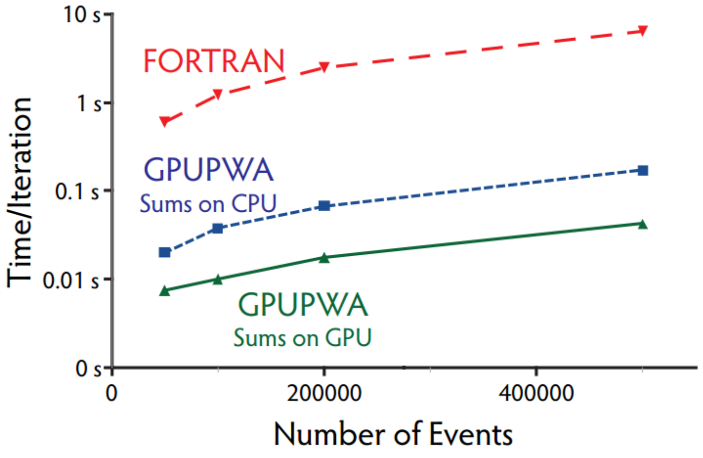
Introduction

- PWA with mass resolution at BESIII
  - Mass resolution should be considered in many analyses
    - Dalitz analyses: $D^+ \to K_S\pi^+\pi^0$, $\eta'/\eta \to 3\pi$, $\eta' \to \eta\pi\pi$, …
    - PWA of $J/\psi \to K^+K^-\pi^0$ ***BAM-114***
    - PWA of $J/\psi(\psi') \to \eta'\pi^+\pi^-$ ***BAM-225***
    - PWA of $J/\psi \to \gamma\eta\eta'$ ***Dr. Liu Beijiang's talk at 2014 BESIII Collab. Meeting [link]***
  - No general framework for PWA with resolution
  - Growing data size ⇒ GPUPWA

Phys. Rev. D 89, 052001
Phys. Rev. D 92, 012014
Phys. Rev. Lett. 118, 012001
BAM-205
…

$J/\psi(\psi', e^+e^-) \to \pi^0\pi^+\pi^-$
$J/\psi \to \gamma\pi^0\pi^+\pi^-$
…

We need GPUPWA with mass resolution ☺



J. Phys. Conf. Ser. 219, 042031

Mass resolution in GPUPWA

How to describe mass resolution in PWA?

$$\frac{d\sigma}{d\Phi} = |\sum F_i A_i|^2 = \sum F_i F_j^* A_i A_j^* = \sum F_i F_j^* T_i T_j P_i P_j^*$$

Propagator
where mass resolution matters

Magnitude/Phase
independent of mass

Orbital Tensor
weak dependency on mass

w/o mass resolution $P_i P_j^* = f_i(x) f_j^*(x) \cdot f_i(y) f_j^*(y) \cdot f_i(z) f_j^*(z) \cdot \ldots$

w/ mass resolution $P_i P_j^* = f_i(x) f_j^*(x) \otimes g(x) \cdot f_i(y) f_j^*(y) \otimes g(y) \cdot f_i(z) f_j^*(z) \otimes g(z) \cdot \ldots$

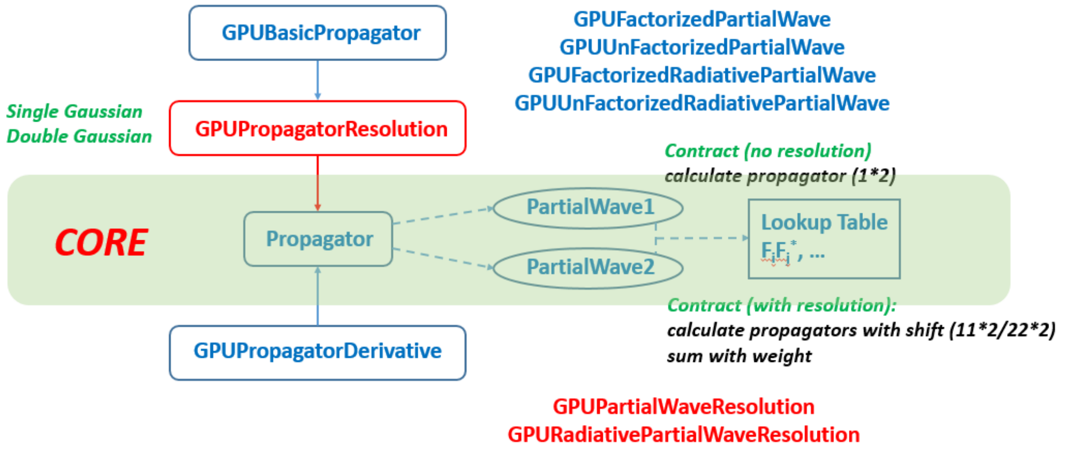
$\equiv h_{ij}(x) \cdot h_{ij}(y) \cdot h_{ij}(z) \cdot \ldots$

How to describe mass resolution in PWA?

- Analytical convolution: apply to only a few functions
- Numerical convolution:
  - $h_{ij}(x) = \int f_i(x-m) f_j^*(x-m) g(m) \mathrm{d}m \simeq \sum_k w(m_k) f_i(x-m_k) f_j^*(x-m_k)$
  - $g(m)$ is Gaussian:
    - Use **Gauss-Hermite quadrature**
      - ***High precision with quite a few sampling points***
      - *http://mathworld.wolfram.com/Hermite-GaussQuadrature.html*
  - $g(m)$ is double Gaussian:
    - Perform two separate Gauss-Hermite quadrature

Code design

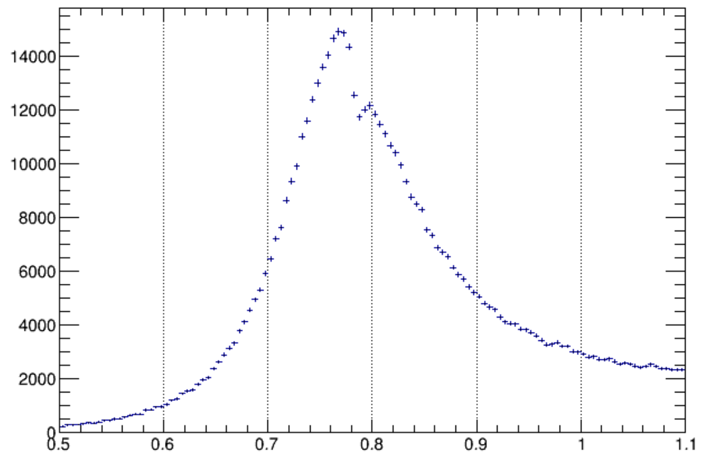
- *GPUPropagatorResolution*
  - *abstract base class* for propagators
  - Sampling & store in memory
    - Support resolution of arbitrary shape
    - Single Gaussian: 11 points Gauss-Hermite quadrature
    - Double Gaussian: 2*11 points Gauss-Hermite quadrature
- *GPUBasicPartialWaveResolution*
  - *abstract base class* for partial waves
  - Provide interface for management of parameters
- *GPUPartialWaveResolution/GPURadiativePartialWaveResolution*
  - *class* for partial waves of non-radiative/radiative decays
  - Inherit from *GPUBasicPartialWaveResolution*
  - Provide interface for contraction
- *Resolution.cl*
  - *__kernel* functions that do actual calculations



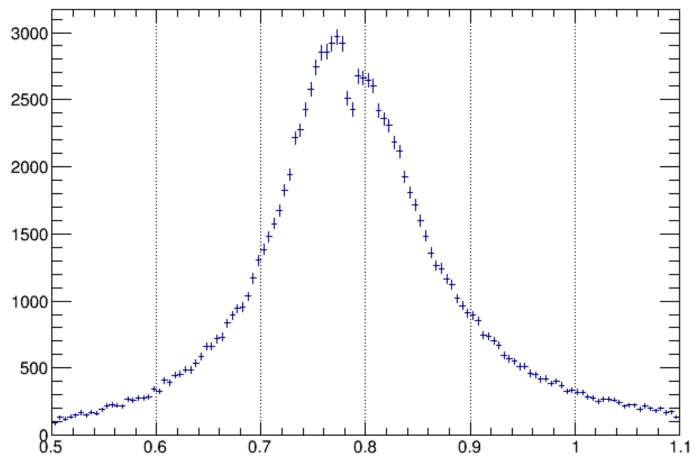
I/O check: $J/\psi \to \pi^0\pi^+\pi^-$

- Input parameters
  - ρ^0
    - M = 775.26 MeV/c^2
    - Γ = 149.1 MeV/c^2
  - ω
    - M = 782.65 MeV/c^2
    - Γ = 8.49 MeV/c^2
    - Magnitude = 0.1
    - Phase = 2.8

Real data of $J/\psi \to \pi^0\pi^+\pi^-$
(BESIII 2009 run)



Data-like MC
✓ $J/\psi \to \pi^0\rho^0$, $\rho^0 \to \pi^+\pi^-$
✓ $J/\psi \to \pi^0\omega$, $\omega \to \pi^+\pi^-$

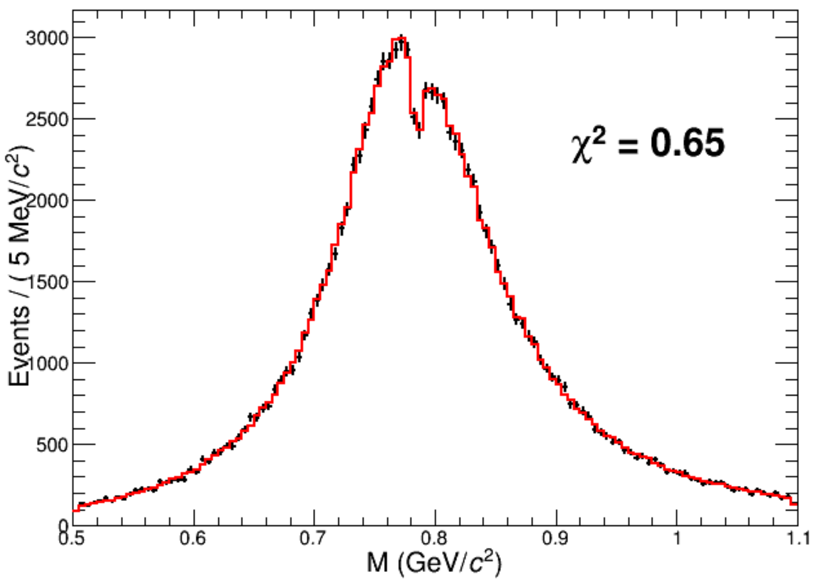


$M[\pi^+\pi^-]$

I/O check: PWA w/o mass resolution

		Input	Output	$\lvert\Delta\rvert/\sigma$
ρ	M (MeV/c^2)	775.26	775.8 ± 0.3	1.8
	Γ (MeV/c^2)	149.1	147.9 ± 0.8	1.5
ω	M (MeV/c^2)	782.65	782.5 ± 0.7	0.2
	Γ (MeV/c^2)	**8.49**	**$13.6^{+1.6}_{-1.4}$**	**3.7**
	Magnitude	**0.1**	**0.129 ± 0.007**	**4.1**
	Phase	2.8	2.73 ± 0.05	1.4
	log likelihood		74521.9	
$\chi^2/N_{dof} \equiv (\sum \lvert\Delta\rvert^2/\sigma^2)/N_{dof}$			**38.2/6**	

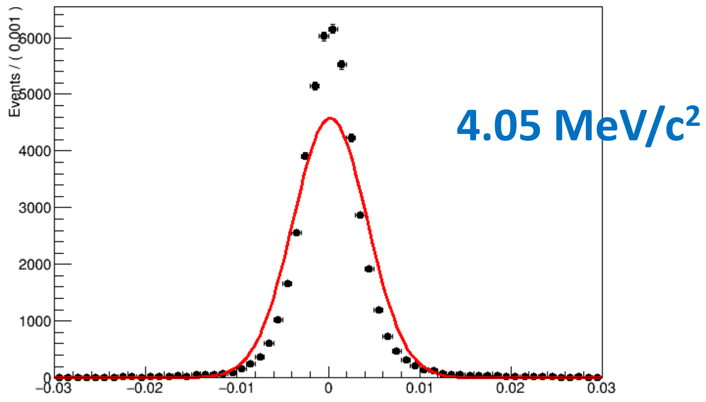
MC Projection v.s. data for $M[\pi^+\pi^-]$

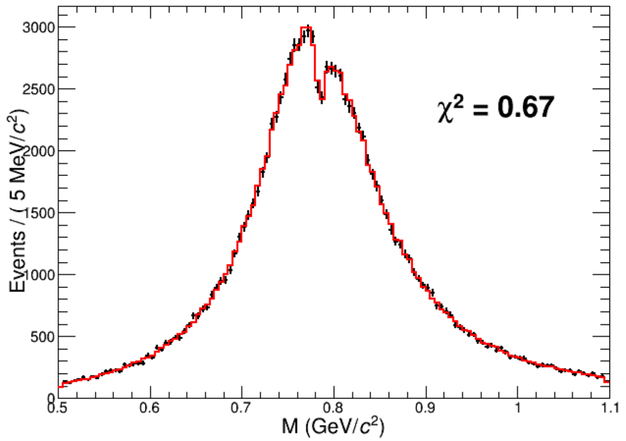


PWA w/o mass resolution ⇒ Large deviation (>3σ)

I/O test: PWA w/ single Gaussian resolution

		Input	Output	$\lvert\Delta\rvert/\sigma$
ρ	M (MeV/c^2)	775.26	775.6 ± 0.3	1.1
	Γ (MeV/c^2)	149.1	148.2 ± 0.8	1.1
ω	M (MeV/c^2)	782.65	782.3 ± 0.8	0.4
	Γ (MeV/c^2)	8.49	$9.2^{+2.0}_{-1.7}$	0.4
	Magnitude	0.1	0.113 ± 0.006	2.2
	Phase	2.8	2.72 ± 0.05	1.6
	log likelihood		74520.4	
	$\chi^2/\mathrm{N}_{dof} \equiv (\sum \lvert\Delta\rvert^2/\sigma^2)/\mathrm{N}_{dof}$		10.1/6	



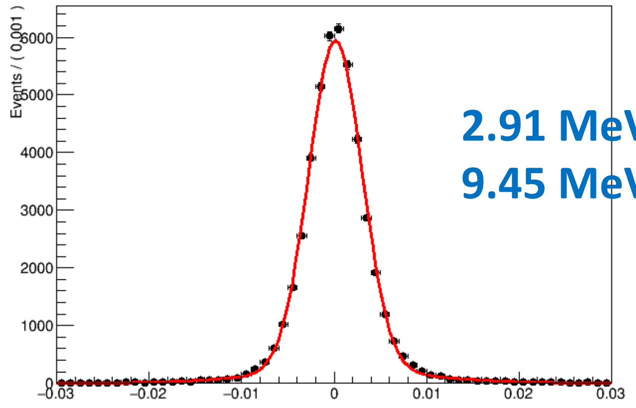


MC Projection v.s. data for M[$\pi^+\pi^-$]

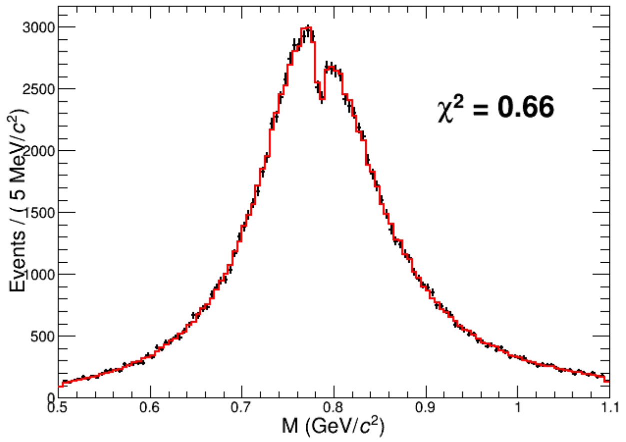
PWA w/ single Gaussian resolution ⇒ reasonable results

I/O test: PWA w/ double Gaussian resolution

		Input	Output	$\lvert\Delta\rvert/\sigma$
ρ	M (MeV/c^2)	775.26	775.5 ± 0.3	0.8
	Γ (MeV/c^2)	149.1	148.3 ± 0.8	1.0
ω	M (MeV/c^2)	782.65	782.6 ± 0.7	0.1
	Γ (MeV/c^2)	8.49	$9.7^{+1.7}_{-1.5}$	0.8
	Magnitude	0.1	0.112 ± 0.006	2.0
	Phase	2.8	2.75 ± 0.05	1.0
	log likelihood		74521.6	
	$\chi^2/\mathrm{N}_{dof} \equiv (\sum \lvert\Delta\rvert^2/\sigma^2)/\mathrm{N}_{dof}$		7.3/6	



2.91 MeV/c^2 * 0.9
9.45 MeV/c^2 * 0.1



PWA w/ double Gaussian resolution ⇒ reasonable results

MC Projection v.s. data for $M[\pi^+\pi^-]$

Conclusion from I/O test

- PWA without mass resolution leads to large deviation between input and output on width/magnitude ($\rightarrow$branching ratio) of narrow resonances
- PWA with mass resolution gives reasonable results
- Those conclusions hold in several other I/O tests
  - $J/\psi \rightarrow \rho^0\pi^0$, $\rho\pi$, $\rho_3\pi \rightarrow \pi^0\pi^+\pi^-$
  - $J/\psi \rightarrow \gamma f_0 \rightarrow \gamma K^+K^-$
  - $J/\psi \rightarrow \gamma f_0$, $\gamma f_0' \rightarrow \gamma K^+K^-$
  - …

Time consumption

	w/o resolution	w/ s.g. res.	w/ d.g. res.
Start up (sec.)	2.17	2.16	2.07
MC integral (sec.)	**0.91**	**1.08**	**1.07**
LUT creation (sec.)	**0.07**	**0.1**	**0.1**
Fit (sec.)	52.49	31.64	32.83
Avg. fit time (sec.)	0.0030	0.0030	0.0025
Total (sec.)	55.57	34.88	35.97

- ✓ ***Time consumption of sampling is negligible***
- ✓ ***Total time consumption decreases because it is easier for fitter to find minimum with correct model than with wrong one***

Summary

- Mass resolution is essential for studying narrow resonances with PWA
  - Width, branching ratio, …
- PWA with mass resolution is implemented within the GPUPWA framework
  - ✓ *Support mass resolution of arbitrary shape/dimension (uncorrelated)*
  - ✓ *I/O checks show good performance*
  - ✓ *Time consumption of sampling is negligible*
  - ✓ *Total time consumption decreases because it is easier for fitter to find minimum with correct model than with wrong one*
  - ✓ *PWA of processes including narrow resonances such as $J/\psi(\psi', e^+e^-) \to \pi^0\pi^+\pi^-$ become possible*
- Future improvements
  - Extreme narrow resonances: more sophisticated sampling method (e.g., FFT)
  - Correlated multi-dimensional resolution
  - …

Backup

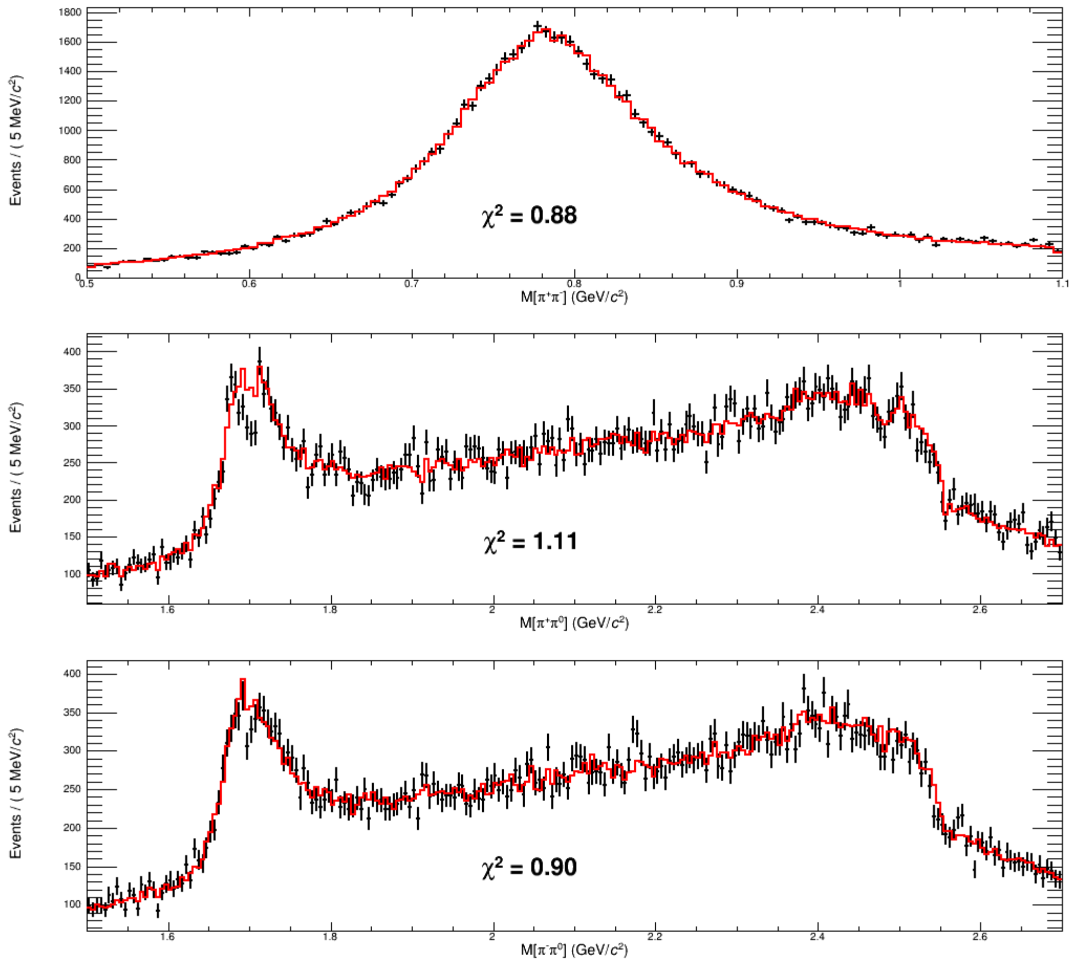
I/O check: ρ^0-ρ-ρ_3 in $J/\psi \rightarrow \pi^0\pi^+\pi^-$

Input

- ρ
  - M = 1701 MeV/c^2
  - Γ = 17 MeV/c^2
  - Magnitude = 19.85
  - Phase = 0.501
- $ρ_3$
  - M = 1687 MeV/c^2
  - Γ = 51 MeV/c^2
  - Magnitude = 89
  - Phase = 0.1213
- $ρ^0$
  - M = 775.26 MeV/c^2
  - Γ = 149.1 MeV/c^2
- 69420 “data” events
- 931384 phsp MC events

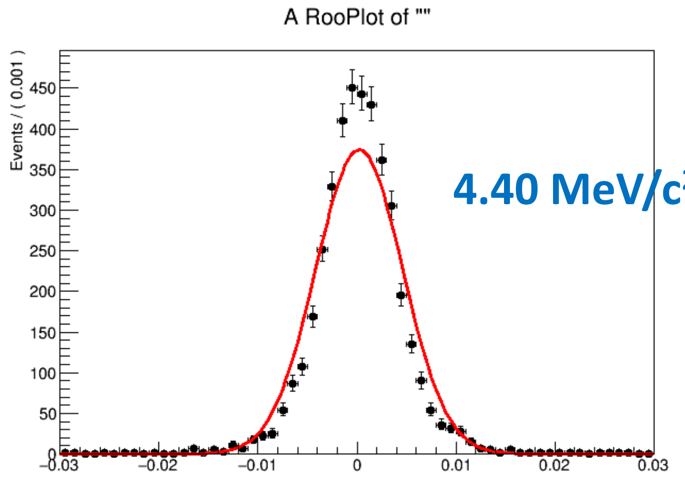
Output: w/o resolution

		Input	Output
ρ	M (MeV/c^2)	1687	1686.7 ± 0.6
	Γ (MeV/c^2)	**51**	**$64.5^{+1.6}_{-1.5}$**
	Magnitude	**89**	**91.42 ± 0.96**
	Phase	**0.1213**	**-0.068 ± 0.015**
ρ_3	M (MeV/c^2)	1701	1700.1 ± 0.2
	Γ (MeV/c^2)	**17**	**15.0 ± 0.3**
	Magnitude	**19.85**	**18.52 ± 0.16**
	Phase	**0.501**	**0.271 ± 0.014**
	log likelihood		45400.5
	χ^2/N_{dof} (p-value)		**650.6/8 (0)**

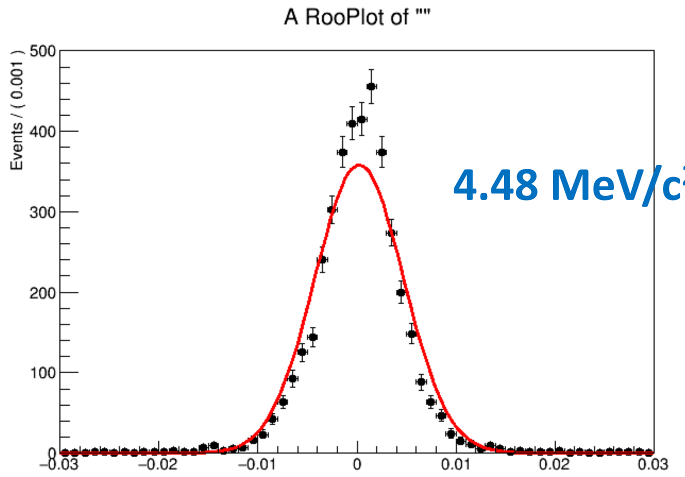


Output: w/ single Gaussian resolution

		Input	Output
ρ	M (MeV/c²)	1687	1686.9±0.5
	Γ (MeV/c²)	**51**	**50.3±1.3**
	Magnitude	**89**	**87.47±0.92**
	Phase	**0.1213**	**0.137±0.017**
ρ_3	M (MeV/c²)	1701	1700.9±0.2
	Γ (MeV/c²)	**17**	**17.0±0.3**
	Magnitude	**19.85**	**19.57±0.22**
	Phase	**0.501**	**0.504±0.016**
	log likelihood		45554.2
	χ^2/N_{dof} (p-value)		**5.8/8 (0.67)**

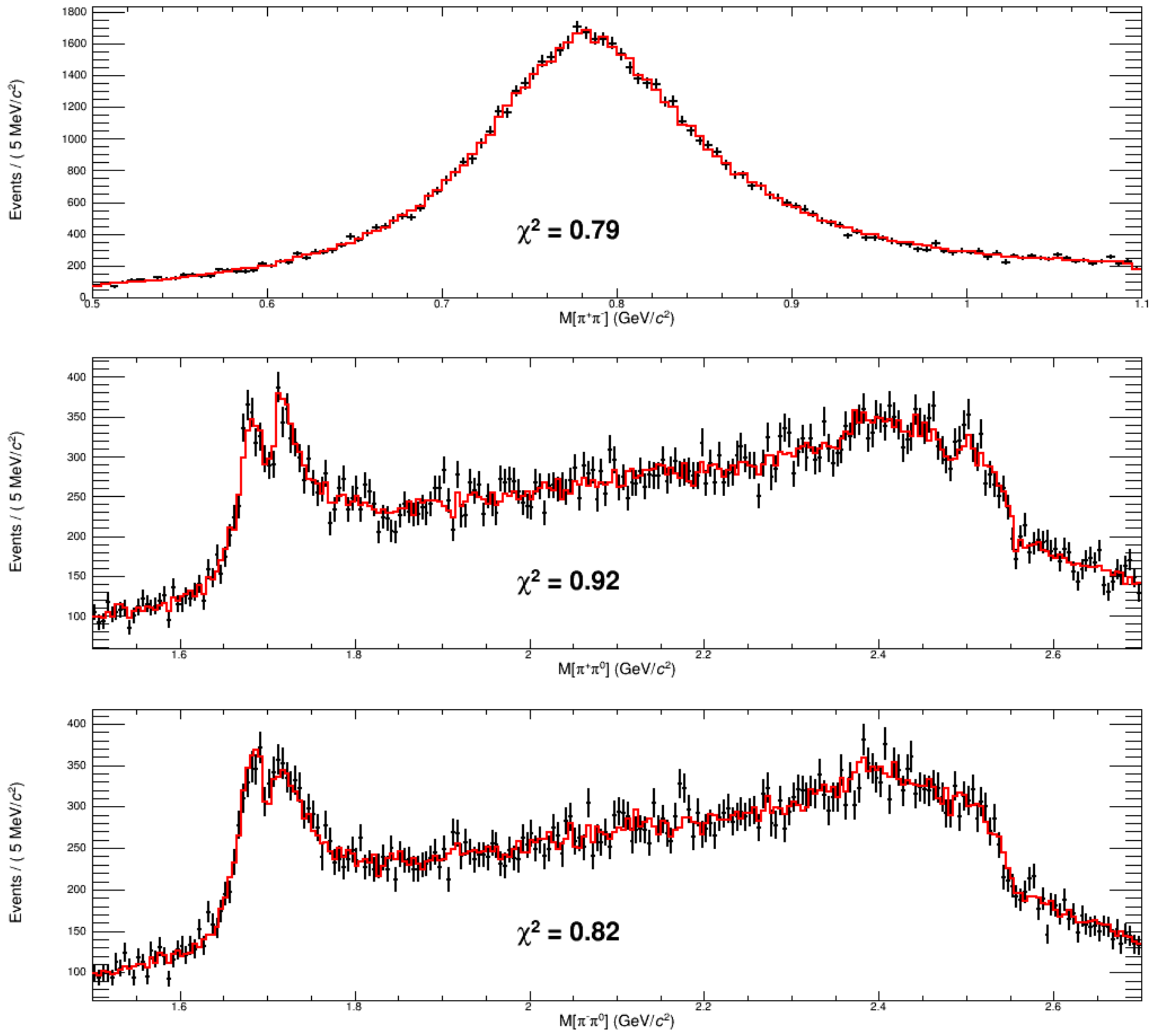


4.40 MeV/c² for M[$\pi^+\pi^0$]



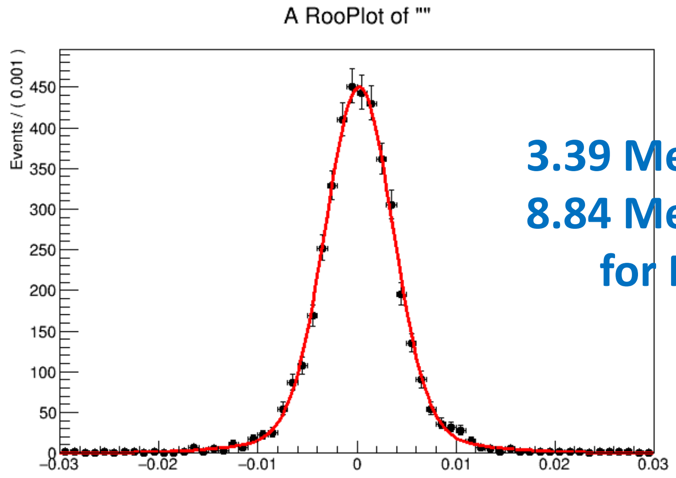
4.48 MeV/c² for M[$\pi^-\pi^0$]

Output: w/ single Gaussian resolution

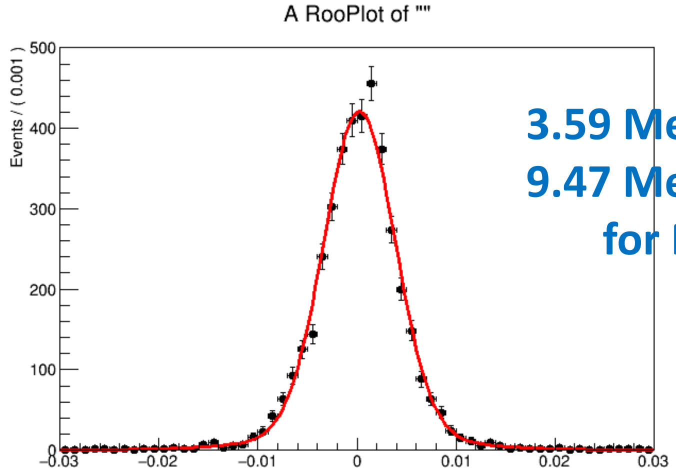


Output: w/ double Gaussian resolution

		Input	Output
ρ	M (MeV/c^2)	1687	1686.9±0.5
	Γ (MeV/c^2)	**51**	**$50.7^{+1.3}_{-1.2}$**
	Magnitude	**89**	**86.88±0.91**
	Phase	**0.1213**	**0.134±0.017**
$ρ_3$	M (MeV/c^2)	1701	1701.0±0.2
	Γ (MeV/c^2)	**17**	**16.9±0.3**
	Magnitude	**19.85**	**19.40±0.21**
	Phase	**0.501**	**0.512±0.016**
	log likelihood		45556.3
	$χ^2/N_{dof}$ (p-value)		**11.3/8 (0.19)**

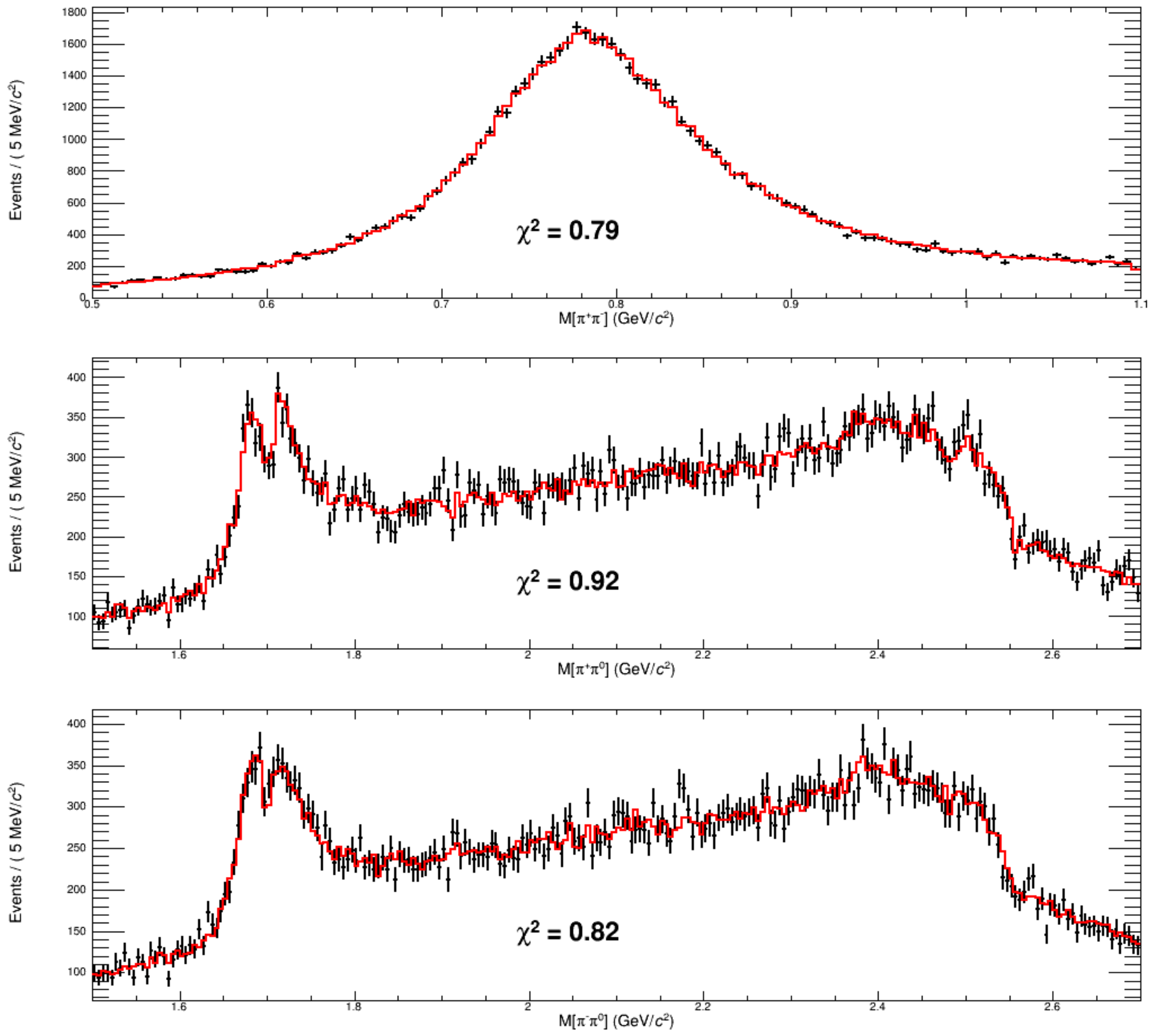


3.39 MeV/c^2 * 0.88
8.84 MeV/c^2 * 0.12
for M[$π^+π^0$]



3.59 MeV/c^2 * 0.90
9.47 MeV/c^2 * 0.10
for M[$π^-π^0$]

Output: w/ double Gaussian resolution



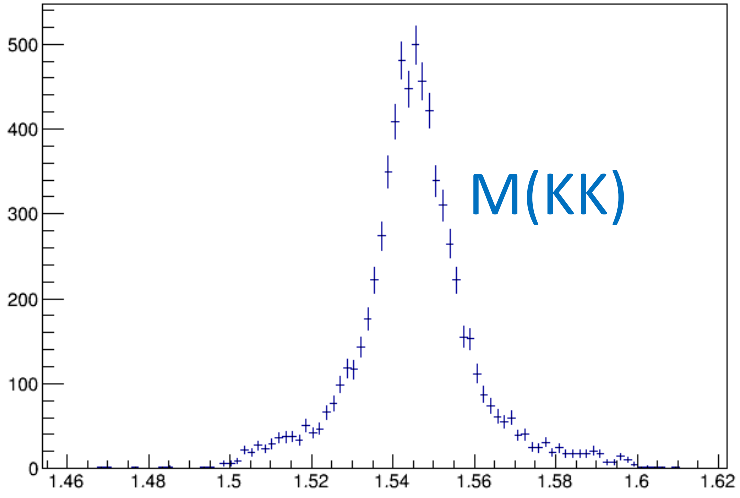
Comparison

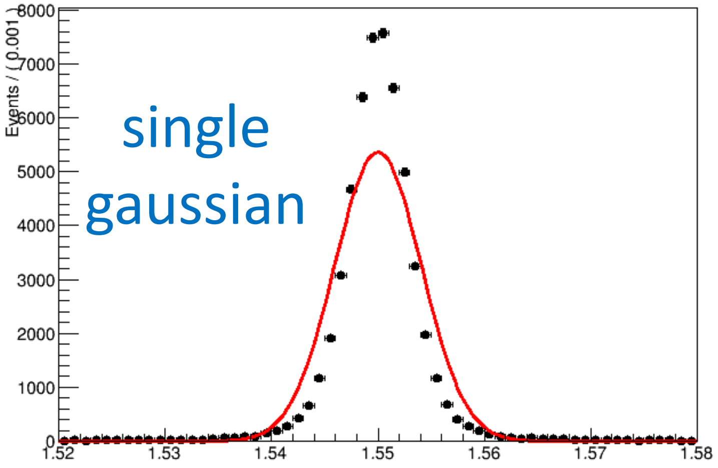
		Input	Output		
			w/o res.	w/ s.g. res.	w/ d.g. res.
ρ	M (MeV/c^2)	1687	1686.7 $\pm$ 0.6	1686.9 $\pm$ 0.5	1686.9 $\pm$ 0.5
	Γ (MeV/c^2)	**51**	**$64.5^{+1.6}_{-1.5}$**	**50.3 $\pm$ 1.3**	**$50.7^{+1.3}_{-1.2}$**
	Magnitude	**89**	**91.42 $\pm$ 0.96**	**87.47 $\pm$ 0.92**	**86.88 $\pm$ 0.91**
	Phase	**0.1213**	**-0.068 $\pm$ 0.015**	**0.137 $\pm$ 0.017**	**0.134 $\pm$ 0.017**
ρ_3	M (MeV/c^2)	1701	1700.1 $\pm$ 0.2	1700.9 $\pm$ 0.2	1701.0 $\pm$ 0.2
	Γ (MeV/c^2)	**17**	**15.0 $\pm$ 0.3**	**17.0 $\pm$ 0.3**	**16.9 $\pm$ 0.3**
	Magnitude	**19.85**	**18.52 $\pm$ 0.16**	**19.57 $\pm$ 0.22**	**19.40 $\pm$ 0.21**
	Phase	**0.501**	**0.271 $\pm$ 0.014**	**0.504 $\pm$ 0.016**	**0.512 $\pm$ 0.016**
	log likelihood		45400.5	45554.2	45556.3
	χ^2/N_{dof} (p-value)		**650.6/8 (0)**	**5.8/8 (0.67)**	**11.3/8 (0.19)**

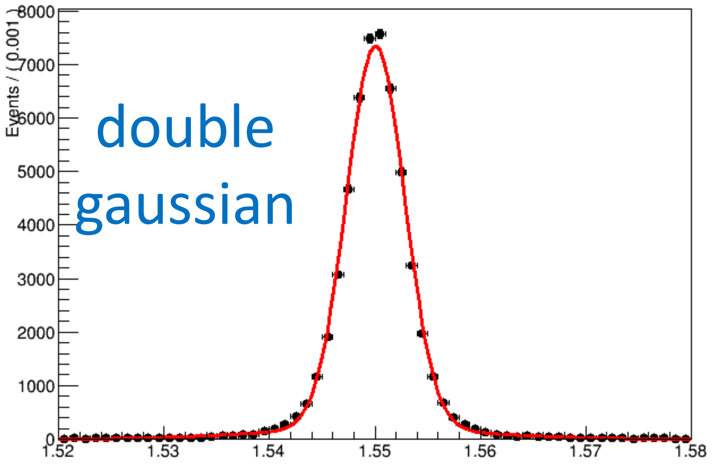
I/O check:
single f_0 in $J/\psi \rightarrow \gamma K^+K^-$

one narrow f_0 state

- Input parameters
  - M = 1554.0 MeV/c^2
  - Γ = 14.7 MeV/c^2
- "Data": 7042 events
- PHSP MC: 218760 events

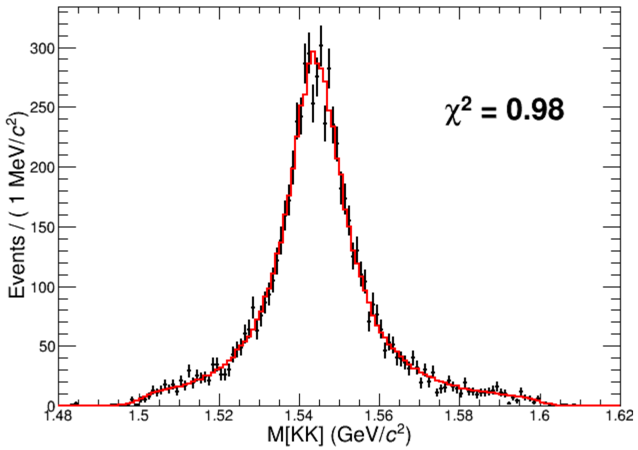


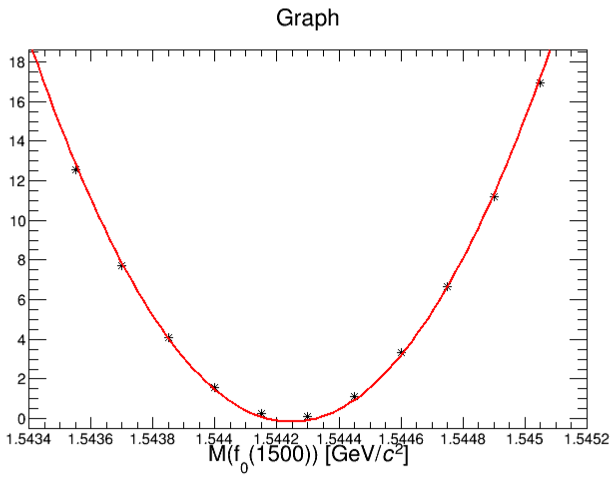


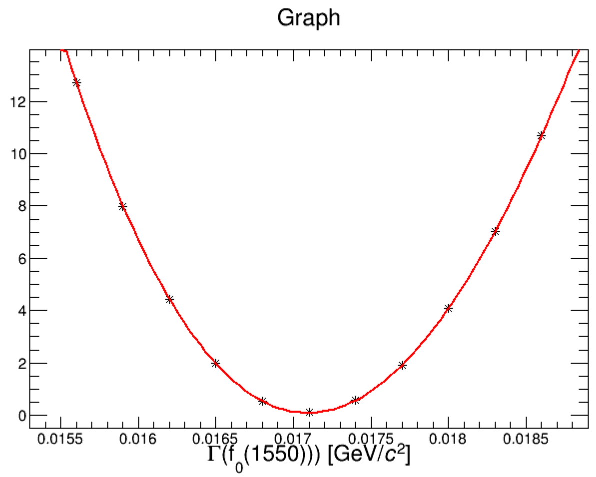


w/o resolution

- Input parameters
  - M = 1554.0 MeV/c^2
  - Γ = 14.7 MeV/c^2
- Output parameters
  - M = $1544.2^{+0.1}_{-0.1}$ MeV/c^2
  - Γ = $17.1^{+0.3}_{-0.3}$ MeV/c^2
  - log L = 3991.0

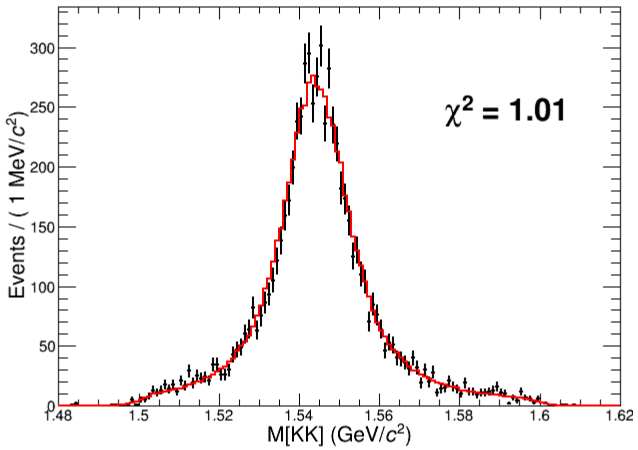


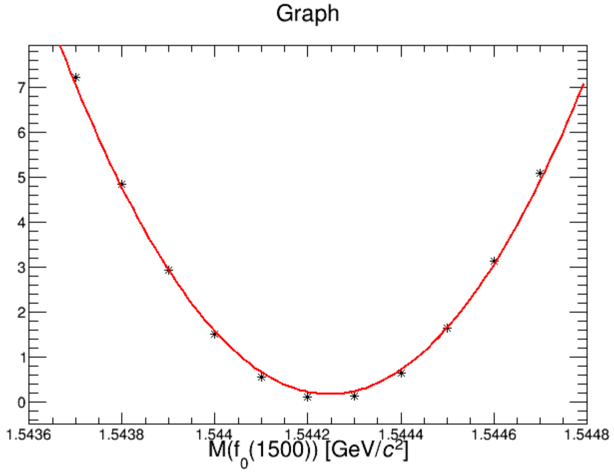


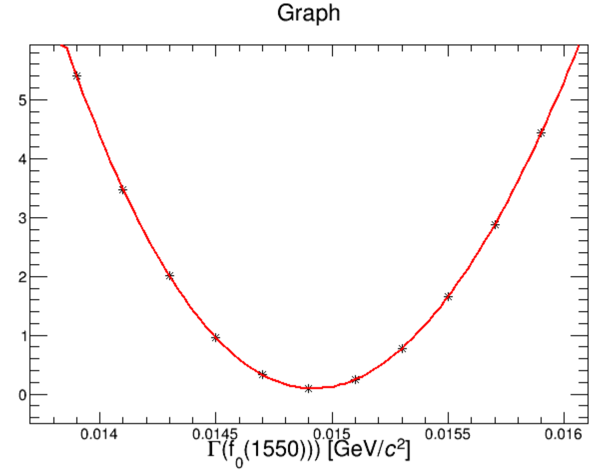


w/ single Gaussian resolution

- Input parameters
  - M = 1554.0 MeV/c^2
  - Γ = 14.7 MeV/c^2
- Output parameters
  - M = $1544.2^{+0.1}_{-0.1}$ MeV/c^2
  - Γ = $14.9^{+0.3}_{-0.3}$ MeV/c^2
  - log L = 3981.7

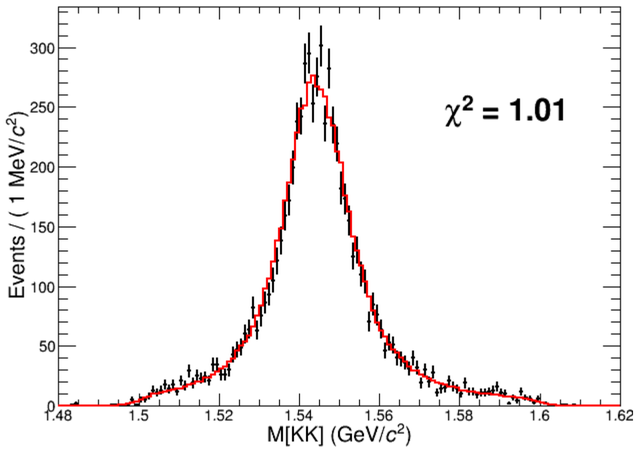


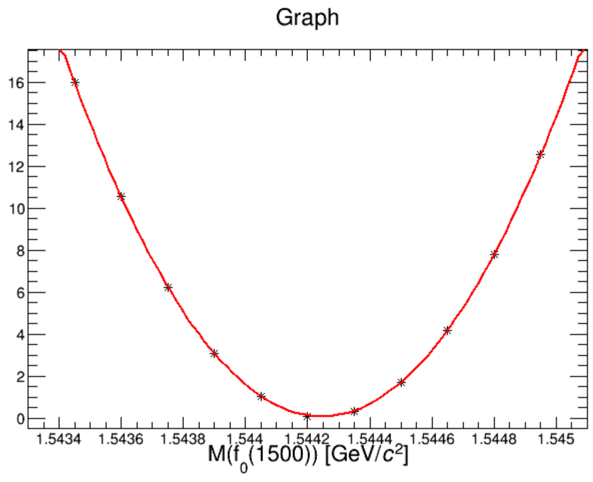


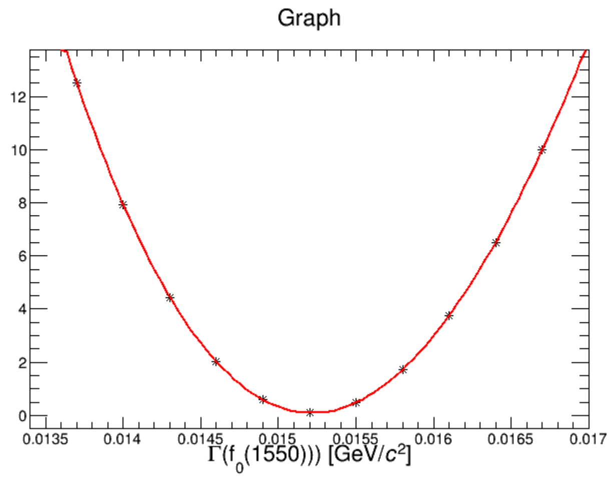


w/ double Gaussian resolution

- Input parameters
  - M = 1554.0 MeV/c^2
  - Γ = 14.7 MeV/c^2
- Output parameters
  - M = $1544.2^{+0.1}_{-0.1}$ MeV/c^2
  - Γ = $15.2^{+0.3}_{-0.3}$ MeV/c^2
  - log L = 3988.8







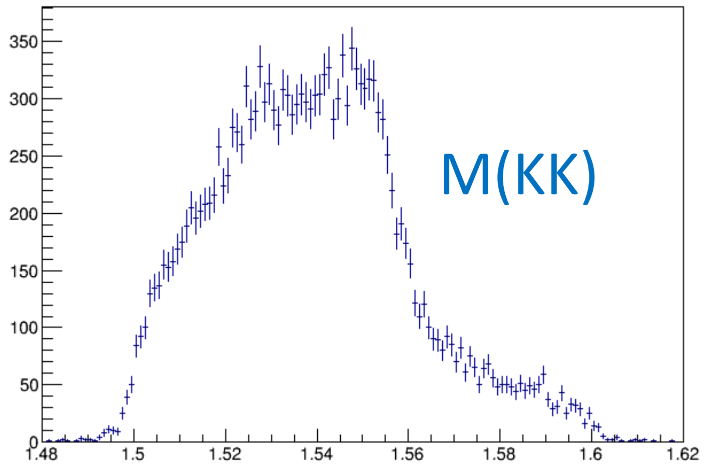
Comparison

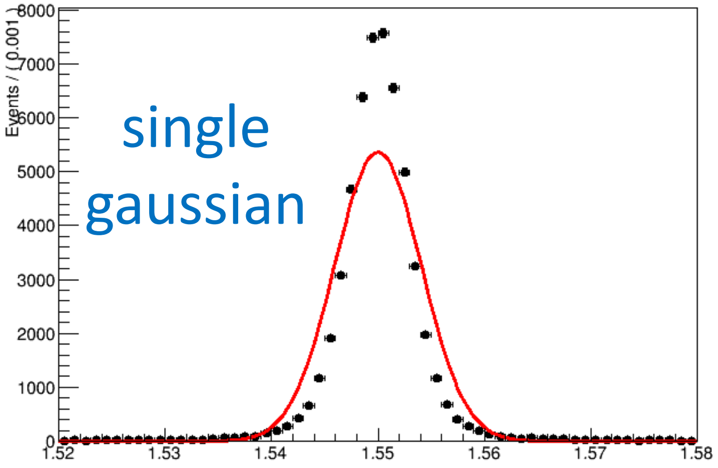
		no resolution	single gaussian	double gaussian
input	mass (MeV/c^2)	1554.0		
	width (MeV/c^2)	14.7		
output	mass (MeV/c^2)	$1544.2^{+0.1}_{-0.1}$	$1544.2^{+0.1}_{-0.1}$	$1544.2^{+0.1}_{-0.1}$
	width (MeV/c^2)	$17.1^{+0.3}_{-0.3}$	$14.9^{+0.3}_{-0.3}$	$15.2^{+0.3}_{-0.3}$
	log L	3991.0	3981.7	3988.8

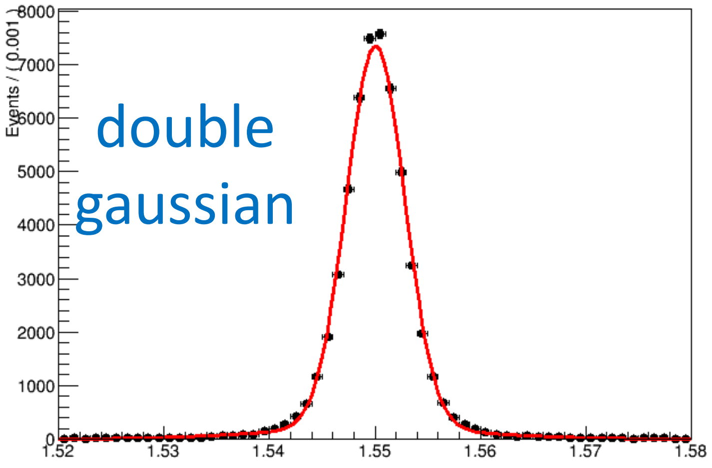
I/O check:
f_0-f_0 interference in $J/\psi \rightarrow \gamma K^+K^-$

one ordinary f_0 and one narrow f_0

- Input parameters
  - M1 = 1530.5 MeV/c^2
  - Γ1 = 52.9 MeV/c^2
  - M2 = 1554.2 MeV/c^2
  - Γ2 = 10.8 MeV/c^2
  - Mag2 = 0.87098
  - Phase2 = 1.6201
- "Data": 17596 events
- PHSP MC: 218760 events







w/o resolution

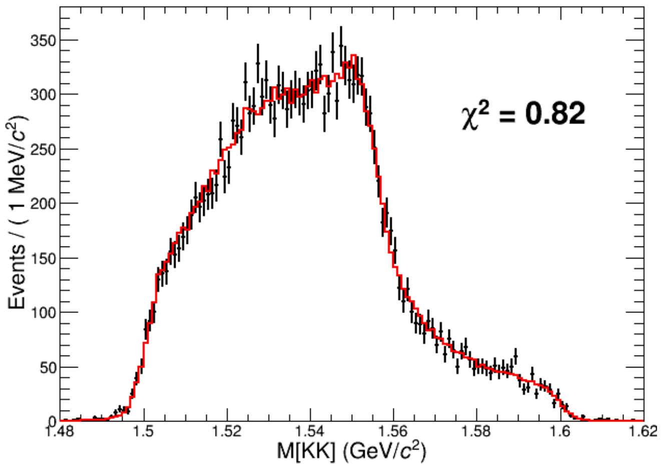
Input parameters

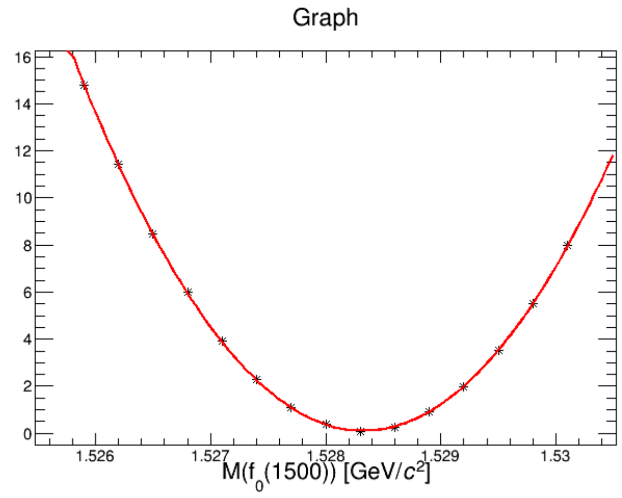
- $M1 = 1530.5\ MeV/c^2$
- $\Gamma 1 = 52.9\ MeV/c^2$
- $M2 = 1554.2\ MeV/c^2$
- $\Gamma 2 = 10.8\ MeV/c^2$
- Mag2 = 0.87098
- Phase2 = 1.6201

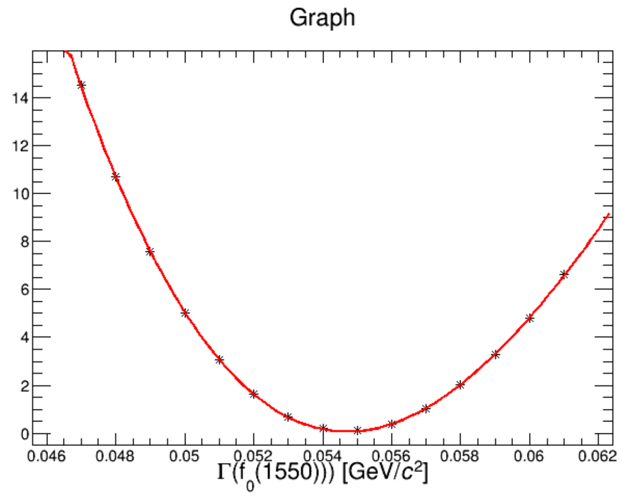
Output parameters

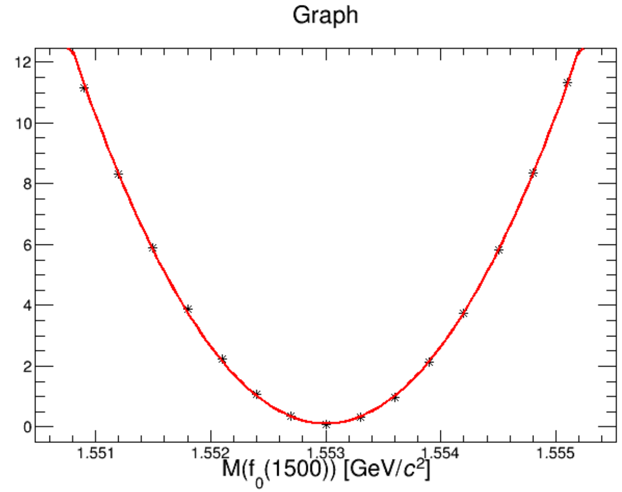
- $M1 = 1528.3^{+0.4}_{-0.4}\ MeV/c^2$
- $\Gamma 1 = 54.7^{+1.6}_{-1.6}\ MeV/c^2$
- $M2 = 1553.0^{+0.4}_{-0.4}\ MeV/c^2$
- $\Gamma 2 = 14.3^{+1.0}_{-0.9}\ MeV/c^2$
- Mag2 = 1.162
- Phase2 = 1.374
- log L = 3949.1

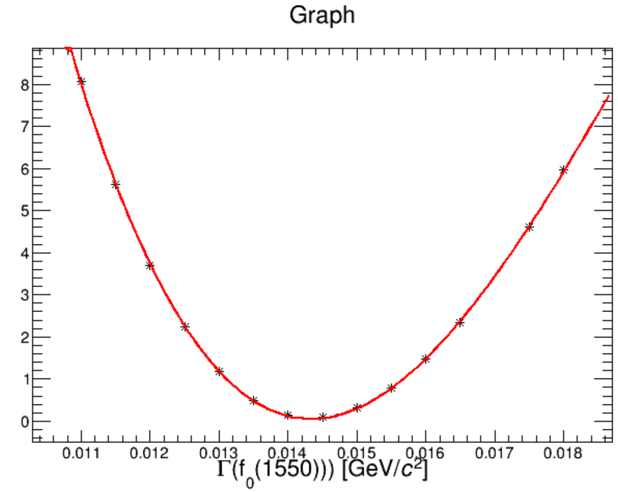
w/o resolution











w/ single Gaussian resolution

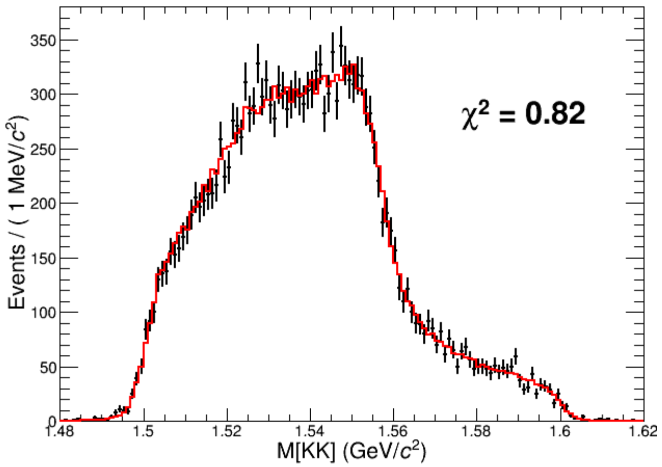
Input parameters

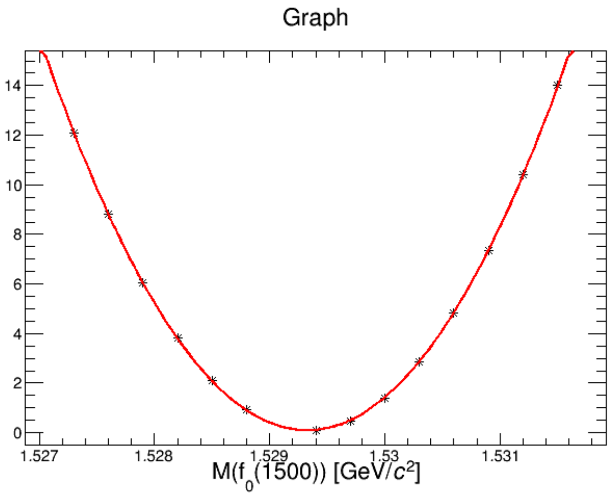
- $M1 = 1530.5$ MeV/c^2
- $\Gamma 1 = 52.9$ MeV/c^2
- $M2 = 1554.2$ MeV/c^2
- $\Gamma 2 = 10.8$ MeV/c^2
- Mag2 = 0.87098
- Phase2 = 1.6201

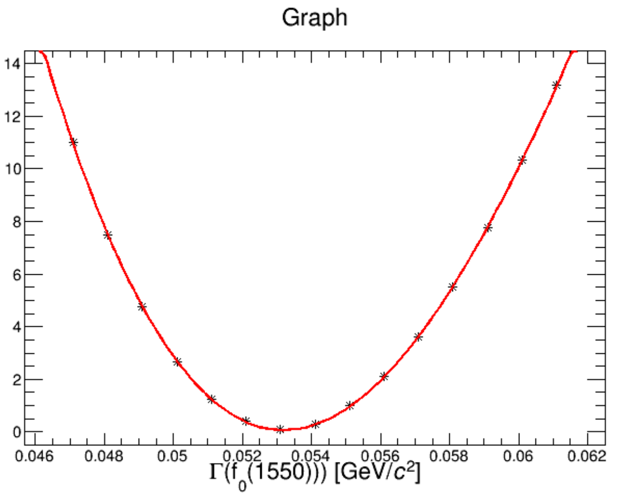
Output parameters

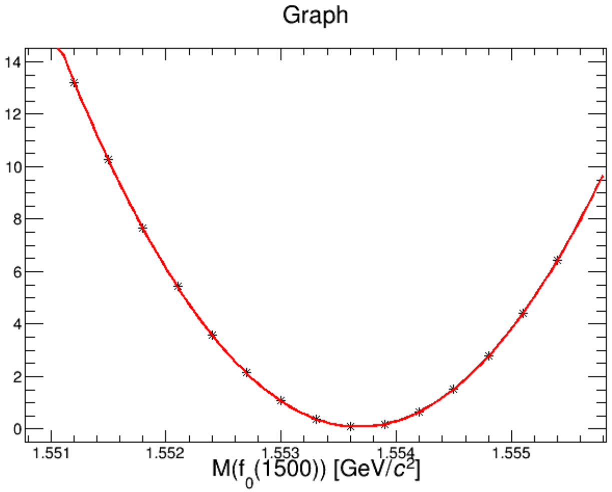
- $M1 = 1529.3^{+0.4}_{-0.4}$ MeV/c^2
- $\Gamma 1 = 53.2^{+1.4}_{-1.4}$ MeV/c^2
- $M2 = 1553.7^{+0.5}_{-0.5}$ MeV/c^2
- $\Gamma 2 = 8.1^{+1.1}_{-1.0}$ MeV/c^2
- Mag2 = 0.766
- Phase2 = 1.501
- log L = 3950.1

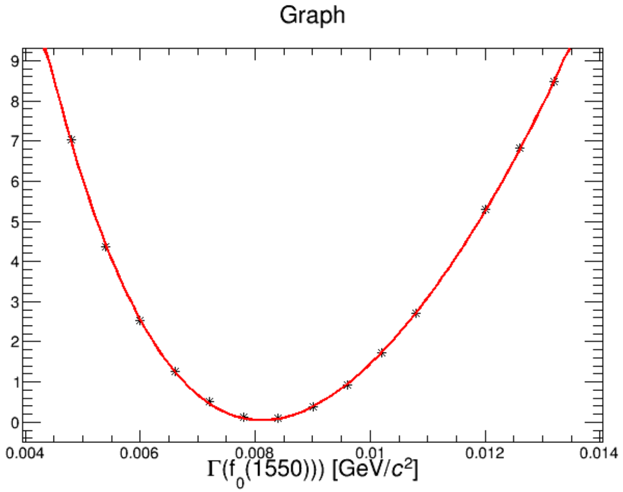
w/ single Gaussian resolution











w/ double Gaussian resolution

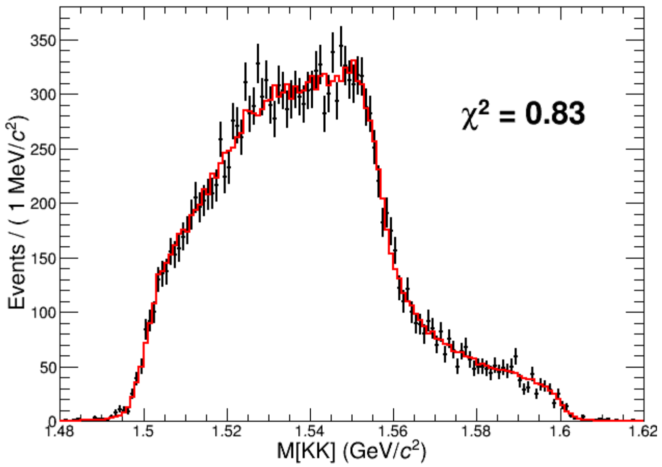
Input parameters

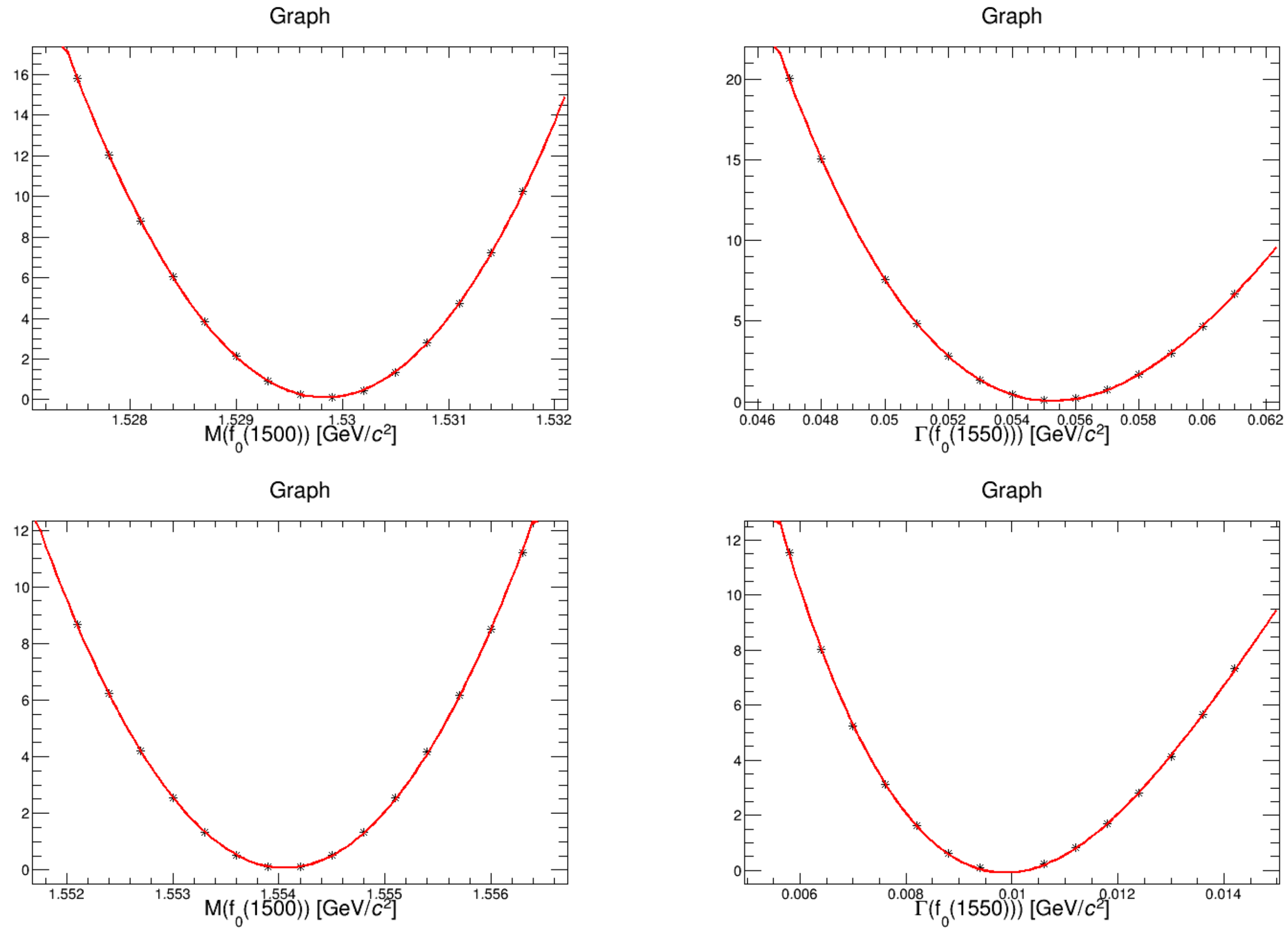
- M1 = 1530.5 MeV/c^2
- Γ1 = 52.9 MeV/c^2
- M2 = 1554.2 MeV/c^2
- Γ2 = 10.8 MeV/c^2
- Mag2 = 0.87098
- Phase2 = 1.6201

Output parameters

- M1 = $1529.8^{+0.4}_{-0.4}$ MeV/c^2
- Γ1 = $55.3^{+1.4}_{-1.4}$ MeV/c^2
- M2 = $1554.1^{+0.5}_{-0.5}$ MeV/c^2
- Γ2 = $9.9^{+1.0}_{-0.9}$ MeV/c^2
- Mag2 = 0.8350
- Phase2 = 1.6124
- log L = 3950.0

w/ double Gaussian resolution



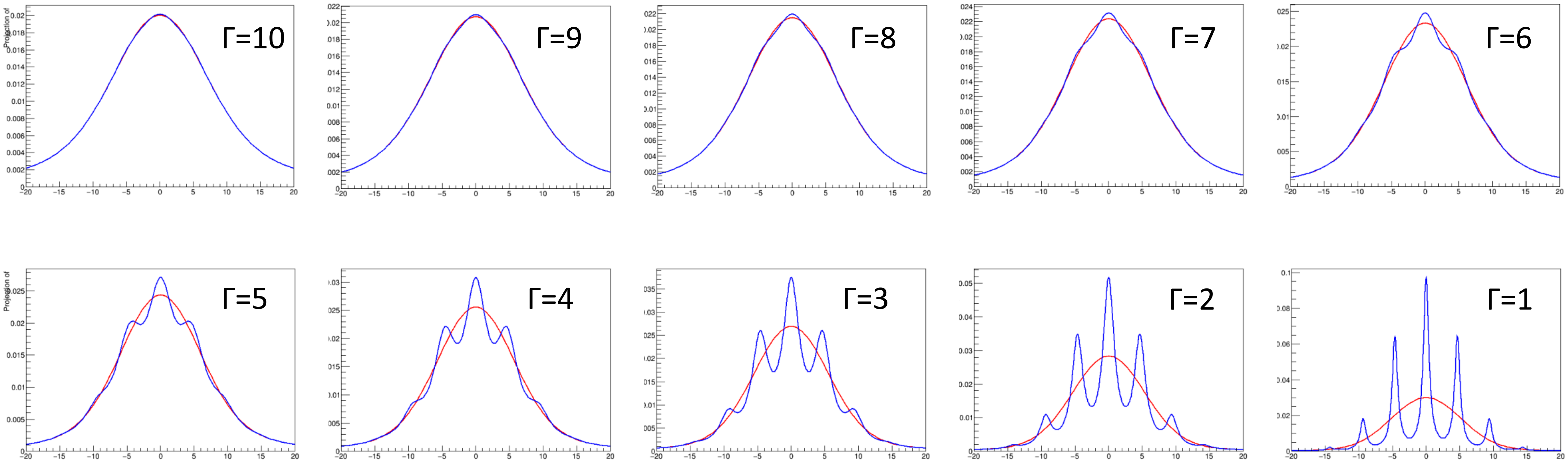


Comparison

		no resolution	single gaussian	double gaussian
input	mass1 (MeV/c^2)	1530.5		
	width1 (MeV/c^2)	52.9		
	mass2 (MeV/c^2)	1554.2		
	width2 (MeV/c^2)	10.8		
	mag2	0.87098		
	phase2	1.6201		
output	mass1 (MeV/c^2)	$1528.3^{+0.4}_{-0.4}$	$1529.3^{+0.4}_{-0.4}$	$1529.8^{+0.4}_{-0.4}$
	width1 (MeV/c^2)	$54.7^{+1.6}_{-1.6}$	$53.2^{+1.4}_{-1.4}$	$55.3^{+1.4}_{-1.4}$
	mass2 (MeV/c^2)	$1553.0^{+0.4}_{-0.4}$	$1553.7^{+0.5}_{-0.5}$	$1554.1^{+0.5}_{-0.5}$
	width2 (MeV/c^2)	$14.3^{+1.0}_{-0.9}$	$8.1^{+1.1}_{-1.0}$	$9.9^{+1.0}_{-0.9}$
	mag2	1.162	0.766	0.8350
	phase2	1.374	1.501	1.6124
	log L	3949.1	3950.1	3950.0

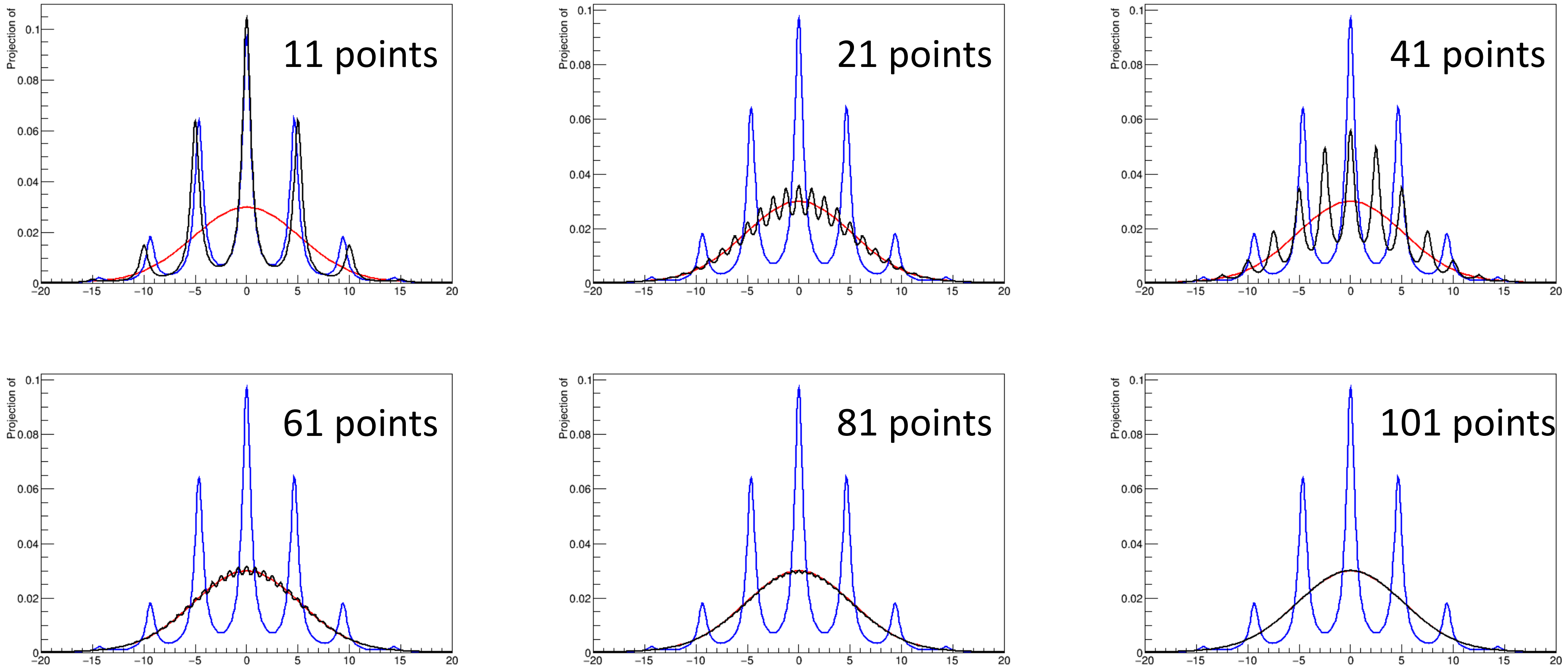
Extreme narrow resonance?

Resolution = 5



Gauss-Hermite quadrature's performance is bad when Γ <= σ

$$\int f(x-y)\,g(y)dy \approx \sum_i f(x-yi)g(yi)$$



Resolution = 5, Γ=1

More sampling points → better performance

Potential problems:

- **Enough memory?**
- **Time consumption?**