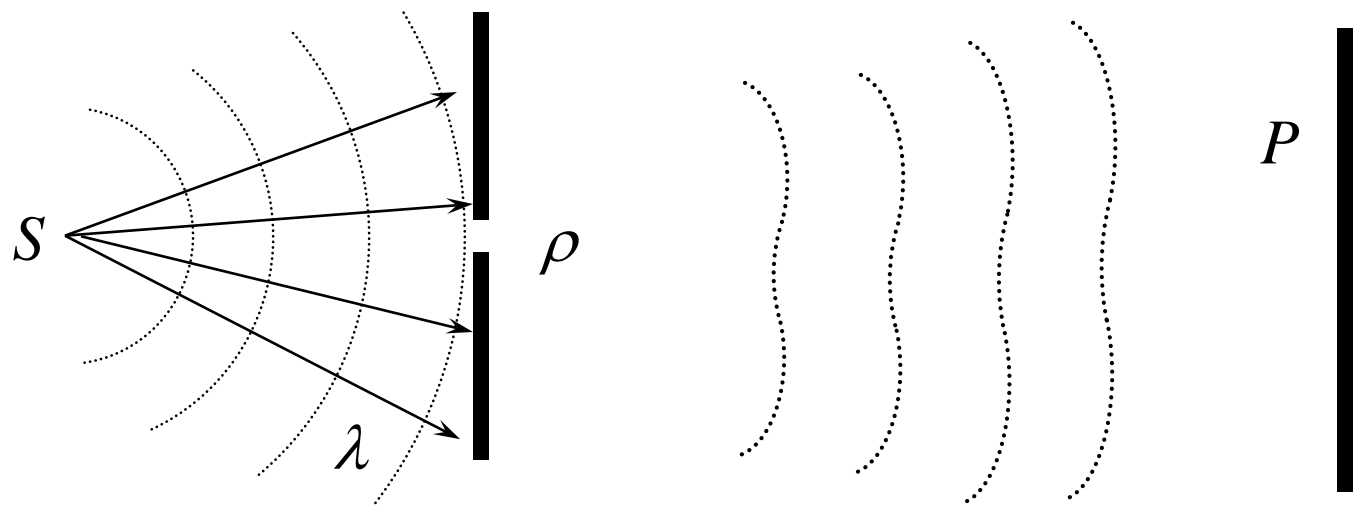


# 圆孔的夫琅禾费衍射

## 费涅尔衍射与夫琅禾费衍射

在光扰动振幅随空间的变化远小于相位空间变化的情况下，我们可以用几何光学的方法来描述光波的传播。而这个条件得不到满足时，光的传播不再满足几何光学的定律，光不再沿直线传播，即发生衍射（**diffraction**）。



光波的衍射

光场是否满足几何光学光束条件取决于障碍物的线度与光波长的比。根据比例的不同，我们可以把光的衍射分成三个区间：

- 1)  $\rho > 10^3 \lambda$ ，几何光学光束条件在大部分场点满足，光按直线传播。衍射很不明显，只在几何投影边缘区有较明显的衍射。
- 2)  $\rho \in [ \lambda , 10^3 \lambda ]$ ，几何光学光束条件不满足，衍射很显著，出现丰富的衍射图样。
- 3)  $\rho < \lambda$ ，衍射效应极端强烈，过渡到光的散射（scattering）。衍射图样趋于消失。

衍射光场的复振幅可以通过费涅尔 — 基尔霍夫衍射积分来计算

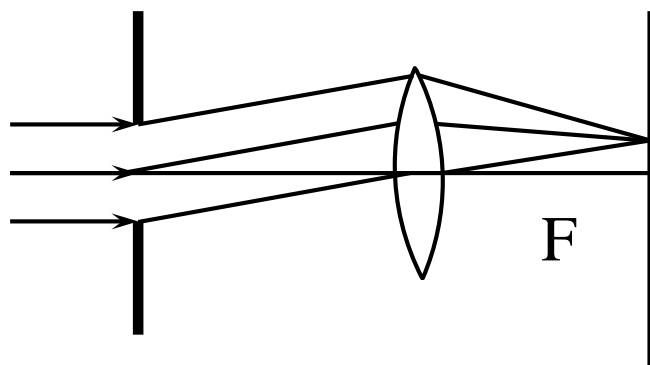
$$\tilde{U}(P) = \frac{-i}{\lambda} \iint_{\Sigma_0} \left( \frac{\cos \theta + \cos \theta_0}{2} \right) \tilde{U}_0(Q) \frac{e^{ikr}}{r} ds$$

在傍轴条件下

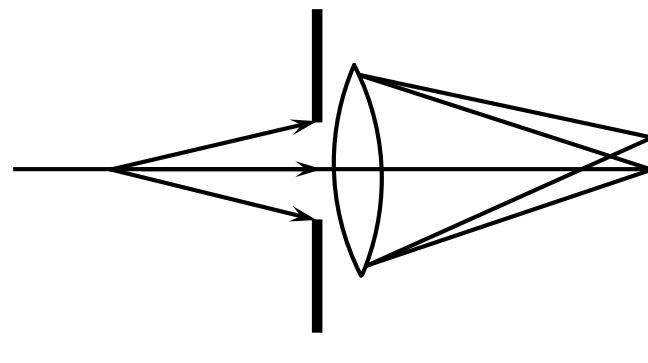
$$\tilde{U}(P) = \frac{-i}{\lambda} \iint_{\Sigma_0} \tilde{U}_0(Q) \frac{e^{ikr}}{r} ds$$

如相对衍射屏接收场和光源均满足远场条件，我们称衍射为夫琅禾费（Fraunhofer）衍射；否则，我们称衍射为费涅尔衍射。

夫琅禾费衍射条件的实现

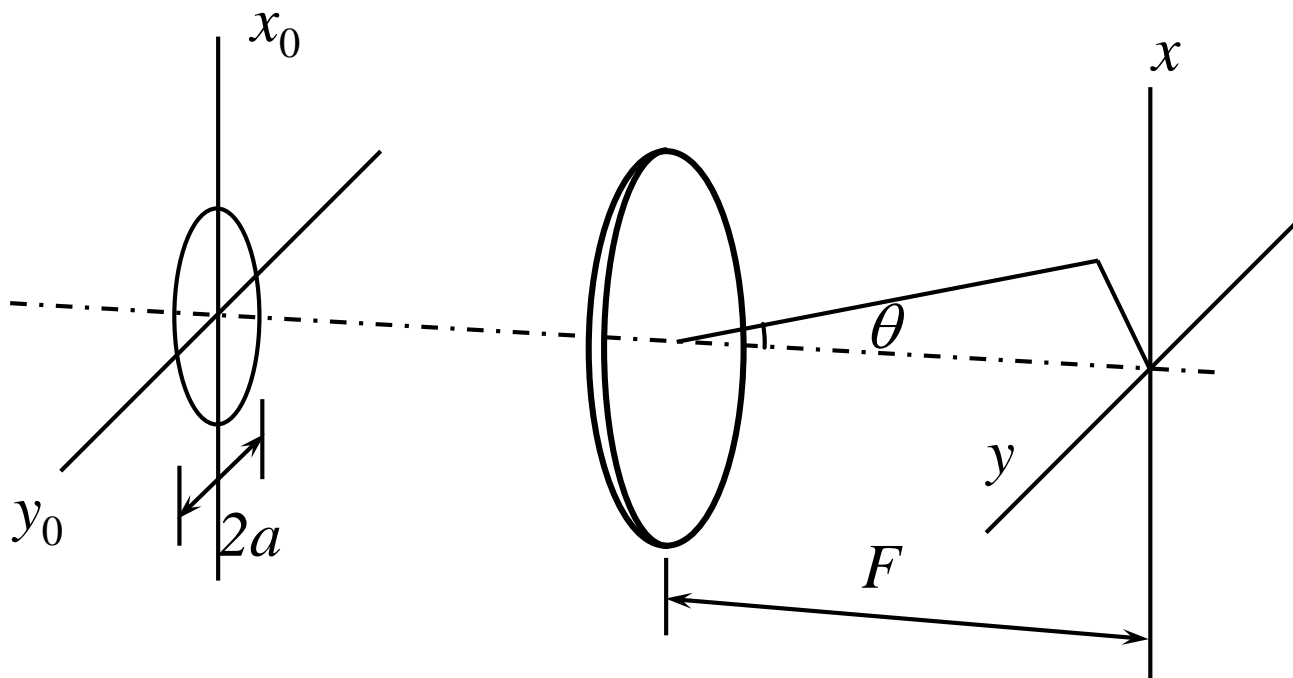


焦面接收

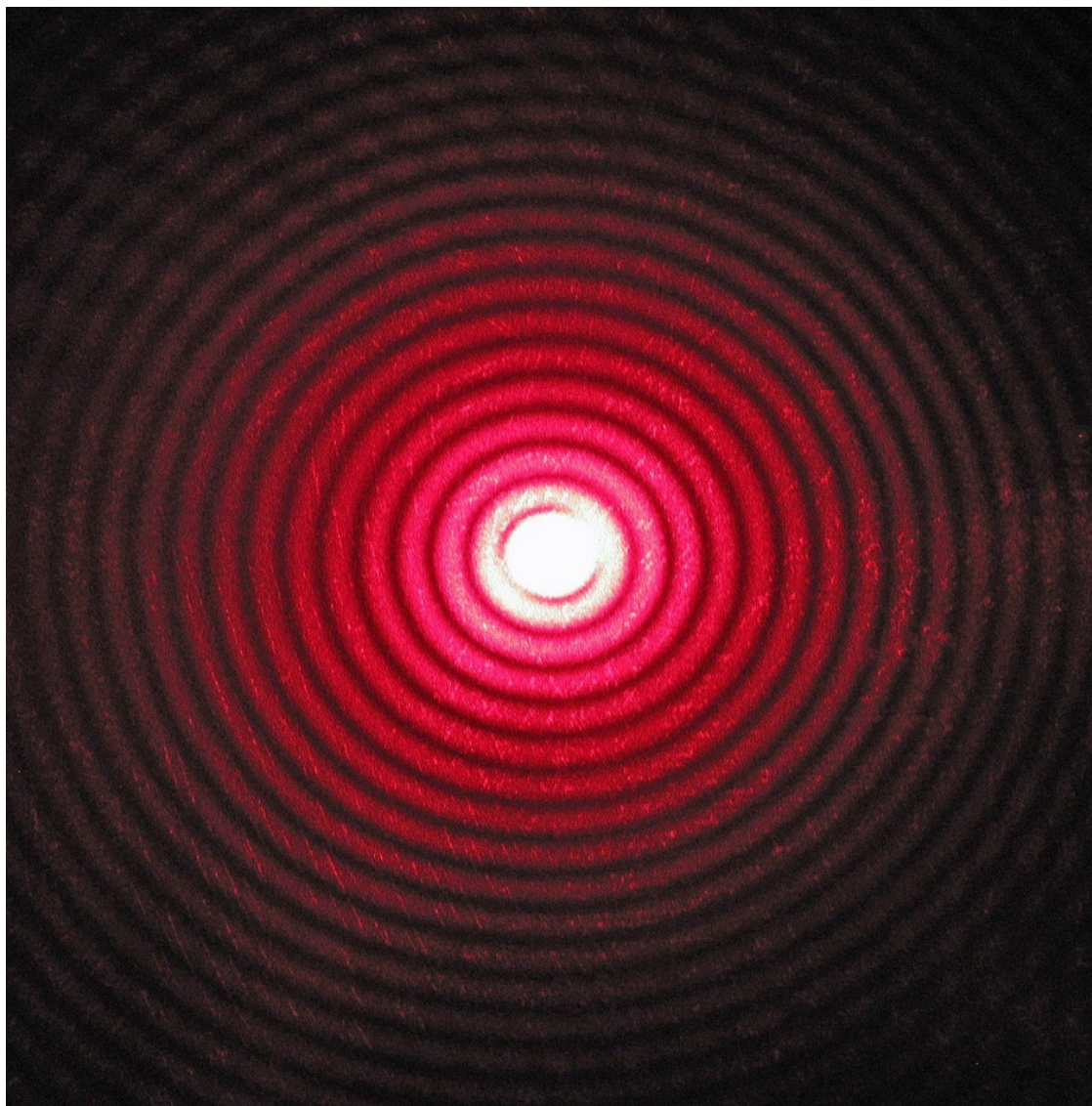


像面接收

# 费涅尔衍射与夫琅禾费衍射



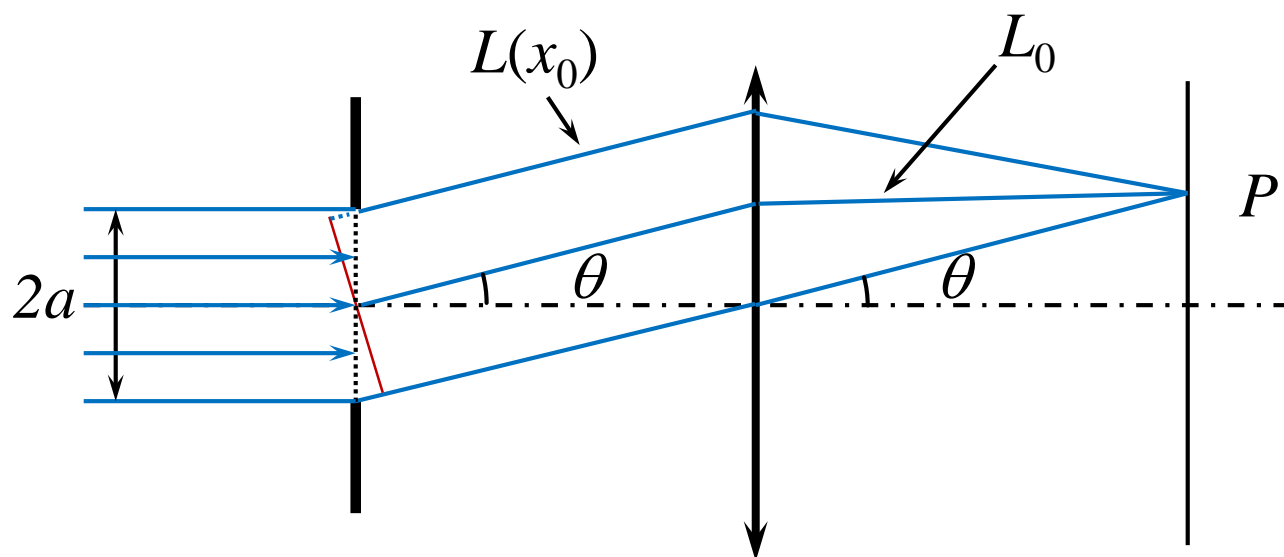
圆孔 (circular aperture) 的夫琅禾费衍射装置



圆孔的夫琅禾费衍射图样 (爱里斑 Airy pattern)



## 衍射场的分析



记经圆孔中心的光程为  $L_0$ ，那么经  $x_0$  点的光程为

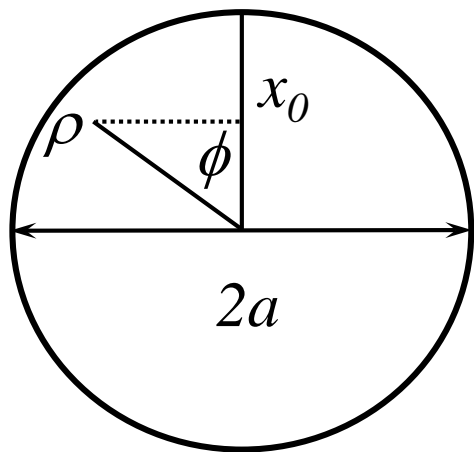
$$L(x_0) = L_0 - x_0 \sin \theta$$

衍射积分

$$\tilde{U}(P) = \frac{-i}{r_0 \lambda} \int_{\Sigma} \tilde{U}_0(Q) e^{ikr} ds$$

我们有以下条件  $\tilde{U}_0(Q) = A$ ,  $r_0 = F$ ,

$$r = L(x_0), \quad x_0 = \rho \cos \phi, \quad ds = \rho d\rho d\phi$$



积分范围:

$$0 \leq \rho \leq a$$

$$0 \leq \phi \leq 2\pi$$

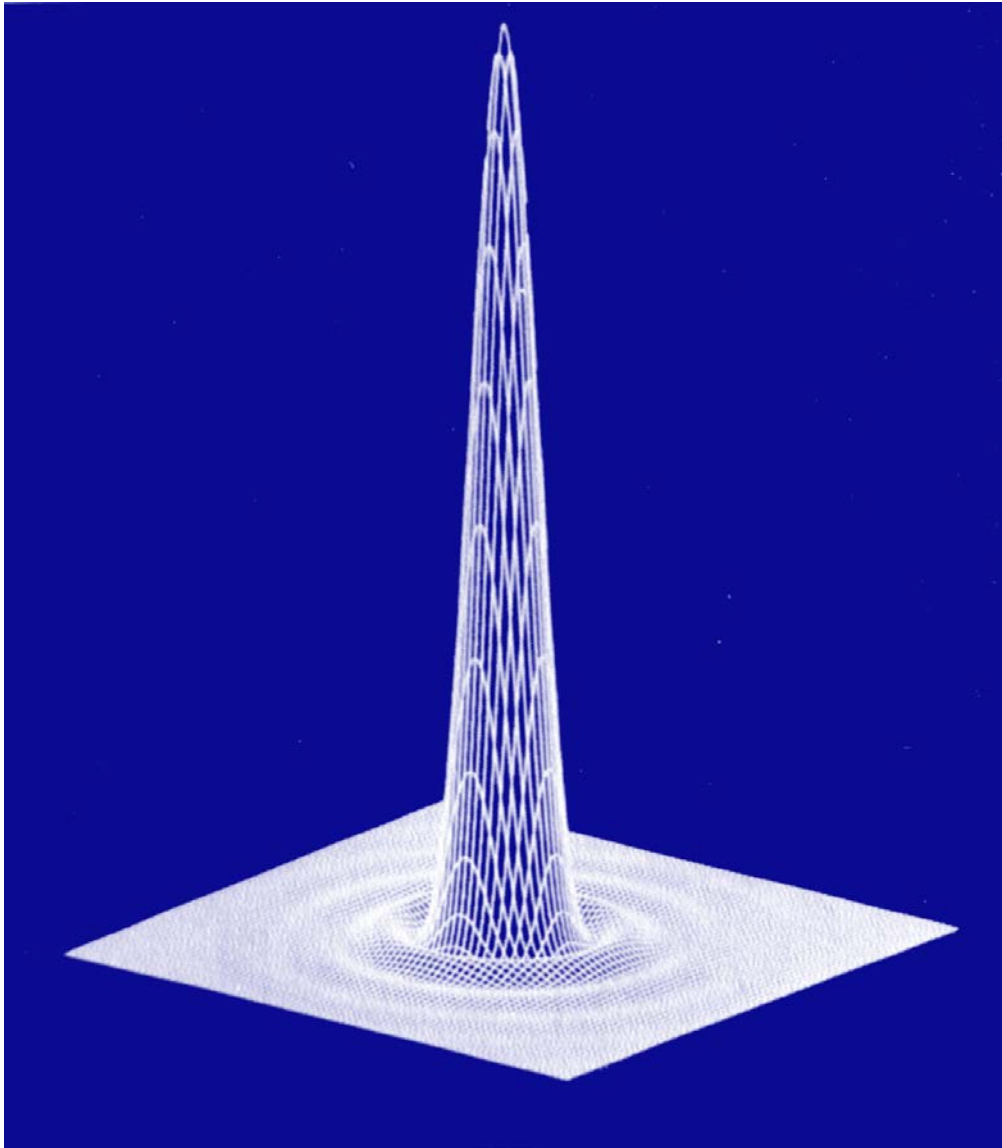
$$\begin{aligned}
\tilde{U}(\theta) &= \frac{-iA}{F\lambda} e^{ikL_0} \int_0^{2\pi} d\phi \int_0^a d\rho \rho \exp(-ik\rho \cos\phi \sin\theta) \\
&= \frac{-iA}{F\lambda} e^{ikL_0} 2\pi \int_0^a d\rho \rho J_0(k\rho \sin\theta) \\
&= \frac{-iA}{F\lambda} (\pi a^2) e^{ikL_0} \left( \frac{2J_1(\gamma)}{\gamma} \right)
\end{aligned}$$

其中  $\gamma = ka \sin\theta$ ， $J_0$  和  $J_1$  为 0 阶和 1 阶 Bessel 函数。

光强分布：

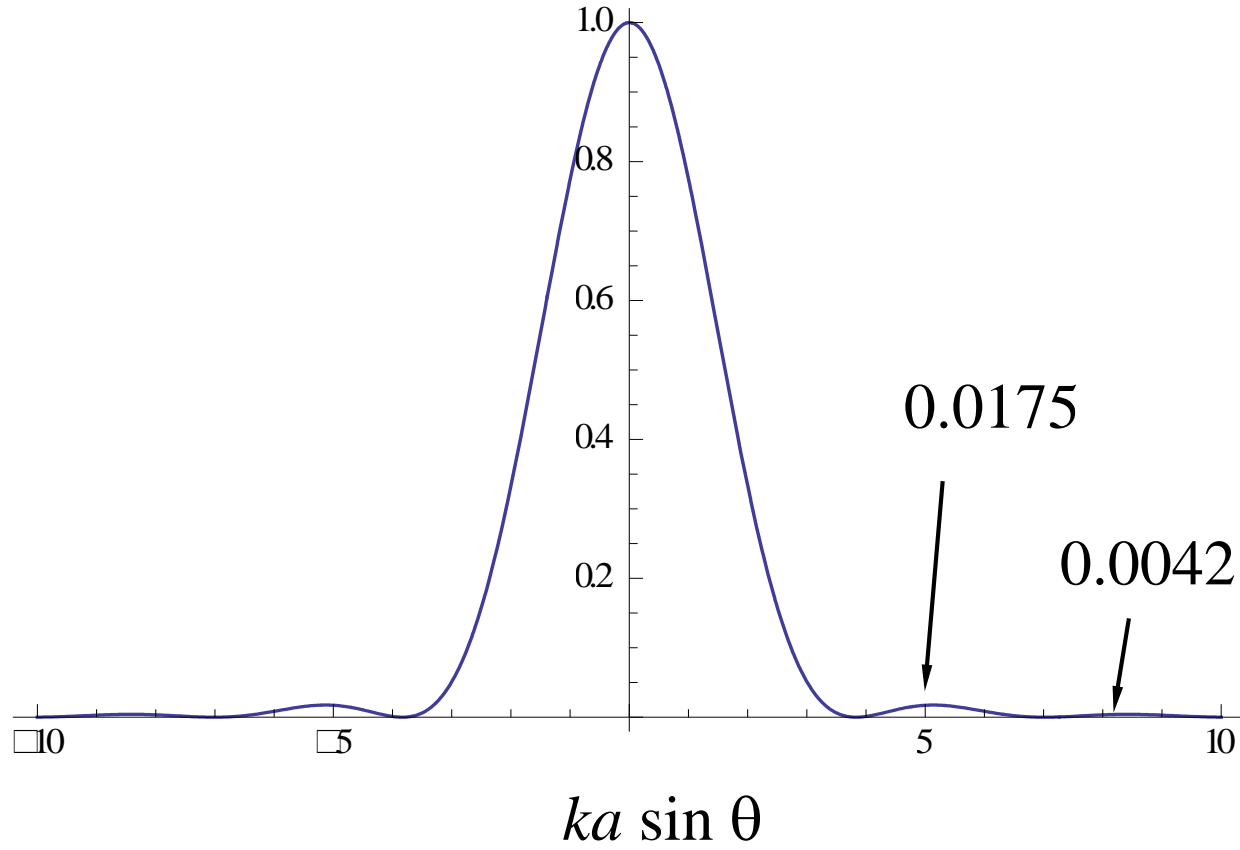
$$I(\theta) = \left( \frac{\pi a^2}{F\lambda} \right)^2 \left( \frac{2J_1(\gamma)}{\gamma} \right)^2 I_0$$

这里  $I_0$  是入射平面波的光强。



圆孔衍射因子

# 圆孔衍射因子



焦点光强：

$$I(0) = \left( \frac{\pi a^2}{F\lambda} \right)^2 I_0$$

与矩形孔的情形一样，焦点光强与入射平面波的光强之比等于光孔面积与焦距和波长乘积之比的平方。

从光强分布图可见能量主要集中在零级斑。第一暗纹的位置在  $\gamma = 1.220 \pi$  处。对应半角宽度：

$$\Delta\theta = 0.61 \frac{\lambda}{a}$$

零级斑占总能量的 84% ( 矩形孔 81%)。

零级斑面积与圆孔面积成反比。零级斑面积与圆孔面积的乘积为：

$$(\pi a^2)(\pi \Delta \theta^2) = (0.61\pi)^2 \lambda^2 = 3.7 \lambda^2$$

( 矩形孔为  $4\lambda^2$ 。 )

衍射斑的大小、亮度主要由光孔的面积决定，与光孔的形状关系不显著。

# 望远镜及其分辨本领

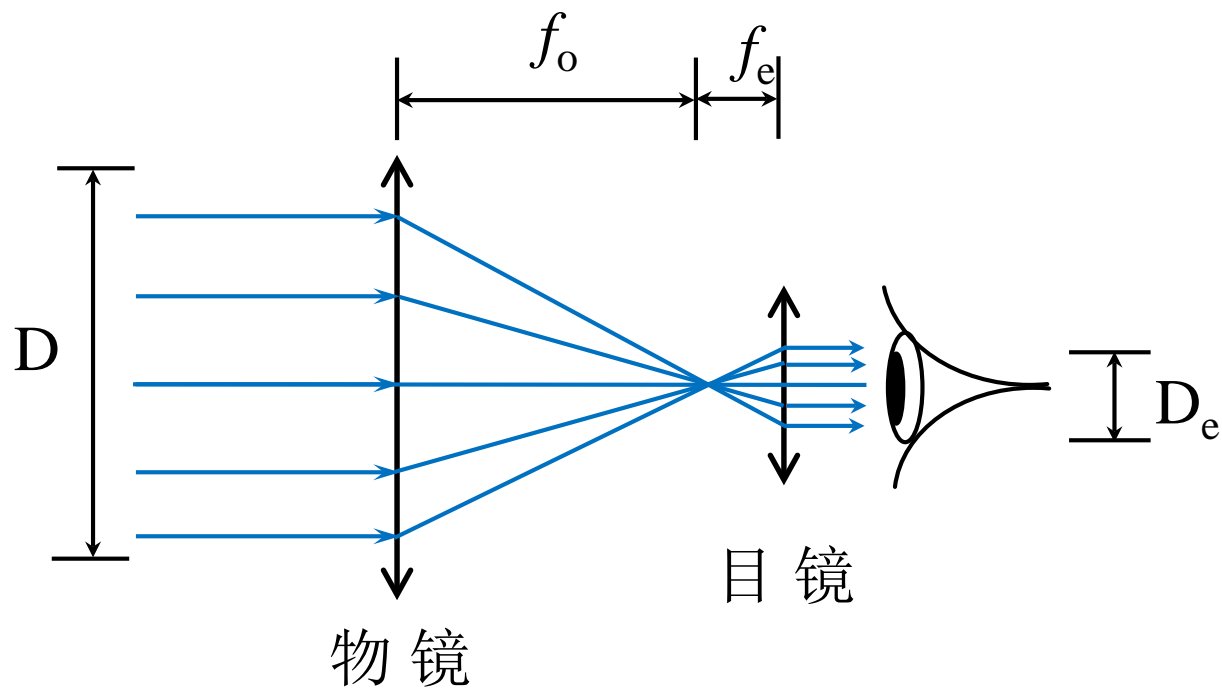


## 1. 望远镜的结构与角放大率

望远镜（telescope）一般由较大孔径和焦距的物镜（或主镜）与较小孔径和焦距的目镜（或多个次级镜）组成。物镜（主镜）将远处的物体的像成于其焦点附近，目镜（次级镜）将这个像放大以便观察或记录。

地面望远镜一般采用透镜（折射式），天文望远镜一般采用反射镜（反射式）。

开普勒 (Kepler) 望远镜由两个凸透镜。成倒立的像。



开普勒望远镜

物镜将远处的物成像于物镜后焦面附近，目镜起放大镜作用，把物镜成的像的视角放大。

原视角为


$$\alpha = \frac{y}{u}$$

经望远镜放大后的视角为

$$\alpha' = \frac{y'}{f_e} = -\frac{f_o}{u} \frac{y}{f_e} = -\alpha \frac{f_o}{f_e}$$

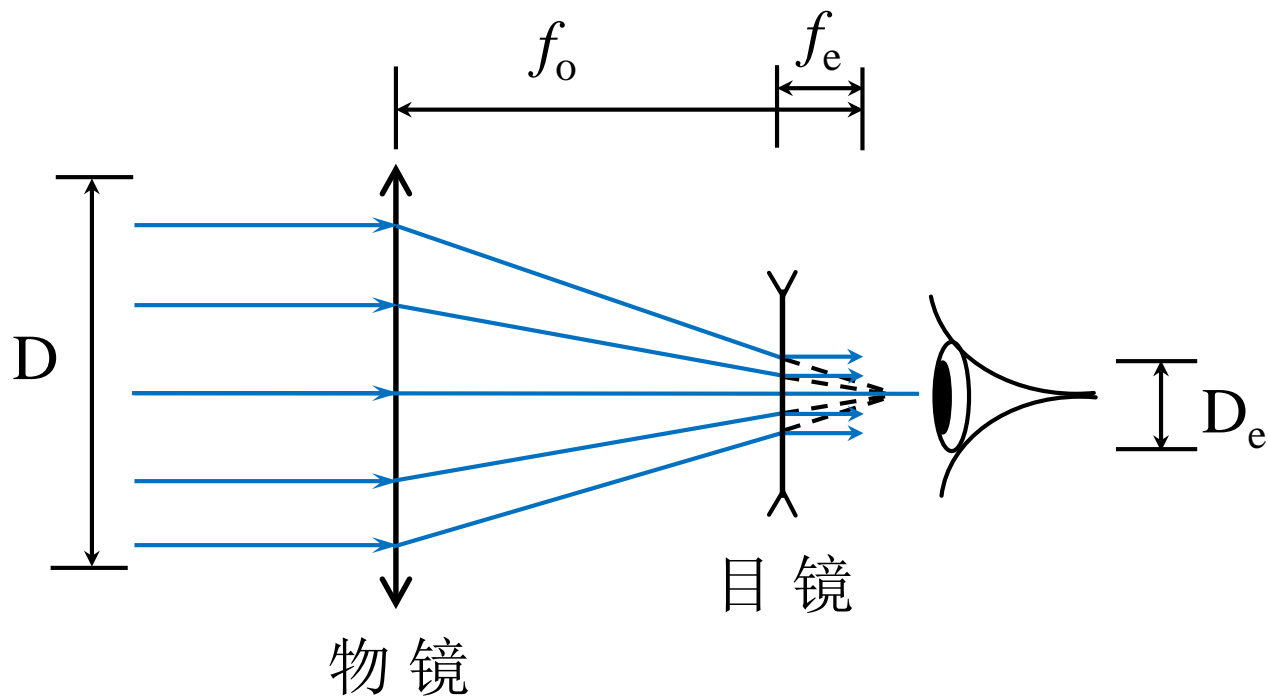
视角放大率为

$$M_T = \frac{\alpha'}{\alpha} = -\frac{f_o}{f_e}$$

An aerial photograph of Venice, Italy, showing the St. Mark's Basilica and its tall brick Campanile tower. The scene is set against a clear blue sky and the Venetian lagoon. A white, triangular pointer originates from the top of the Campanile tower and points towards a rectangular, stone-like plaque in the upper right corner of the image. The plaque contains text in Italian commemorating Galileo Galilei's use of his telescope in 1609.

GALILEO GALILEI  
CON IL SUO CANNOCCHIALE  
DA QUI IL 21 AGOSTO 1609  
ALLARGAVA GLI ORIZZONTI  
DELL'UOMO  
NEL QUARTO CENTENARIO

伽利略 (Galileo) 望远镜由作为物镜的凸透镜和作为目镜的凹透镜组成。成正立的像。



伽利略望远镜



观剧镜（伽利略望远镜）





望远镜的焦距

$$F = \frac{f_o f_e}{f_o + f_e - d}$$

望远镜中

$$f_e + f_o - d \leq 0$$

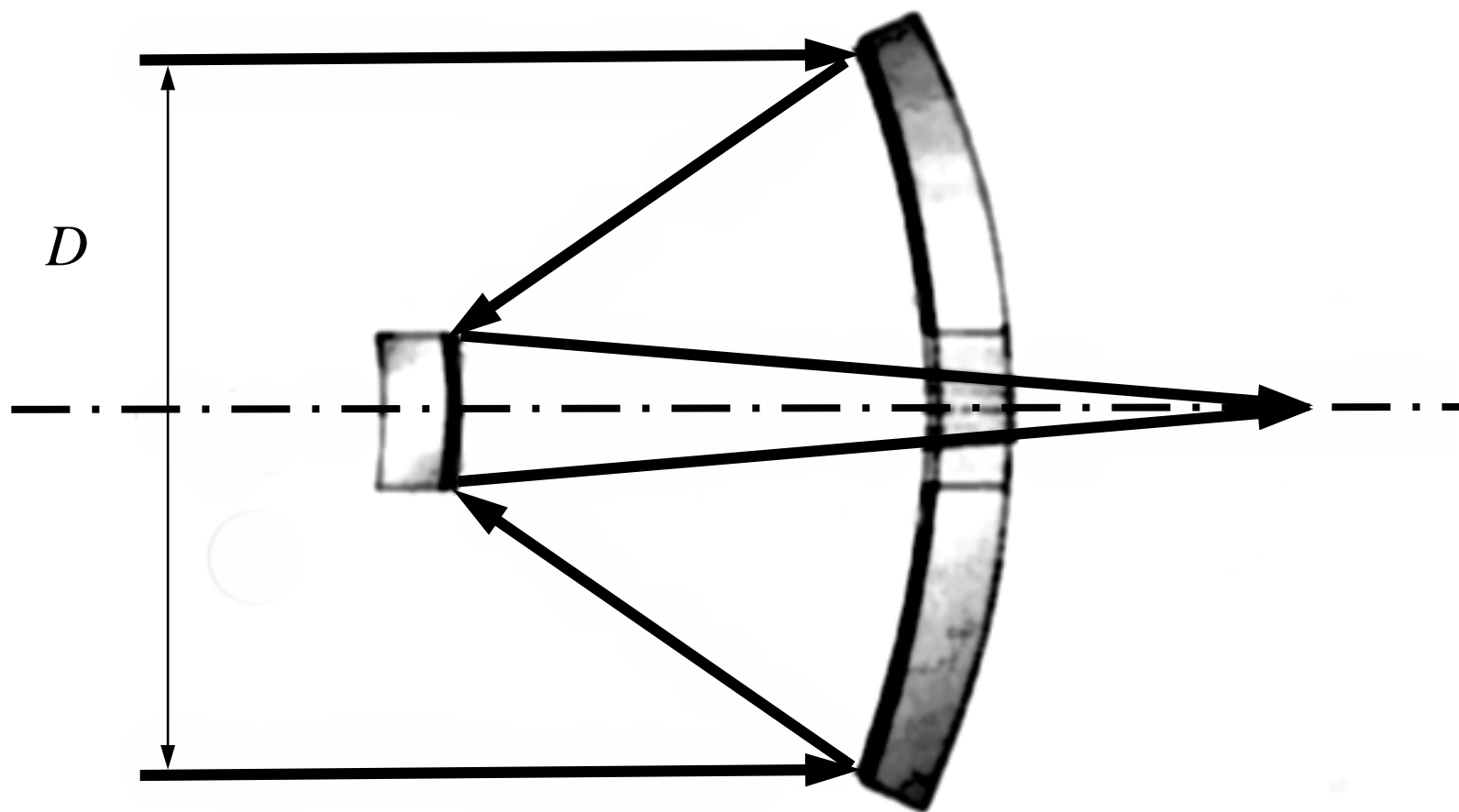
所以对于伽利略望远镜有

$$F > 0$$

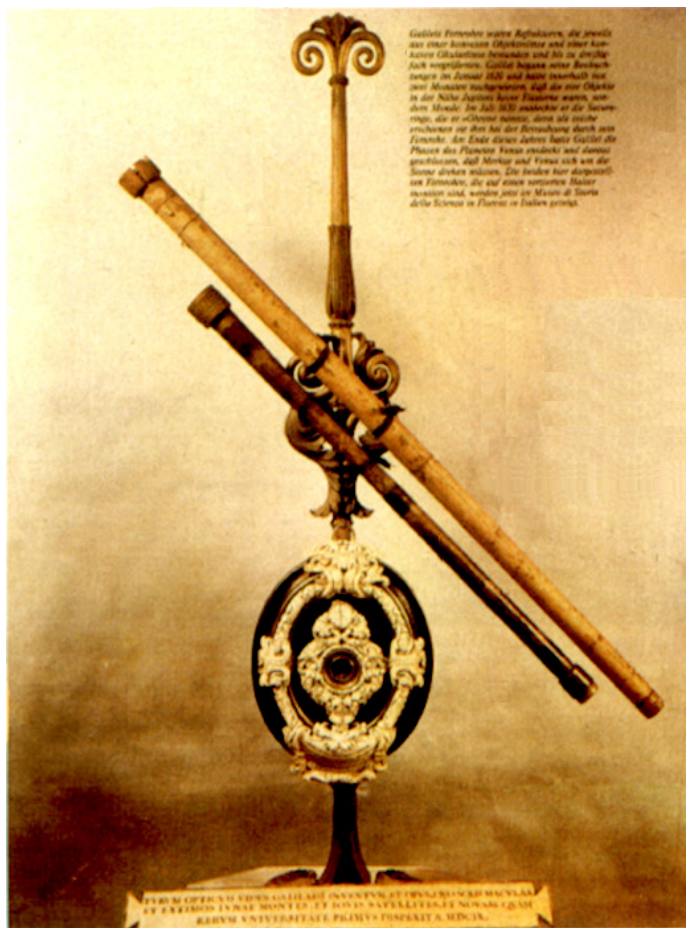
而对于开普勒望远镜则有

$$F < 0$$





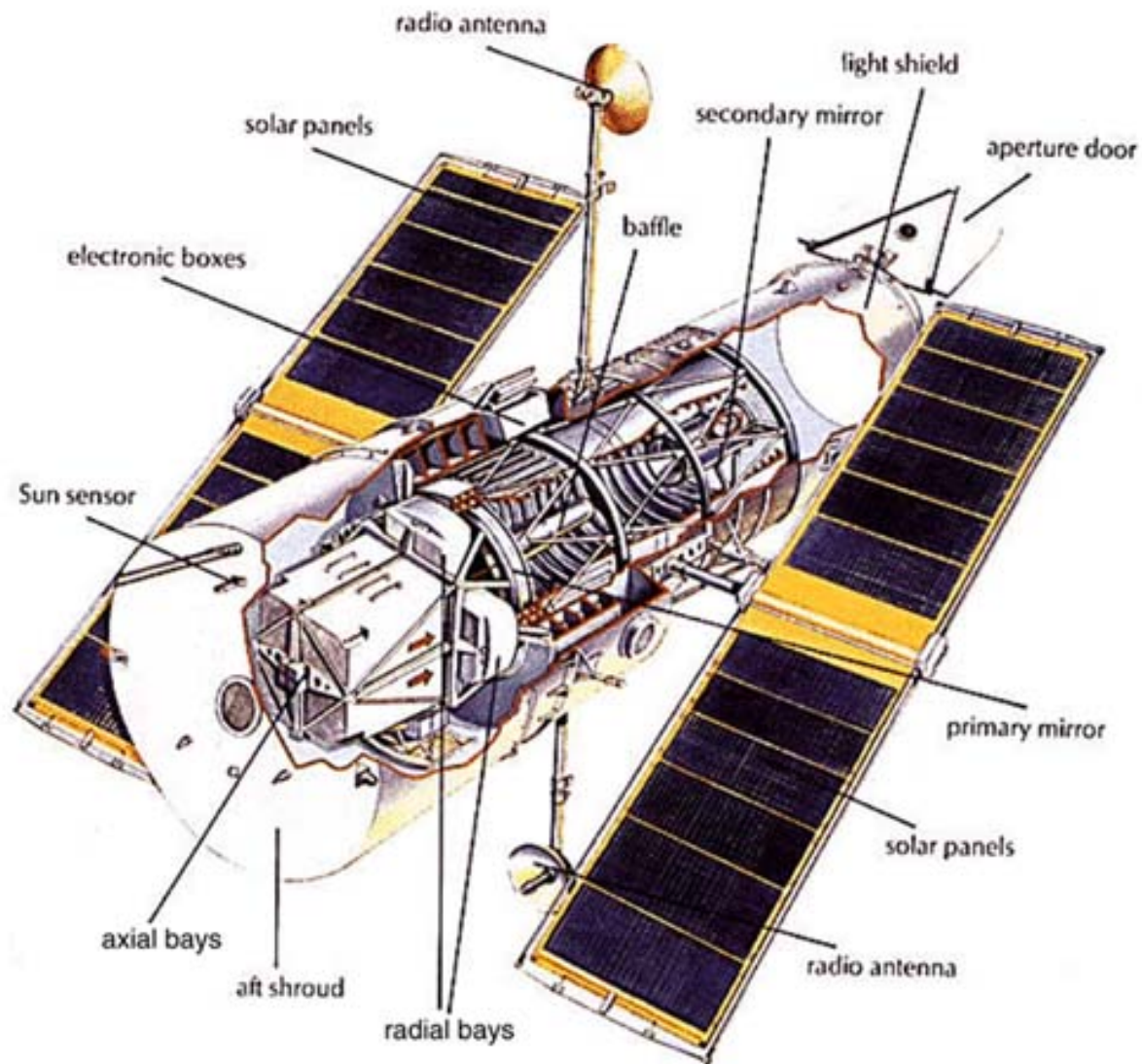
反射式天文望远镜



伽利略望远镜

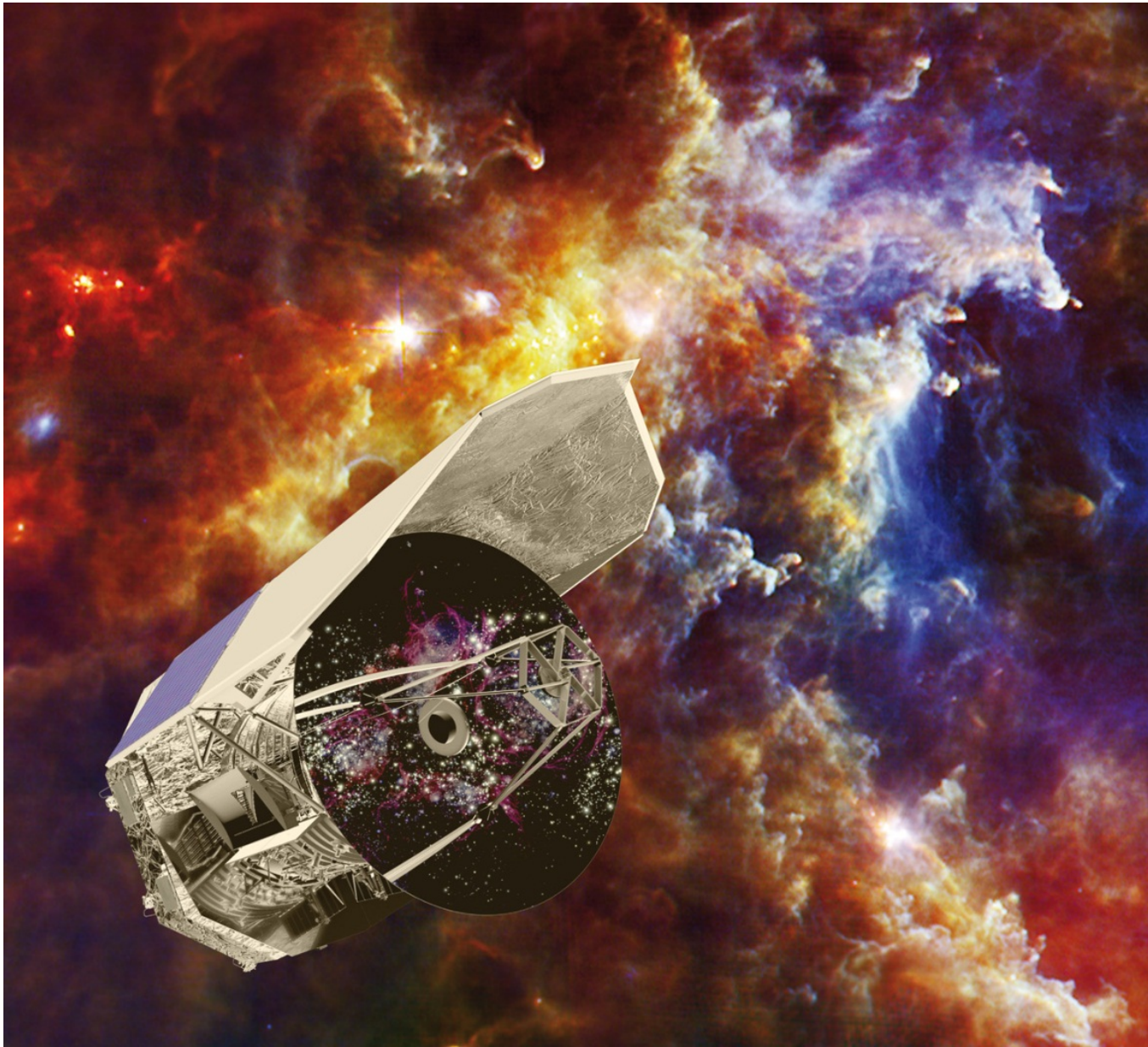


牛顿的反射式望远镜

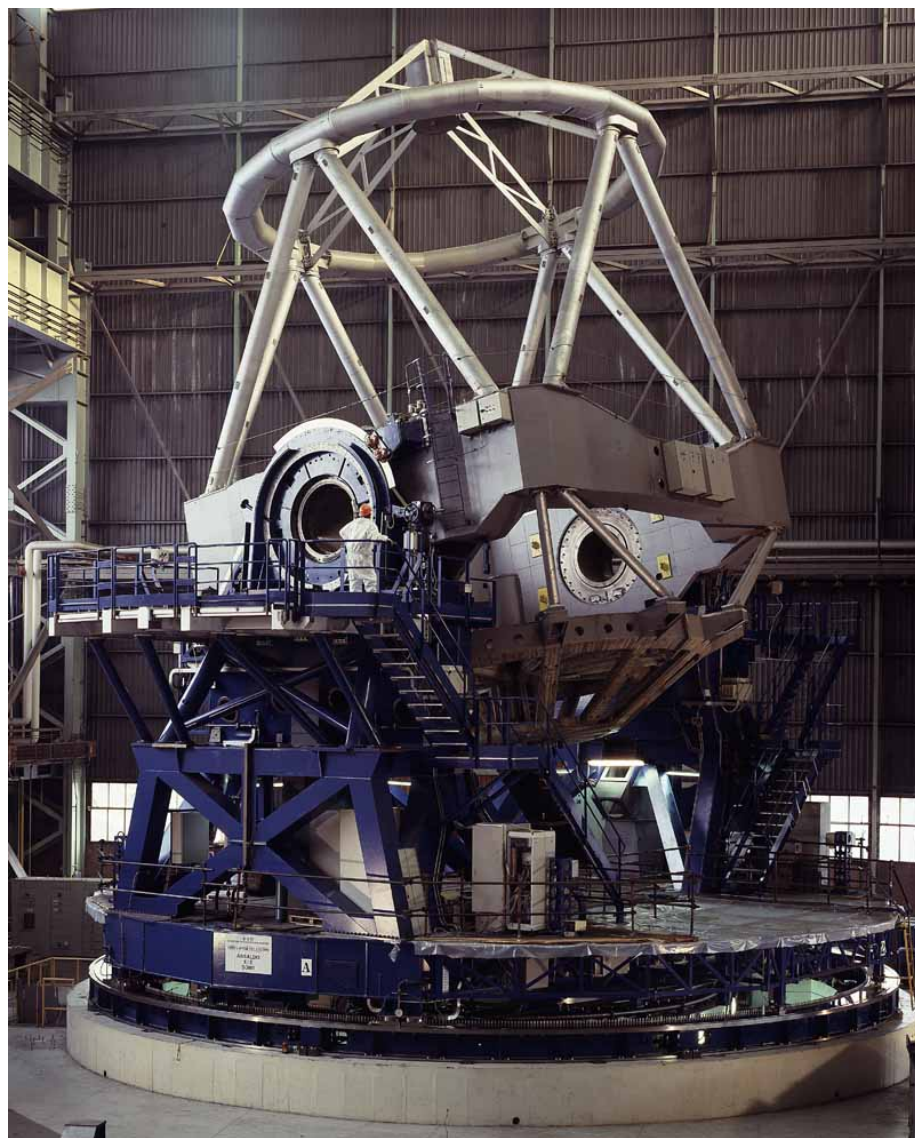


哈勃太空望远镜





主镜  
直径达3.5  
米的“赫  
歇尔”远  
红外望远  
镜。由欧  
洲航天局  
于2009年  
5月14日发  
射。

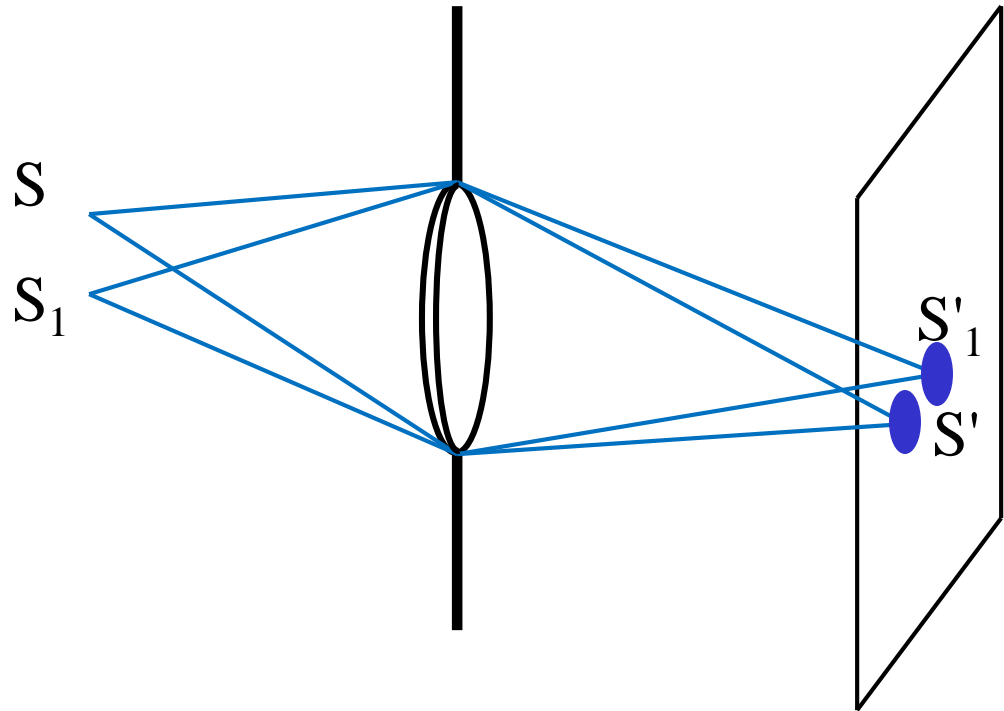


VLT天文望远镜

## 2. 瑞利判据 (Rayleigh criterion)

考虑两个点光源  $S$ 、 $S_1$ ，经光学系统成像于  $S'$ 、 $S'_1$ 。

由于光学系统的孔径有限，点光源的像扩散为一定大小的光斑。

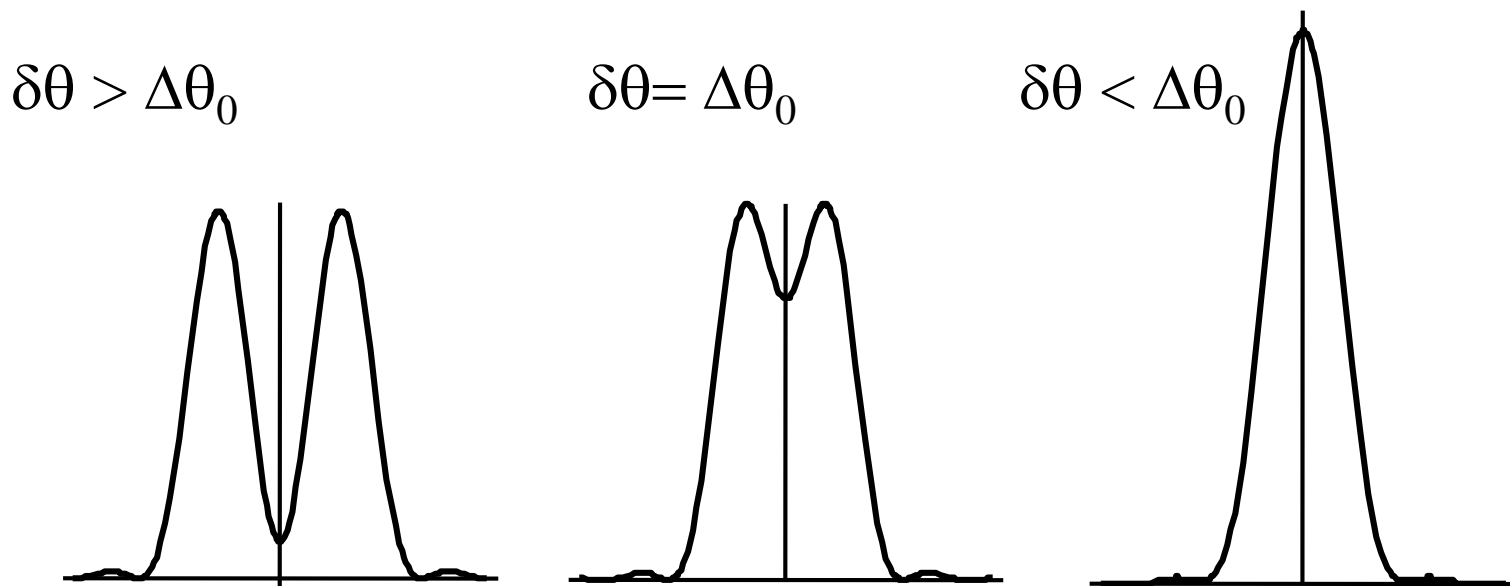


从光波衍射的角度看问题，成像系统是一个像面接收的夫琅禾费衍射系统。成像系统的有限孔径起到与衍射屏相同的对波前的限制作用，而在像面上观察到的衍射图样为光孔的夫琅禾费衍射图样。由几何像点扩散而成的光斑就是夫琅禾费衍射的零级斑。

一般光学成像系统的光孔为圆形，而另一方面夫琅禾费衍射的零级斑的大小主要与光孔的面积有关(成反比)，而与光孔的形状关系不很大，所以我们用圆孔夫琅禾费衍射衍射的爱里斑 来描述像点的光斑。用  $\Delta\theta_0$  表示爱里斑的角半径。

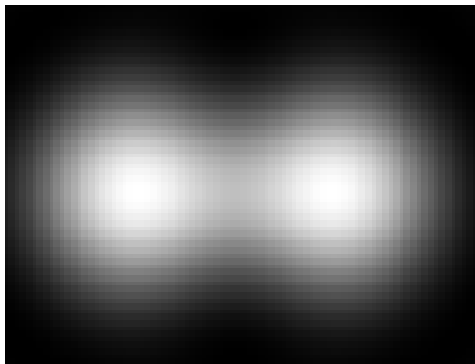
如  $S'$ 、 $S'_1$  中心距离较大，则两像可分辨，否则两像重叠成一个亮斑，不能分辨出两个像。爱里斑的角半径  $\Delta\theta_0$  由系统参数确定。

设两点光源像角的间距为  $\delta\theta$ ， $\Delta\theta_0$  与  $\delta\theta$  关系反映光斑的重叠程度。

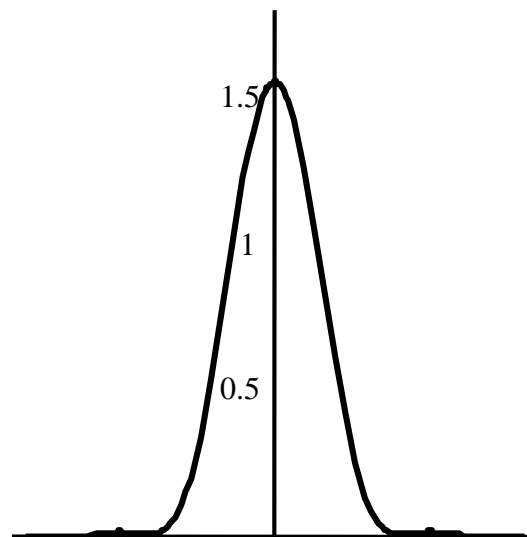
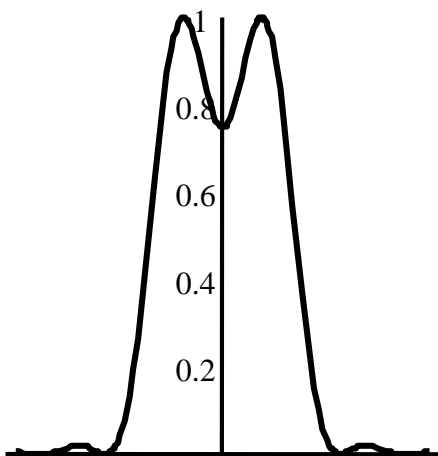
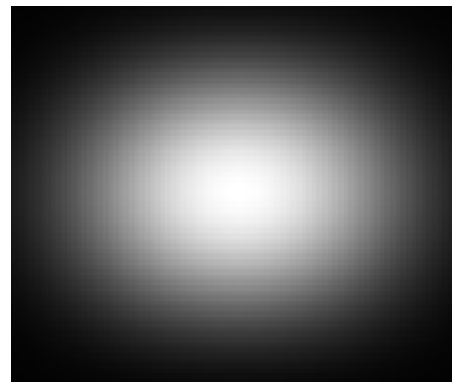




$$\delta\theta = \Delta\theta_0$$



$$\delta\theta < \Delta\theta_0$$



瑞利提出以  $\delta\theta = \Delta\theta_0$  为可分辨的极限。在  $\delta\theta = \Delta\theta_0$  时，光强有大于 20% 的起伏。这样光学系统像的角分辨本领为

$$\delta\theta = 1.22 \frac{\lambda}{D}$$

$D$  为光学系统的孔径 (直径)。孔径越大，所成像的分辨率越高，波长越短，所成像的分辨率也越高。

人眼的角分辨率： $D \sim 3\text{mm}$ ,  $\lambda \sim 555\text{nm}$

$$\delta\theta = 1.22 \frac{\lambda}{D} = 2.3 \times 10^{-4} \text{ rad} \cong 1'$$

### 3. 望远镜的角分辨本领

望远镜的角分辨本领由物镜的分辨角决定。物镜的分辨角为

$$\delta\theta = 1.22 \frac{\lambda}{D}$$

其中  $D$  为物镜（主镜）的直径。为充分利用物镜（主镜）的分辨能力，接收装置应满足分辨匹配条件，即接收装置的分辨角应不大于望远镜的角放大倍数与物镜的分辨角的乘积。即有效放大率为

$$M_{eff} = \frac{\delta\theta_e}{\delta\theta_o} = \frac{D}{D_e}$$

其中  $D_e$  为接收装置的孔径。

例：  $D=20\text{ cm}$ ,  $\lambda = 555\text{ nm}$ , 最小分辨角为

$$\delta\theta_0 = 1.22 \frac{\lambda}{D} = 3.4 \times 10^{-6} \text{ rad}$$

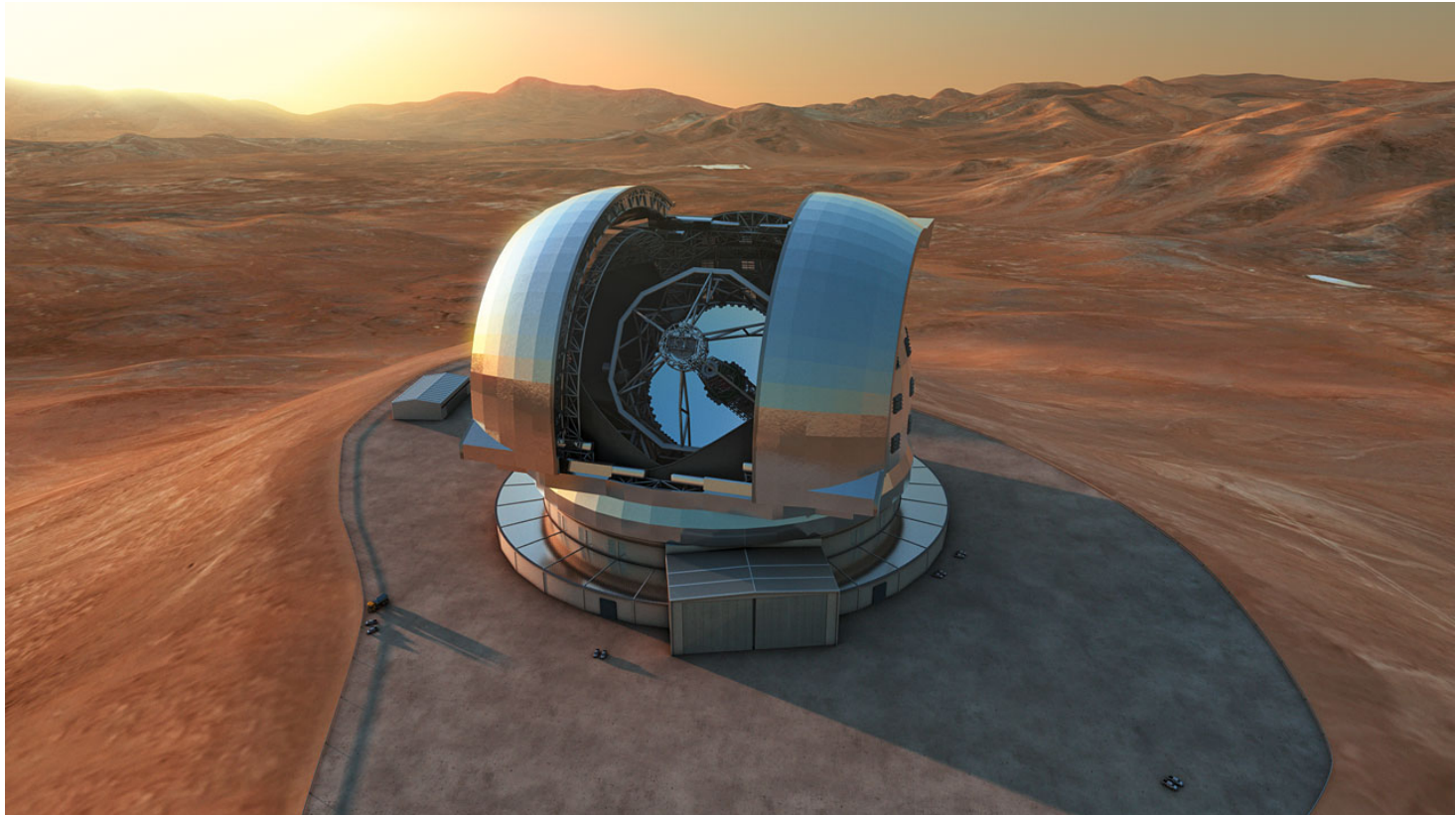
如接收装置为人眼，

$$\delta\theta_e = 2.3 \times 10^{-4} \text{ rad}$$

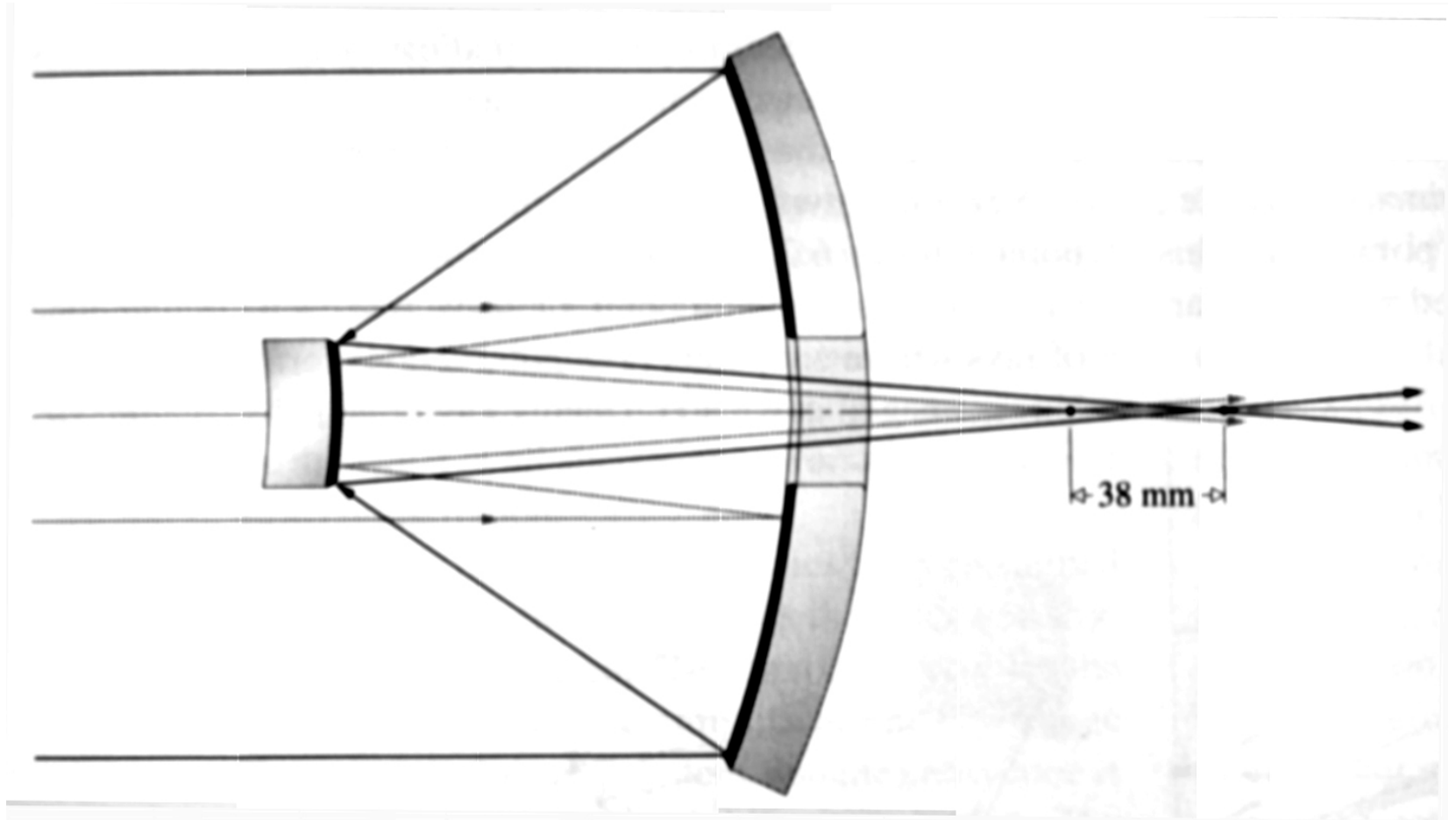
匹配的角放大率为：

$$M_{eff} = \frac{\delta\theta_e}{\delta\theta_0} = 68$$

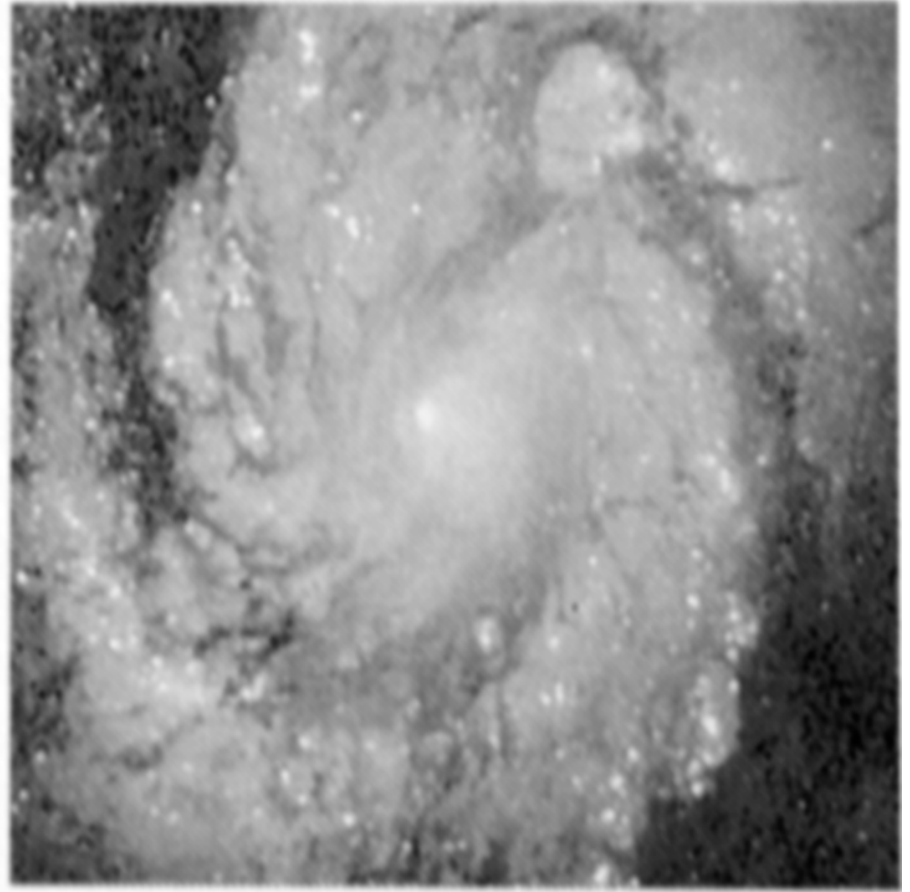
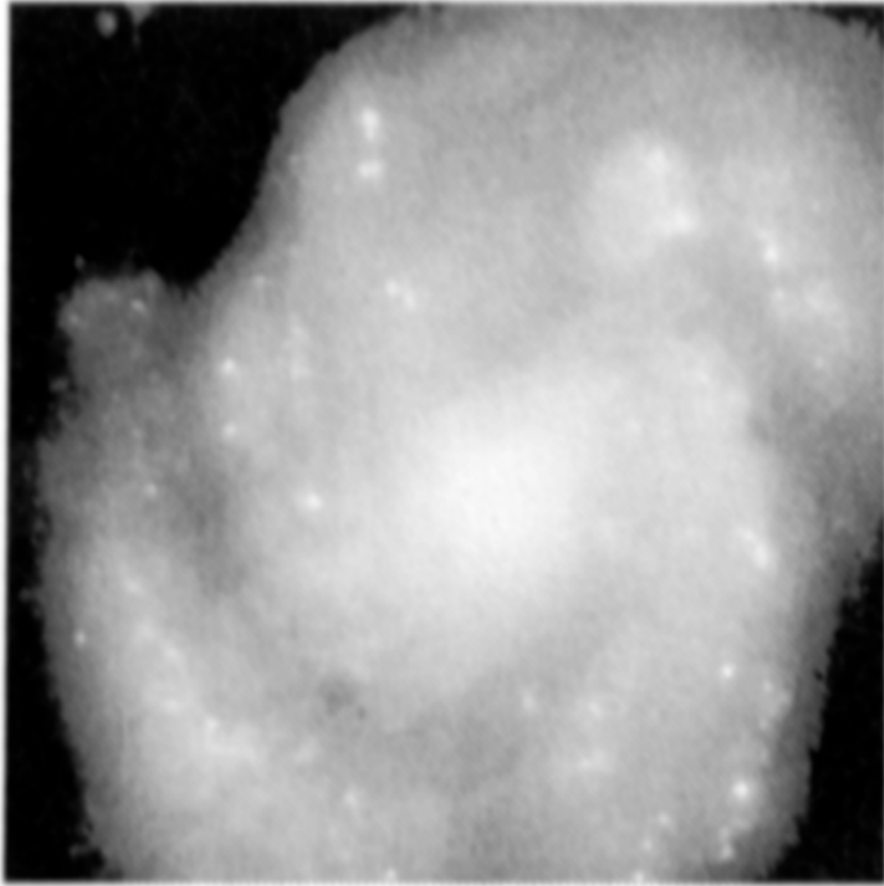
预计2025年建成的欧洲南方天文台的“欧洲特大天文望远镜” (E-ELT)直径达到 39m，其最小分辨角则为 $1.6 \times 10^{-8} \text{ rad}$ 。



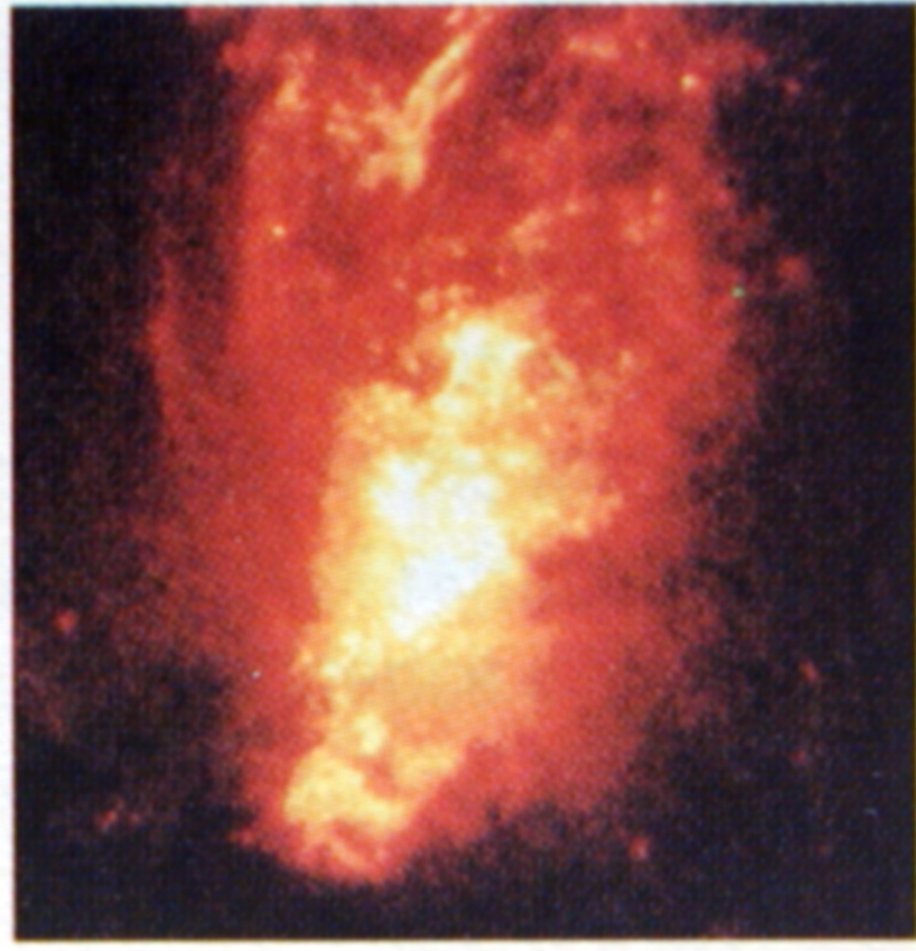
望远镜的实际分辨本领还与非衍射因素有关，如像差。



哈勃太空望远镜的像差



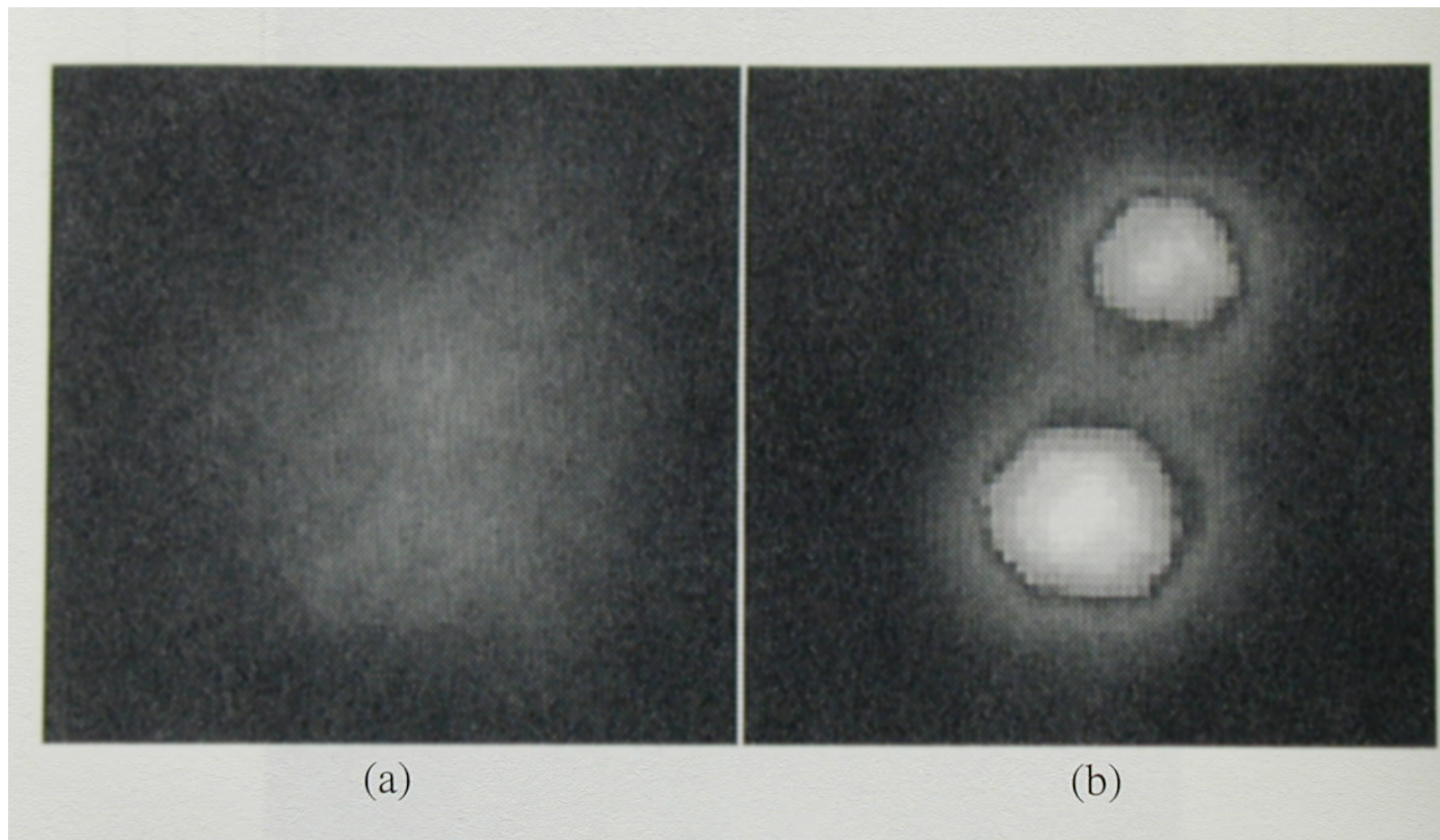
哈勃太空望远镜的像差修正前后拍的照片



哈勃太空望远镜的像差修正前后拍摄的照片

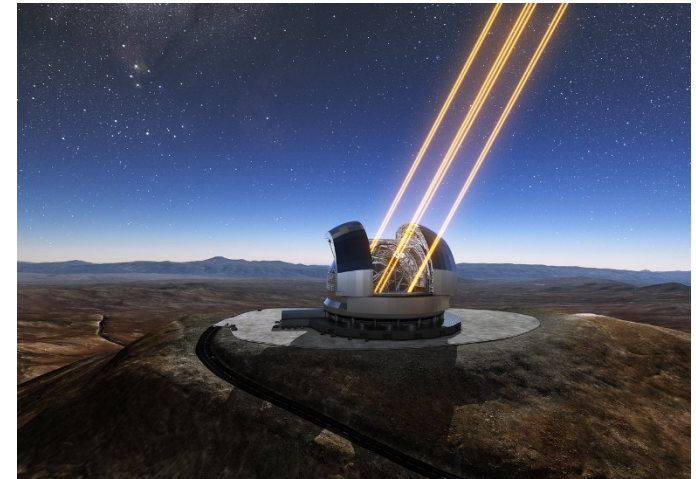
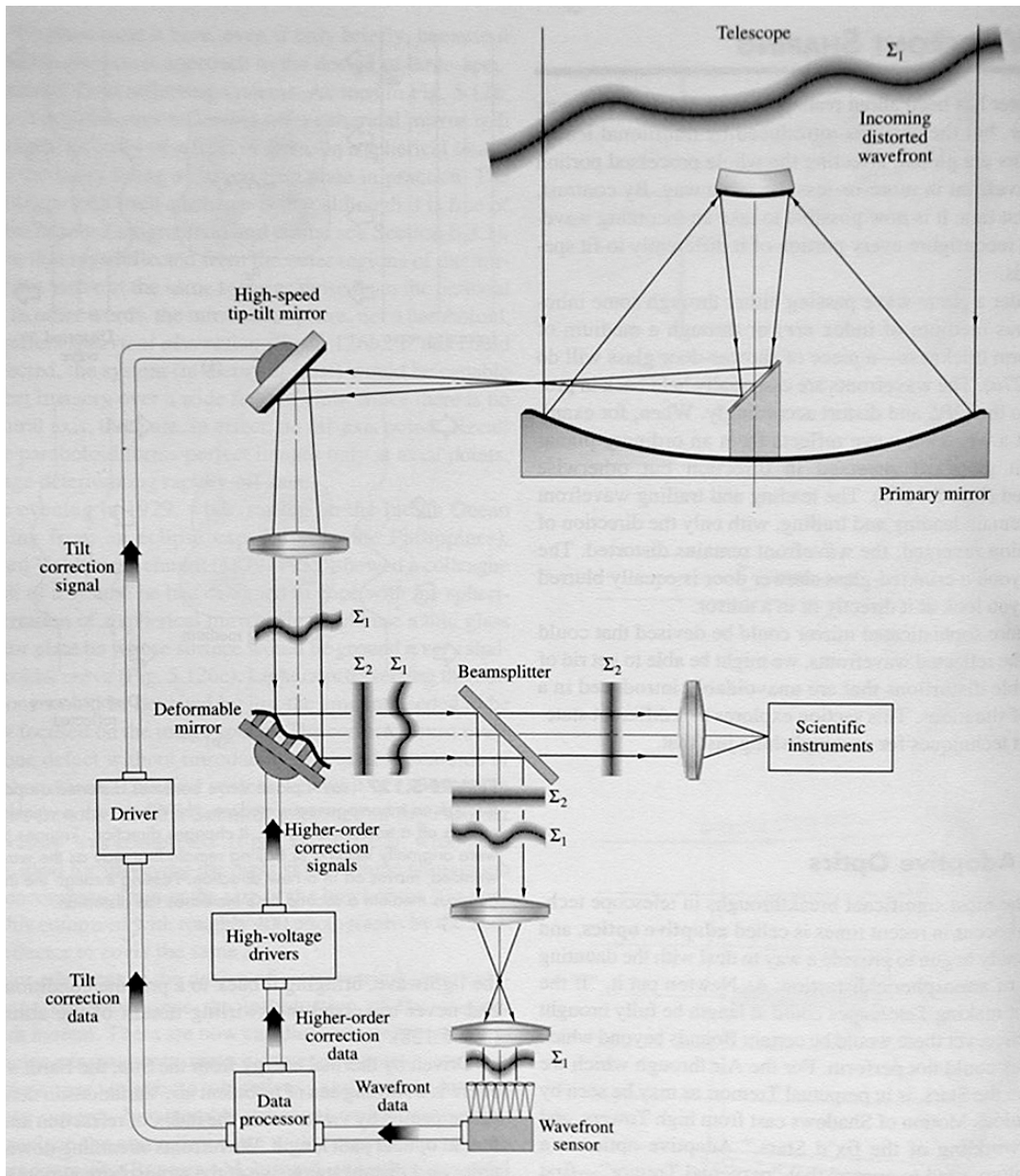


天文望远镜还受大气扰动的影响。



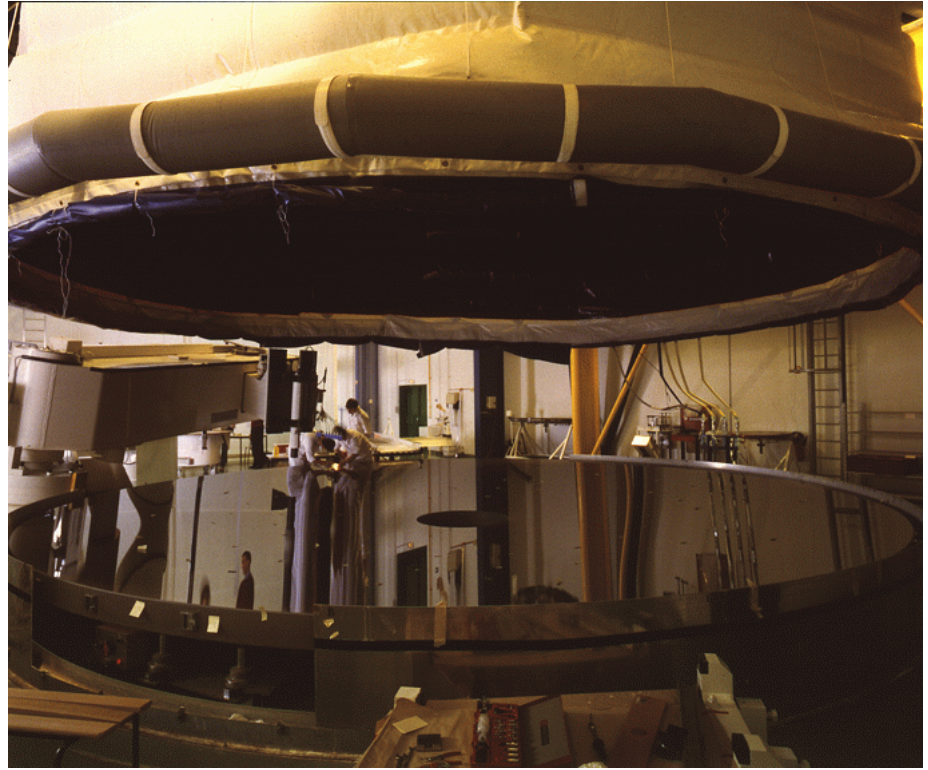
1.5m 直径天文望远镜拍摄的照片。

(a)未用自适应光学系统；(b)采用自适应光学系统



自适应天文望远镜及激光导星系统





欧洲南方天文台的 VLT 天文望远镜阵列和 VLT 天文望远镜的8.2 米直径的主反射镜。



## THE CRAB NEBULA

This impressive image of the Crab Nebula in the constellation Taurus (The Bull) was obtained on November 10, 1999, with the FORS2 (FOcal Reducer and Spectrograph) instrument mounted at the Cassegrain focus of the second 8.2-m VLT Unit Telescope (KUEYEM) at the ESO Paranal Observatory.

It is the remnant of a supernova explosion at a distance of about 6,000 light-years, observed almost 1000 years ago, in the year 1054. It contains a neutron star at its center that spins 30 times per second around its axis. This "pulsar" is an exceedingly dense object that represents an extreme state of matter - it weighs as much as the Sun, but measures only about 30 km across.

The different colours of this photo have different origins: the blue light is predominantly emitted by very high-energy electrons that are being accelerated in

a strong magnetic field. The green and red colours correspond to the emission by hydrogen atoms and sulphur ions, respectively.

This image of the Crab Nebula was obtained just a few days after FORS2 saw "First Light". The FORS project is carried out under ESO contract by a consortium of three German astronomical institutes: the Heidelberg State Observatory and the University Observatories of Göttingen and Munich.

FORS2, with its twin (FORS1 at VLT ANTU), is the product of one of the most thorough and advanced technological studies ever made of a ground-based astronomical instrument. It measures 3 x 1.5 metres and weighs 2.5 tonnes.

Puffing from the large mirror area and the excellent optical properties of the VLT

telescope, the FORS instruments have been designed to investigate a great variety of objects in the universe, including stars and nebulae in the Milky Way and some of the most distant galaxies. These powerful astronomical instruments are real workhorses for advanced astronomical studies.

Technical information: This photo of the Crab Nebula is based on a composite of three CCD images taken behind three different filters: B (wavelength: 429 nm; 5 min exposure; 0.80 arcsec seeing), R (657 nm; 1 min; 0.65 arcsec) and JS (H $\alpha$  656 nm; 5 min; 0.65 arcsec). The field measures about 6.8 x 6.5 arcmin. North is up, East is to the left.

More information about ESO can be found at URL: <http://www.eso.org>

## VLT 拍摄的 蟹状星云





## SPIRAL GALAXY NGC 1232

This spectacular image of the large spiral galaxy NGC 1232 was obtained on September 21, 1996, with the FORS1 (FOcal Reducer and Spectrograph) instrument mounted at the Cassegrain focus of the first 8.2-m VLT Telescope (UT1) at the ESO Paranal Observatory. The colours of the different regions are well visible: the central areas contain older stars of reddish colour, while the spiral arms are populated by young, blue stars and many star-forming regions.

NGC 1232 is located south of the celestial equator, in the constellation Eridanus (The River). The distance is about 100 million light-years, but the excellent optical quality of the VLT and FORS allows us to see an incredible wealth of details.

At the indicated distance, the edge of the field corresponds to about 170,000 light-years, or almost twice the size of the Milky Way galaxy.

The image of NGC 1232 was obtained just a few days after FORS1 saw "First Light". The FORS project is carried out under ESO contract by a consortium of three German astronomical institutes: the Heidelberg State Observatory and the University Observatories of Göttingen and Munich.

FORS1, with its future twin (FORS2), is the product of one of the most thorough and advanced technological studies ever made of a ground-based astronomical instrument. It measures 5 x 1.5 metres and weighs 2.3 tonnes.

Profiling from the large mirror area and the excellent optical properties of the UT1, FORS1 has been specifically designed to investigate the faintest and most remote objects in the universe. This powerful astronomical instrument is a real workhorse for the study of the distant universe.

Technical information: The photo of NGC 1232 is based on a composite of three images taken behind three different filters: U (360 nm; 10 min), B (420 nm; 6 min) and R (650 nm; 2:30 min) during a period of 0.7 arcsec seeing. The field measures about 5 x 7 arcmin. North is up; East is to the left.

More information about ESO can be found at URL: <http://www.eso.org>



# VLT 拍摄的 螺旋星系

# 显微镜及其分辨本领

## 1. 显微镜的基本结构与角放大率

显微镜（microscope）一般由较小孔径和焦距的物镜与较大孔径和焦距的目镜组成。

物镜将物镜前焦面附近的物以很高的横向放大倍数成像于目镜的前焦面。目镜起放大镜作用，把物镜成的像的视角放大。

原视角为

$$\alpha = \frac{y}{l_0}$$

经显微镜放大后的视角为

$$\alpha' = \frac{y'}{f_e} = -\frac{l}{f_o} \frac{y}{f_e}$$

其中  $l$  为镜筒长度。

显微镜的视角放大率为

$$M_M = \left| \frac{\alpha'}{\alpha} \right| = \frac{l}{f_o} \frac{l_0}{f_e} = M_o M_e$$



显微镜的焦距为

$$F = \frac{f_o f_e}{f_o + f_e - d}$$

其中  $d$  为物镜到目镜的距离。我们有

$$d = l + f_e$$

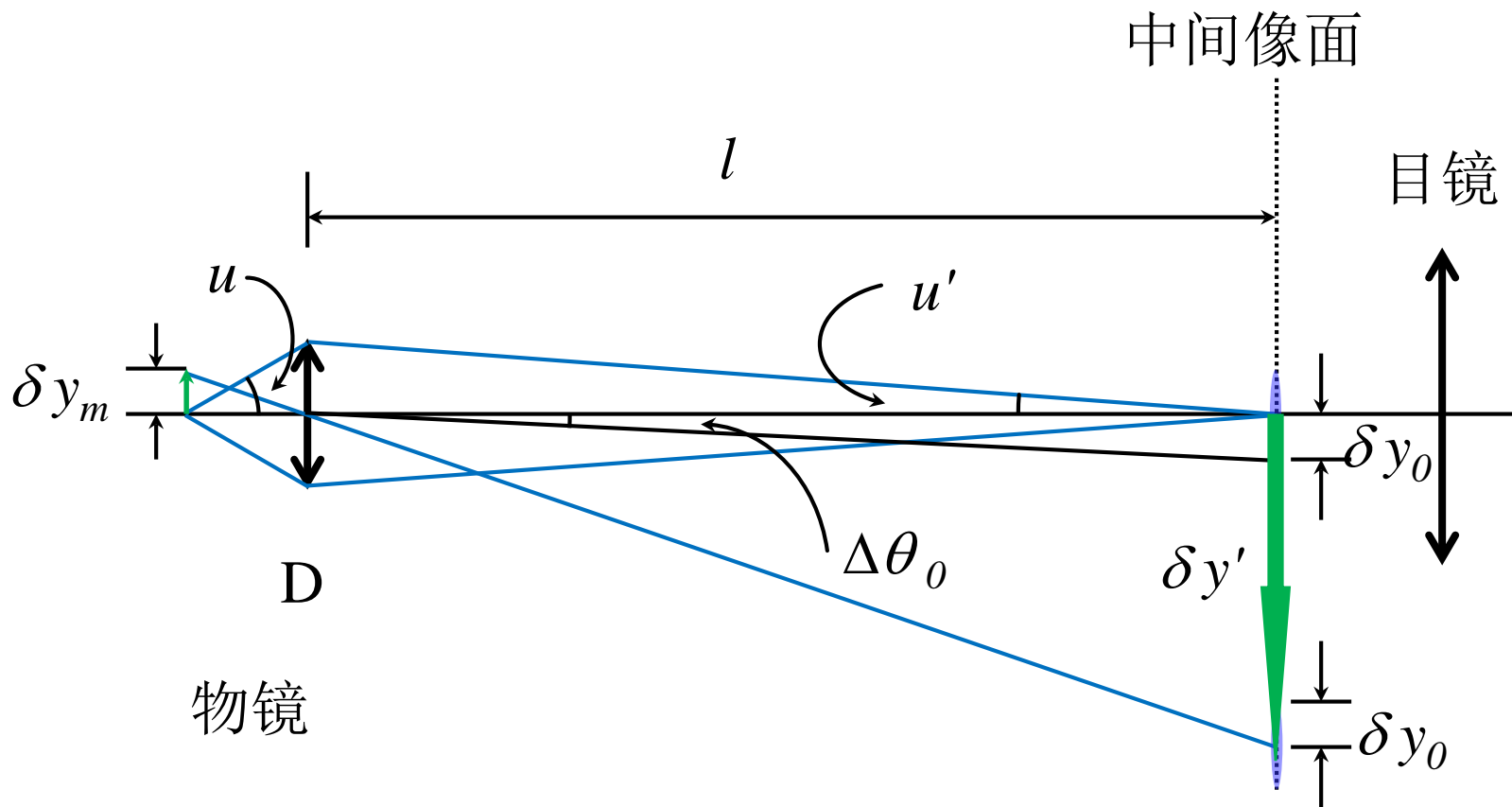
所以

$$F = \frac{f_o f_e}{f_o - l} \approx -\frac{f_o f_e}{l} = -\frac{l_0}{M_M}$$



老式和现代光学显微镜

## 2. 显微镜的最小分辨距离 $\delta y_m$



物上一点发出的球面光波经显微镜物镜衍射，在中间像面形成夫琅禾费衍射斑。

两个相距  $\delta y$  的物点在中间像面形成两个中心间距为  $\delta y'$ 、半径为  $\delta y_0$  的爱里斑。

采用瑞利判据， $\delta y' = \delta y_0$  为可分辨的极限，此时的  $\delta y$  即为显微镜的最小分辨距离  $\delta y_m$ 。

$$\delta y_0 = l\Delta\theta_0, \quad \Delta\theta_0 = 1.22 \frac{\lambda}{D}$$

所以

$$\delta y' = 1.22 \frac{l\lambda}{D} = 1.22 \frac{l\lambda_0}{Dn'}$$

$\delta y'$  与  $\delta y_m$  之间存在一定关系。

显微镜物镜成像为非傍轴成像，物点与像点为一对  
齐明点，满足阿贝 (Abbe) 正弦条件

$$n_0 \sin u \delta y_m = n' \sin u' \delta y'$$

像方满足傍轴条件：

$$\sin u' = \frac{D}{2l}$$

于是

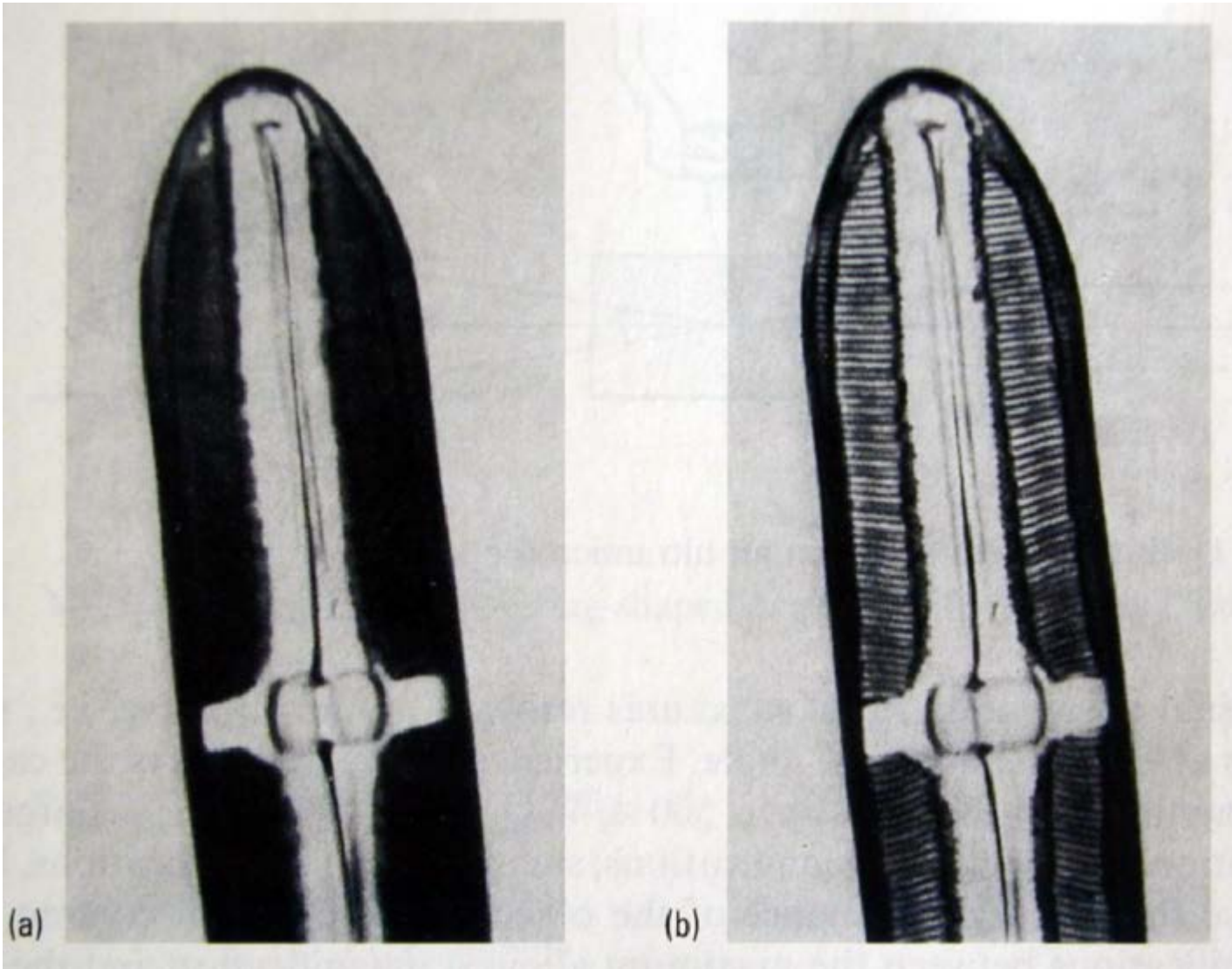
$$\delta y_m = \frac{n' \sin u'}{n_0 \sin u} \delta y' = \frac{n' \frac{D}{2l}}{n_0 \sin u} 1.22l \frac{\lambda_0}{n' D} = 0.61 \frac{\lambda_0}{n_0 \sin u}$$

或

$$\delta y_m = 0.61 \frac{\lambda_0}{NA}$$

$NA = n_0 \sin u$  是显微镜的数值孔径。

显然， $NA < n_0$ 。对于干镜头  $n_0 = 1$ ，最小分辨距离极限约为 400nm。对于油镜头，这一极限约为 200nm。



海藻的显微像，照明光波长为：(a) 680 nm；(b) 458 nm



物镜放大率的匹配：

$$M_o^{eff} = \frac{\delta y_e}{\delta y_m}$$

其中  $\delta y_e$  为接收装置在中间像面上的最小分辨距离。

如接收装置为人眼，则有

$$\delta y_e = f_e \delta \theta_e$$

那么匹配的角放大率为：

$$M_{eff} = M_o^{eff} M_e = \frac{f_e \delta \theta_e}{\delta y_m} \cdot \frac{l_0}{f_e} = \frac{\delta y'_e}{\delta y_m}$$

其中

$$\delta y'_e = l_0 \delta \theta_e$$

为人眼在明视距上的最小分辨间隔。

在明视距上，人眼的最小分辨间隔约为 0.075mm，干物镜显微镜在紫光波长的最小分辨距离极限约为 240nm，因此显微镜放大倍数应不大于 300。但瑞利判据不是绝对判据，且在较大放大倍数下观看更为舒适，所以实际上显微镜最大倍数一般做到 1000。

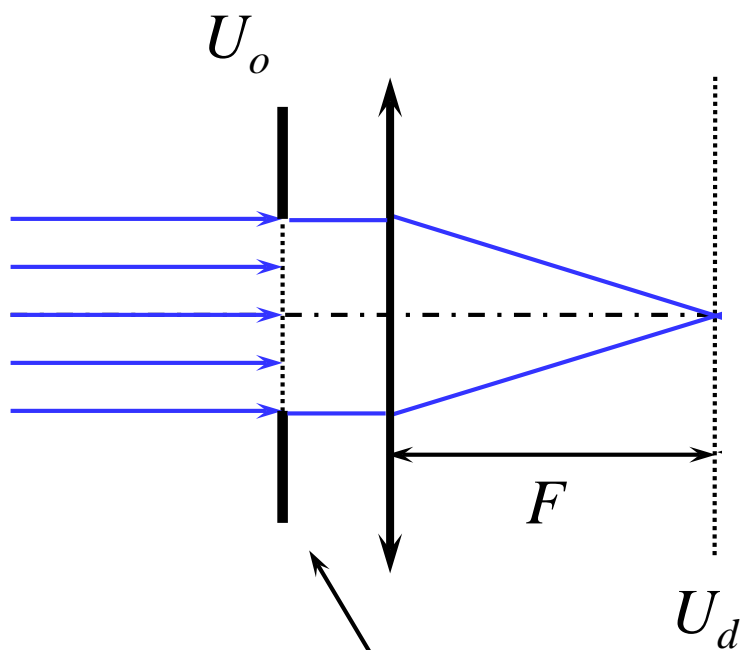
物镜放大倍数一般取 5、10、25、50、100，以方便利用测微目镜测长。而目镜的放大倍数一般取 5、10 或 15。

# 夫琅禾费衍射的基本特性

衍射屏的一些简单变化会引起夫琅禾费衍射场的简单变化。

1— 缩放特性：如光孔形状不变，尺度放大 $1/m$ 倍，则衍射图形不变，尺度放大 $m$ 倍。

衍射图形尺度放大 $m$ 倍



衍射屏尺度放大  $1/m$  倍

2— 平移特性：如衍射屏平移，则衍射场发生相移，光强分布不变。

衍射屏平移时屏上各点间的光程差并不发生改变，改变的只是屏上参照点的光程，因此衍射场发生只会改变一定的相位。

由

$$\begin{aligned}\Delta L &= x_0 \cos \alpha_x + y_0 \cos \alpha_y \\ &= x_0 \sin \theta_1 + y_0 \sin \theta_2\end{aligned}$$

可得，平移 $(x_0, y_0) \rightarrow (x_0 + a, y_0 + b)$ 引起的参照点的光程改变量为

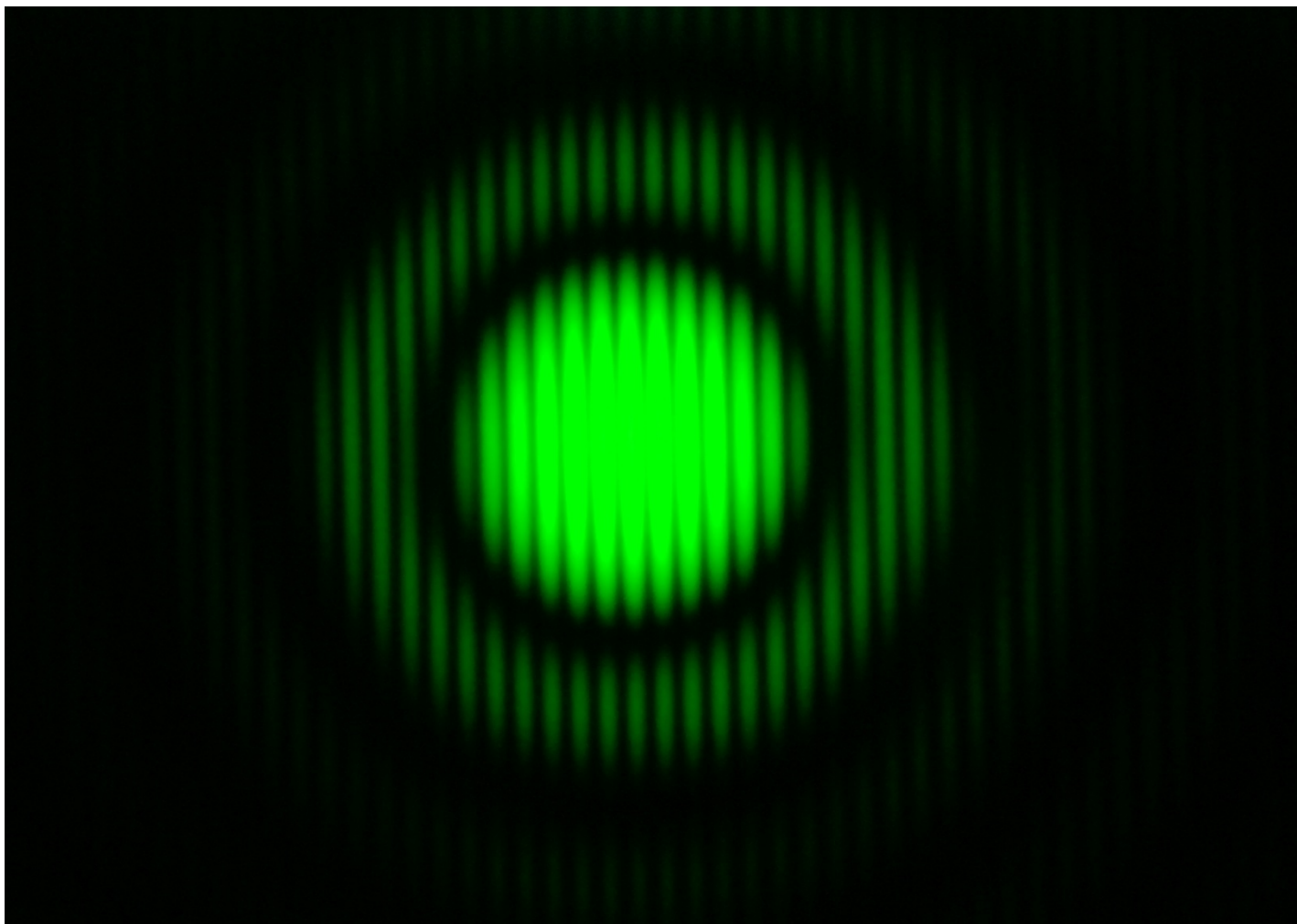
$$\Delta L_0 = -a \sin \theta_1 - b \sin \theta_2$$

而衍射场发生以下变化

$$U'_d(\theta_1, \theta_2) = e^{-ik(a \sin \theta_1 + b \sin \theta_2)} U_d(\theta_1, \theta_2)$$



# 双圆孔干涉图样



3— 旋转特性：如光孔旋转，则衍射场发生相应旋转。

夫琅禾费衍射接收装置具有旋转对称性，所以光孔旋转等效于整个系统旋转。