Better Time Measurement

Will Imoehl
Indiana University
Email: wimoehl@iu.edu

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Issue

- Fit with
$$a + \frac{be^{-c(x-d)}}{1 + e^{-f(x-t)}}$$
- Currently use parameter t
- This is unstable - similar shapes give different t values
- Large interplay between parameters
- Other attempts were
  - Time at half max
- Function fits data well - no need to change functions

Better Time: Time at Maximum?

- Issue with half max is error propagation not correct
- Similar solution - time at maximum
- Can solve analytically (steps in backup):

$$\frac{dfunc}{dx} = 0 \implies t_{max} = -\frac{1}{f} \ln\left(\frac{c}{f - c}\right) + t$$

- This should be more stable
- Error propagation possible with error matrix from fit

Calculating Maximum

$$\frac{d}{dx}\left(a+\frac{be^{-c(x-d)}}{1+e^{-f(x-t)}}\right)=0$$

$$\frac{-bce^{-c(x_{max}-d)}(1+e^{-f(x_{max}-t)})+fbe^{-c(x_{max}-d)}e^{-f(x_{max}-t)}}{(1+e^{-f(x_{max}-t)})^2}=0$$

$$-c(1+e^{-f(x_{max}-t)})+fe^{-f(x_{max}-t)}=0$$

$$c=(f-c)e^{-f(x_{max}-t)}$$

$$\ln\left(\frac{c}{f-c}\right)=-f(x_{max}-t)$$

$$-\frac{1}{f}\ln\left(\frac{c}{f-c}\right)+t=x_{max}$$

Error Calculation

$$\sigma_{t_{max}}^2 = \sigma_t^2 \partial_t^2 + \sigma_c^2 \partial_c^2 + \sigma_f^2 \partial_f^2 + 2\partial_c \partial_f \sigma_{cf} + 2\partial_c \partial_t \sigma_{ct} + 2\partial_f \partial_t \sigma_{ft}$$

$$t_{max} = -\frac{1}{f} \ln\left(\frac{c}{f-c}\right) + t$$

$$\partial_c = \frac{\partial t_{max}}{\partial c} = \frac{1}{c^2 - cf}$$

$$\partial_t = \frac{\partial t_{max}}{\partial t} = 1$$

$$\partial_f = \frac{\partial t_{max}}{\partial f} = \frac{\left(\frac{f}{f-c}\right) + \ln\left(\frac{c}{f-c}\right)}{f^2}$$

- $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$
- Error calculated with formula above
- There are large correlations between the parameters

Do calculation for half max?

Define $t_1 =$ time at half max, t_2 time at max

$$f(t_1) = 0.5 * f(t_{max})$$

$$a + \frac{be^{-c(t_1-d)}}{1+e^{-f(t_1-t)}} = 1/2\left(a + \frac{be^{-c(t_2-d)}}{1+e^{-f(t_2-t)}}\right)$$

$$\frac{be^{-c(t_1-d)}}{1+e^{-f(t_1-t)}} = \frac{-a}{2} + \frac{0.5be^{-c(t_2-t)}}{1+e^{-f(t_2-t)}}$$

$$be^{-c(t_1-d)} = (1+e^{-f(t_1-t)})\left(\frac{-a}{2} + \frac{0.5e^{-c(t_2-t)}}{1+e^{-f(t_2-t)}}\right)$$

$$be^{-c(t_1-d)} + e^{-f(t_1-t)}\left(\frac{a}{2} - \frac{0.5be^{-c(t_2-t)}}{1+e^{-f(t_2-t)}}\right) = \frac{-a}{2} + \frac{0.5be^{-c(t_2-t)}}{1+e^{-f(t_2-t)}}$$

No closed form solution