Cosmological Phase Transition and Gravitational Waves in an Expanding Universe

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电弱相变与Higgs物理专题研讨会

In collaboration with Kuver Sinha, Daniel Vagie, Graham White(arxiv:2007.08537)

Prelude

Gravitational Waves from Cosmological Phase Transition

- A new tool to search for physics beyond the (particle and cosmology) Standard Model
- Shed light on understanding both Bayon Asymmetry and Dark Matter
- Collider - Gravitational Wave Complementarity

Alves, Goncalves, Ghosh, Guo, Sinha, JHEP03,053(2020)

Alves, Ghosh, Guo, Sinha, Vagie, JHEP04,052(2019)

Alves, Ghosh, Guo, Sinha, JHEP12,070(2018)

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Di-Higgs Blind Spots in Gravitational Wave Signals, Alves, Goncalves, Ghosh, Guo, Sinha (to appear on Monday)

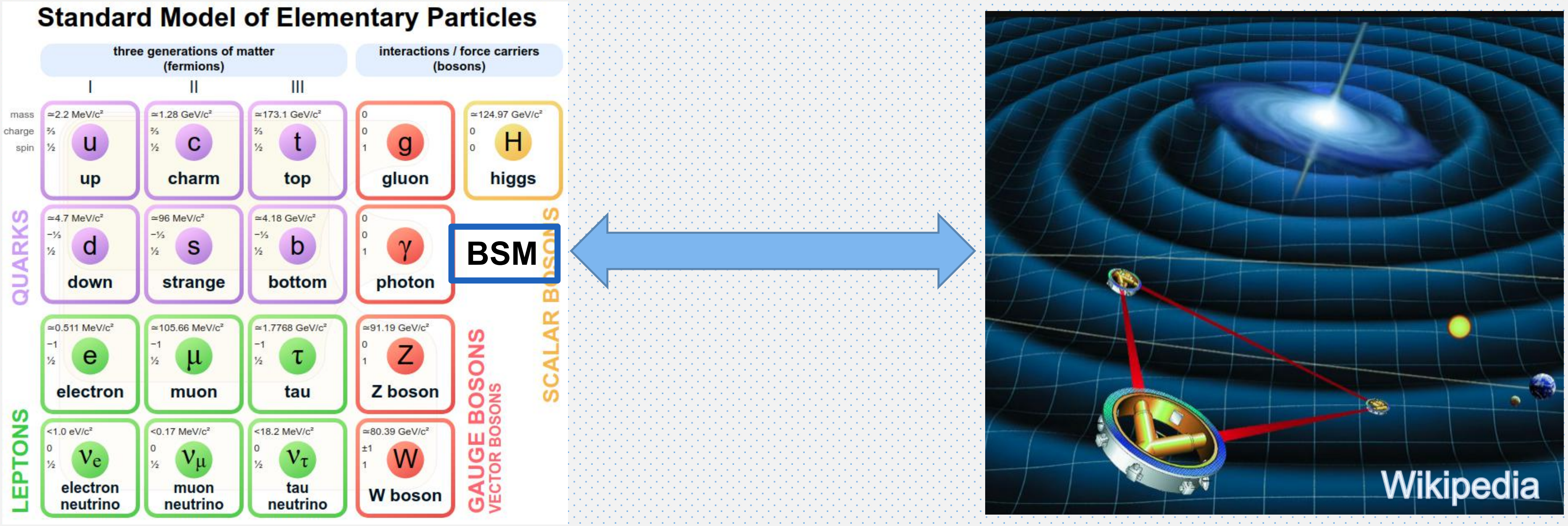
Motivations

- Precision calculation of the gravitational wave spectrum

  Lay out the framework for modelling GW production in an expanding universe

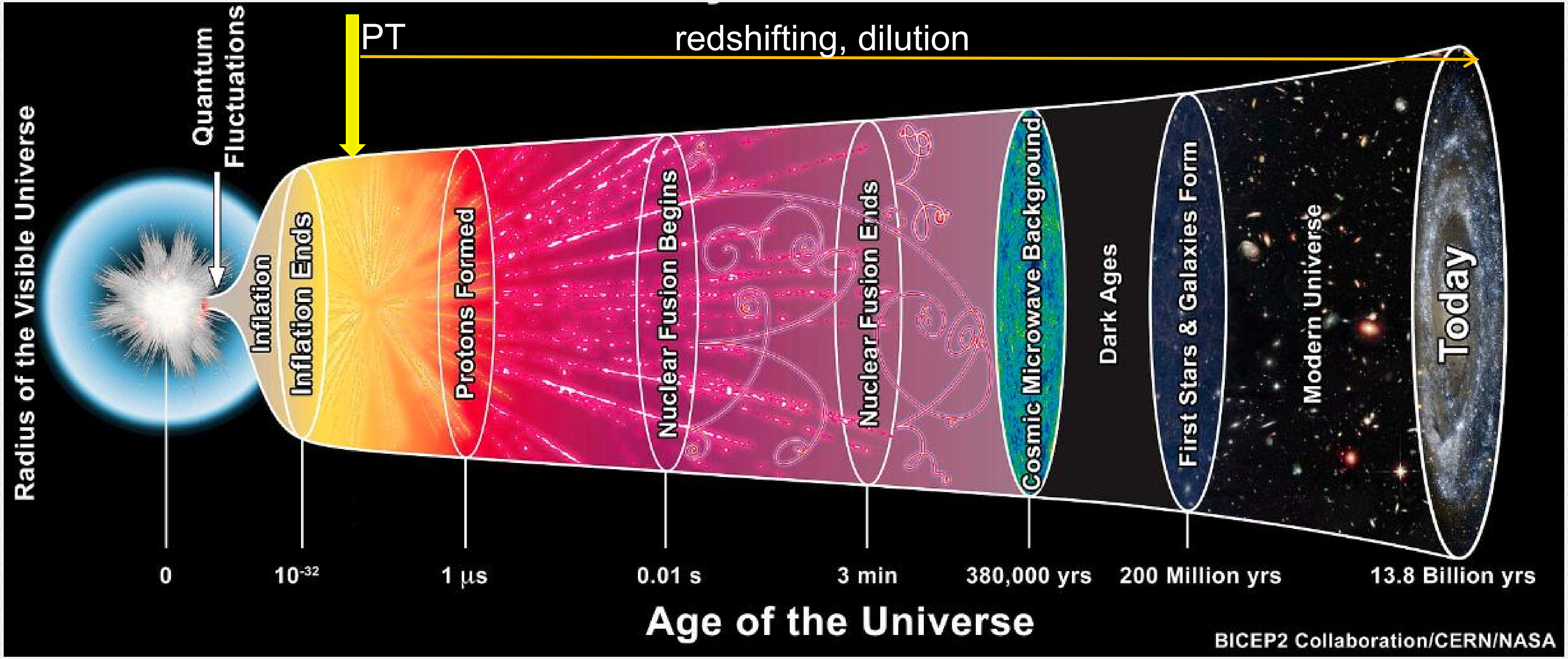
- Gravitational waves as cosmic witnesses (PT, cosmic strings, etc)

  Early matter domination(string moduli), Kination, Intermediate Inflationary stage(supercooling), etc

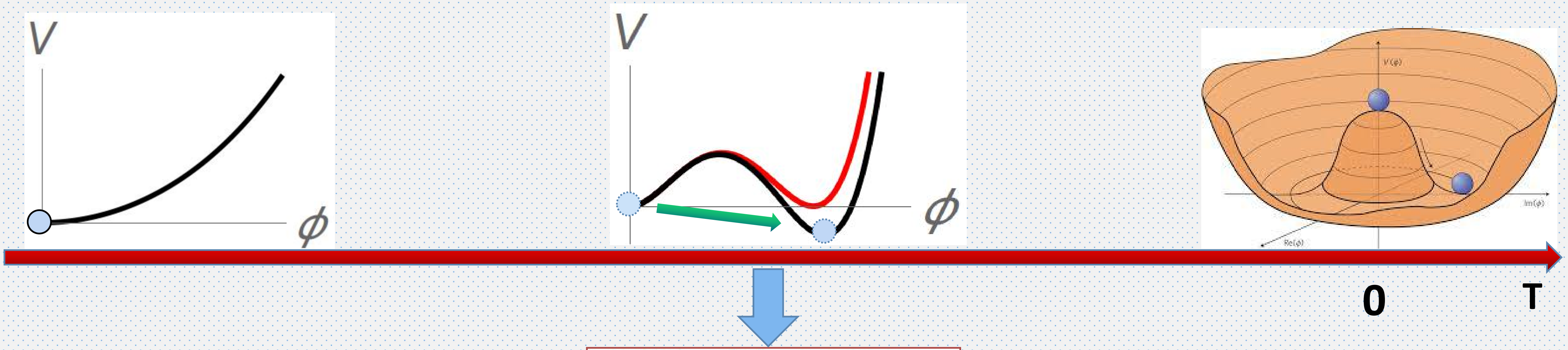


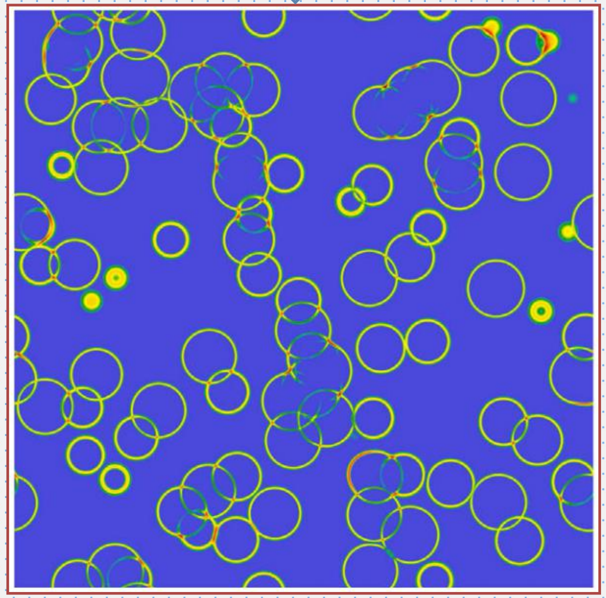
Main Questions

- How will the properties of the PT and GW be modified?
- Do we need a new simulation?



The Picture

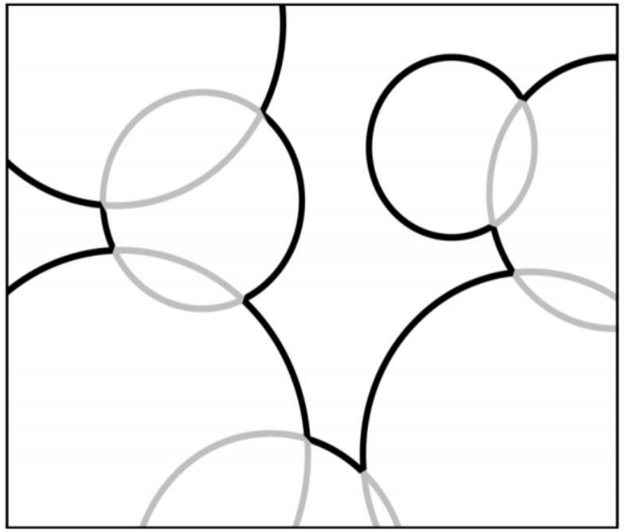


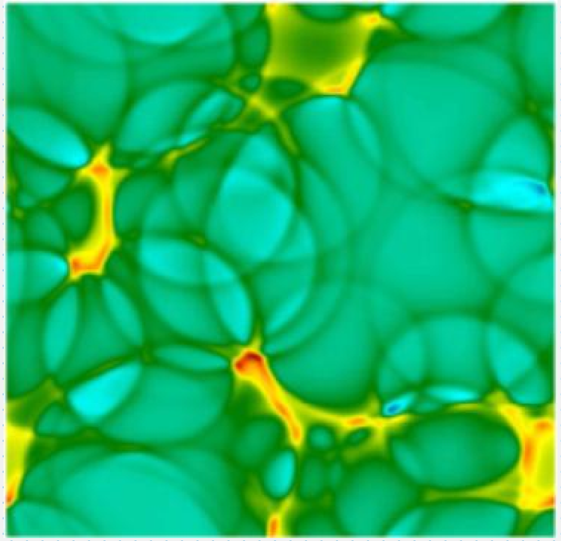


Hindmarsh, et al, 2015

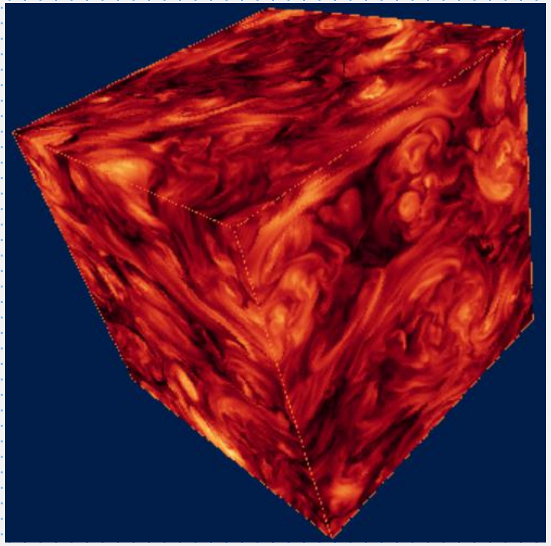
Sources for Gravitational Wave Production

- Bubble Collisions
- Sound Waves in Plasma → dominant in a thermal plasma
- MagnetoHydrodynamic Turbulence





Hindmarsh, et al,PRL112,041301(2013)



https://home.mpcdf.mpg.de/~wcm/projects/homog-mhd/mhd.html

How to Calculate Gravitational Waves?

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$$

Tensor Mode

$$\langle \dot{h}_{ij}(t,\mathbf{q})\dot{h}_{ij}(t,\mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k}+\mathbf{q})P_{\dot{h}}(k,t)$$

$$\frac{d\rho_{\mathrm{GW}}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t,k)$$

GW Spectrum

Einstein equation

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

Source evolutions

Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM
Energy-momentum conservation (hydrodynamic limit)

How numerical simulations is done?

- Realized only several years ago (Hindmarsh, et al,PRL112,041301,2013)

$$T^{\mu\nu}{}_{;\mu}|_{\text{field}} = (\partial^2\phi)\partial^\nu\phi + \frac{1}{\sqrt{g}}(\partial_\mu\sqrt{g})(\partial^\mu\phi)(\partial^\nu\phi) - \frac{\partial V}{\partial\phi}\partial^\nu\phi = \delta^\nu,$$

$$T^{\mu\nu}{}_{;\mu}|_{\text{fluid}} = \partial_\mu\left[(e+p)U^\mu U^\nu\right] + \left[\frac{1}{\sqrt{g}}(\partial_\mu\sqrt{g})g^\nu_\lambda + \Gamma^\nu_{\mu\lambda}\right](e+p)U^\mu U^\lambda + g^{\mu\nu}\partial_\mu p + \frac{\partial V}{\partial\phi}\partial^\nu\phi = -\delta^\nu.$$

bubble generation → bubble, fluid, metric evolution → gravitational wave measurement

bubble, fluid, metric evolution ↓

partial differential equation solving on a lattice, difficult

How to Calculate Gravitational Waves Analytically?

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$$

Tensor Mode

$$\langle \dot{h}_{ij}(t,\mathbf{q})\dot{h}_{ij}(t,\mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k}+\mathbf{q})P_{\dot{h}}(k,t)$$

$$\frac{d\rho_{\mathrm{GW}}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t,k)$$

GW Spectrum

neglect backreaction
solve with Green's function

Einstein equation

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

Source evolutions

Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM
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The flow of calculations

$$T_{ij} = a^2 \left[p\delta_{ij} + (p+e)\gamma^2 v^i v^j \right]$$

$$\langle \pi^T_{ij}(\eta_1, \mathbf{k}) \pi^T_{ij}(\eta_2, \mathbf{q}) \rangle$$

$$\pi^T_{ij}(\mathbf{q}, \eta) = \frac{a_s^4}{a^4(\eta)} \tilde{\pi}^T_{ij}(\mathbf{q}, \eta)$$

$$v^i = dx^i / d\eta$$

$$\langle \tilde{v}^i_{\mathbf{q}}(\eta_1) \tilde{v}^{j*}_{\mathbf{k}}(\eta_2) \rangle = \delta^3(\mathbf{q} - \mathbf{k}) \hat{q}^i \hat{k}^j G(q, \eta_1, \eta_2)$$

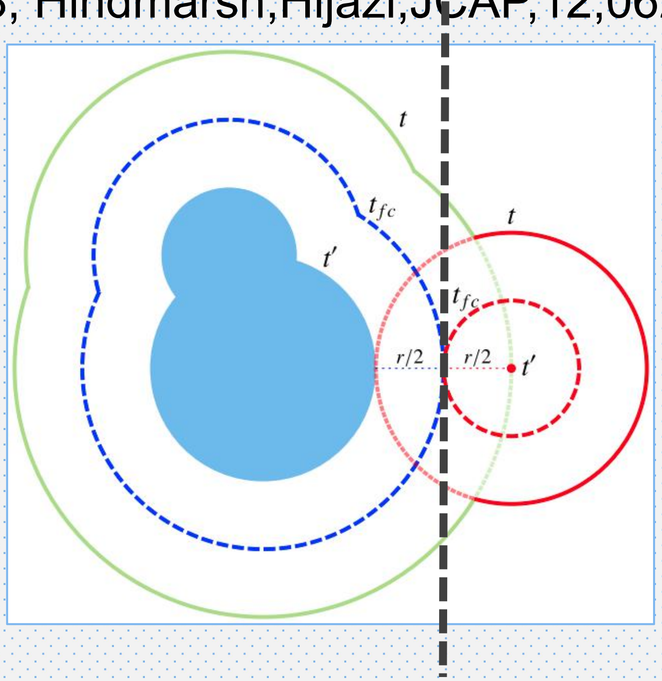
the key part

The Sound Shell Model

- The velocity field is a linear superposition of the contributions from all the bubbles

Hindmarsh, PRL,120,071301,2018, Hindmarsh,Hijazi,JCAP,12,062,2019

contribution from the red bubble



before collision: velocity profile

after collision: sound waves

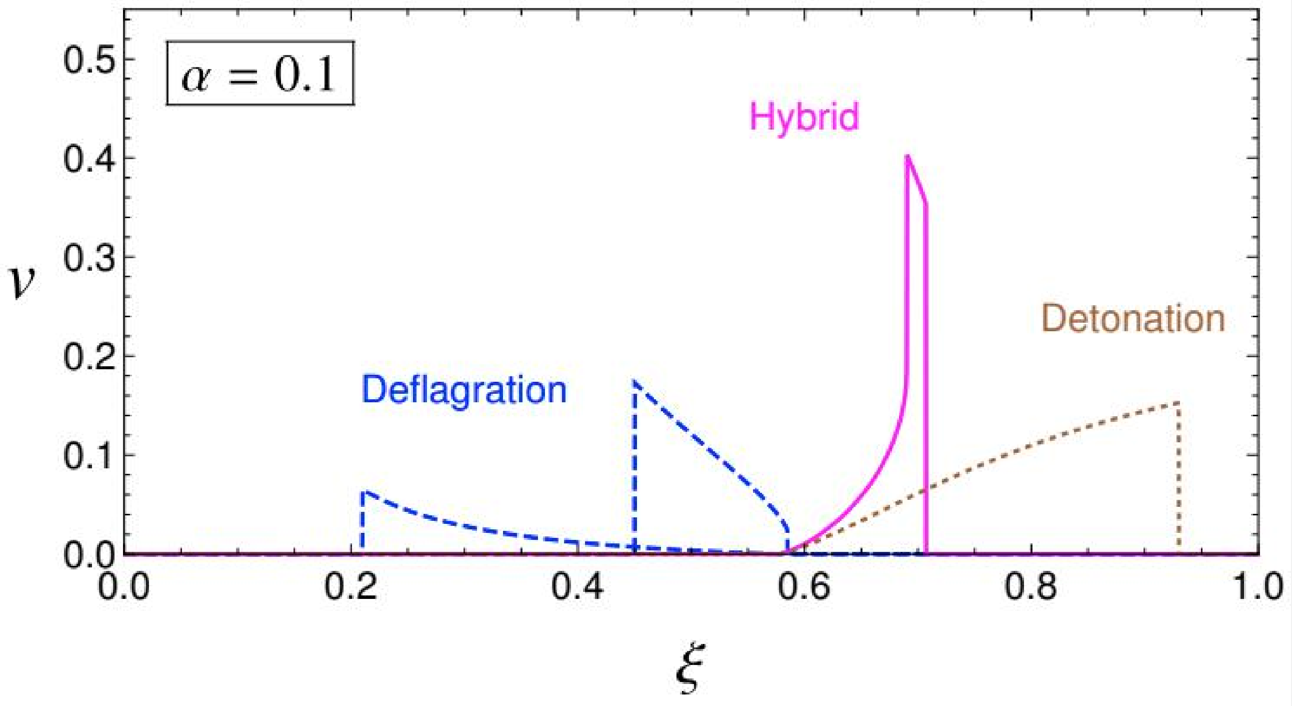
$$v^i(\eta < \eta_{fc}, \mathbf{x}) = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\left[\tilde{v}^i_{\mathbf{q}}(\eta)e^{i\mathbf{q}\cdot\mathbf{x}} + \tilde{v}^{i*}_{\mathbf{q}}(\eta)e^{-i\mathbf{q}\cdot\mathbf{x}}\right]$$

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}}e^{-i\omega\eta+i\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}}e^{i\omega\eta-i\mathbf{q}\cdot\mathbf{x}}\right]$$

$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

Velocity Profile Around a Single Bubble

- We have shown velocity profile remains the same form, when appropriate variables substitution is performed.



Sound Waves when bubbles have all disappeared

- Equations of motion can be obtained by simply rescaling of Minkowski conterpart
- Sound waves(fluctuations of energy, pressure, velocity)

$$(a^4 S^i)' + \nabla \cdot (a^4 S^i \mathbf{v}) + \partial_i (a^4 p) = 0, \quad S^i = \gamma^2(\epsilon + p) v^i$$

$$(a^4 \epsilon \gamma)' + [\gamma' + \nabla \cdot (\gamma \mathbf{v})](a^4 p) + \nabla \cdot (a^4 \epsilon \gamma \mathbf{v}) = 0,$$

$$\gamma^2 (v' + \frac{1}{2} \hat{\mathbf{v}} \cdot \nabla v^2)[a^4(\epsilon + p)] + v(a^4 p)' + \hat{\mathbf{v}} \cdot \nabla (a^4 p) = 0$$

conformal time

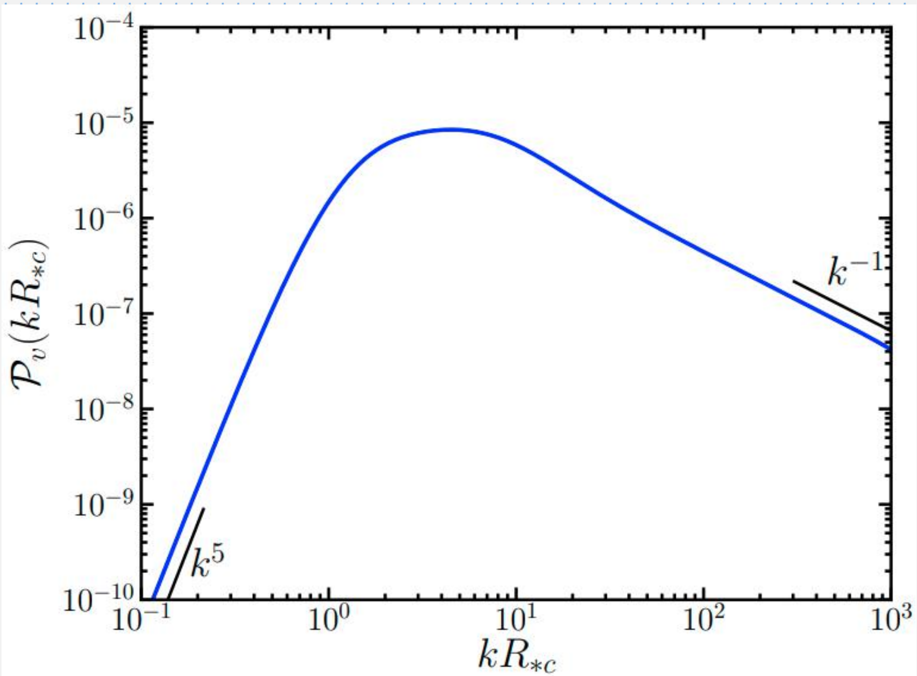
reduces to special relativistic Hydrodynamics when using rescaled quantities

Velocity Field Power Spectrum

stochastic field: bubble position, formation time, collision time(final size)

after averaging, depends only on conformal lifetime distribution of the bubbles

$$\langle \tilde{v}^i_{\mathbf{q}}(\eta_1)\tilde{v}^{j*}_{\mathbf{k}}(\eta_2)\rangle = \delta^3(\mathbf{q}-\mathbf{k})\hat{q}^i\hat{k}^j G(q,\eta_1,\eta_2)$$



Gravitational Wave Power Spectrum

$$(\kappa_M y + 1 - \kappa_M)\frac{d^2 h_q}{dy^2} + \left[\frac{5}{2}\kappa_M + \frac{2(1-\kappa_M)}{y}\right]\frac{dh_q}{dy} + \tilde{\tilde{q}}^2 h_q = \frac{16\pi G a(y)^2 \pi_q^T(y)}{(a_s H_s)^2}$$

$$y \equiv a/a_s$$

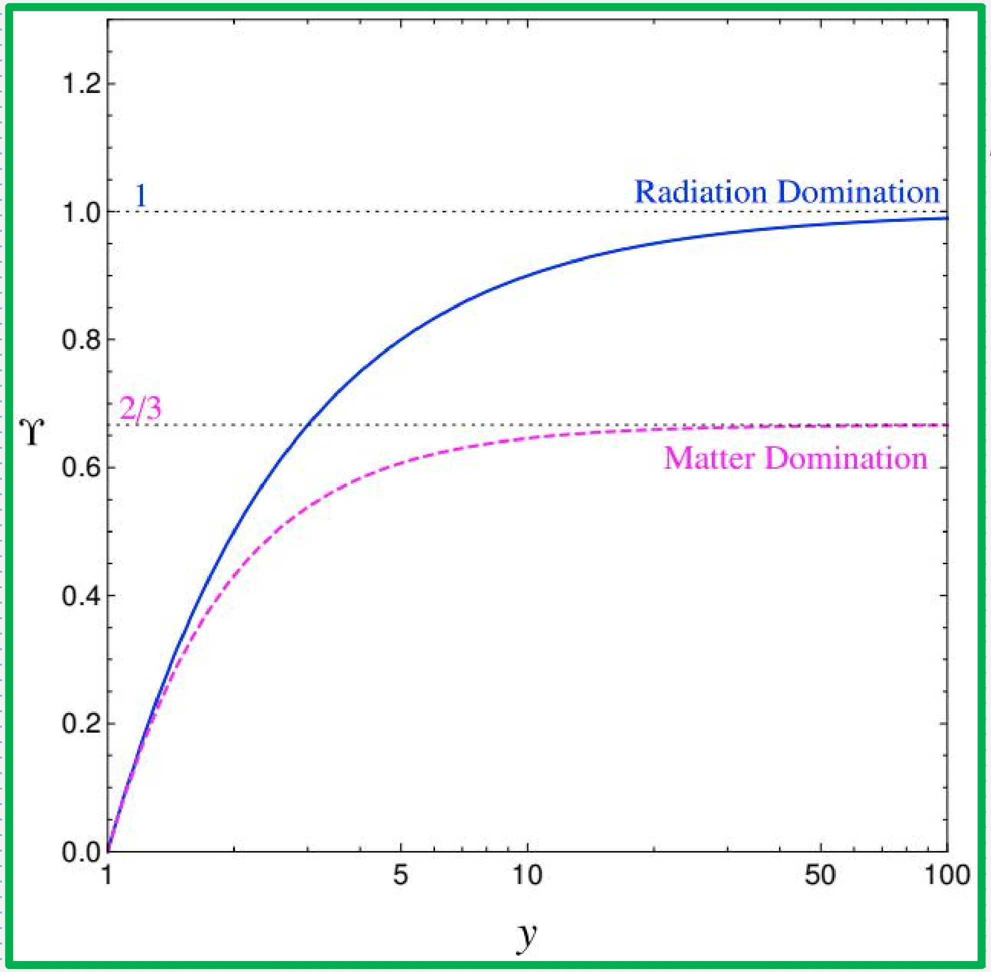
$$h_{ij}(\tilde{y}, \mathbf{q}) = \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}' G(\tilde{y}, \tilde{y}') \frac{16\pi G a(\tilde{y}')^2 \pi_{ij}^T(\tilde{y}', \mathbf{q})}{q^2}$$

$$\mathcal{P}_{\mathrm{GW}}(y, kR_{*c}) = 3\Gamma^2 \, \bar{U}_f^4 \frac{H_{R,s}^4}{H^2 H_s}(a_s R_{*c}) \frac{(kR_{*c})^3}{2\pi^2} \tilde{P}_{\mathrm{gw}}(kR_*) \times \frac{1}{y^4} \Upsilon(y)$$

Gravitational Wave Power Spectrum

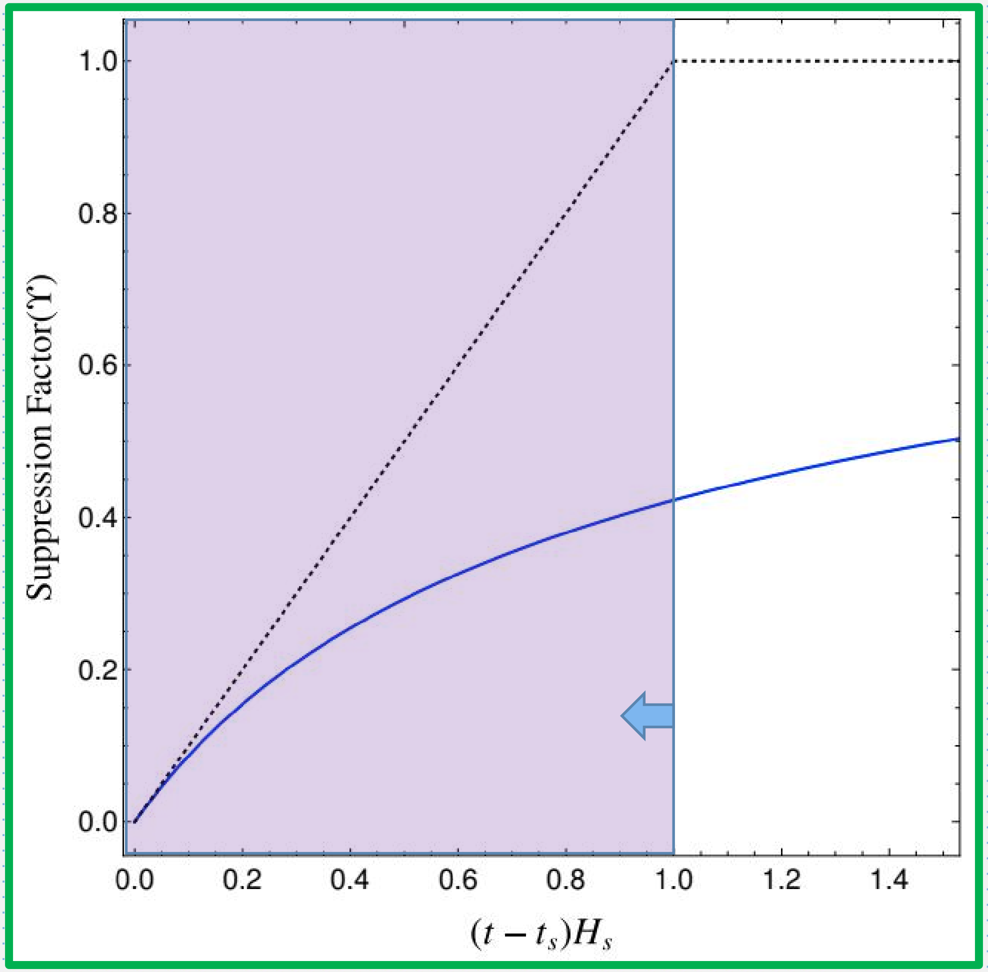
use spectrum from numerical simulation

$$h^2\Omega_{\rm GW}(f) = 8.5 \times 10^{-6} \left(\frac{100}{g_s(T_e)}\right)^{1/3} \Gamma^2 \bar{U}_f^4 \left[\frac{H_s}{\beta(v_w)}\right] v_w \mathcal{S}_{\rm SW}(f) \times \Upsilon(y)$$



Lifetime of the Source

- Shocks, turbulence, dissipative processes all disrupt the source
- So lifetime is usually less than a Hubble time, meaning a suppression



Dynamics of Phase Transition

- Bubble Nucleation Rate
- False Vacuum Fraction
- Unbroken Wall Area
- Bubble Lifetime Distribution
- Bubble Number Density and Mean Bubble Separation(R*)
- Relation between beta and R*

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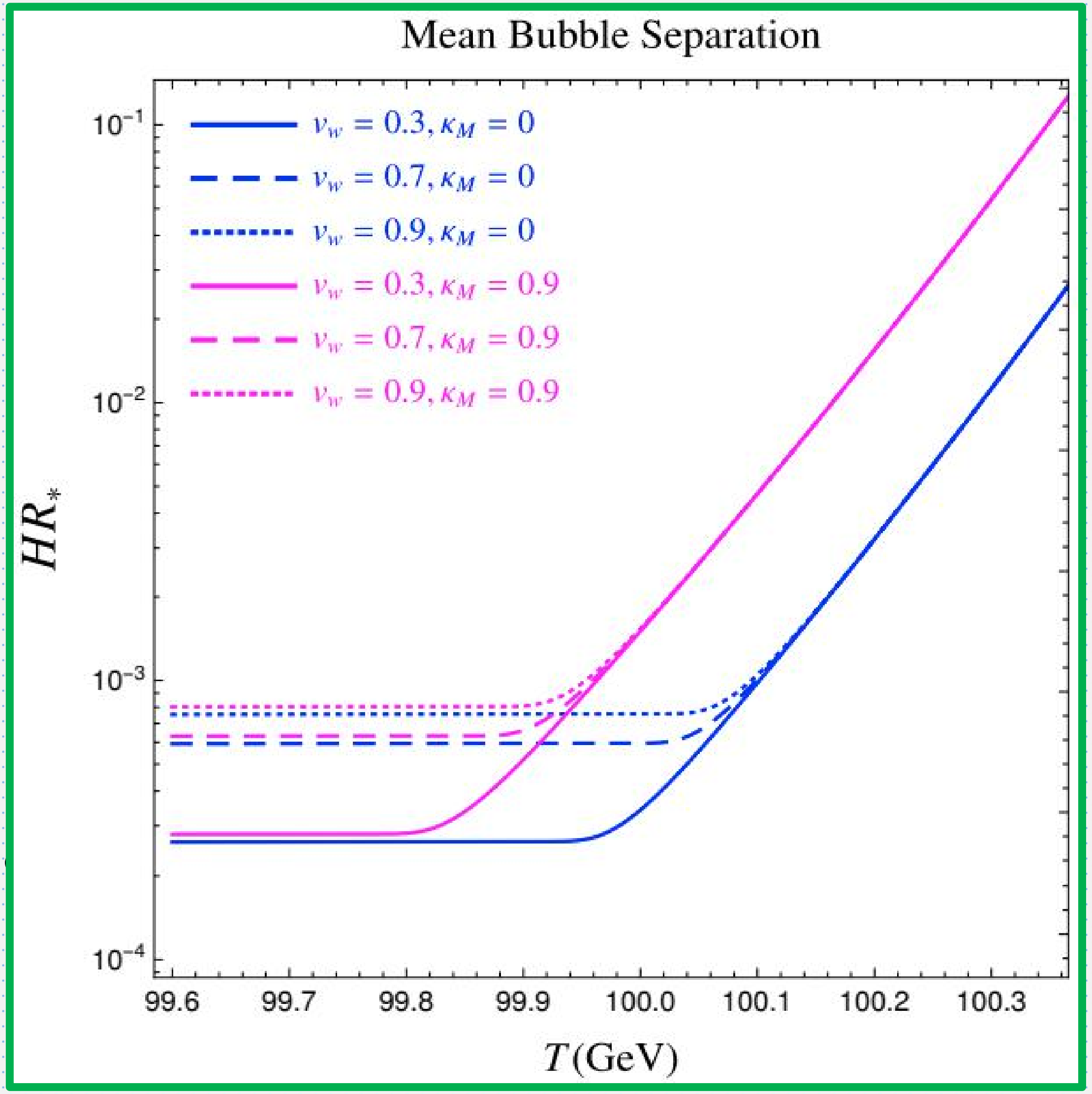

Summary

- We have set up the framework for modelling the GW from sound waves, in an expanding unvierse

  generally no need for new simulations, rescaled quantities need to be used

  PT and GW in matter domination
- A suppression factor is found and needs to be included to the generally used spectrum
- Details of the PT process is analyzed in standard and non-standard cosmic histories

Thanks!