Strongly first-order electroweak phase transition, electroweak sphaleron, and SGWB

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电弱相变与希格斯物理专题研讨会@IHEP

July, 31 2020

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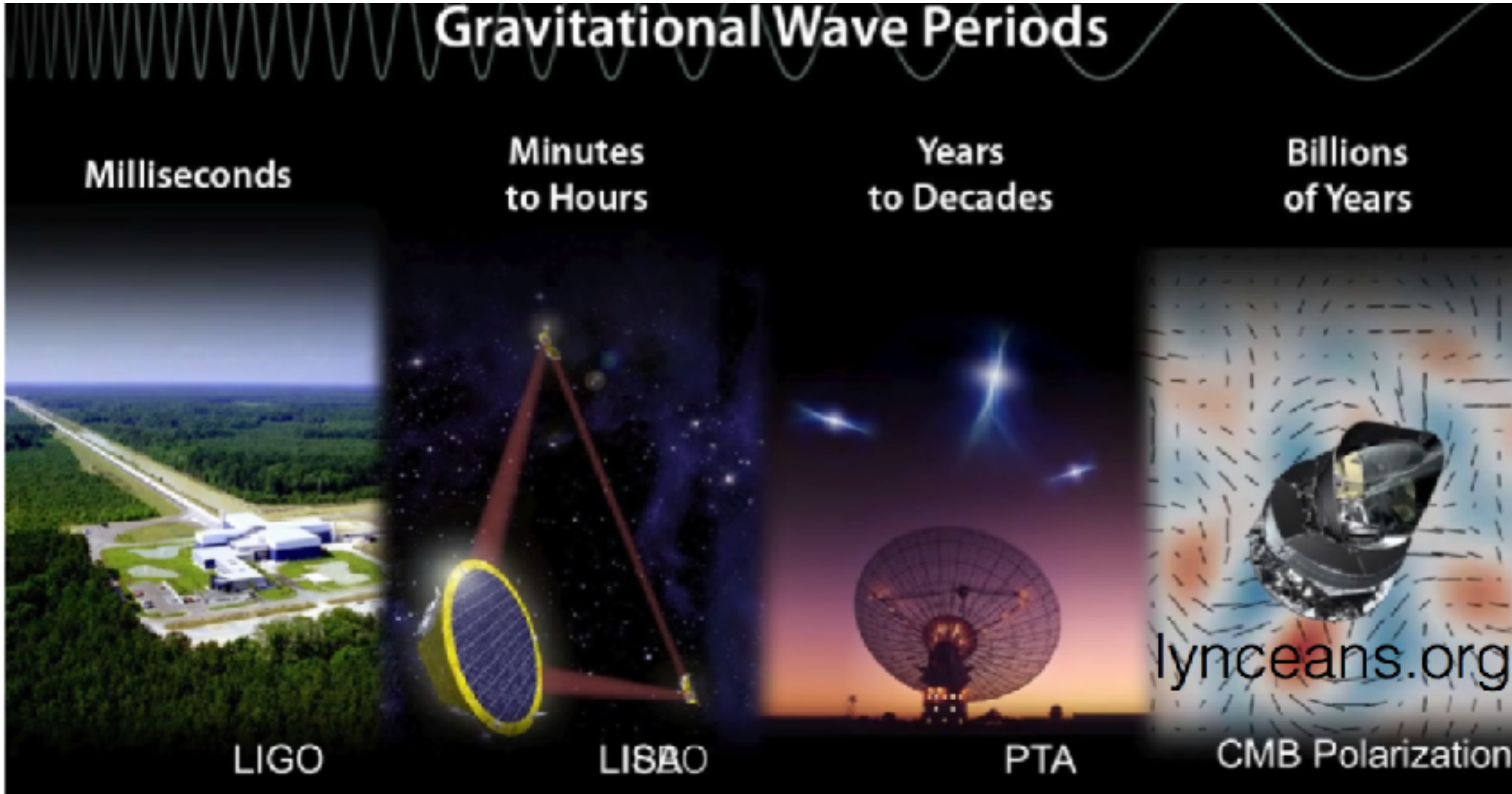
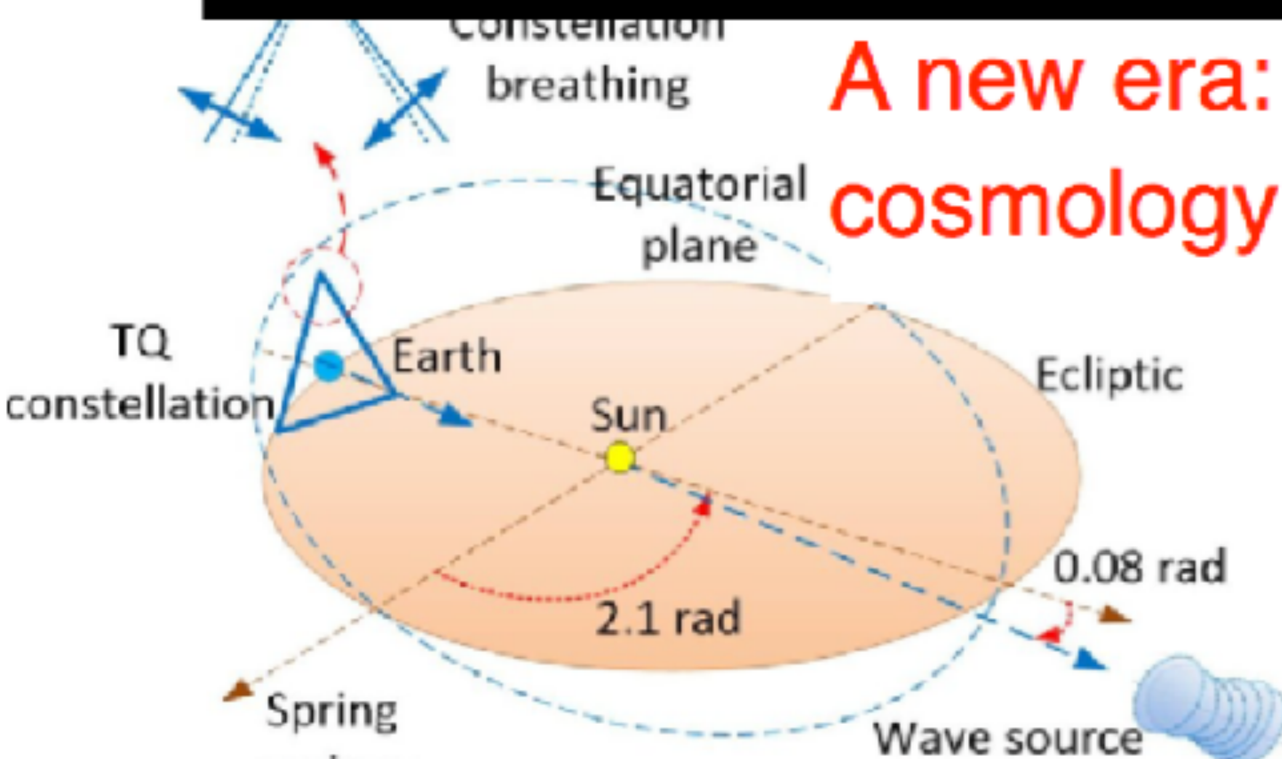
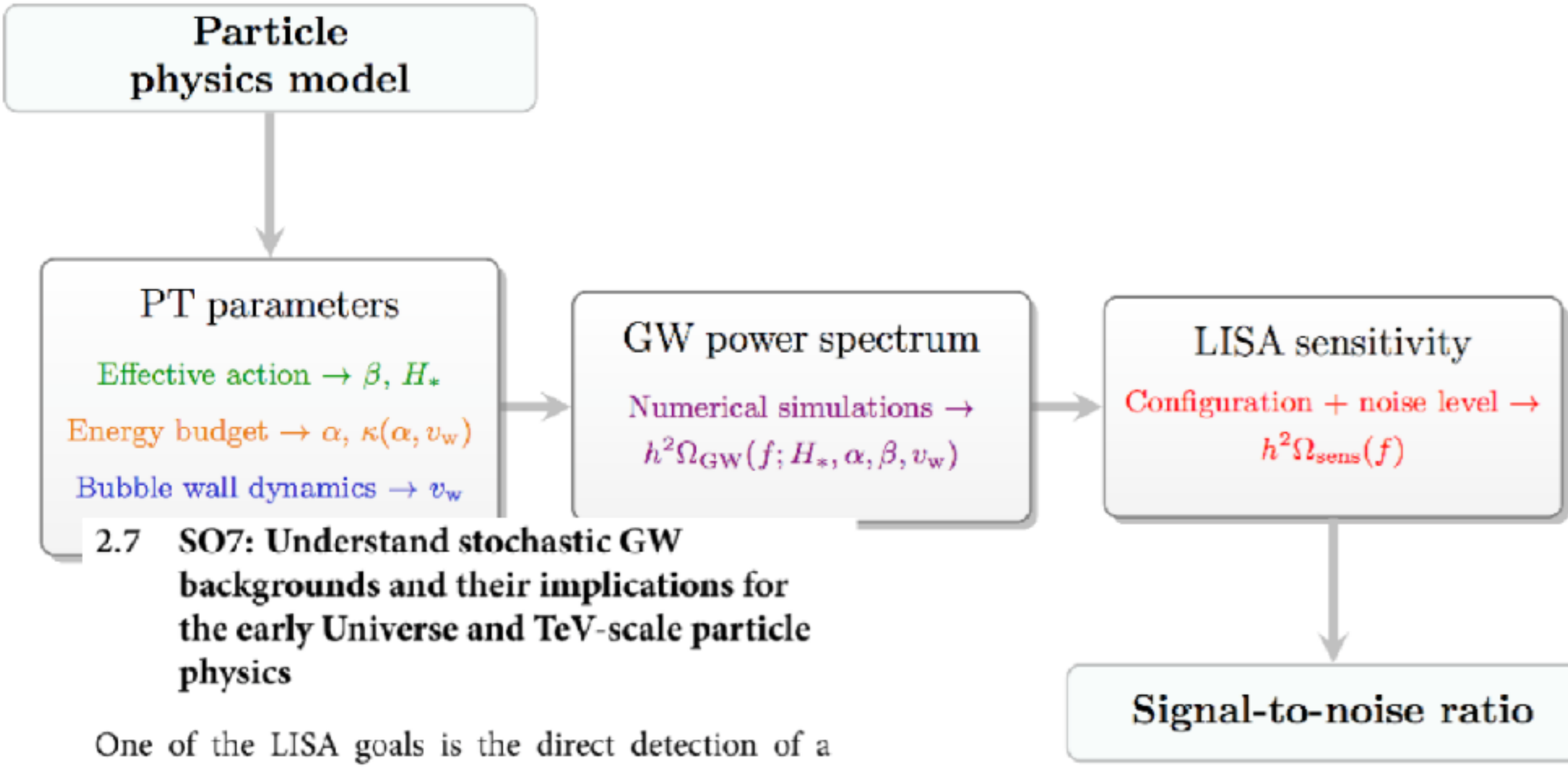




A new era: GWs as a new tool for probing cosmology and high energy physics

Current: LIGO,PTA,…

Future GW experiments:
LISA, Taiji, TianQin, BBO, DECIGO, ET, CE,…



2.7 SO7: Understand stochastic GW backgrounds and their implications for the early Universe and TeV-scale particle physics

One of the LISA goals is the direct detection of a stochastic GW background of cosmological origin (like for example the one produced by a first-order phase transition around the TeV scale) and stochastic foregrounds. Probing a stochastic GW background of cosmological origin provides information on new physics in the early Universe. The shape of the signal gives an indication of its origin, while an upper limit allows to constrain models of the early Universe and particle physics beyond the standard model.

1702.00786

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\rm min}}^{f_{\rm max}} \mathrm{d}f \left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm Sens}(f)}\right]^2}$$

JCAP03(2020)024

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}(\frac{d\phi_b}{dr})^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0 \,, \qquad \frac{d\phi_b}{dr}|_{r=0} = 0$$

Bubble nucleation:

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

Latent heat:

$$\alpha = \frac{1}{\rho_R}\left[-(V_{\rm EW} - V_f) + T\left(\frac{dV_{\rm EW}}{dT} - \frac{dV_f}{dT}\right)\right]\Big|_{T=T_*}$$

phase transition inverse duration:

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}|_{T=T_n}$$

GW from FOPT

$$\Omega_{\rm GW}(f)h^2 \approx \Omega_{\rm sw}(f)h^2 + \Omega_{\rm turb}(f)h^2$$

Sound Wave:

$$\Omega h^2_{\rm sw}(f) = 2.65 \times 10^{-6}(H_* \tau_{sw}) \left(\frac{\beta}{H}\right)^{-1} v_b \left(\frac{\kappa_\nu \alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

Hindmarsh ’17 Ellis ’18

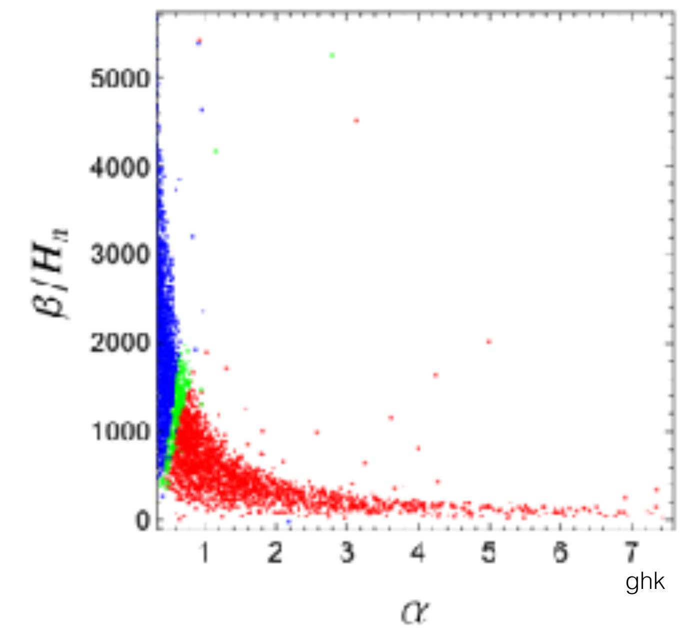
phase transition duration:

$$\tau_{sw} = min\left[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f}\right], \; H_* R_* = v_b (8\pi)^{1/3}(\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma

$$\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu \alpha}{1+\alpha}$$

$$f_{\rm sw} = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} {\rm Hz}$$



MHD turbulence:

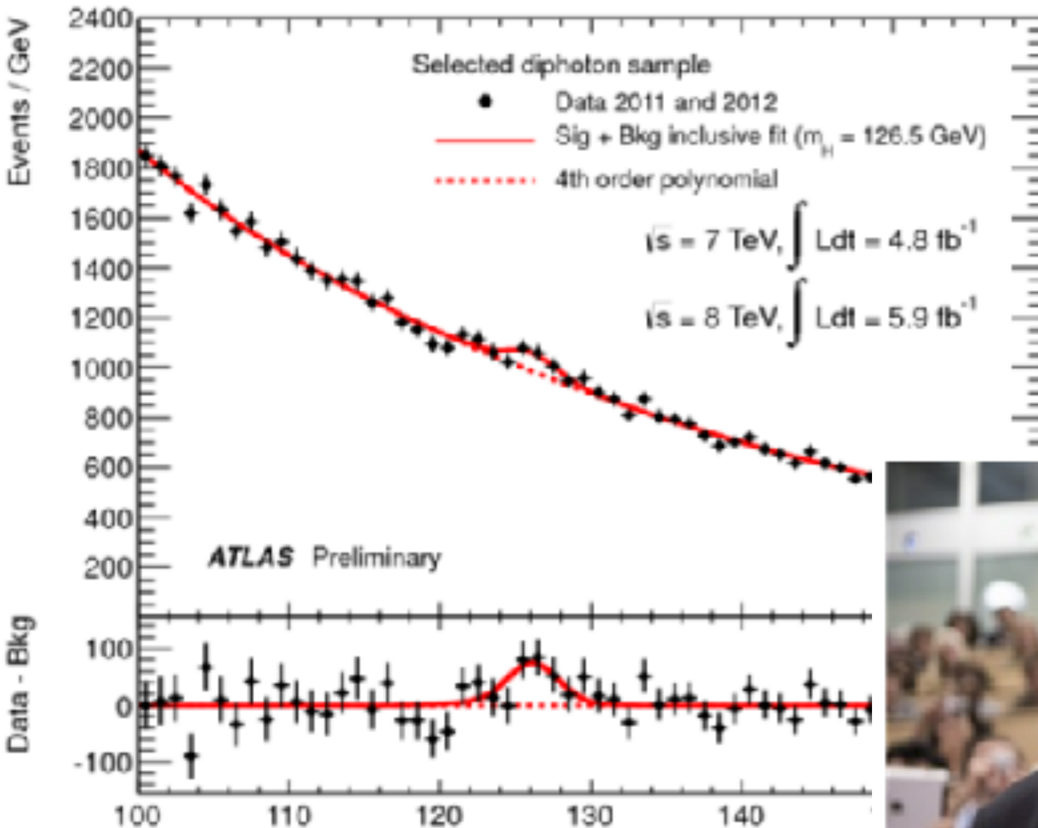
$$\Omega h^2_{\rm turb}(f) = 3.35 \times 10^{-4} \left(\frac{\beta}{H}\right)^{-1} \left(\frac{\epsilon \kappa_\nu \alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b \frac{(f/f_{\rm turb})^3 (1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_* H_*)]}$$

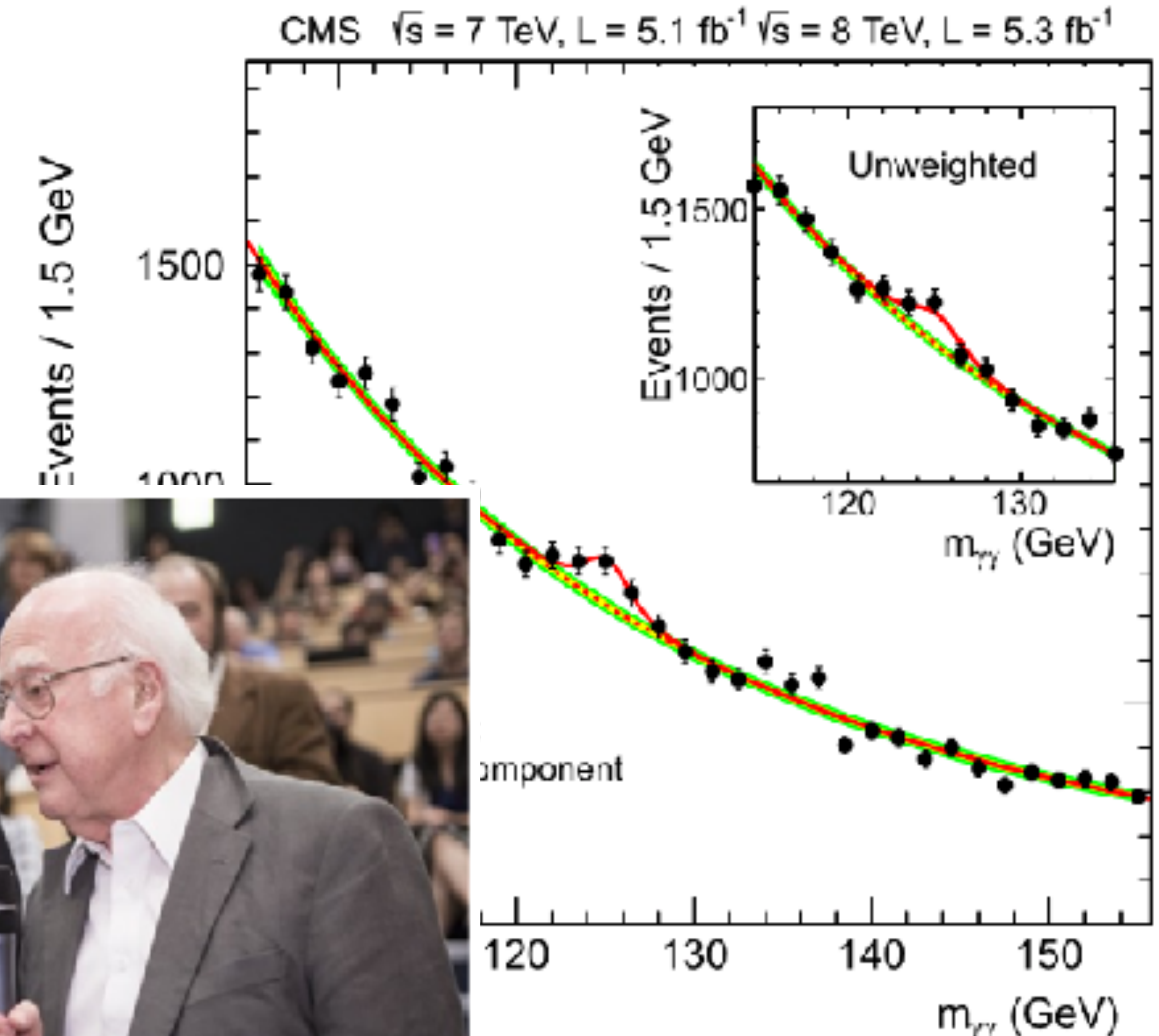
Caprini ‘09

$$f_{\rm turb} = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} {\rm Hz}$$

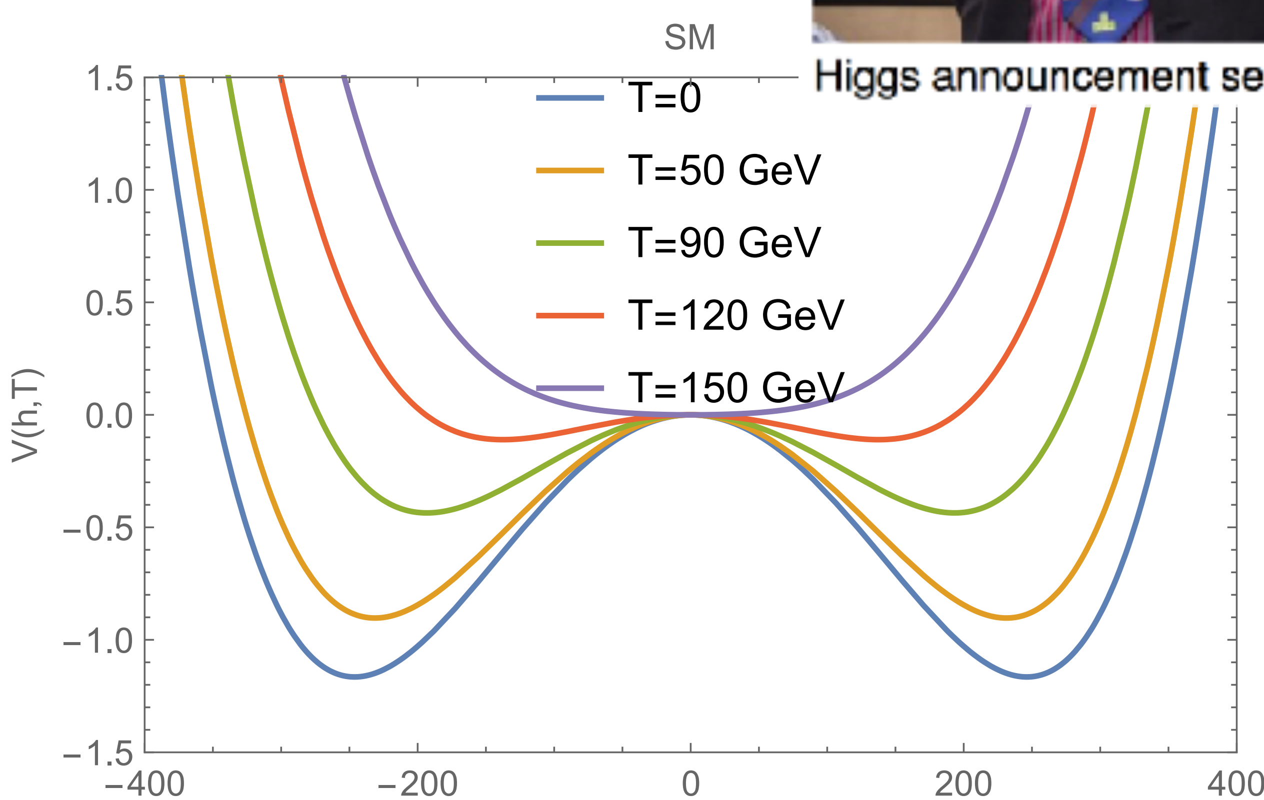
$\kappa_\nu(v_b, \alpha)$: the fraction of released energy going to the kinetic energy of the plasma

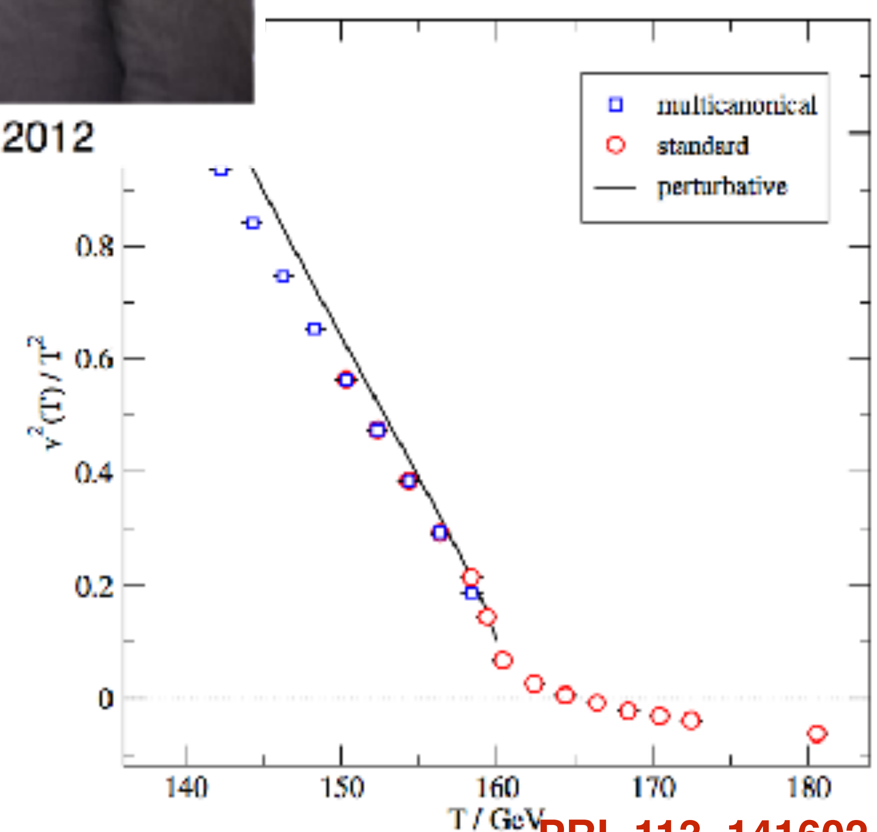
Implication of 125 GeV Higgs






Higgs announcement seminar on 4 July 2012





PRL 113, 141602 (2014)

BSM for EWPT

SM+Scalar Singlet

Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, Jiang, Bian, Huang, Shu 15, Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17,Cheng, Bian 17, Bian, Tang 18,Chen, Li, Wu, Bian,19…

SM+Scalar Doublet

Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, Bernon, Bian, Jiang 17, Bian, Liu 18,…

SM + Scalar Triplet

Profumo, Ramsey-Musolf 12, Chiang 14, Zhou, Cheng, Deng, Bian, Wu 18,Zhou, Bian, Guo,Wu 19,…

NMSSM

Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17,…

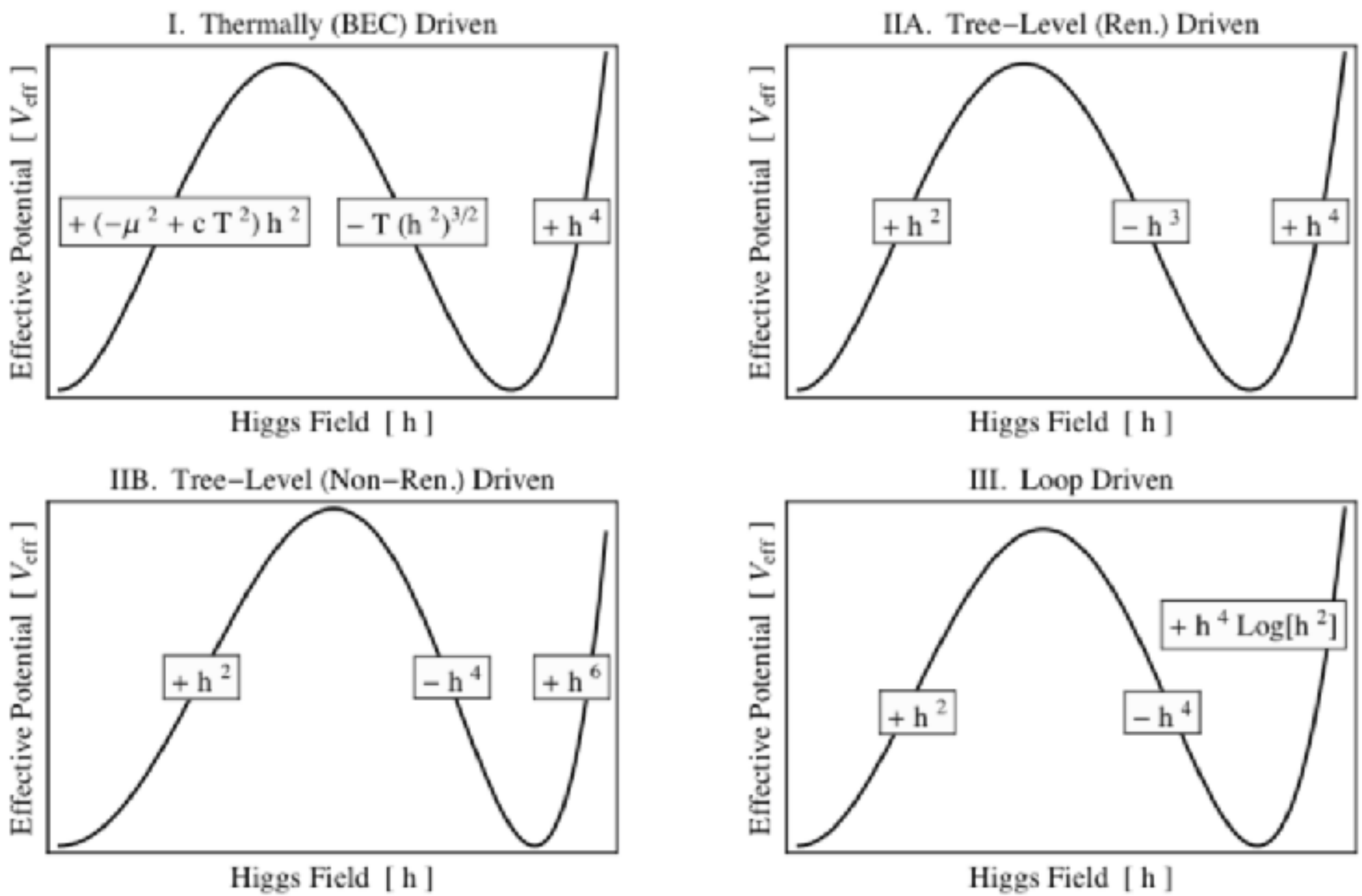
Composite Higgs

Espinosa, Gripaios, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, Bian,Wu,Xie 19, De Curtis, Delle Rose, Panico 19, Bian,Wu,Xie 20,…

EFT

Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15,Cai,Sasaki ,Wang17,Zhou, Bian, Guo 19, …

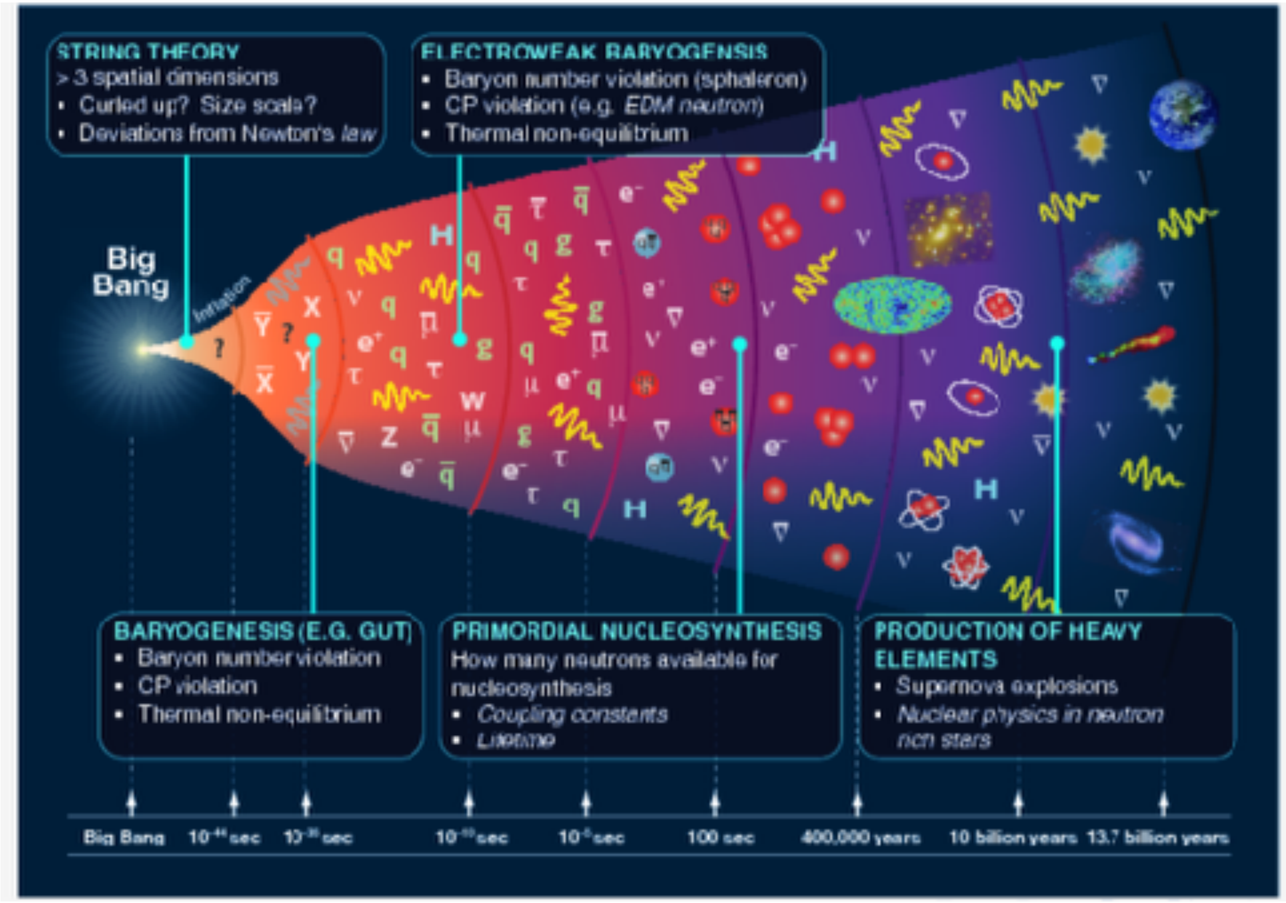
Model classes for catalyzing a strongly first order electroweak phase transition



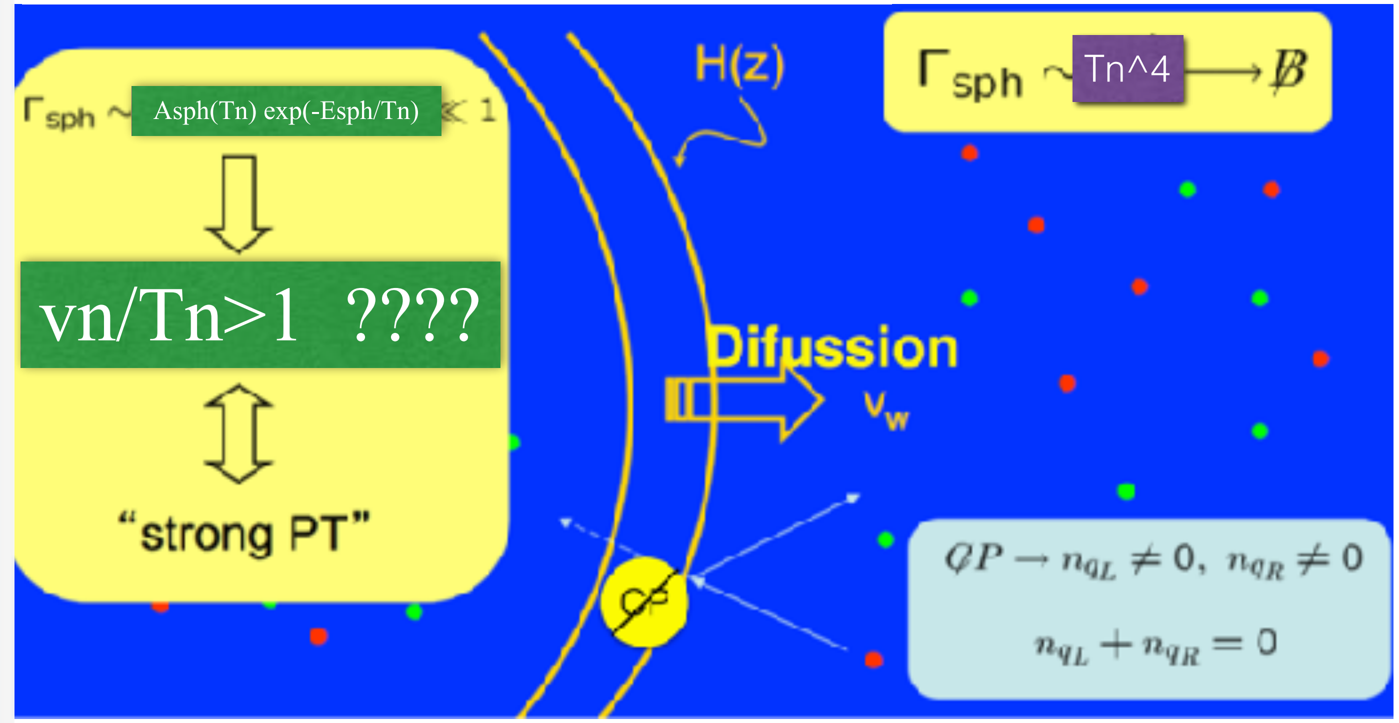
Daniel J H. Chung, Andrew J. Long, and Lian-Tao Wang PRD87, 023509 (2013)

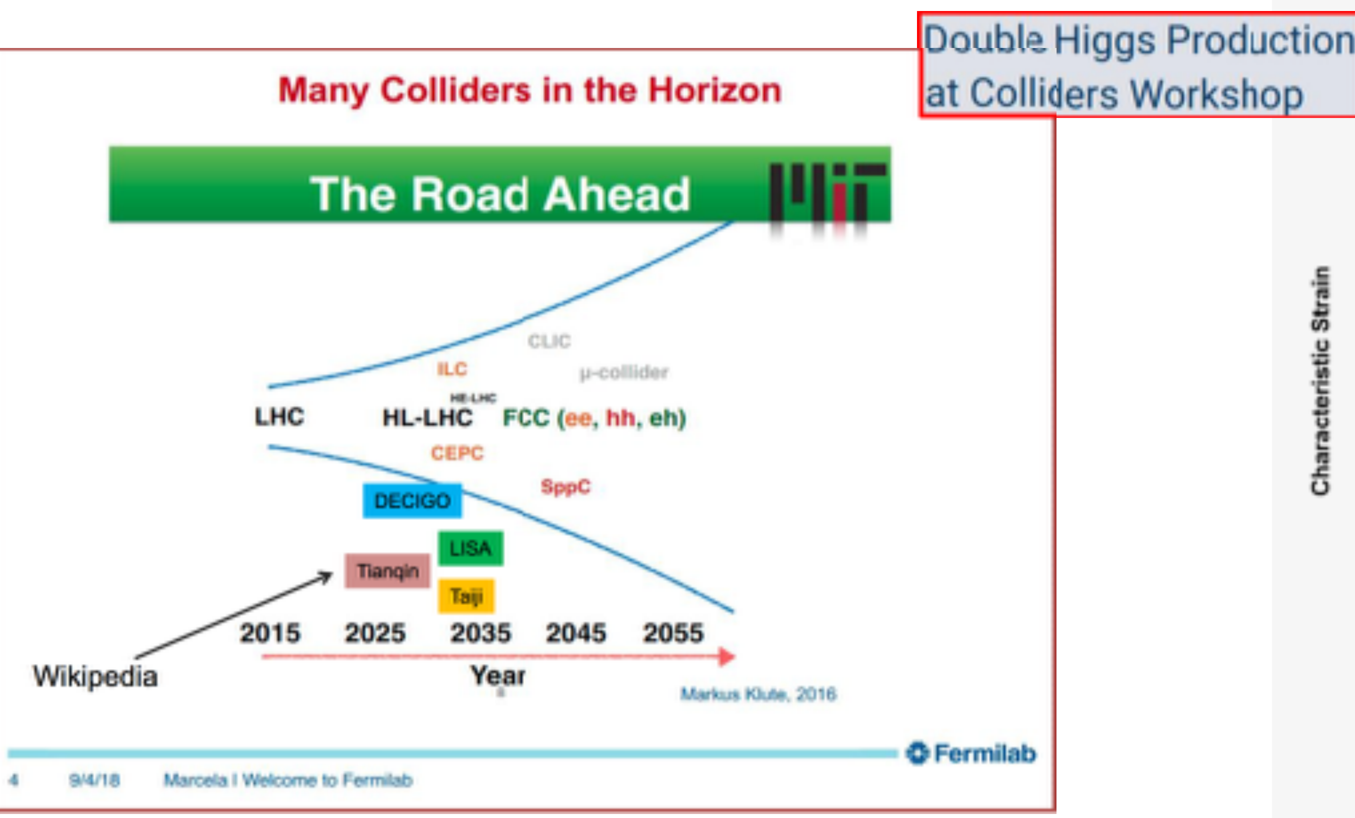
Why SFOEWPT

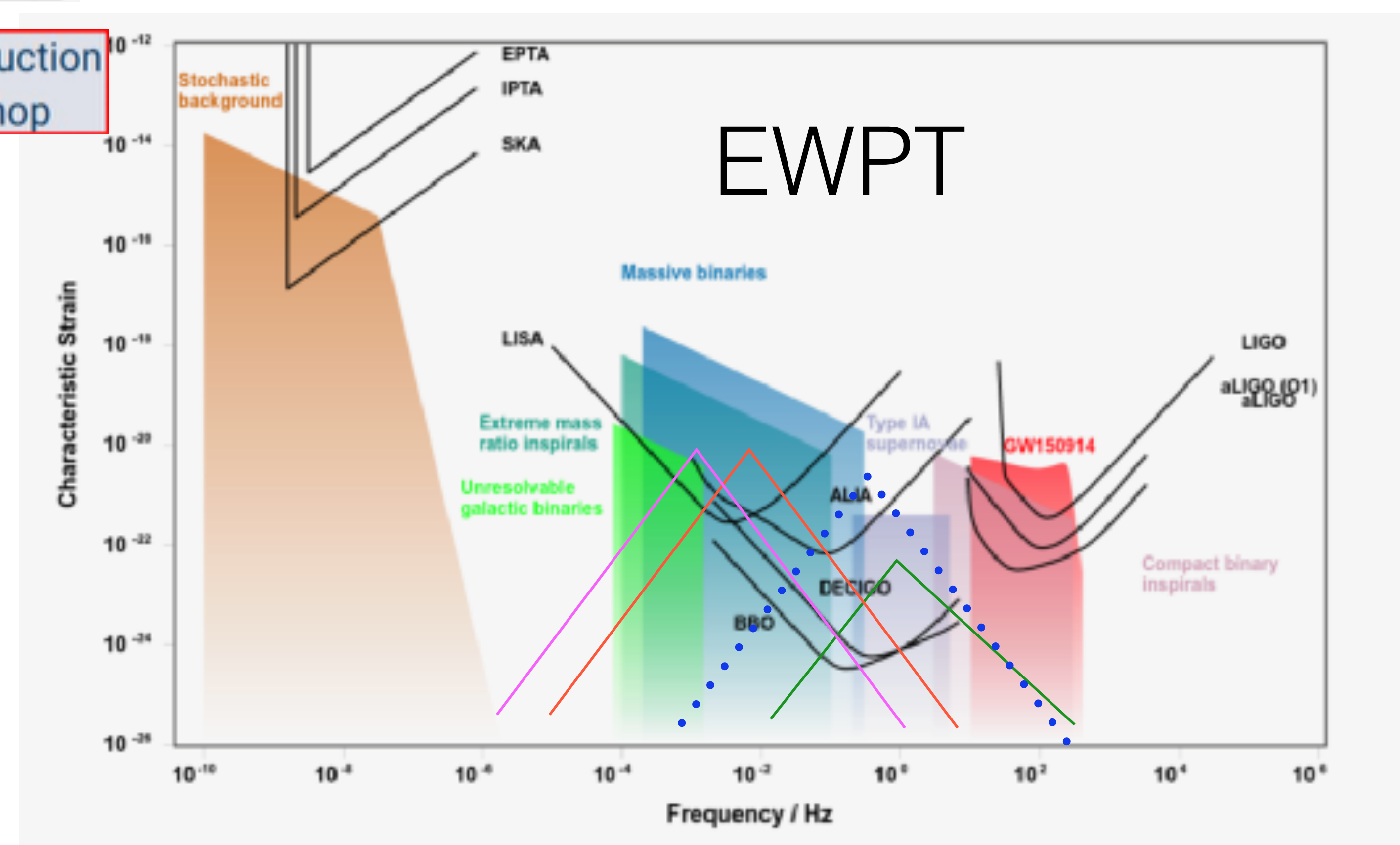
Tn~ 10^2 GeV



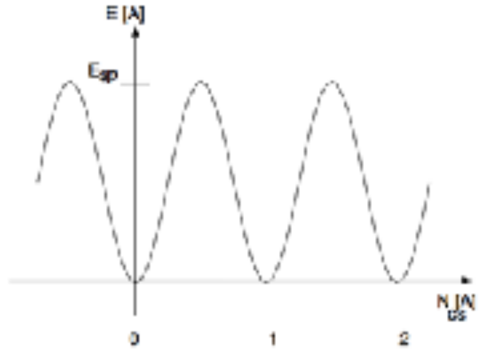
$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$







C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.



Sphaleron decay rate

$$\Gamma / V = \frac{\omega_-}{2\pi} \mathcal{N}_{tr} (\mathcal{N}V)_{rot} \left[\frac{\alpha_W T}{4\pi} \right]^3 \alpha_3^{-6} e^{-E_{sp}/T} \kappa$$

$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left(-g_2^2 F^{a\mu\nu} \widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu} \widetilde{f}_{\mu\nu} \right),$$

$$\Delta B = N_F (\Delta N_{CS} - \Delta n_{CS}),$$

$$N_{CS} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \, \mathrm{Tr} \left[\partial_i A_j A_k + i \frac{2}{3} g_2 A_i A_j A_k \right],$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \, \partial_i B_j B_k,$$

$$A_i \to U A_i U^{-1} + \frac{i}{g_2} (\partial_i U) U^{-1},$$

$$\delta N_{CS} = \frac{1}{24\pi^2} \int d^3x \, \mathrm{Tr} \left[(\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right] \epsilon^{ijk}.$$

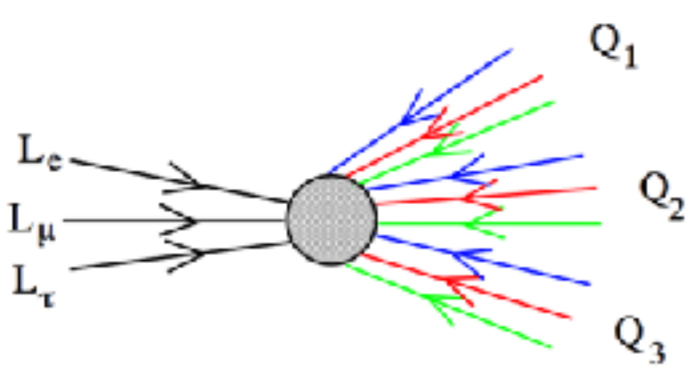
w-: negative-mode frequency, rate of decay in small fluctuations around the sphaleron

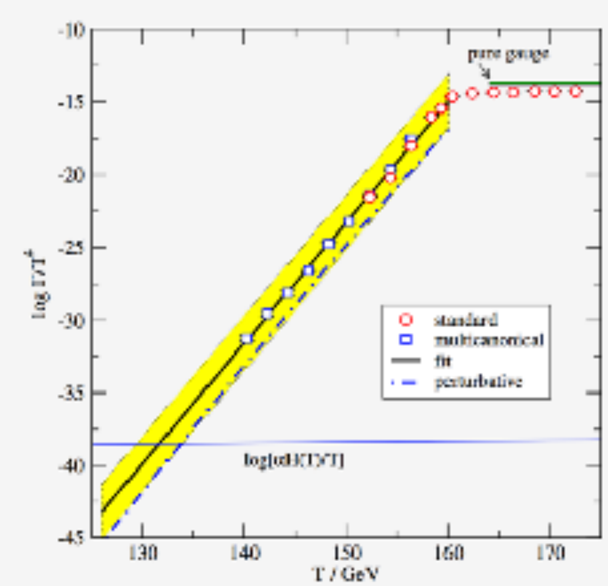
k: the sphaleron zero modes

Ntr and Nrot are the translations and rotations of the sphaleron of these zero mode.

PETER ARNOLD AND LARRY McLERRAN, 1987

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleros" is Greek for "ready to fall").





Lattice result, $T_C = (159.5 \pm 1.5)$GeV, Phys.Rev.Lett,113, 141602 (2014).

$$\Gamma^{sym} \approx 6 \times (18 \pm 3) \alpha_W^5 T^4, \qquad \Gamma^{brok} \sim T^4 \exp(-\frac{E_{sph}}{T})$$

Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)

BNPC , v/T and EW sphaleron

$$\frac{dn_B}{dt} = -\frac{13n_f}{2}\frac{\Gamma_{sph}}{VT^3}n_B$$

PETER ARNOLD AND LARRY McLERRAN, 1987

$$(\Gamma_{sph}/V) < \alpha H T^3$$

$$\frac{E_{sph}(T)}{T} > \ln\left[2\mathcal{N}_{tr}\mathcal{N}_{rot}\mathcal{V}_{rot}\frac{\omega_-}{gv(T)}\right] + 4\ln\left(\frac{\alpha_W}{4\pi}\right) + 7\ln\left(\frac{4\pi v(T)}{gT}\right) + \ln\kappa - \ln\alpha - \frac{1}{2}\ln\left(\frac{\pi^2}{90}g_*\right) - \ln\frac{T}{M_{pl}} .$$

$$\frac{E_{sph}(T)}{T} > (35.9 - 42.8) + 7\ln\frac{v(T)}{T} - \ln\frac{T}{100\ \text{GeV}}$$

$$E_{sph}(T) \approx E_{sph,0}\frac{v(T)}{v}$$

$$\frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{sph,0}}{1.916\times 4\pi v/g}\right)^{-1}$$

Xucheng Gan, Andrew J. Long, Lian-Tao Wang, 17'

$$PT_{sph} \equiv \frac{E_{sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ \text{GeV}}$$

SFOEWPT condition

$$PT_{sph} > (35.9 - 42.8)$$

$$\frac{n_B(\Delta t_{EW})}{n_B(0)} = \exp\left[-\frac{13n_f}{2}\int_0^{\Delta t_{EW}} dt\,\frac{\Gamma_{sph}(T(t))}{VT^3(t)}\right]$$

$$\frac{n_B(\Delta t_{EW})}{n_B(0)} > e^{-X}$$

$$\frac{4\pi B}{g}\frac{\bar{v}(T_C)}{T_C} - 6\ln\frac{\bar{v}(T_C)}{T_C} > -\ln X - \ln\left(\frac{\Delta t_{EW}}{t_H}\right) + \ln\mathcal{Z} + \hbar\ln\kappa$$

$$\mathcal{Z} = \left(\frac{13n_f}{2}\right)\mathcal{N}_{tr}(\mathcal{N}\mathcal{V})_{rot}\left(\frac{\omega_- t_H}{\pi}\right)$$

Δt_{EW}: the phase transition duration

$$S = \frac{n_B(\Delta t_{EW})}{n_B(0)}$$

baryon asymmetry erase during the phase transition

X=-lnS

Hiren H. Patel and Michael J. Ramsey-Musolf, 15'

Class IIB

Dim. six operator, SMEFT

Higgs potential
$$V(H) = -m^2(H^\dagger H) + \lambda (H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

Finite temperature potential
$$V_T(h,T) = V(h) + \frac{1}{2} c_{hT} h^2$$

Thermal correction
$$c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$$

Electroweak minimum being the global one
$$\Lambda \geq v^2/m_h$$

Potential barrier requirement
$$\Lambda < \sqrt{3}v^2/m_h$$

Class IIA (1) no extra EWSB: xSM

For the "xSM" model, the gauge invariant finite temperature effective potential is found to be:

$$V(h,s,T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad \text{(C1)}$$

with the thermal masses given by

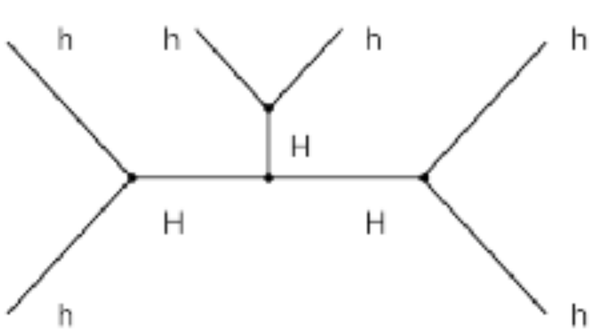
$$\Pi_h(T) = \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24}\right)T^2,$$

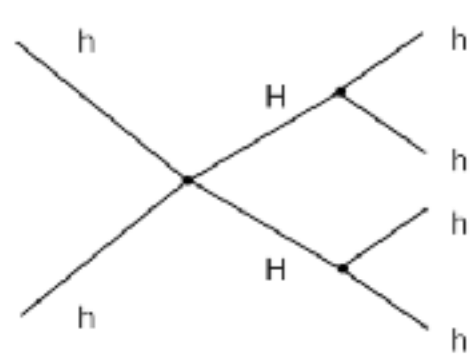
$$\Pi_s(T) = \left(\frac{a_2}{6} + \frac{b_4}{4}\right)T^2, \quad \text{(C2)}$$

PT strength

$$v^{\rm xSM}/T \equiv \frac{v_h(T)}{T} = \frac{\sqrt{v_h^2(T) + v_s^2(T)}\cos\theta(T)}{T},$$

$$\cos\theta(T) \equiv \frac{v_h(T)}{\sqrt{v_h^2(T) + v_s^2(T)}},$$





For small mixing limit between the extra Higgs and the SM Higgs, one have

$$c_4^{xSM} = -\frac{a_1^2 - 8b_2\lambda}{32b_2} + \frac{\theta^2(a_1^2(6b_2 - \mu^2) - 8a_1b_2b_3 + 8b_2^2(a_2 - 2\lambda))}{32b_2^2} + O(\theta^3)$$

$$c_6^{xSM} = -\frac{a_1^2(a_1b_3 - 3a_2b_2)}{192b_2^3} - \frac{\theta^2 a_1}{256b_2^4}(a_1^3b_2 + 4a_1^2b_3(\mu^2 - 3b_2) + 4a_1b_2(a2(11b_2 - 2\mu^2) - 6b_2(b4 + \lambda) + 4b_3^2) - 32a_2b_2^2b_3) + O(\theta^3)$$

$$c_8^{xSM} = \frac{a_1^4 b_4}{1024b_2^4} + \frac{a_1^3\theta^2}{1024b_2^5}(a_1(a_2b_2 + 4b_4(\mu^2 - 3b_2)) + 16b_2b_3b_4) + O(\theta^3)$$

Class III 2HDM Finite-T potential in 2HDM

$$V(h_1,h_2,T) = V_0(h_1,h_2) + V_{\rm CW}(h_1,h_2) + V_{\rm CT}(h_1,h_2) + V_{\rm th}(h_1,h_2,T) + V_{\rm daisy}(h_1,h_2,T)$$

Tree-level

$$V_0(h_1,h_2) = \frac{1}{2}m_{12}^2 t_\beta \left(h_1 - h_2 t_\beta^{-1}\right)^2 - \frac{v^2}{4}\frac{\lambda_1 h_1^2 + \lambda_2 h_2^2 t_\beta^2}{1+t_\beta^2} - \frac{v^2}{4}\frac{\lambda_{345}(h_1^2 t_\beta^2 + h_2^2)}{1+t_\beta^2} + \frac{1}{8}\lambda_1 h_1^4 + \frac{1}{8}\lambda_2 h_2^4 + \frac{1}{4}\lambda_{345} h_1^2 h_2^2$$

One-loop at zero temperature:

$$V_{\rm CW}(h_1,h_2) = \sum_i (-1)^{2s_i} n_i \frac{\hat{m}_i^4(h_1,h_2)}{64\pi^2}\left[\ln\left(\frac{\hat{m}_i^2(h_1,h_2)}{Q^2}\right) - C_i\right]$$ [Coleman, Weinberg '73]

One-loop at finite temperature:

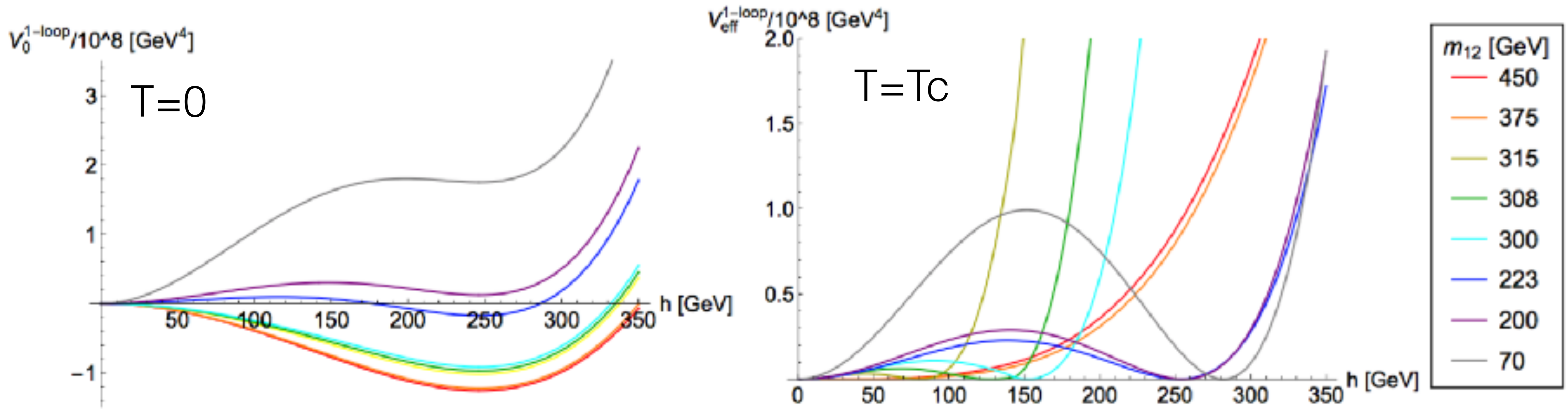
$$V_{\rm th}(h_1,h_2,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{B,F}\left(\frac{m_i^2(h_1,h_2)}{T^2}\right)$$ [Dolan, Jackiw '74]

$$J_{B,F}(y) = \mp \sum_{l=1}^{\infty} \frac{(\pm 1)^l y}{l^2} K_2(\sqrt{y}\,l)$$ [Anderson, Halle '92]

$$V_{\rm daisy}(h_1,h_2,T) = -\frac{T}{12\pi}\sum_i n_i \left[\left(M_i^2(h_1,h_2,T)\right)^{\frac{3}{2}} - \left(m_i^2(h_1,h_2)\right)^{\frac{3}{2}}\right]$$

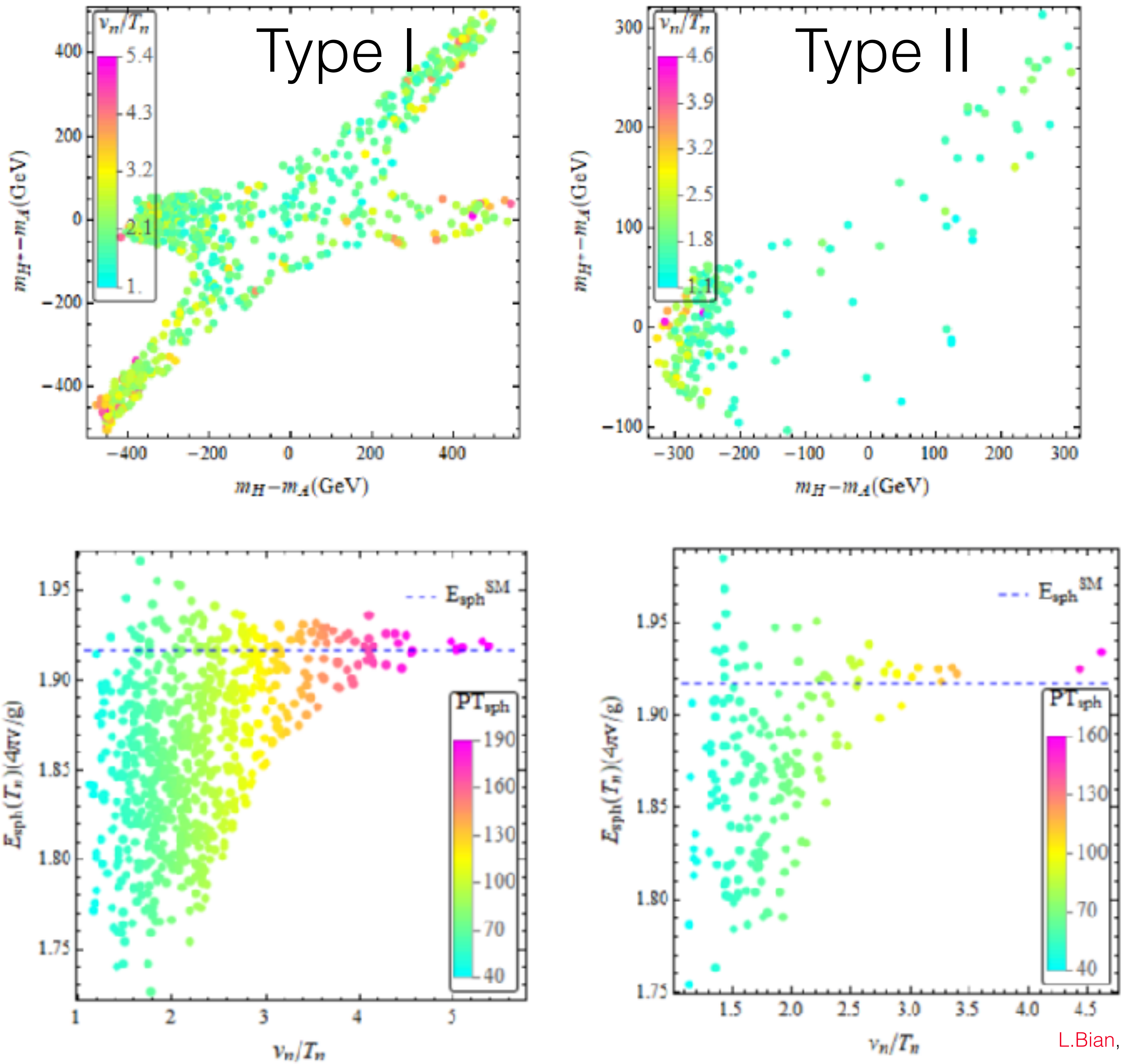
Jérémy Bernon, Ligong Bian, Yun Jiang, JHEP 05 (2018) 151 [Carrington '92; Arnold, Espinosa '93; Delaunay, Grojean, Wells '07]

Class III 2HDM The potential shape



$m_{H^\pm} = m_A = m_H = 600$ GeV, $\tan\beta = 1$ and $\sin(\beta - \alpha) = 1$

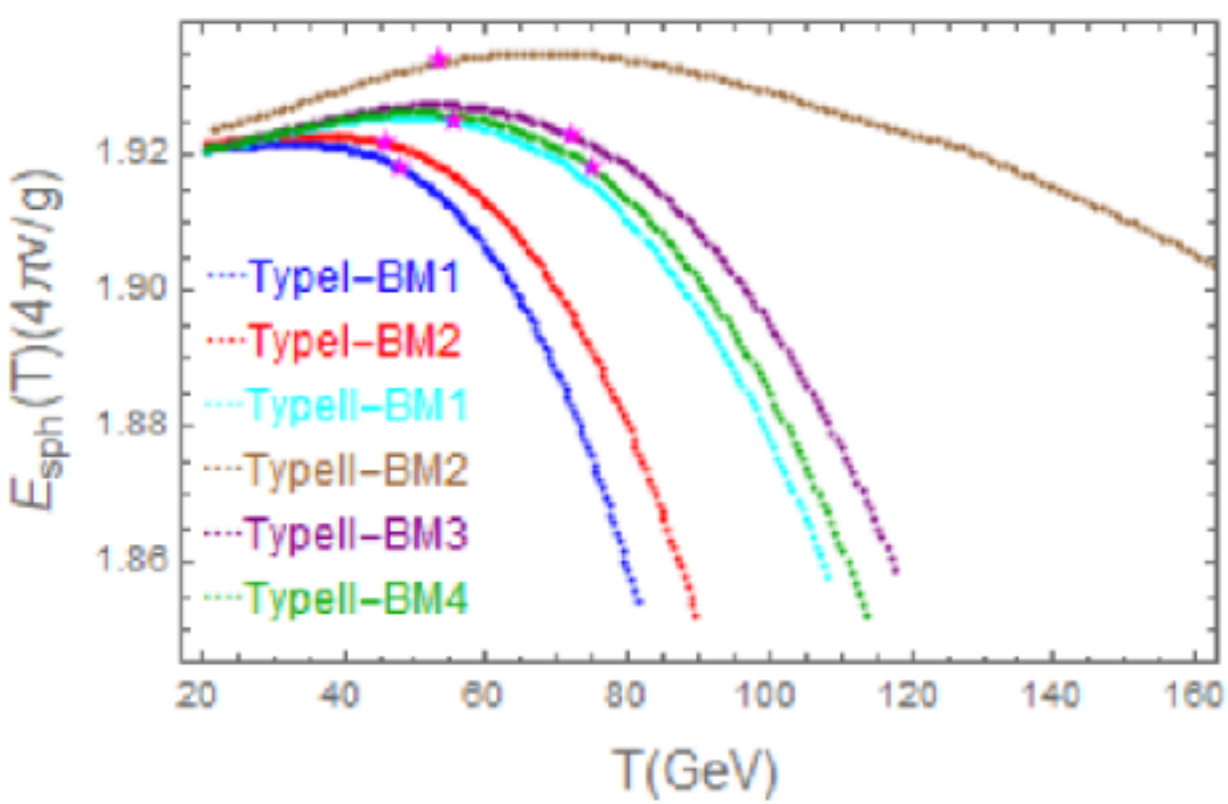
Class III 2HDM **Sphaleron energy and SFOEWPT condition**

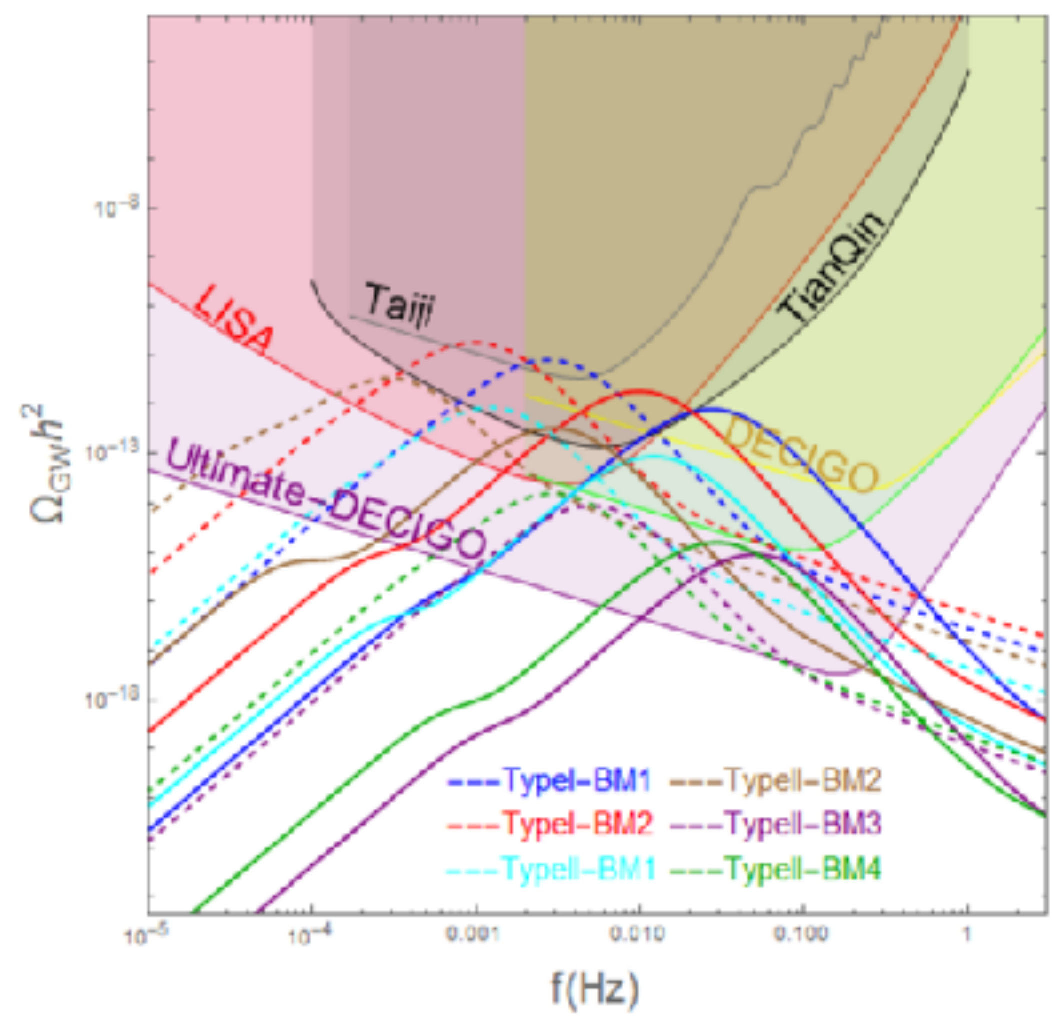


L.Bian, R.Zhou, 2001.01237

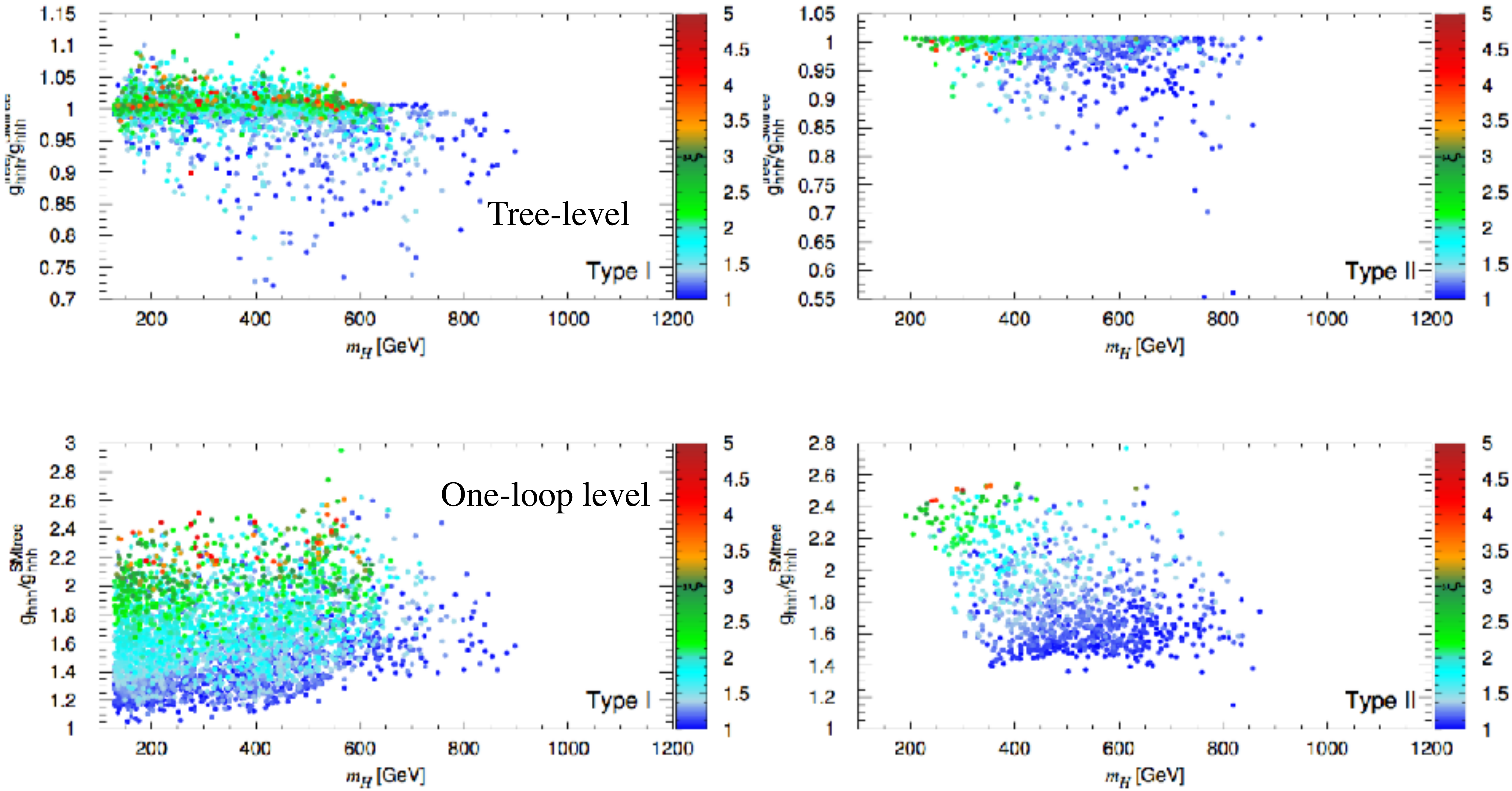
Sphaleron and GWs

type I/II	m_H	$m_{H^\pm}$	m_A	m_{12}^2	$\tan\beta$
I/ BM 1	164.61	168.05	555.28	533.12	50.621
I/ BM 2	605.91	168.03	158.06	8924.4	41.104
II/BM 1	300.02	622.17	616.82	14967	5.8655
II/BM 2	497.19	766.95	751.77	112180	0.99212
II/BM 3	311.79	621.21	619.95	13956	6.8395
II/BM 4	294.33	621.83	605.56	17366	4.6623



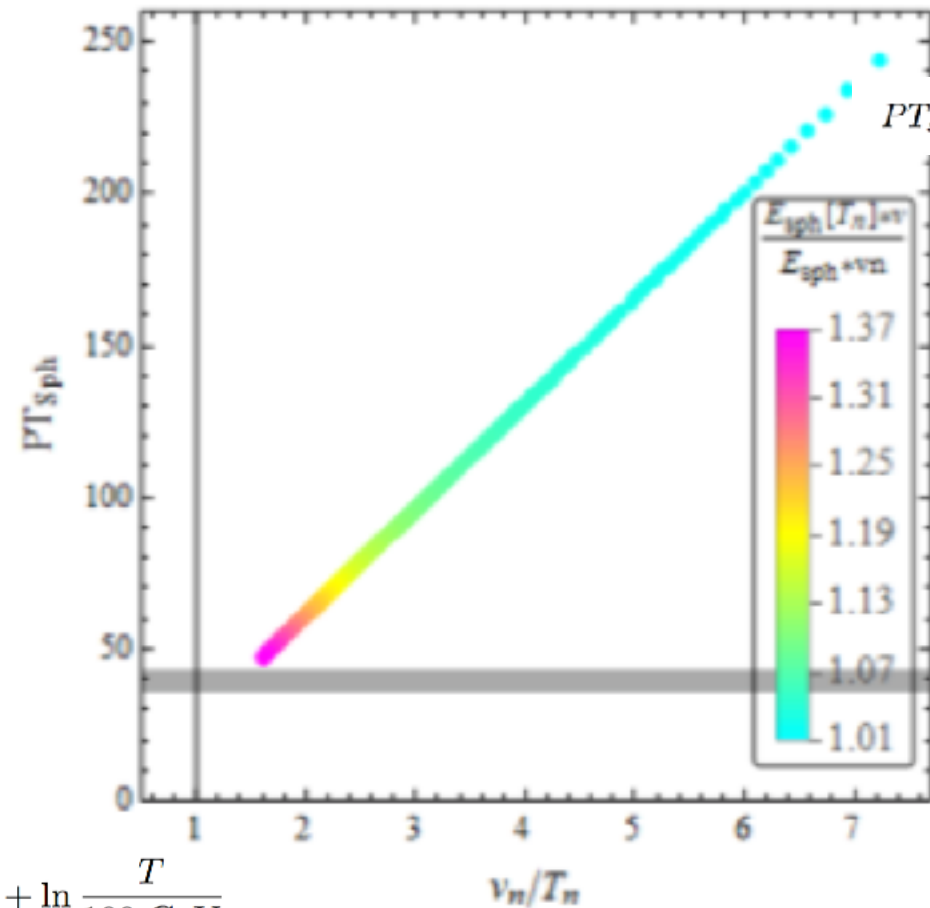


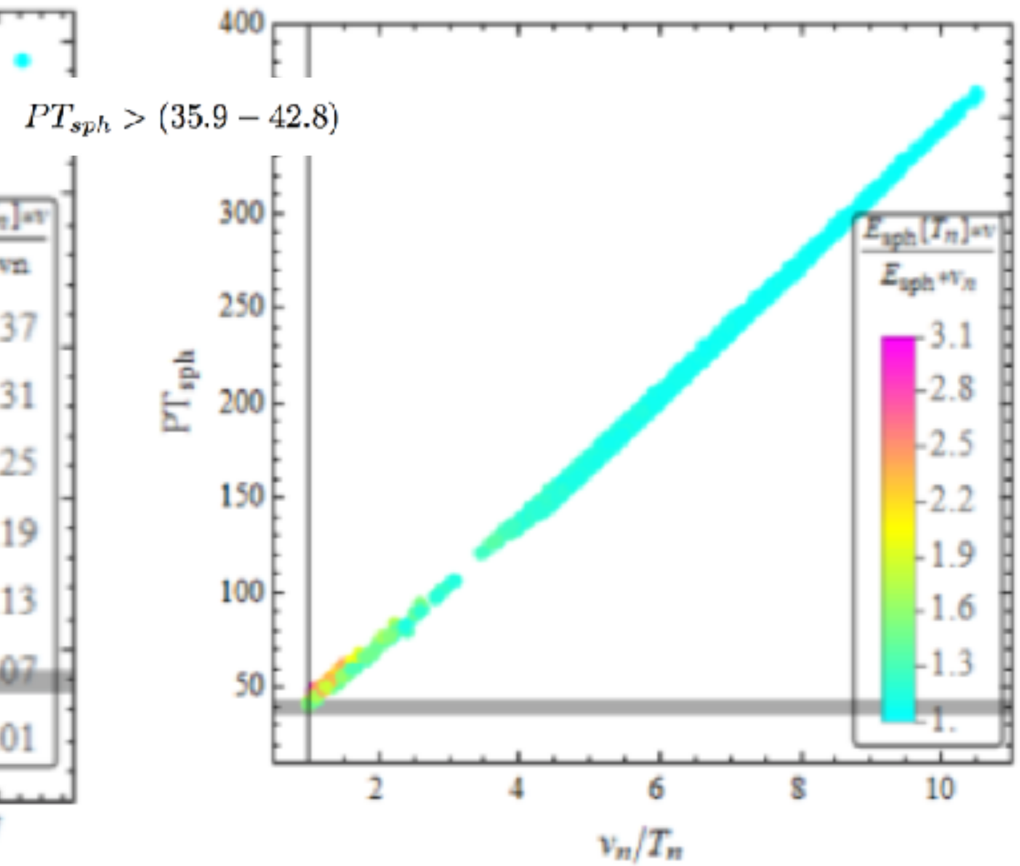
Class III 2HDM **Triple Higgs coupling**



Jérémy Bernon, Ligong Bian, Yun Jiang,JHEP 05 (2018) 151

Sphaleron energy and SFOEWPT condition



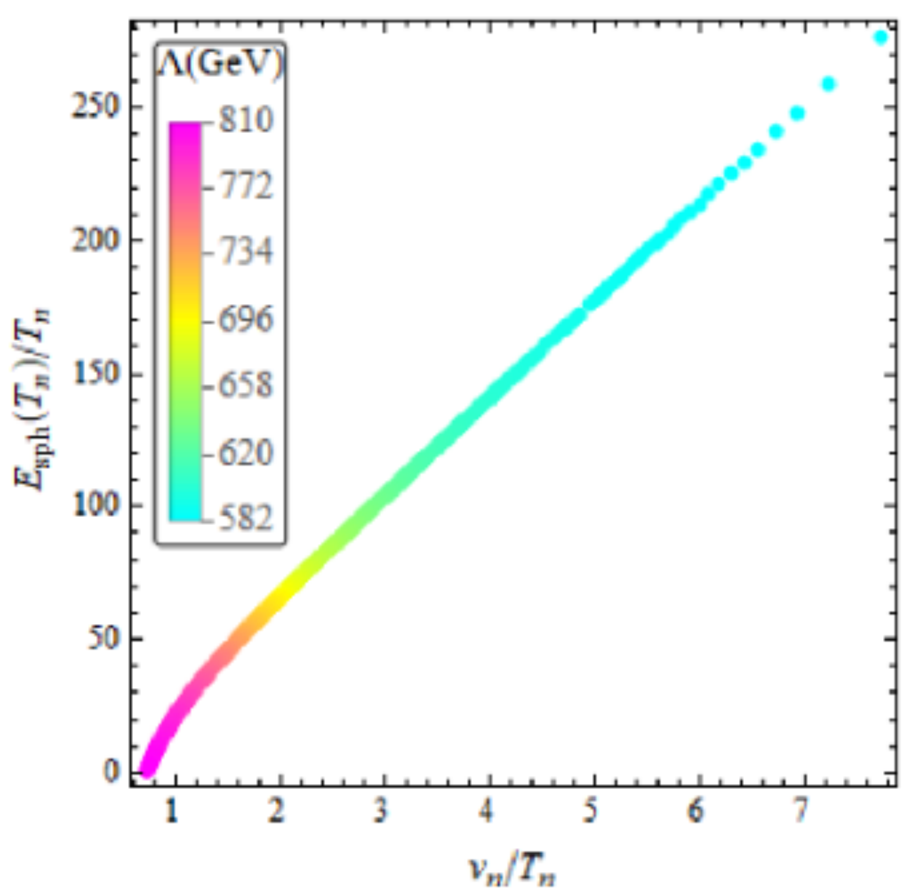


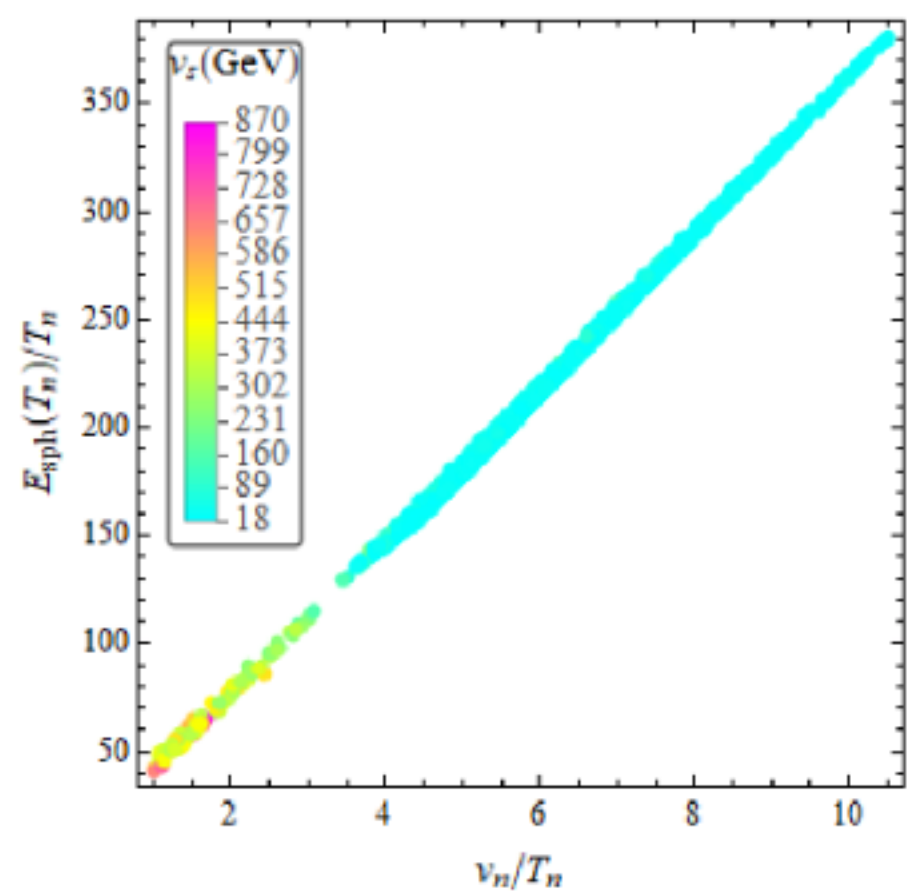
$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ {\rm GeV}}$$

SMEFT

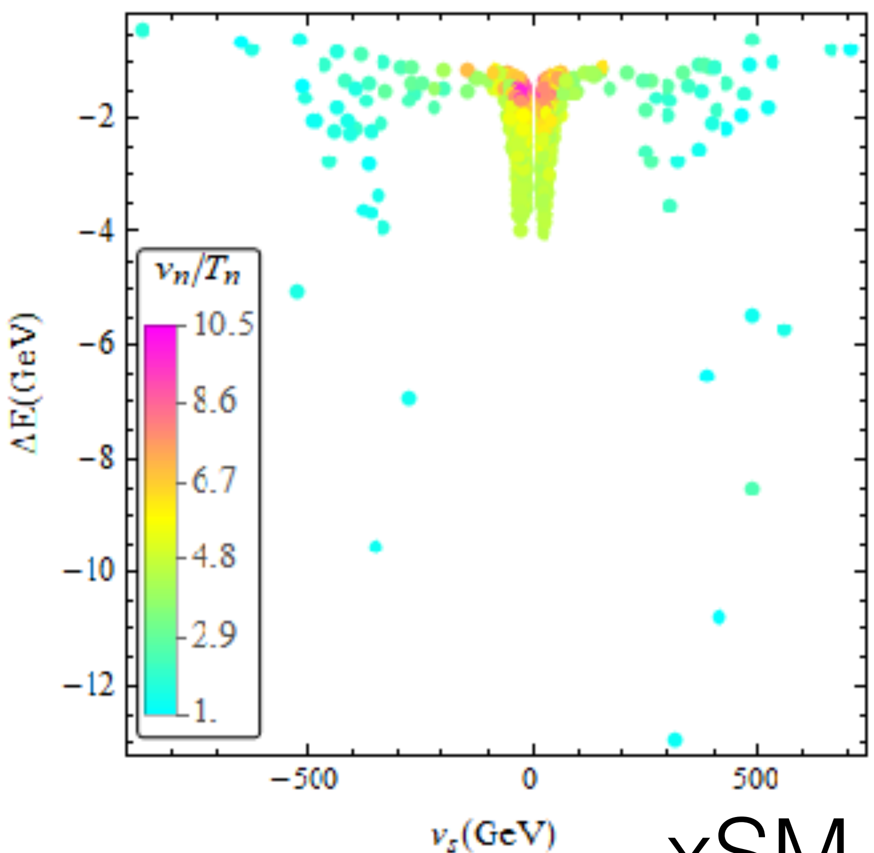
xSM

$$PT_{sph} > (35.9 - 42.8)$$

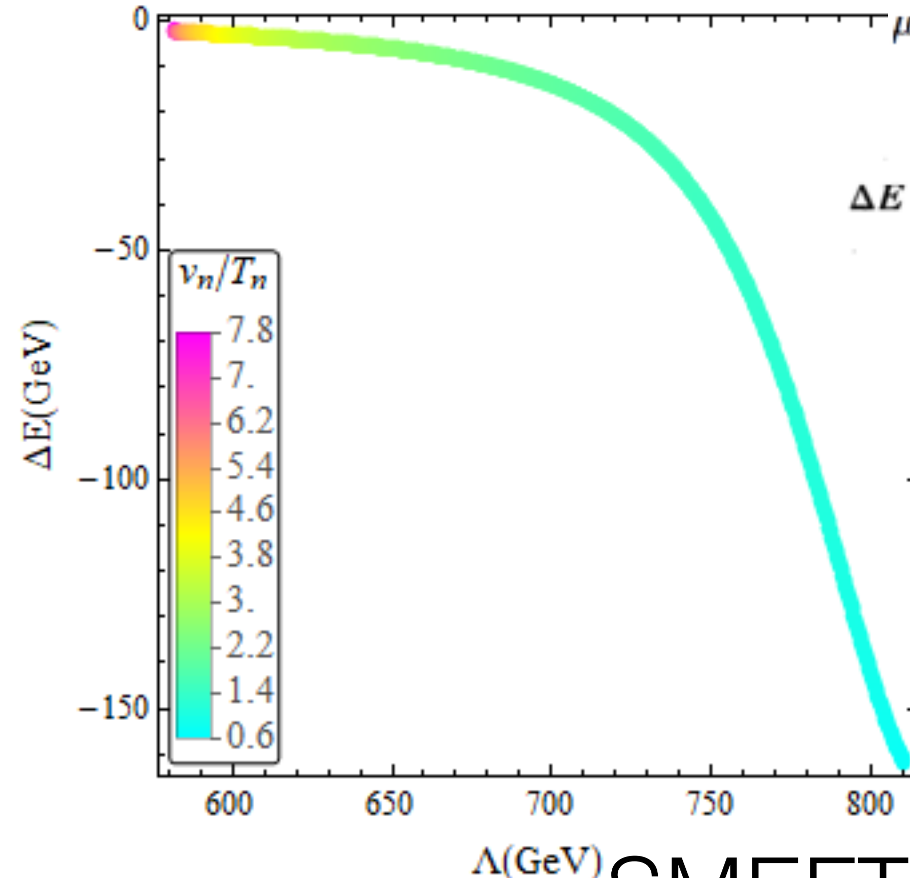




Sphaleron energy and magnetic dipole moment



xSM



SMEFT

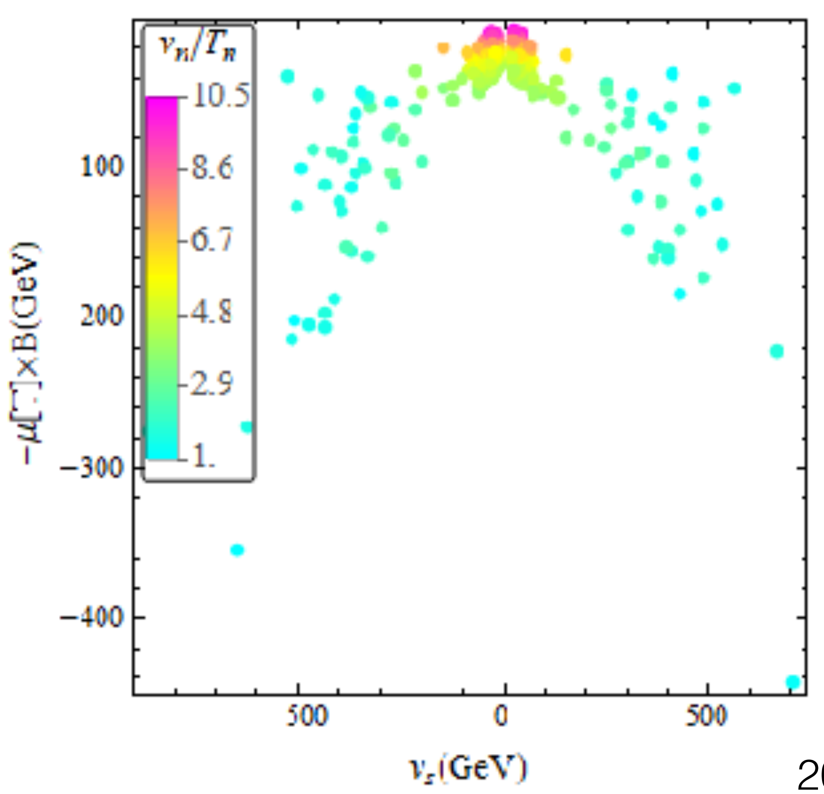
$$\mu(T) = \frac{2\pi}{3}\frac{g'}{g^3 v(T)}\int_0^\infty d\xi\, \xi^2 h^2(\xi)[1-f(\xi)]$$

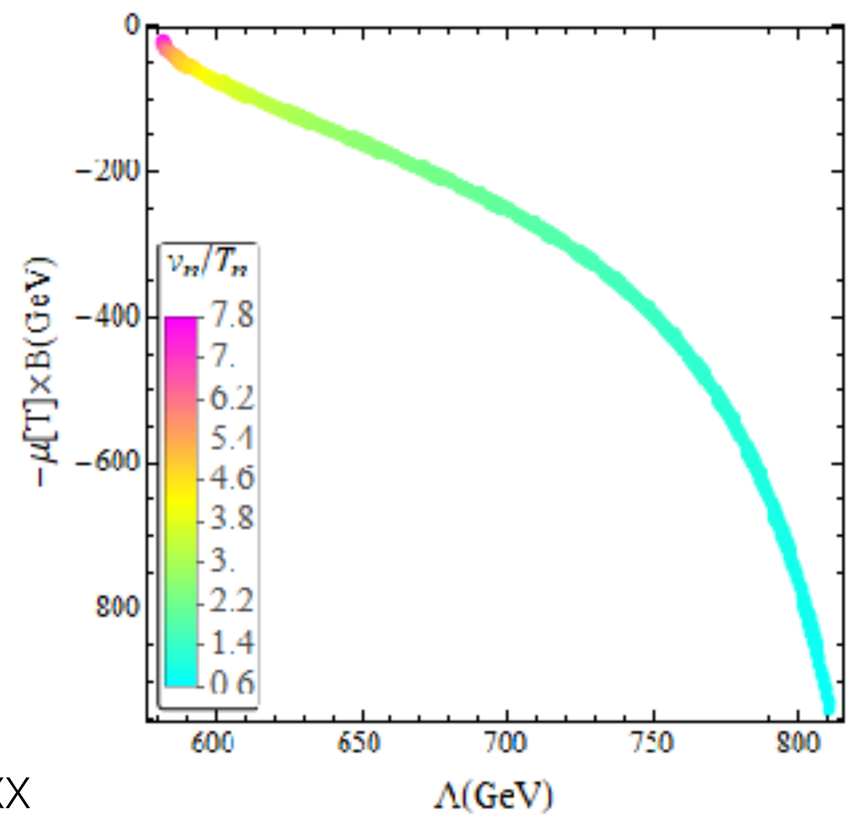
$$\Delta E = -\frac{\pi}{3}\frac{g'^2 v}{g^3}\int_0^\infty \xi^2 h^2(\xi)[1-f(\xi)]p(\xi)d\xi$$

$$p(\xi) = \frac{1}{3\xi^3}\int_0^\xi d\eta\, \eta^2 h^2(\eta)[1-f(\eta)] + \int_\xi^\infty d\eta \frac{1}{3\eta}h^2(\eta)[1-f(\eta)]$$

Considering small Winberg angle

Klinkhamer and Manton, 1984





$$E^{(1)}_{\rm dipole}(T,B) = -\mu(T)\,B$$

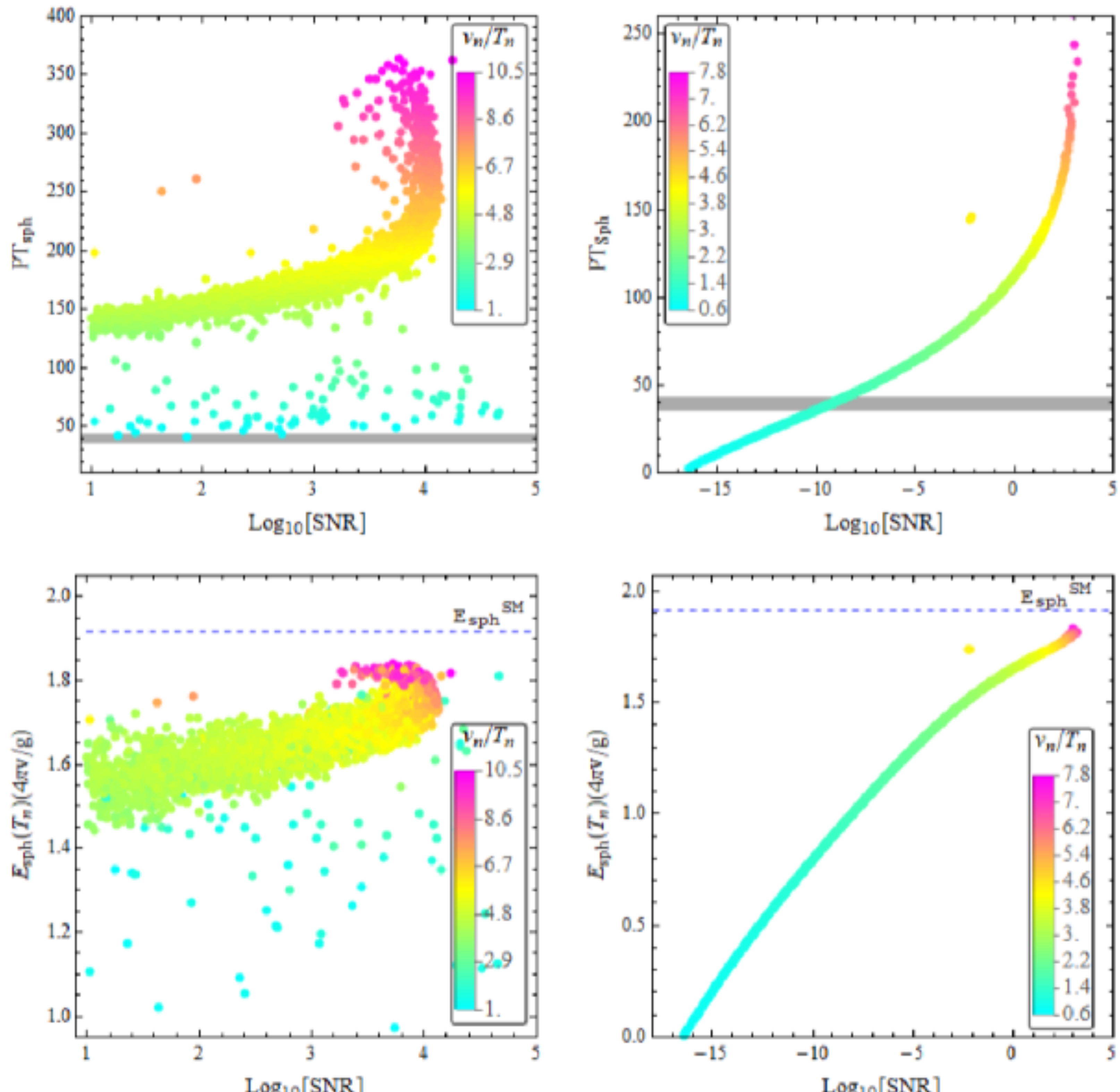
$$B = bT^2$$

Taking b=0.1 for SFOEWPT

Comelli etal, 1999
Simone etal, 2011

2008.XXXXX

Portray sphaleron with GW ???



xSM

SMEFT

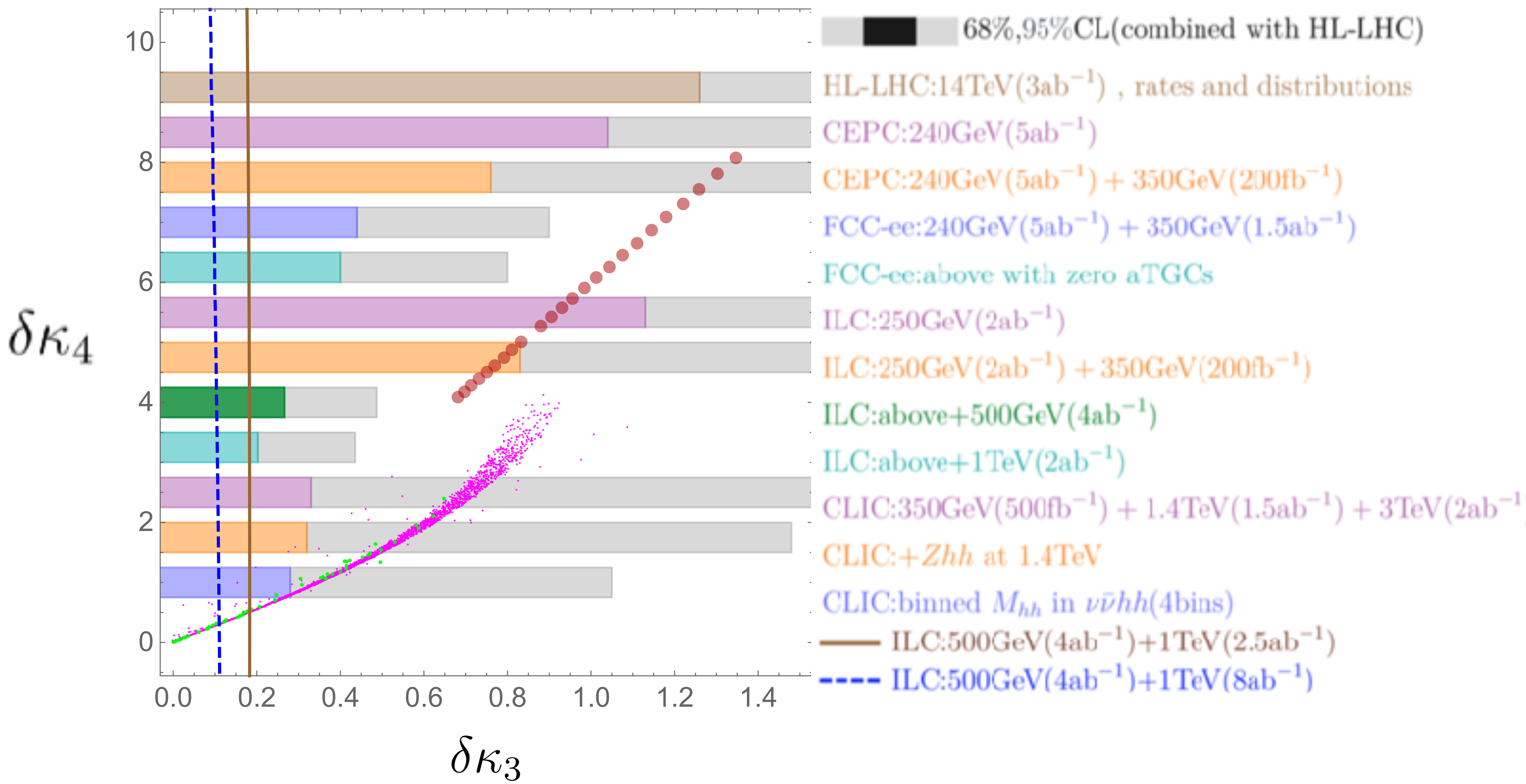
Gravitational waves can be searched for by cross-correlating outputs from two or more detectors, with the resulting signal-to- noise ratio(SNR)

$$\mathrm{SNR} = \sqrt{\mathcal{T}\int df \left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm exp}(f)}\right]^2}$$

where T is the duration of the data in years and Ωexp the power spectral density of the detector.

L. Bian, H. Guo,R.Zhou, Phys.Rev.D 101 (2020) 9, 091903

Triple and quartic Higgs coupling deviation, GW



$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$

$$\delta\kappa_3^{h^6} = \frac{2v^4}{\Lambda^2 m_h^2}\,,\ \delta\kappa_4^{h^6} = \frac{12v^4}{\Lambda^2 m_h^2}$$

$$\delta\kappa_3^{\rm xSM} = \alpha^2\left[-\frac{3}{2} + \frac{2m_H^2 - 2b_3 v_s - 4b_4 v_s^2}{m_h^2}\right]$$

$$\delta\kappa_4^{\rm xSM} = \alpha^2\left[-3 + \frac{5m_H^2 - 4b_3 v_s - 8b_4 v_s^2}{m_h^2}\right]$$

Related interesting topics

1. Dark matter with phase transition

   1712.03962,Michael J. Baker et al.

   1810.03172, L.Bian, Y. Tang

   …

2. Leptogenesis with phase transition

   A. strumia, T. Hambye, …

3. Wall velocity

   T.Konstandin,G. Moore, J Kozaczuk, …

4. Nonperturbative evaluation of EWPT

   Kari Rummukainen, Anders Tranberg,Michael Ramsey-Musolf, Lauri Niemi, M. Laine, …

5. Sphaleron calculation and simulations

   Manton, Klinkhamer, L. Carson, L. Mclerran, G. D. Moore, Mark Hindmarsh, X.M.Zhang, L. Wang, L.Bian,…

Thanks!

GW sources

$$\Omega_{GW}(f) = \begin{cases} \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW1}} & \text{for } f < f_*, \\ \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW2}} & \text{for } f > f_*, \end{cases}$$

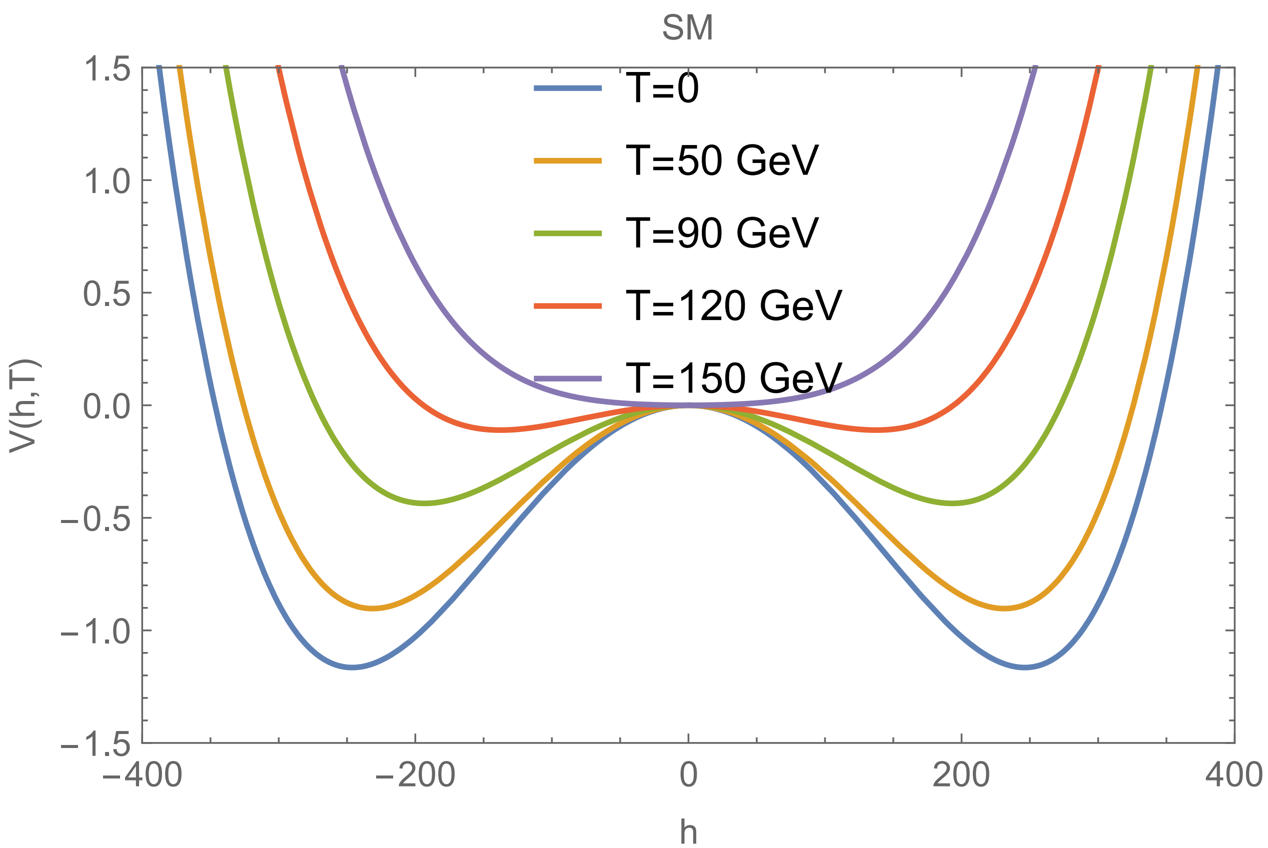
1807.00786

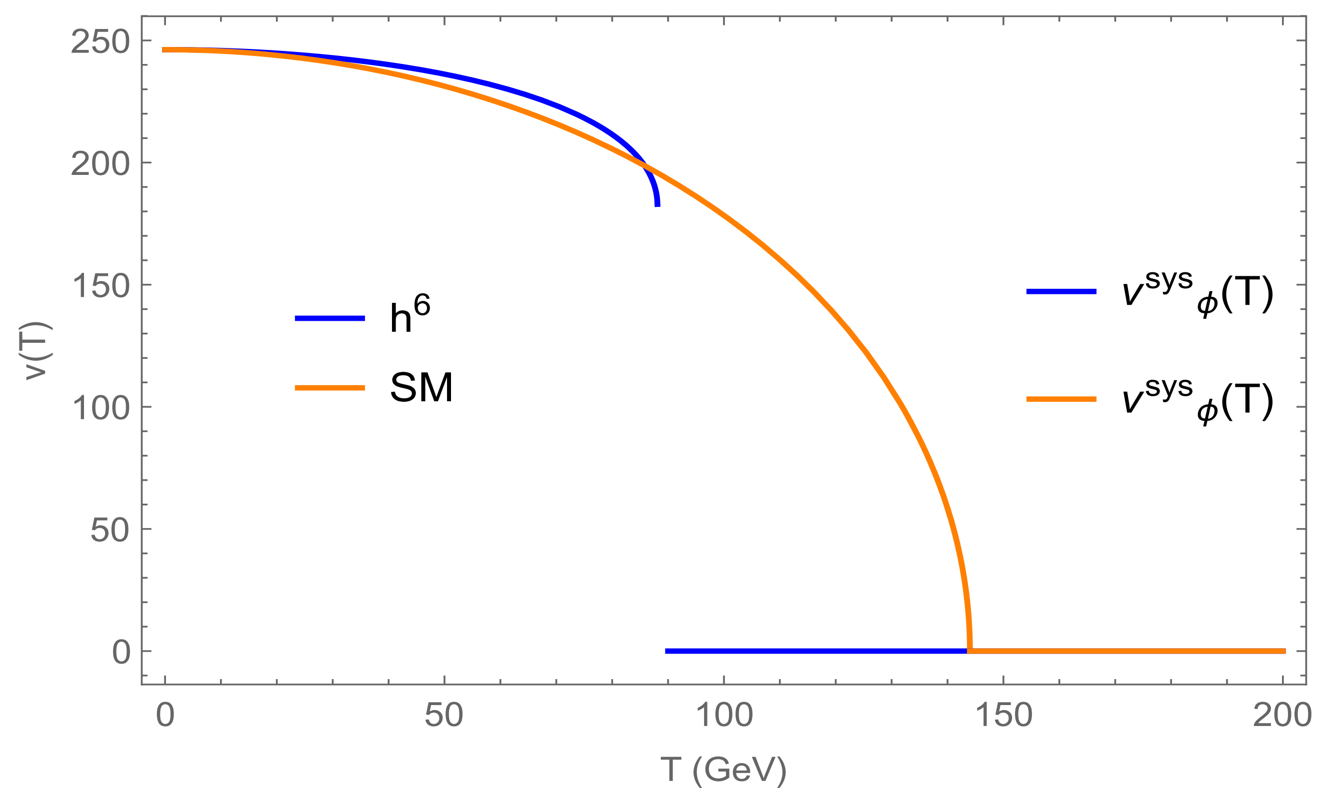
Table 1. Cosmolbgical GW sources

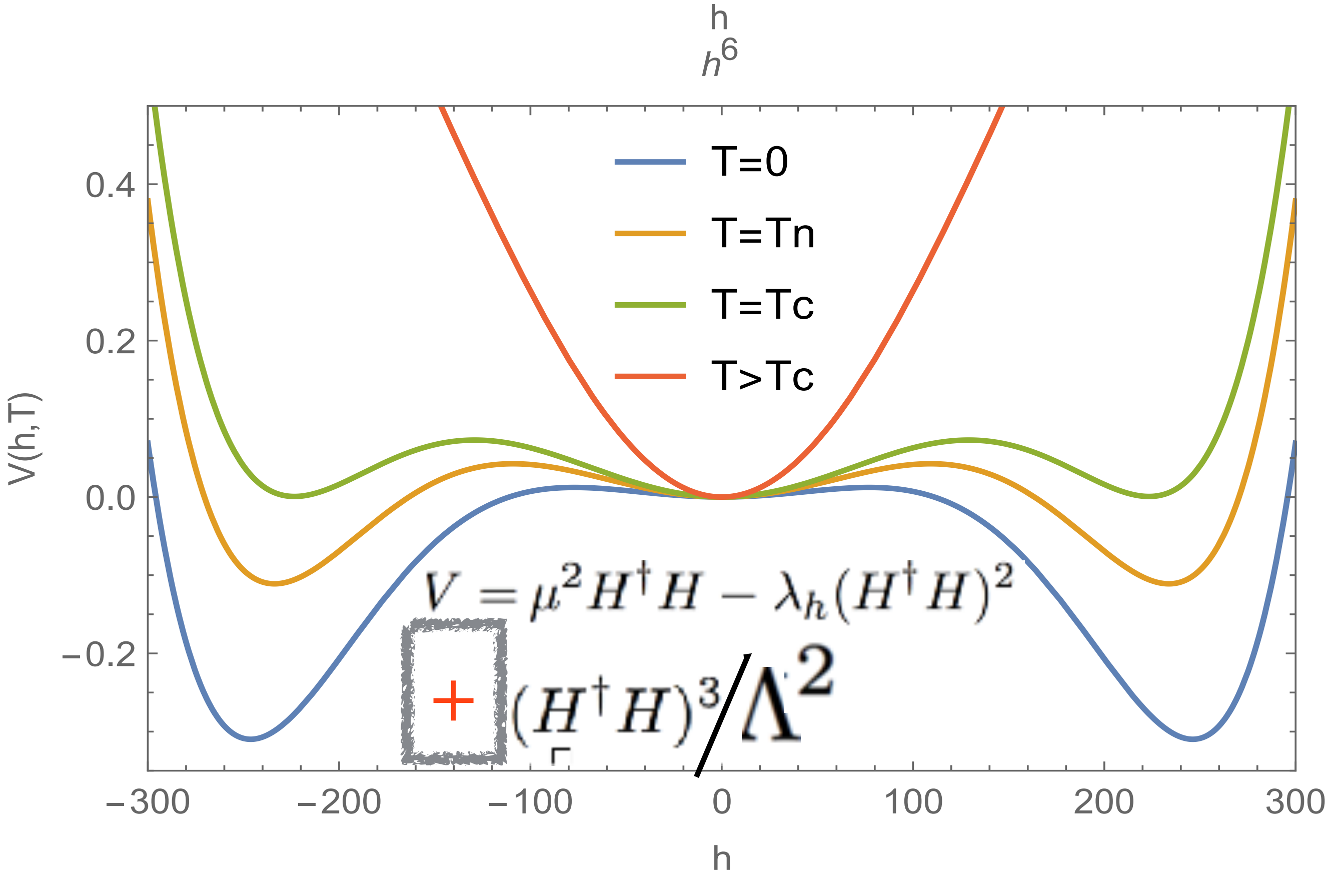
source	n_{GW1}	n_{GW2}	f_* [Hz]	Ω_{GW}
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{PT}}{\beta}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 3\times 10^{-4}\left(\frac{H_{PT}}{\beta}\right)\left(\frac{\kappa_{turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 3\times 10^{-6}\left(\frac{H_{PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{pl}}\right)^2$
Cosmic strings (loops 1)	[1,2]	[−1, −0.1]	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	[−1, −0.1]	0	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	[0, 0.2]	[0, 0.2]	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{ann}}{10^{-2}\text{GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1\text{TeV}^3}\right)^2\left(\frac{T_{ann}}{10^{-2}\text{GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{rad}\left(\frac{v}{M_{pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim \frac{511}{N}\Omega_{rad}\left(\frac{v}{M_{pl}}\right)^4$
Magnetic fields	3	α_B+1	$\sim 10^{-6}\left(\frac{T_*}{10^2\text{GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}\text{G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi-6)(\epsilon-\eta)$	—	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times 10^5\left(\frac{T_{reh}}{10^9\text{ GeV}}\right)^{1/3}\left(\frac{M_{inf}}{10^{16}\text{ GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{reh}}{10^9\text{ GeV}}\right)^{-4/3}\left(\frac{M_{inf}}{10^{16}\text{ GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times 10^{-2}\left(\frac{M_{PBH}}{10^{20}\text{ g}}\right)^{-1/2}$	$\sim 7\times 10^{-9}\left(\frac{A^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3-2\mu$	—	$\sim 1.4\times 10^{-6}\left(\frac{H_s}{0.15M_{pl}}\right)^4$

Higgs Potential Shape??? EFT or ???

First or second order







Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner (2015)
F.P. Huang, Gu, Yin, Yu, Zhang (2015) F.P. Huang, Wan, Wang, Cai, Zhang (2016) Cao, F.P. Huang, Xie, & Zhang (2017)

LHC say the quantum fluctuation (quadratic oscillation) around h=v with mh=126 GeV, not sensitive to the specifically potential shape

$$A_i(r,\theta,\phi) = -\frac{i}{g} f(r)\partial_i U(\theta,\phi)(U(\theta,\phi))^{-1}, \quad (11)$$

$$\Phi_1(r,\theta,\phi) = \frac{v_1}{\sqrt{2}} h_1(r) U(\theta,\phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (12)$$

$$\Phi_2(r,\theta,\phi) = \frac{v_2}{\sqrt{2}} h_2(r) U(\theta,\phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (13)$$

where A_i are SU(2) gauge fields, $A_i = \frac{1}{2} A_i^a \tau^a$, $v = \sqrt{v_1^2 + v_2^2}$, and $U(\theta,\phi)$ is defined as

$$U(\theta,\phi) = \begin{pmatrix} \cos\theta & e^{i\phi}\sin\theta \\ -e^{-i\phi}\sin\theta & \cos\theta \end{pmatrix}, \quad (14)$$

Adopting the $A_0 = 0$ gauge, the Electroweak sphaleron energy function can be obtained as:

$$\begin{aligned} E_{\rm sph}[f,h_1,h_2] = \frac{4\pi v}{g} \int_0^\infty d\xi \Bigg[& 4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f-f^2)^2 \\ & + \frac{\xi^2}{2}\frac{v_1^2}{v^2}\left(\frac{dh_1}{d\xi}\right)^2 + \frac{\xi^2}{2}\frac{v_2^2}{v^2}\left(\frac{dh_2}{d\xi}\right)^2 \\ & + \left(\frac{v_1^2}{v^2}h_1^2 + \frac{v_2^2}{v^2}h_2^2\right)(1-f)^2 \\ & + \frac{\xi^2}{g^2 v^4} V(h_1,h_2) \Bigg], \end{aligned} \quad (15)$$