Better Time Measurement

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Issue

- Fit with

$$a + \frac{be^{-c(x-d)}}{1 + e^{-f(x-t)}}$$

- Currently use parameter t
- This is unstable - similar shapes give different t values
- Large interplay between parameters
- Other attempts were
  - Time at half max
- Function fits data well - no need to change functions

Better Time: Time at Maximum?

- Issue with half max is error propagation not correct
- Similar solution - time at maximum
- Can solve analytically (steps in backup):

$$\frac{dfunc}{dx} = 0 \implies t_{max} = -\frac{1}{f} \ln\left(\frac{c}{f-c}\right) + t$$

- This should be more stable
- Error propagation possible with error matrix from fit

Error Calculation

$$\sigma_{t_{max}}^2 = \sigma_t^2 \partial_t^2 + \sigma_c^2 \partial_c^2 + \sigma_f^2 \partial_f^2 + 2\partial_c \partial_f \sigma_{cf} + 2\partial_c \partial_t \sigma_{ct} + 2\partial_f \partial_t \sigma_{ft}$$

$$t_{max} = -\frac{1}{f} \ln\left(\frac{c}{f-c}\right) + t$$

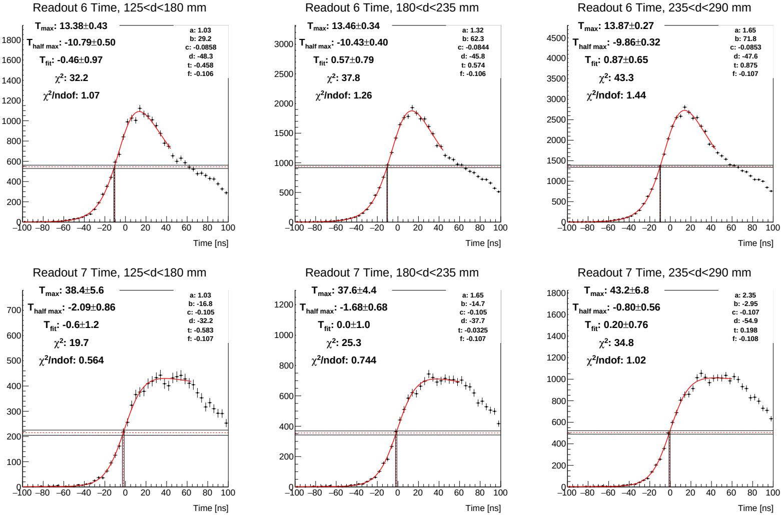
$$\partial_c = \frac{\partial t_{max}}{\partial c} = \frac{1}{c^2 - cf}$$

$$\partial_t = \frac{\partial t_{max}}{\partial t} = 1$$

$$\partial_f = \frac{\partial t_{max}}{\partial f} = \frac{\left(\frac{f}{f-c}\right) + \ln\left(\frac{c}{f-c}\right)}{f^2}$$

- $\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$
- Error calculated with formula above
- There are large correlations between the parameters

Sample Fits



- Error on t_{max} smaller for clean peaks, larger for distributions that plateau

Backup Slides

Calculating Maximum

$$\frac{d}{dx}\left(a+\frac{be^{-c(x-d)}}{1+e^{-f(x-t)}}\right)=0$$

$$\frac{-bce^{-c(x_{max}-d)}(1+e^{-f(x_{max}-t)})+fbe^{-c(x_{max}-d)}e^{-f(x_{max}-t)}}{(1+e^{-f(x_{max}-t)})^2}=0$$

$$-c(1+e^{-f(x_{max}-t)})+fe^{-f(x_{max}-t)}=0$$

$$c=(f-c)e^{-f(x_{max}-t)}$$

$$\ln\left(\frac{c}{f-c}\right)=-f(x_{max}-t)$$

$$-\frac{1}{f}\ln\left(\frac{c}{f-c}\right)+t=x_{max}$$

$\frac{\partial t_{max}}{\partial t}$

$$\frac{\partial}{\partial t}\left[-\frac{1}{f}\ln\left(\frac{c}{f-c}\right)+t\right]=1$$

$\frac{\partial t_{max}}{\partial c}$

$$\frac{\partial}{\partial c}\left[-\frac{1}{f}\ln\left(\frac{c}{f-c}\right)+t\right]$$

$$\frac{\partial}{\partial c}\left[-\frac{1}{f}\left(\ln c-\ln(f-c)\right)\right]$$

$$-\frac{1}{cf}-\frac{1}{f(f-c)}$$

$$-\frac{(f-c)}{cf(f-c)}-\frac{c}{cf(f-c)}$$

$$-\frac{f}{cf(f-c)}=-\frac{1}{c(f-c)}=\frac{1}{c^2-cf}$$

$\frac{\partial t_{max}}{\partial f}$

$$\frac{\partial}{\partial f}\left[-\frac{1}{f}\ln\left(\frac{c}{f-c}\right)+t\right]$$

$$\frac{\partial}{\partial f}\left[-\frac{1}{f}(\ln c-\ln(f-c))\right]$$

$$\frac{1}{f^2}\ln c+\frac{\partial}{\partial f}\frac{\ln(f-c)}{f}$$

$$\frac{1}{f^2}\ln c+\frac{\frac{f}{f-c}-\ln(f-c)}{f^2}$$

$$\frac{1}{f^2}\left[\ln\left(\frac{c}{f-c}\right)+\frac{f}{f-c}\right]$$