

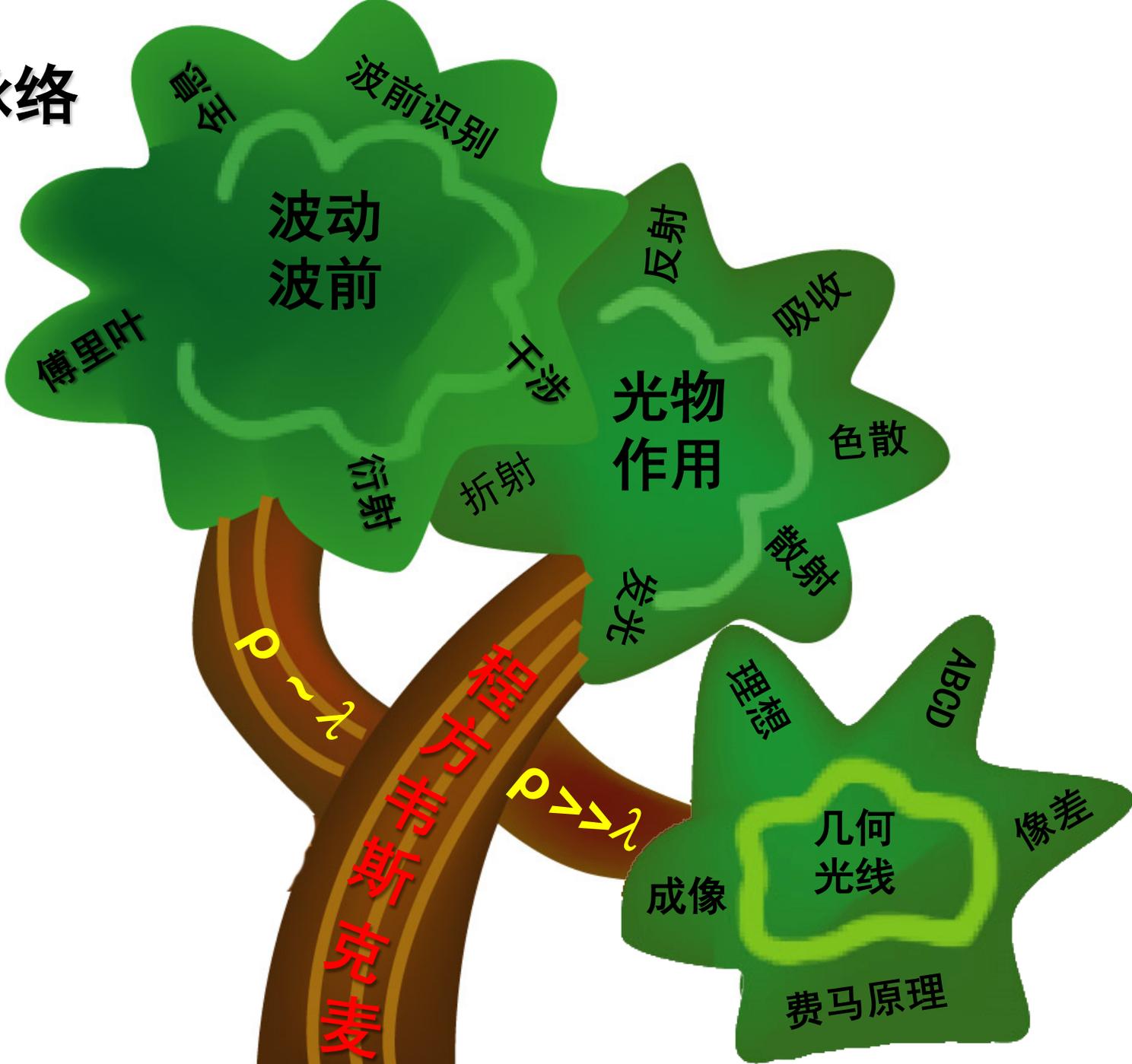
光波的复数描述

The background features a vibrant, multi-colored gradient with wavy, flowing lines. The colors transition from bright yellow and orange at the top left, through green and cyan, to deep blue and purple at the bottom right. The overall effect is dynamic and fluid, resembling a liquid or light wave pattern.

教学内容

1. 引言
2. 光波电磁理论和定态波描述
3. 界面光学（介质和金属）
4. 几何光学
5. 光波干涉和相干性
6. 衍射光学
7. 信息光学引论
8. 晶体光学
9. 吸收、色散和散射
10. 光源和光度学基本概念
11. 光量子性

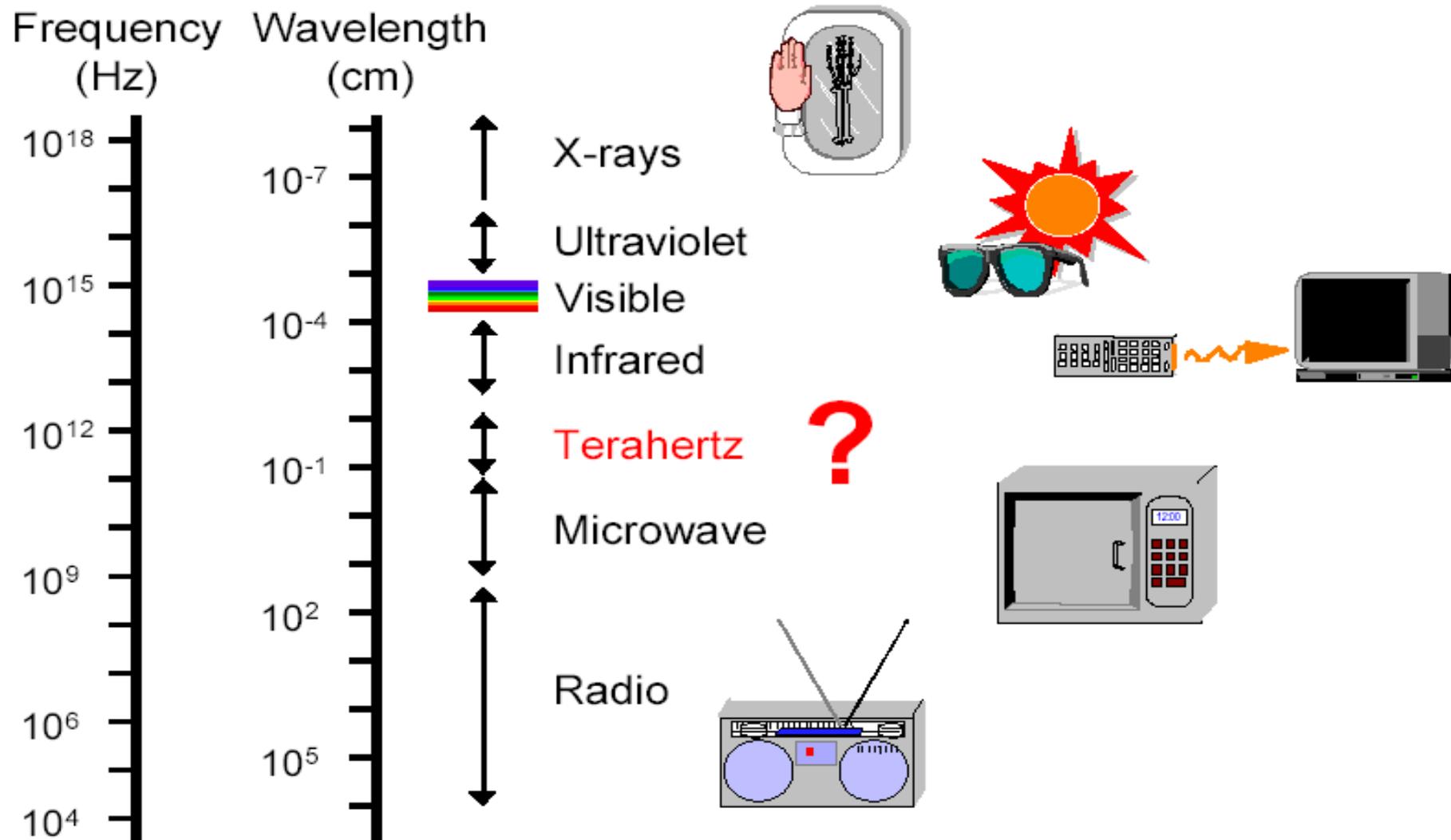
光学主脉络



第一节：光波的电磁理论

光波是特定波段的电磁波

可见光波长约为380-760nm，光频为 8×10^{14} - 4×10^{14} Hz。



麦克斯韦方程组：

麦克斯韦那个时代电磁场的基本规律可概括如下；

(1) 电场的高斯定理：

$$\oiint \vec{D} \cdot d\vec{S} = q_0$$

(2) 静电场的环路定理：

$$\oint \vec{E} \cdot d\vec{l} = 0$$

(3) 磁场的高斯定理

$$\oiint \vec{B} \cdot d\vec{S} = 0$$

(4) 安培环路定理

$$\oint \vec{H} \cdot d\vec{l} = I_0$$

(5) 法拉第电磁感应定律

$$\xi = -\frac{\partial \Phi_B}{\partial t}$$

麦克斯韦分析了各个定律的使用条件，特别是安培环路定理，发现将它应用到非恒定情形时遇到了矛盾。为了克服这一矛盾，他提出了最重要的假设——“位移电流”。总结了著名麦克斯韦方程组：

$$\left\{ \begin{array}{l} \oiint \vec{D} \cdot d\vec{S} = q_0 \\ \oint \vec{E} \cdot d\vec{l} = - \iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \\ \oiint \vec{B} \cdot d\vec{S} = 0 \\ \oint \vec{H} \cdot d\vec{l} = I_0 + \iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \end{array} \right.$$

位移电流

微分形式方程：

$$\left\{ \begin{array}{l} \nabla \cdot \vec{D} = \rho_{e0} \\ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} \end{array} \right.$$

其中：

$$\left\{ \begin{array}{l} \vec{D} = \vec{\varepsilon} \varepsilon_0 \cdot \vec{E} \\ \vec{B} = \vec{\mu} \mu_0 \cdot \vec{H} \\ \vec{j}_0 = \vec{\sigma} \cdot \vec{E} \end{array} \right.$$

这里 ε 、 μ 和 σ 分别是(相对)介电常量、(相对)磁导率和电导率。对于各向同性线性介质来，它们退化为标量，有：

$$\left\{ \begin{array}{l} \vec{D} = \varepsilon \varepsilon_0 \vec{E} \\ \vec{B} = \mu \mu_0 \vec{H} \\ \vec{j}_0 = \sigma \vec{E} \end{array} \right.$$

电磁波主要性质

自由空间的麦克斯韦方程, $\rho_{e0}=0, j_0=0$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{E} = 0 \\ \nabla \times \vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t} \\ \nabla \cdot \vec{H} = 0 \\ \nabla \times \vec{H} = \varepsilon\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{array} \right. \xrightarrow{\nabla \times (...)} \quad (2.1) \quad \left\{ \begin{array}{l} \nabla \times (\nabla \times \vec{E}) = -\mu\mu_0 \frac{\partial}{\partial t} \nabla \times \vec{H} \\ \nabla \times (\nabla \times \vec{H}) = \varepsilon_0 \frac{\partial}{\partial t} \nabla \times \varepsilon \vec{E} \end{array} \right.$$

$$\nabla = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right)$$

$$\nabla \cdot \vec{A} = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \cdot \{A_x, A_y, A_z\} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) i - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) j + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) k$$

由上式得：

$$\frac{\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}}{\left\{ \begin{array}{l} \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu\mu_0 \frac{\partial}{\partial t} \nabla \times \vec{H} \\ \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \varepsilon\varepsilon_0 \frac{\partial}{\partial t} \nabla \times \vec{E} \end{array} \right.}$$

麦克斯韦方程组

$$\left\{ \begin{array}{l} \nabla^2 \vec{E} - \varepsilon\varepsilon_0\mu\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \nabla^2 \vec{H} - \varepsilon\varepsilon_0\mu\mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \end{array} \right. \quad (2.2)$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$$

(2.2) 是标准的波动方程，表明了自由空间交变的电场和磁场的运动和变化具有波动形式，形成电磁波，其传播速度为：

$$\varepsilon\varepsilon_0\mu\mu_0 = \left(\frac{1}{v}\right)^2 \rightarrow v = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}}$$

真空中电磁波的速度：

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$\varepsilon_0 \approx 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2, \mu_0 = 4\pi \times 10^{-7} \text{ N} / \text{A}^2,$$

求得真空中电磁波传播速度为： $c \approx 3 \times 10^8 \text{ m} / \text{s}$,

和光速相同，再次证明光波就是电磁波。

根据折射率的定义：

$$n = \frac{c}{v} = \sqrt{\epsilon\mu}$$

上式为折射率的深层微观机理和性质。

问题： $n = \frac{c}{v} = \sqrt{\epsilon\mu}$ 一定大于0？？

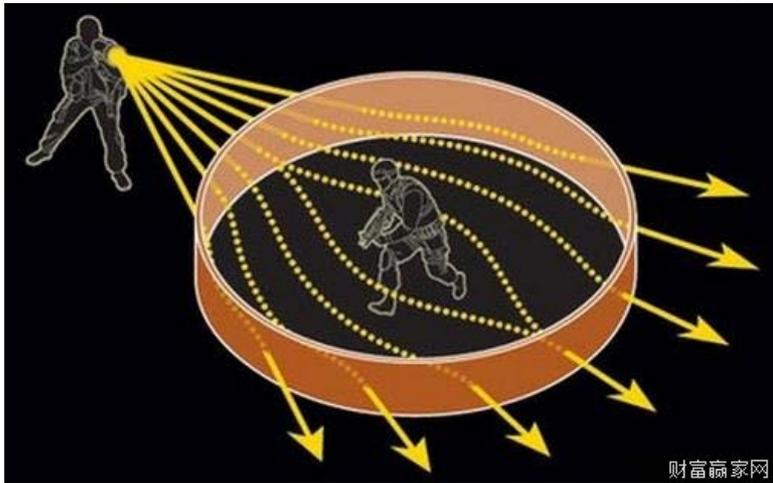
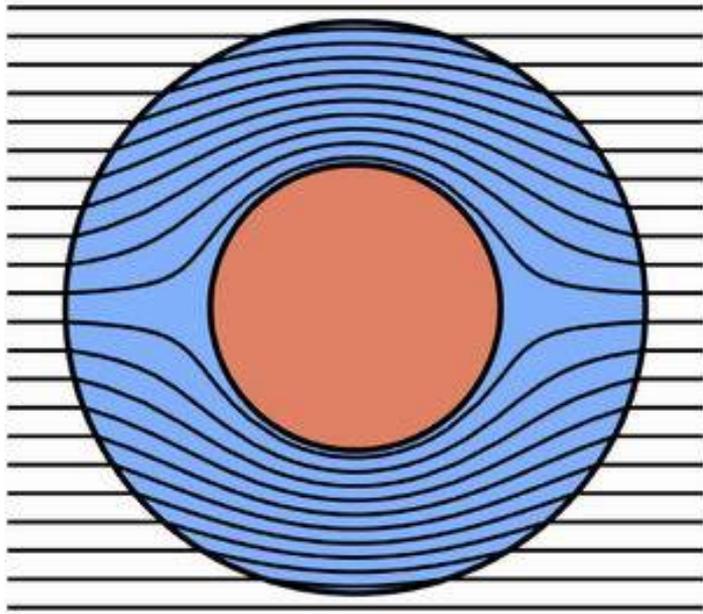
有没有 $n = \frac{c}{v} = -\sqrt{\epsilon\mu}$ 的情况？？

比如2003年美国《科学》十大新闻：多个研究小组证实，特定介质能使光和其他电磁波以负折射率偏转。这种所谓“左手”性物质有可能用于制造性能更好的透镜。



美国UCSD的NIM研究小组已经设计制成了具有负折射率的材料。这种材料是由铜质方形裂环振荡器和一条细铜线嵌在玻璃纤维的底板上形成的。铜质方形裂环振荡器（split ring resonator）和铜线分别嵌在底板的两面。（如图所示）。将用这样的材料制成的棱镜与用聚四氟乙烯（Teflon）制成的棱镜对比后发现，经两者折射的波偏离主轴的方向相反。由此证明了这种材料具有负折射率的性质。

隐形技术



transformation optics and metamaterials

nature materials | VOL 9 | MAY 2010

光波的性质：

平面简谐电磁波是自由空间电磁波的一基元成分。平面电磁波函数：

$$\vec{E}(r, t) = \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \varphi_E)$$

$$\vec{H}(r, t) = \vec{H}_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \varphi_H)$$

满足波动方程2.2

k 称为波矢，其方向与等相面的正交，即为波面的法线方向，其大小为：

$$k^2 = \varepsilon\varepsilon_0\mu\mu_0\omega^2 \Rightarrow k = \sqrt{\varepsilon\mu} \cdot \sqrt{\varepsilon_0\mu_0} \cdot \omega$$

$$k = \frac{n\omega}{c} \quad \Longrightarrow \quad k = \frac{2\pi}{\lambda}$$

(1) 光波是横波。

将平面电磁波函数代入 $\nabla \cdot \mathbf{E} = 0$

$$\begin{aligned}\nabla \cdot \vec{E}(r, t) &= \nabla \cdot \left(\vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \phi_E) \right) \\ &= \left(E_{0x} \cdot k_x + E_{0y} \cdot k_y + E_{0z} \cdot k_z \right) \sin(\omega t - \vec{k} \cdot \vec{r} + \phi_E) \\ &= \left(\vec{E}_0 \cdot \vec{k} \right) \sin(\omega t - \vec{k} \cdot \vec{r} + \phi_E) \equiv 0\end{aligned}$$

$$\therefore \vec{E}_0 \cdot \vec{k} = 0$$

将平面电磁波函数代入 $\nabla \cdot \mathbf{H} = 0$.

$$\begin{aligned}\nabla \cdot \vec{H}(r, t) &= \nabla \cdot \left(\vec{H}_0 \cos(\omega t - \vec{k} \cdot \vec{r} + \phi_H) \right) \\ &= \left(H_{0x} \cdot k_x + H_{0y} \cdot k_y + H_{0z} \cdot k_z \right) \sin(\omega t - \vec{k} \cdot \vec{r} + \phi_H) \\ &= \left(\vec{H}_0 \cdot \vec{k} \right) \sin(\omega t - \vec{k} \cdot \vec{r} + \phi_H) \equiv 0\end{aligned}$$

$$\therefore \vec{H}_0 \cdot \vec{k} = 0$$

$\mathbf{E} \perp \mathbf{k}$ 和 $\mathbf{H} \perp \mathbf{k}$ ，即电磁场振荡方向与波矢方向正交，在与波矢正交的横平面中振荡，即自由空间中光波为横波。

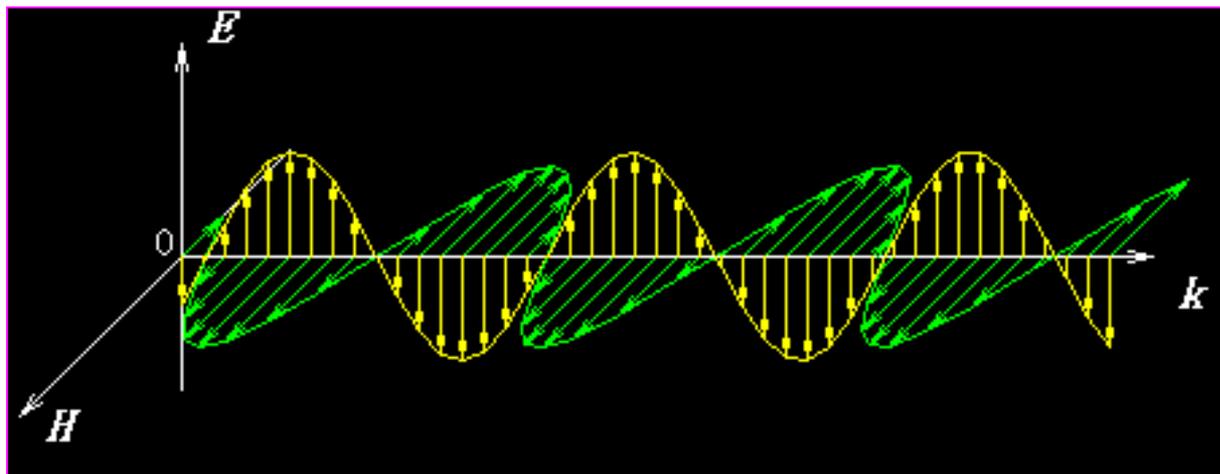
(2) 电场和磁场之间正交和同步。

将平面波函数代入： $\nabla \times \vec{E} = -\mu\mu_0 \frac{\partial \vec{H}}{\partial t}$

得：
$$\mu\mu_0 \vec{H} = \frac{1}{\omega} \vec{k} \times \vec{E}$$

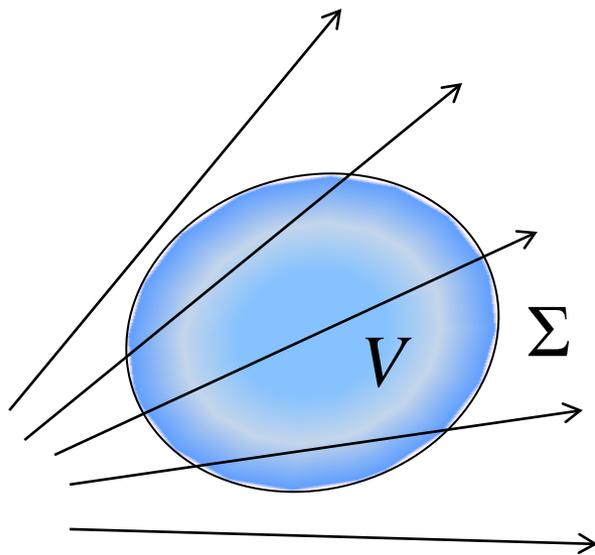
可知：

$$\vec{H} \perp \vec{E}, \quad \phi_H = \phi_E, \quad \sqrt{\mu\mu_0} H_0 = \sqrt{\varepsilon\varepsilon_0} E_0$$



(3) 光强

电磁波能流密度—坡印亭矢量 $\vec{S} = \vec{E} \times \vec{H}$



体积 V 内的电磁能为：

$$W = \frac{1}{2} \iiint_V (D \cdot E + B \cdot H) dV$$

在非恒定情况下，各场量随时间变化，体积V内的电磁能W也将随时间变化，其变化率为：

$$\frac{dW}{dt} = \frac{1}{2} \iiint_V \frac{\partial}{\partial t} (D \cdot E + B \cdot H) dV \xrightarrow{D=\epsilon\epsilon_0 E, B=\mu\mu_0 H}$$

$$= \iiint_V \left(E \cdot \frac{\partial D}{\partial t} + H \cdot \frac{\partial B}{\partial t} \right) dV$$

自由空间： $\nabla \times H = \frac{\partial D}{\partial t}, \quad \nabla \times E = -\frac{\partial B}{\partial t}$

$$E \cdot \frac{\partial D}{\partial t} + H \cdot \frac{\partial B}{\partial t} = E \cdot \nabla \times H - H \cdot \nabla \times E$$

由 $\nabla \cdot (A \times B) = B \cdot \nabla \times A - A \cdot \nabla \times B$ 得:

$$\begin{aligned} E \cdot \frac{\partial D}{\partial t} + H \cdot \frac{\partial B}{\partial t} &= E \cdot \nabla \times H - H \cdot \nabla \times E \\ &= -\nabla \cdot (E \times H) \end{aligned}$$

所以:

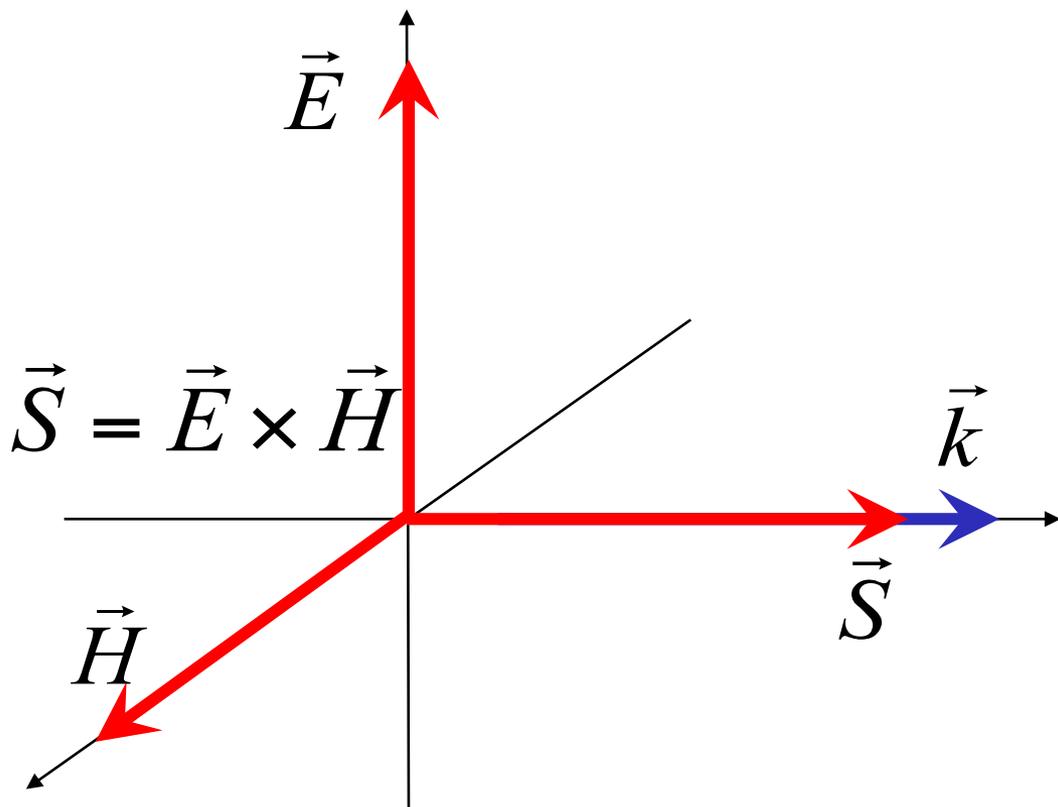
$$\frac{dW}{dt} = -\iiint_V \nabla \cdot (E \times H) dV = -\oiint_{\Sigma} (E \times H) \cdot d\Sigma$$

定义一个新的矢量—坡印亭矢量: $\vec{S} = \vec{E} \times \vec{H}$

其单位为: W / m^2 , $\vec{S} // \vec{k}$

$$\frac{dW}{dt} = -\oiint_{\Sigma} \mathbf{S} \cdot d\mathbf{\Sigma}$$

从能量守恒来看， \mathbf{S} 的大小代表单位时间流过与之垂直的单位面积的电磁能量，其方向代表电磁能传递的方向，即 \mathbf{S} 是**电磁能流密度矢量**。



光强—平均电磁能流密度

$$\begin{aligned} I = \bar{S} &= \frac{1}{T} \left| \int_0^T \vec{E} \times \vec{H} dt \right| = \frac{1}{T} \int_0^T |\vec{E}| |\vec{H}| dt \\ &= \frac{1}{2} E_0 H_0 = \frac{1}{2} \sqrt{\frac{\epsilon \epsilon_0}{\mu \mu_0}} E_0^2 \end{aligned}$$

在可见光波段，介质分子的磁化机构几乎冻结，

所以： $\mu \approx 1$ ， $n \approx \sqrt{\varepsilon}$

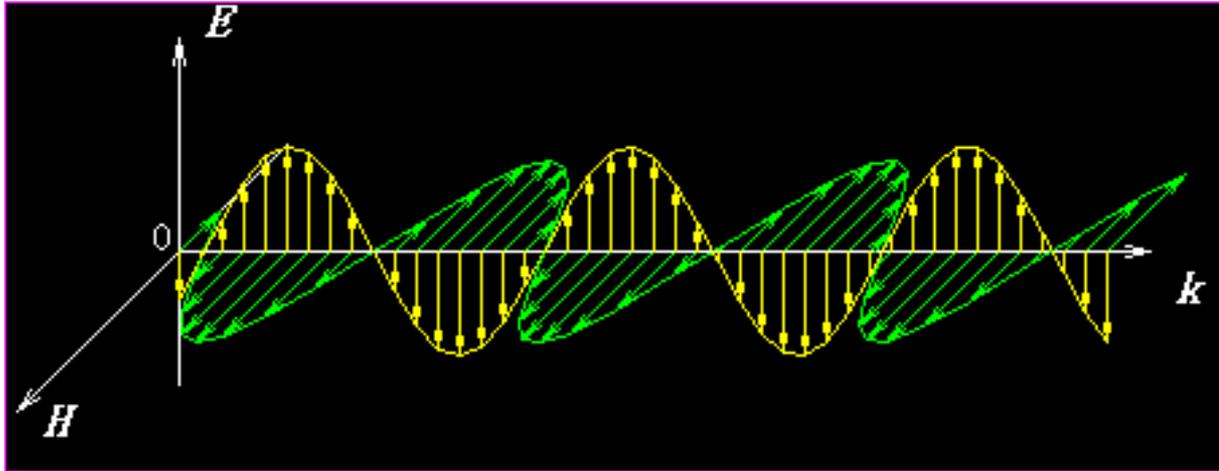
于是光强表示为：

$$I = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} n E_0^2 \propto n E_0^2$$

或写成：

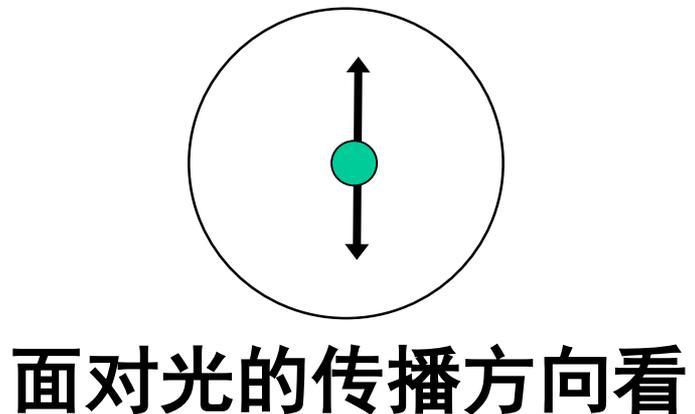
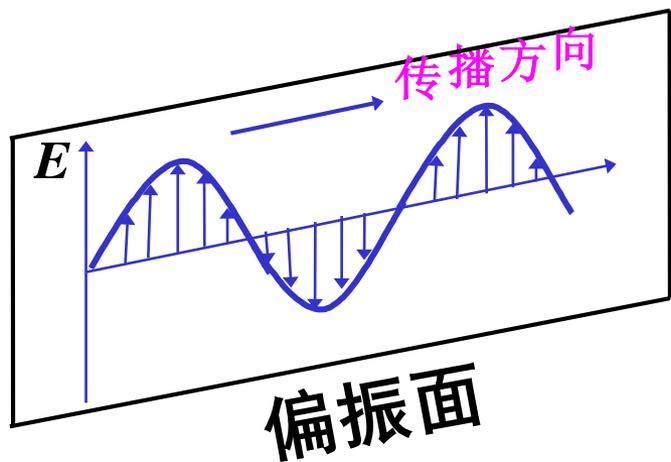
$$I = n E_0^2$$

(4) 光的偏振面和光的偏振态

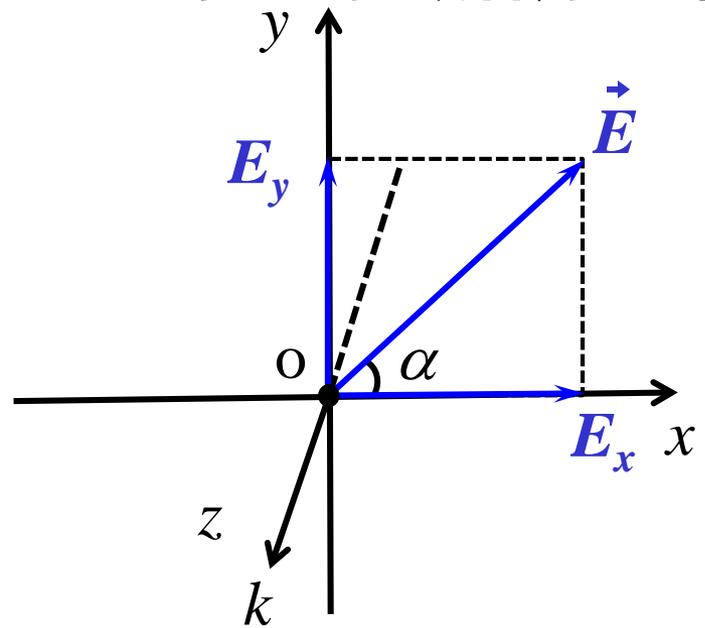


E 和 k 组成的平面成为偏振面。

(4.1)、线偏振光



面对光的传播方向看



线偏振光可分解成同向、同频、偏振面相互垂直的两线偏振光的叠加。

$$\begin{cases} E_x = E_{0x} \cos[(\omega t - \vec{k} \cdot \vec{r}) + \pi] \\ E_y = E_{0y} \cos(\omega t - \vec{k} \cdot \vec{r}) \end{cases}$$

$$\begin{cases} E_{0x} = E_0 \cos \alpha \\ E_{0y} = E_0 \sin \alpha \end{cases}$$

线偏振光的表示法：



光振动垂直板面



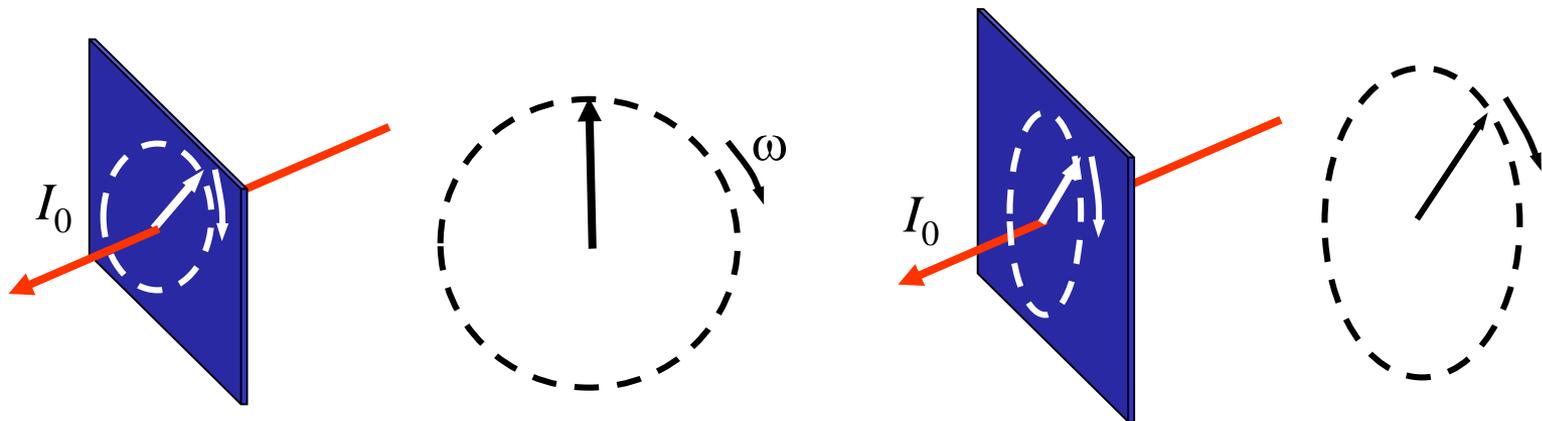
光振动平行板面

(4. 2). 圆偏振光和椭圆偏振光

在垂直于光的传播方向的平面内，光矢量以一定的频率旋转。

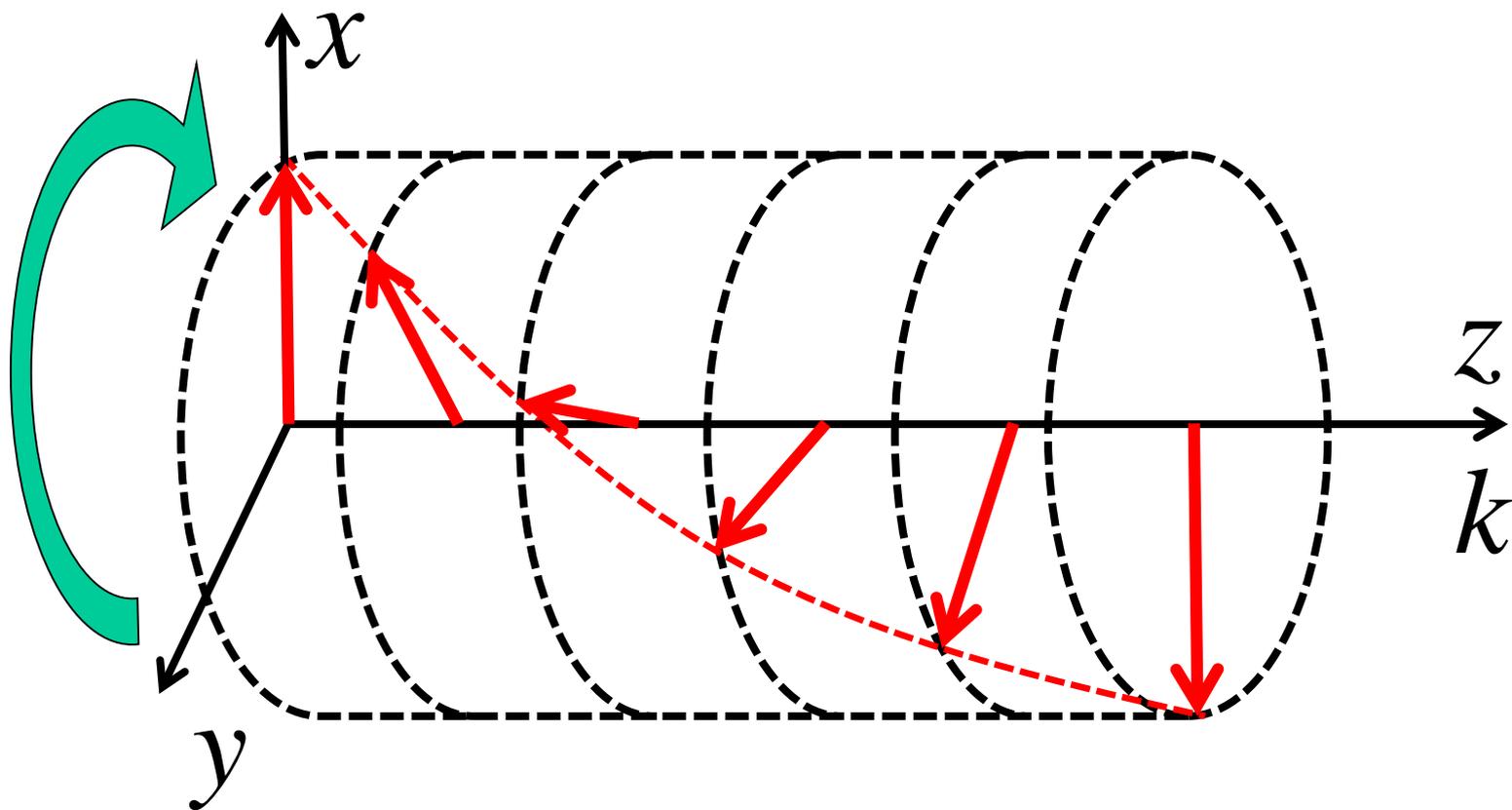
圆偏振光： 矢量端点轨迹为圆

椭圆偏振光： 矢量端点轨迹为椭圆



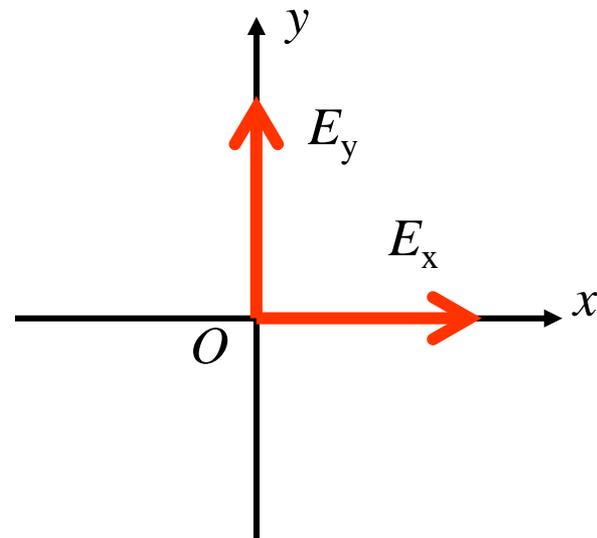
空间某一点的右旋圆和椭圆偏振光

同一时刻，空间不同点，圆偏振光电场方向的变化



某时刻右旋圆偏振光 \vec{E} 随 z 的变化

圆偏振光和椭圆偏振光可以看成是两个同频、同向、振动方向相互垂直，并且有稳定的相位关系的线偏振光合成的结果。即任何一个圆偏振光和椭圆偏振光可以分解成两个同频，振动方向相互垂直，并且有稳定的相位关系的线偏振光。



$$\begin{cases} E_x = E_{0x} \cos(\omega t - \vec{k} \cdot \vec{r}) \\ E_y = E_{0y} \cos(\omega t - \vec{k} \cdot \vec{r} + \Delta\phi) \end{cases} \quad \text{当: } E_{0x} \neq E_{0y}$$

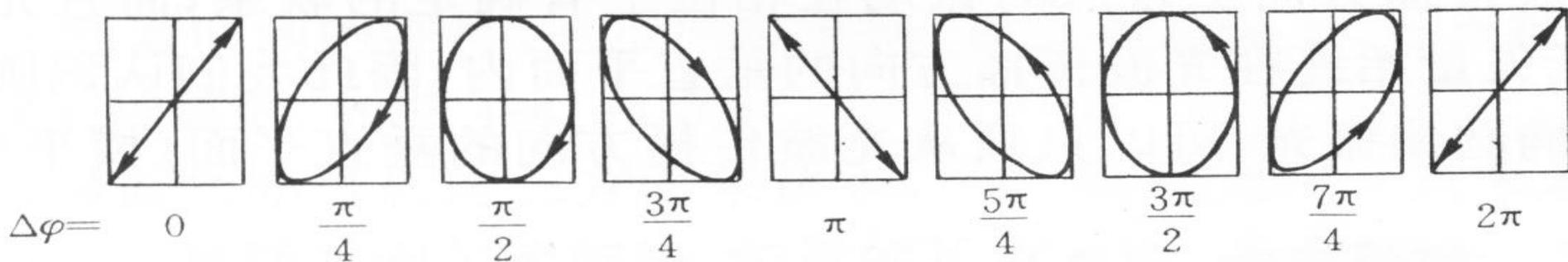
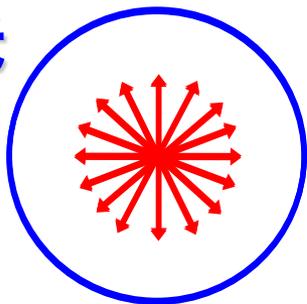
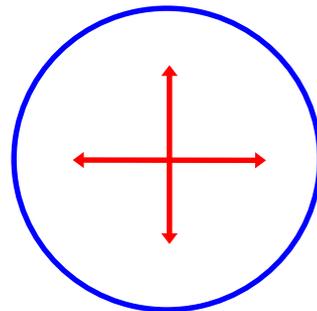


图 4-5 两个互相垂直的简谐振动合成椭圆振动

(4.3). 自然光



没有优势方向



自然光的分解

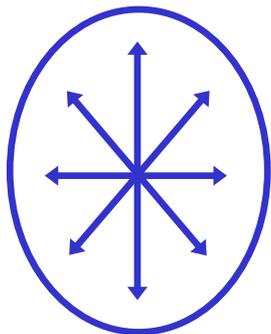
一束自然光可分解为两束同频、同向、振动方向相互垂直的、等幅的、无固定相位关系的线偏振光。

$$\begin{cases} E_x = E_{0x} \cos(\omega t - \vec{k} \cdot \vec{r}) \\ E_y = E_{0y} \cos(\omega t - \vec{k} \cdot \vec{r} + \Delta\phi) \end{cases} \quad \Delta\phi \text{ 随机变化}$$

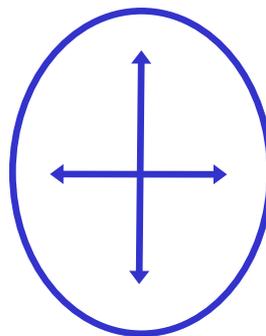
$$I = I_x + I_y \quad E_{0x} = E_{0y} \quad I_x = I_y$$

自然光的表示法：

(4.4). 部分偏振光



部分偏振光



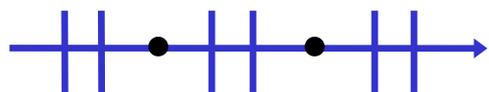
部分偏振光的分解

部分偏振光可分解为两束振动方向相互垂直的、不等幅的、无固定相位关系线偏振光。

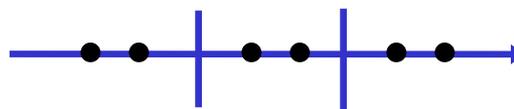
$$\begin{cases} E_x = E_{0x} \cos(\omega t - \vec{k} \cdot \vec{r}) \\ E_y = E_{0y} \cos(\omega t - \vec{k} \cdot \vec{r} + \Delta\phi) \end{cases} \quad \Delta\phi \text{ 随机变化}$$

$$\mathbf{I} = \mathbf{I}_x + \mathbf{I}_y \quad E_{0x} \neq E_{0y} \quad I_x \neq I_y$$

部分偏振光的表示法：



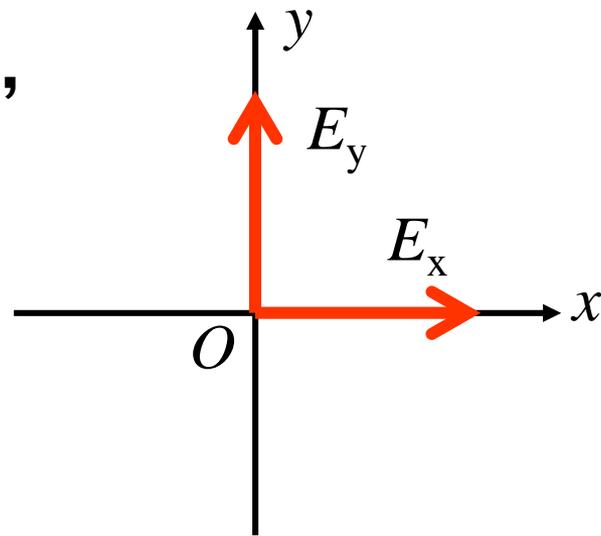
平行板面的光振动较强



垂直板面的光振动较强

任何一种偏振态可以分解成两个同频，振动方向相互垂直的线偏振光，相位关系和 x 、 y 轴的分量大小决定了这种光的偏振态。

$$\begin{cases} E_x = E_{0x} \cos(\omega t - \vec{k} \cdot \vec{r}) \\ E_y = E_{0y} \cos(\omega t - \vec{k} \cdot \vec{r} + \Delta\phi) \end{cases}$$



$\Delta\phi$: 0 、 π 为线偏振光

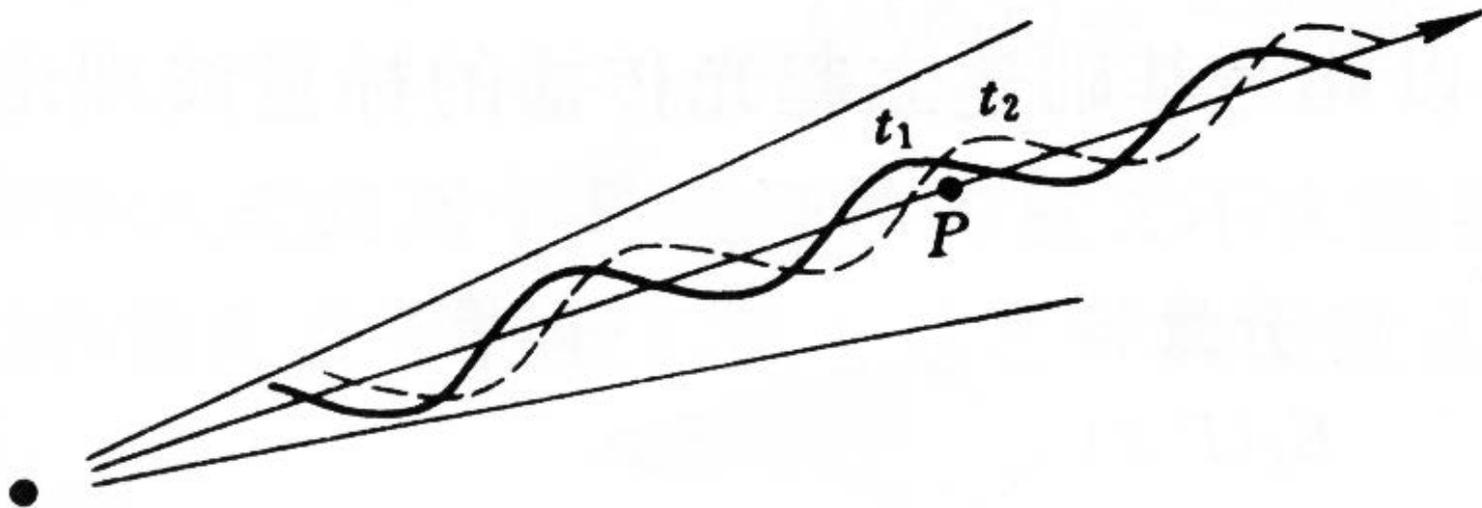
$\pi/2$ 、 $3\pi/2$ ， $E_{0x}=E_{0y}$ ，圆偏振， $E_{0x}\neq E_{0y}$ 椭圆偏振

确定的任意值，椭圆偏振

随机值， $E_{0x}=E_{0y}$ ，自然光， $E_{0x}\neq E_{0y}$ 部分偏振

第二节、定态光波 复振幅描述

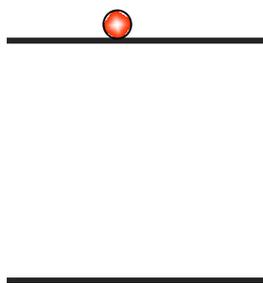
定态光波



(1) 空间各点的扰动是同频率的简谐振荡（频率与振源相同）；

(2) 波场中各点扰动的振幅不随时间变化，在空间形成一个稳定的振幅分布。严格的定态光波要求波列无限长。

一般自发辐射的光是否可以看成定态波？

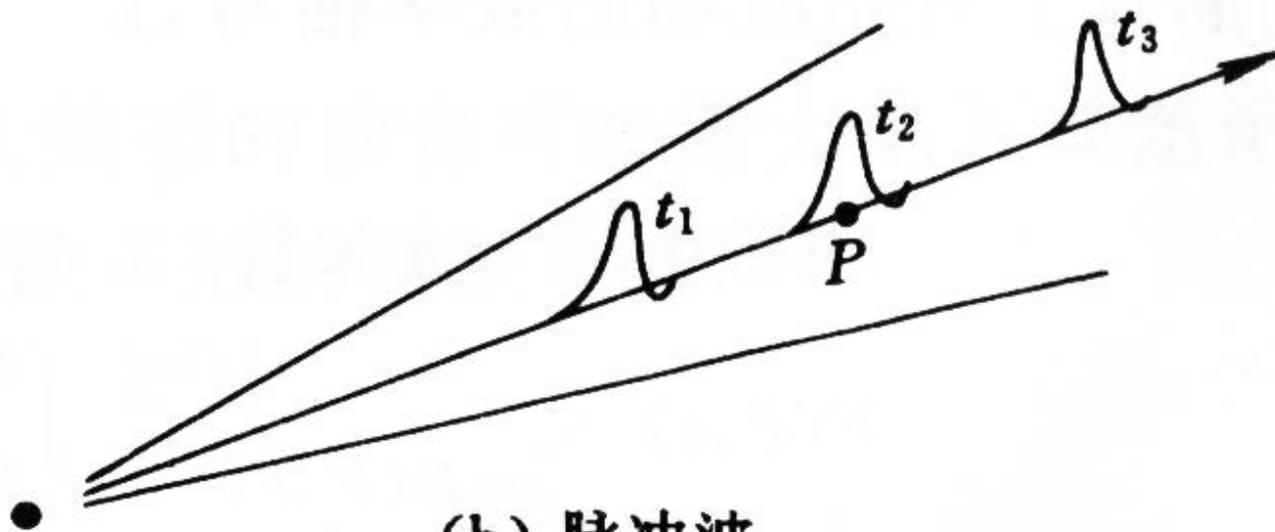


一次发光的持续时间： $\tau \sim 10^{-8} s$ ，可见光的周期：

$T \sim 10^{-14} s = 10 fs$ ，一次发光波列包含 $\frac{\tau}{T} \sim 10^6$ ，

可以看成定态波。

脉冲光波



(b) 脉冲波

超短脉冲激光ps, fs, as激光, 在生物、化学、物理等领域有广泛的应用。

Freezing Time: Six Millennia of History—to the Femtosecond



4000 BC

Calendar

(year / day)



1500 BC

Sundial

(hour)



1500 AD

Mechanical Clock

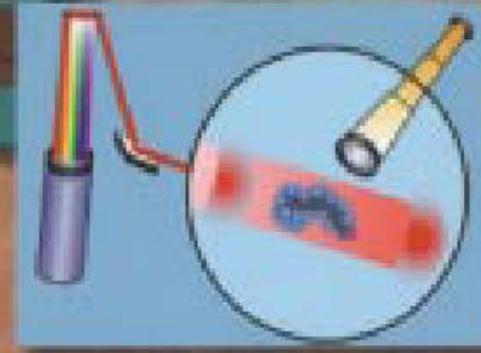
(minute / second)



1880

Fast Photography

(thousandth of a second)



1980

Laser Femtoscopes

(millionth of a billionth
of a second)

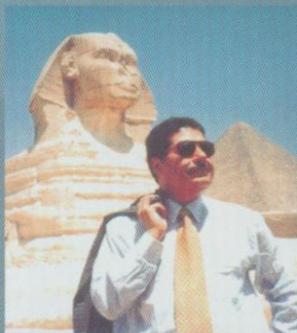
The decisive moments in the life of molecules

Ahmed Zewail receives the 1999 Nobel Prize in Chemistry for being the first to reveal the decisive moments of a chemical reaction – the moments when chemical bonds are broken and formed.

Zewail's technique uses what can be thought of as the world's fastest camera. The "shutter speed" of such a camera must be extremely high since molecules are very small (about 10^{-9} m) and move extremely rapidly (1 000 m/s). To obtain a sharp "image" of the molecules in the course of a chemical reaction requires a femtosecond (10^{-15} s) shutter speed.

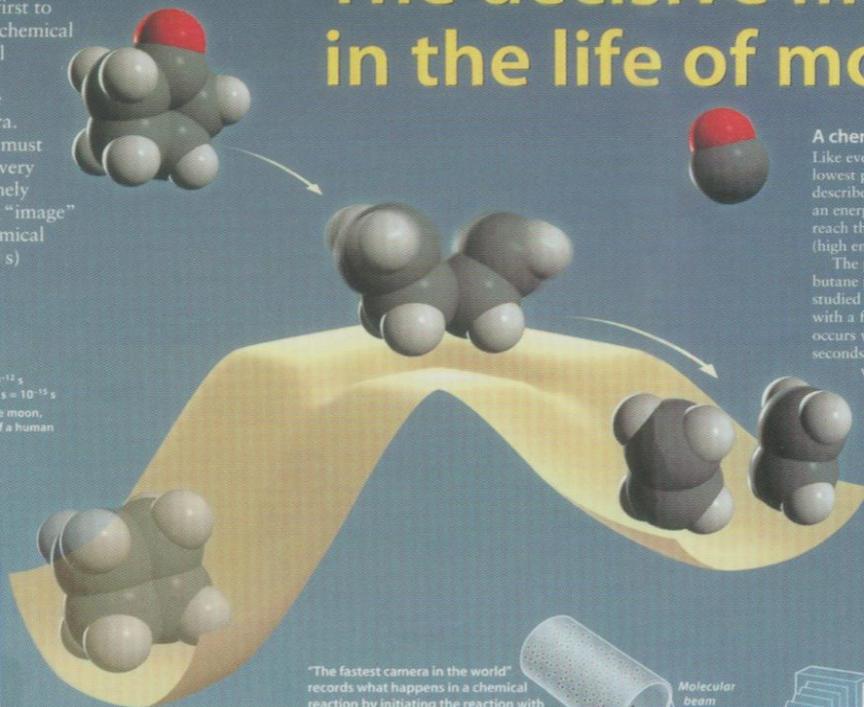
1 ms 1 millisecond = $0.001 \text{ s} = 10^{-3} \text{ s}$
 1 μs 1 microsecond = $0.000\,001 \text{ s} = 10^{-6} \text{ s}$
 1 ns 1 nanosecond = $0.000\,000\,001 \text{ s} = 10^{-9} \text{ s}$
 1 ps 1 picosecond = $0.000\,000\,000\,001 \text{ s} = 10^{-12} \text{ s}$
 1 fs 1 femtosecond = $0.000\,000\,000\,000\,001 \text{ s} = 10^{-15} \text{ s}$

In one second light travels from the earth to the moon, while in one femtosecond it travels a fraction of a human hair's-breadth.



Zewail – King of Femtoland

Ahmed H. Zewail was born near Alexandria in Egypt. He has now been working for many years at Caltech, Pasadena, USA, where he directs a large Laser Femtochemistry laboratory, called Femtoland. He is also Director of the Laboratory for Molecular Sciences (LMS).



A chemical reaction – up hill and down dale

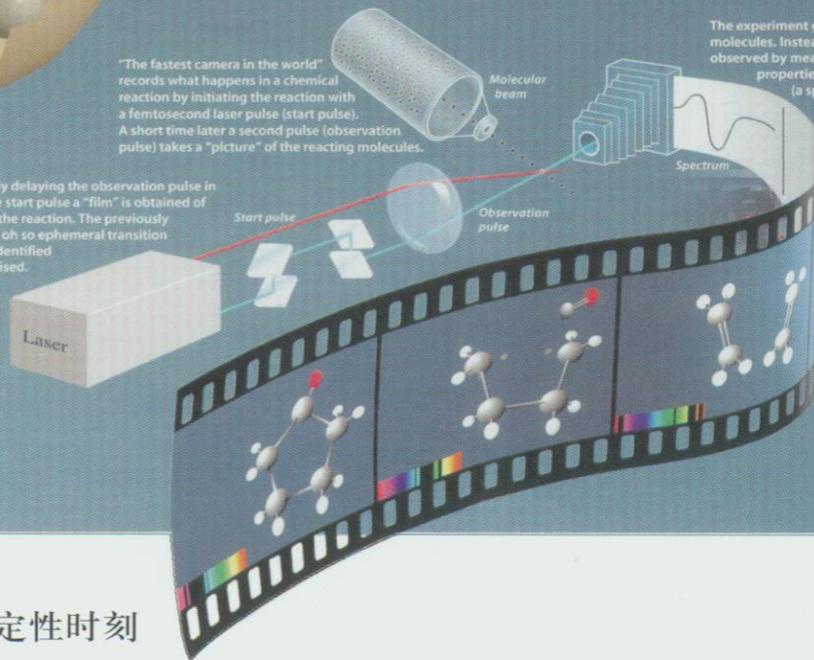
Like everything in nature, molecules strive to reach the lowest possible energy state. This makes it practical to describe reactions using energy surfaces. A molecule on an energy surface tries, like a child in a water-slide, to reach the lowest point. You need enough speed (high energy) to get up over the crests.

The picture to the left shows the ring opening of a cyclopentane molecule to form two ethylene molecules. Zewail studied this reaction by exciting cyclopentanone molecules with a femtosecond pulse. He could show that this reaction occurs via a transition state living a few hundred femtoseconds. This experiment settled an old argument over whether the reaction takes place in one step with simultaneous breaking of both bonds or in two steps, one bond breaking before the other.

We need to know the properties of the transition state if we are to understand, predict and perhaps modify the course of a reaction. For almost a hundred years the transition state remained a hypothetical species that few chemists believed could ever be observed. But this is precisely what Zewail has succeeded in doing.

"The fastest camera in the world" records what happens in a chemical reaction by initiating the reaction with a femtosecond laser pulse (start pulse). A short time later a second pulse (observation pulse) takes a "picture" of the reacting molecules.

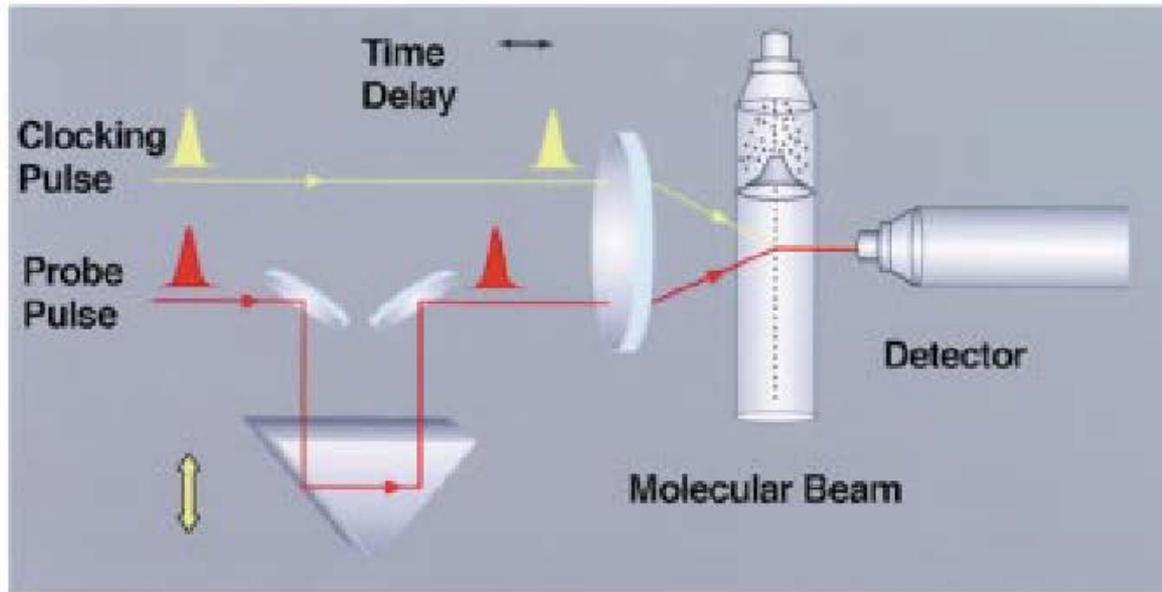
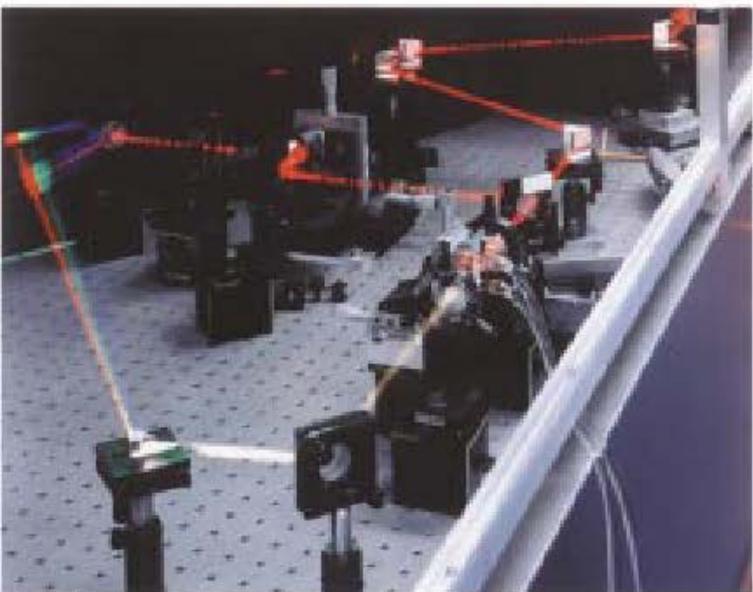
By successively delaying the observation pulse in relation to the start pulse a "film" is obtained of the course of the reaction. The previously predicted but oh so ephemeral transition state can be identified and characterised.



The experiment gives no direct image of the molecules. Instead, the reacting molecules are observed by measuring certain characteristic properties, e.g. an optical property (a spectrum is obtained) or by recording the molecular masses (mass spectrometry).



The picture shows part of Zewail's "camera". It is a complex array of lasers, mirrors, lenses, prisms, molecular beams, detection equipment and more.



飞秒 (10^{-15} S) ---- 万亿分之一秒 (超短脉冲)

超快 (3.9fs) ---- 超快测量手段

超强 (100TW)

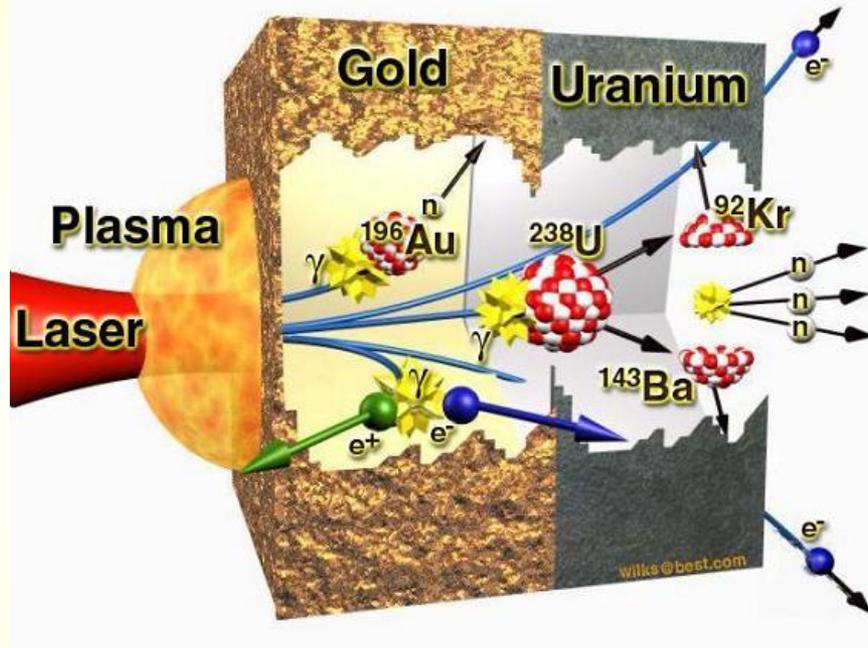
自然界存在

| | | |
|------|-------------------------------|--|
| 聚焦强度 | $10^{20}\text{W}/\text{cm}^2$ | $3.5 \times 10^{16}\text{W}/\text{cm}^2$ |
| 光 压 | 10^{12}bar | 1bar |
| 加速度 | 10^{21}g | g |
| 磁 场 | 10^9Gauss | 0.5Gauss |

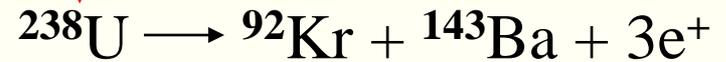


提供了极端实验条件

1、Fs Laser-Induced Fission



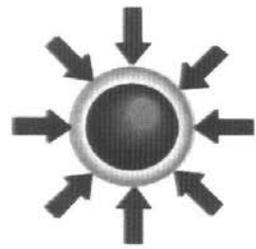
高能电子



- A petawatt laser beam ($10^{20}\text{W}/\text{cm}^2$) strikes a gold foil results in*
- (1) the highest energy electrons (up to 100MeV)*
 - (2) the first laser-induced fission*
 - (3) the first creation of antimatter (positrons) using light*

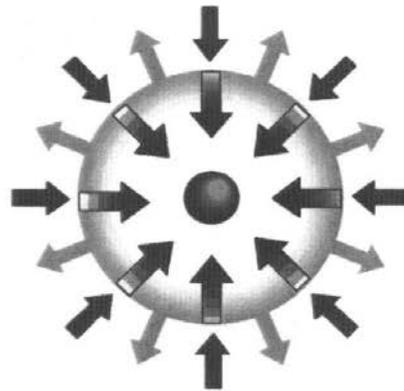
2、 Fast Ignition Concept of ICF

(ICF --- inertial confinement fusion)



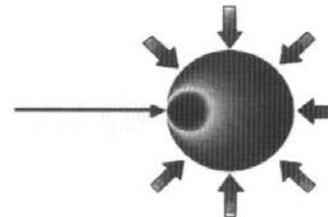
Atmosphere Formation

Laser or particle beams rapidly heat the surface of the fusion target forming a surrounding plasma envelope



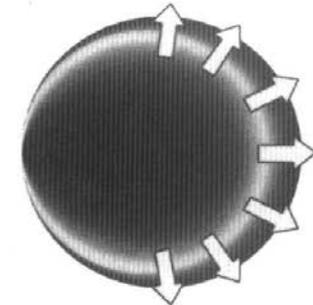
Compression

Fuel is compressed by rocket-like blowoff of the surface material



Ignition

At the moment of maximum compression a short (1-10 ps) high intensity (10^{19} W/cm²) pulse ignites the capsule



Burn

Thermonuclear burn spreads rapidly through the compressed fuel, yielding many times the driver input energy

Laser Energy →

Inward transported thermal energy →

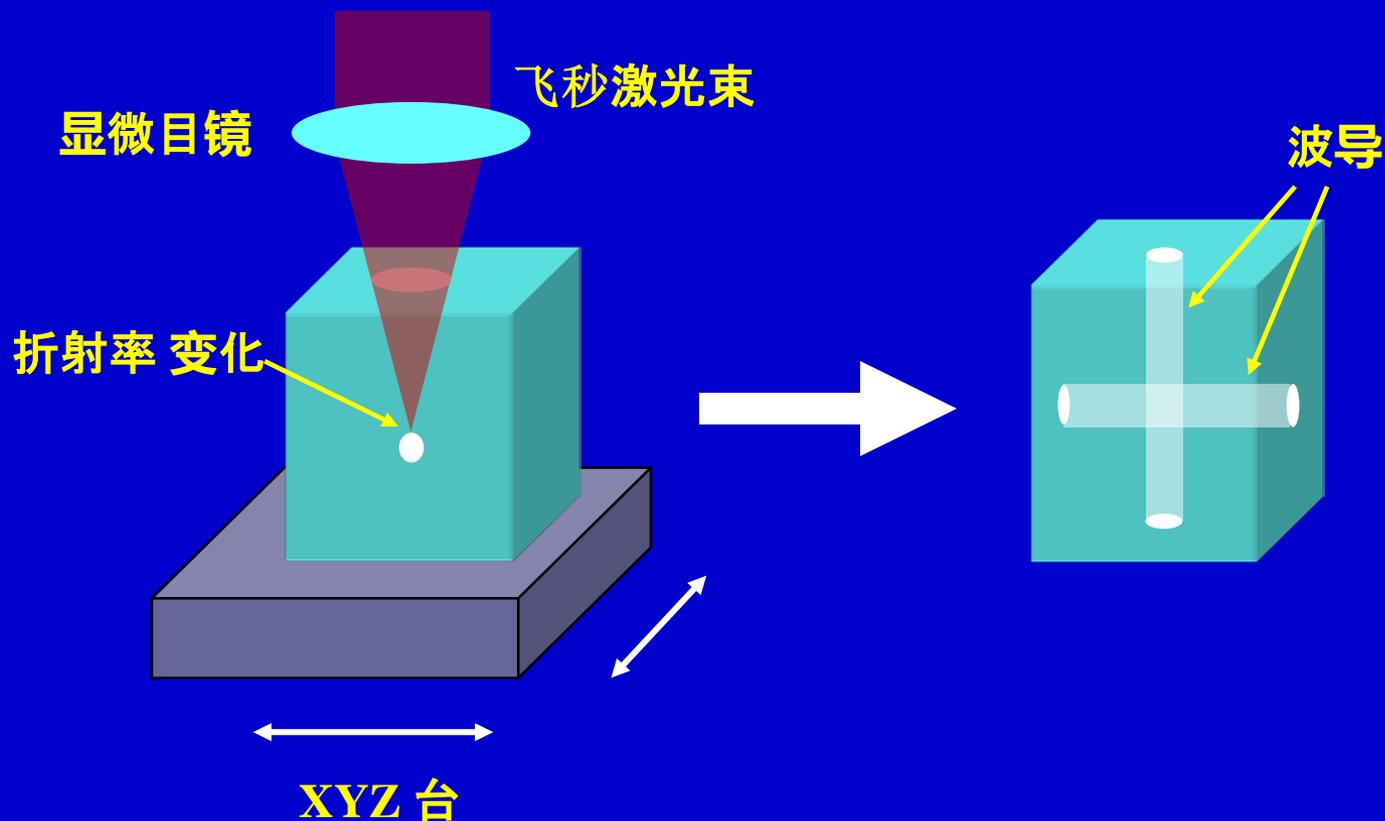
Interaction area

ablation,
microexplosion,
densification,
compaction,
Polymerization
.....

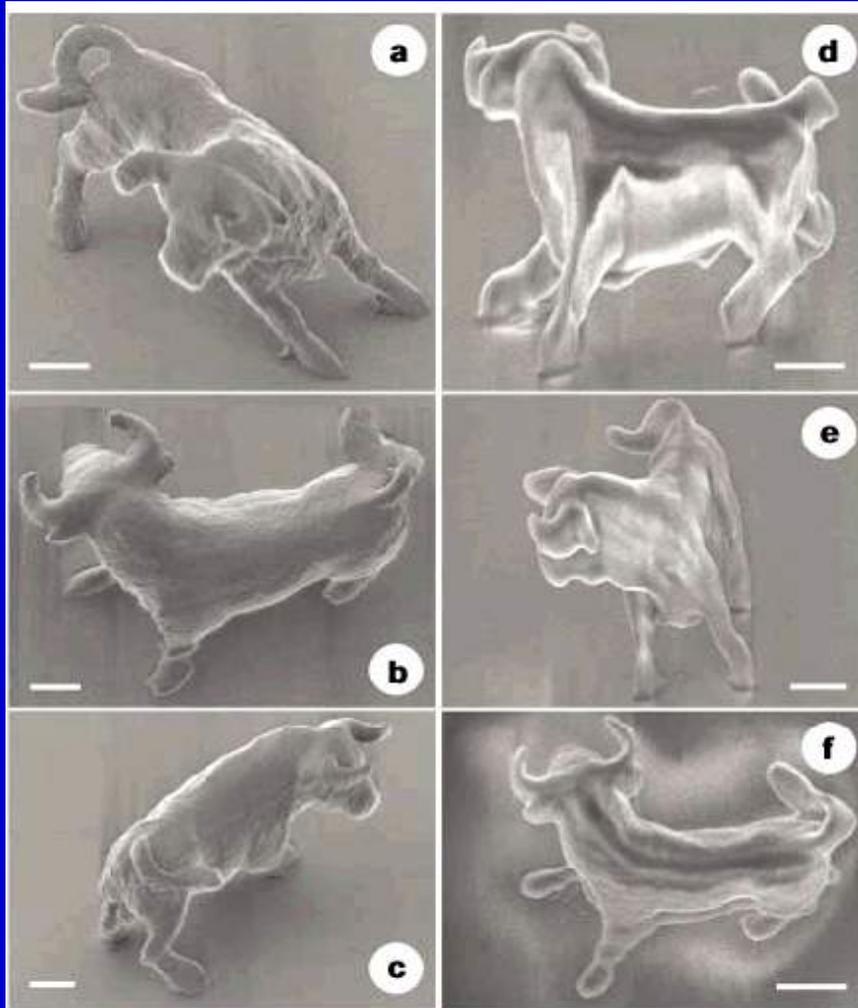
refractive index
change



waveguides, couplers,
gratings, photonic crystals,
holes, channels, voids,
Date storage
.....



Nano bull



Ti:S 780nm, 150fs, 76MHz

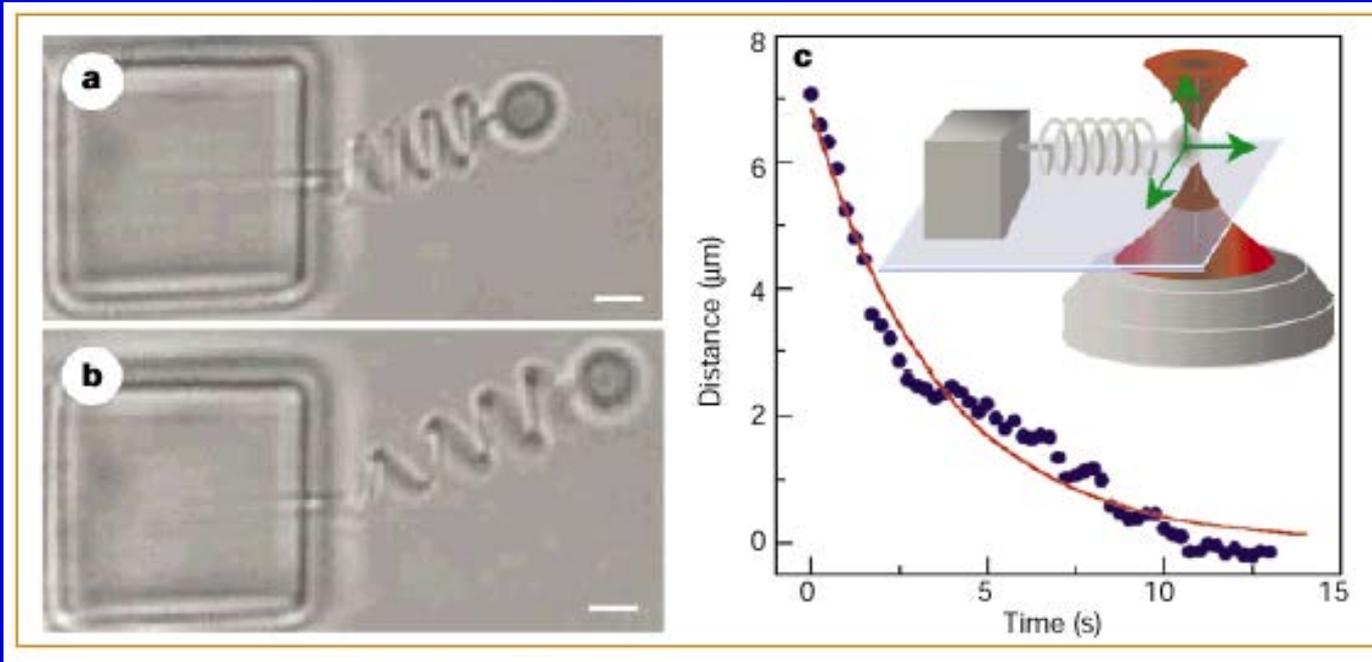
1.4NA microscopic

SCR500 resin (JSR, Japan)
+ urethane acrylate

resolution 120nm

Bull size: $10\mu\text{m} \times 7\mu\text{m}$

Nature, 2001 (Osaka Univ. Sun, Kawada et al)



820nm, 19mW

3pN

$k = 8.2\text{nN/m}$

a. 原始状态

b. 伸展状态

c. 阻尼振荡恢复曲线

Attosecond metrology

M. Hentschel^{*†}, R. Kienberger^{*†}, Ch. Spielmann^{*}, G. A. Reider^{*}, N. Milosevic^{*}, T. Brabec^{*}, P. Corkum[‡], U. Heinzmann[§], M. Drescher[§] & F. Krausz^{*}

** Institut für Photonik, Technische Universität Wien, Gusshausstr. 27, A-1040 Wien, Austria*

‡ Steacie Institute of Molecular Sciences, NRC Canada, Ottawa, Canada K1A 0R6

§ Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

† These authors contributed equally to this work

The generation of ultrashort pulses is a key to exploring the dynamic behaviour of matter on ever-shorter timescales. Recent developments have pushed the duration of laser pulses close to its natural limit—the wave cycle, which lasts somewhat longer than one femtosecond ($1 \text{ fs} = 10^{-15} \text{ s}$) in the visible spectral range. Time-resolved measurements with these pulses are able to trace dynamics of molecular structure, but fail to capture electronic processes occurring on an attosecond ($1 \text{ as} = 10^{-18} \text{ s}$) timescale. Here we trace electronic dynamics with a time resolution of $\leq 150 \text{ as}$ by using a subfemtosecond soft-X-ray pulse and a few-cycle visible light pulse. Our measurement indicates an attosecond response of the atomic system, a soft-X-ray pulse duration of $650 \pm 150 \text{ as}$ and an attosecond synchronism of the soft-X-ray pulse with the light field. The demonstrated experimental tools and techniques open the door to attosecond spectroscopy of bound electrons.

定态光波的标量表示

在自由空间

$$(1) \left. \begin{array}{l} \vec{E}(P, t) \\ \vec{H}(P, t) \end{array} \right\} \begin{array}{l} \vec{H} \perp \vec{E}, \quad \varphi_H = \varphi_E, \\ \sqrt{\mu\mu_0} H_0 = \sqrt{\varepsilon\varepsilon_0} E_0 \end{array} \longrightarrow \vec{E}(P, t)$$

$$(2) \vec{E}(P, t) \Rightarrow \begin{cases} E_x(P, t) \\ E_y(P, t) \\ E_z(P, t) \end{cases} \Rightarrow U(P, t)$$

标量

$$\text{波动方程 } \nabla^2 \vec{E} - \varepsilon \varepsilon_0 \mu \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$\vec{E}(P, t) = \{E_x, E_y, E_z\}$ 各个分量都遵从相同的波动函数。

$$\frac{\partial^2 E_i}{\partial x^2} + \frac{\partial^2 E_i}{\partial y^2} + \frac{\partial^2 E_i}{\partial z^2} - \varepsilon \varepsilon_0 \mu \mu_0 \frac{\partial^2 E_i}{\partial t^2} = 0 \Rightarrow \nabla^2 E_i - \varepsilon \varepsilon_0 \mu \mu_0 \frac{\partial^2 E_i}{\partial t^2} = 0$$

$$i = x, y, z$$

把矢量方程转化为标量方程：

$$\nabla^2 U - \varepsilon\varepsilon_0\mu\mu_0 \frac{\partial^2 U}{\partial t^2} = 0$$

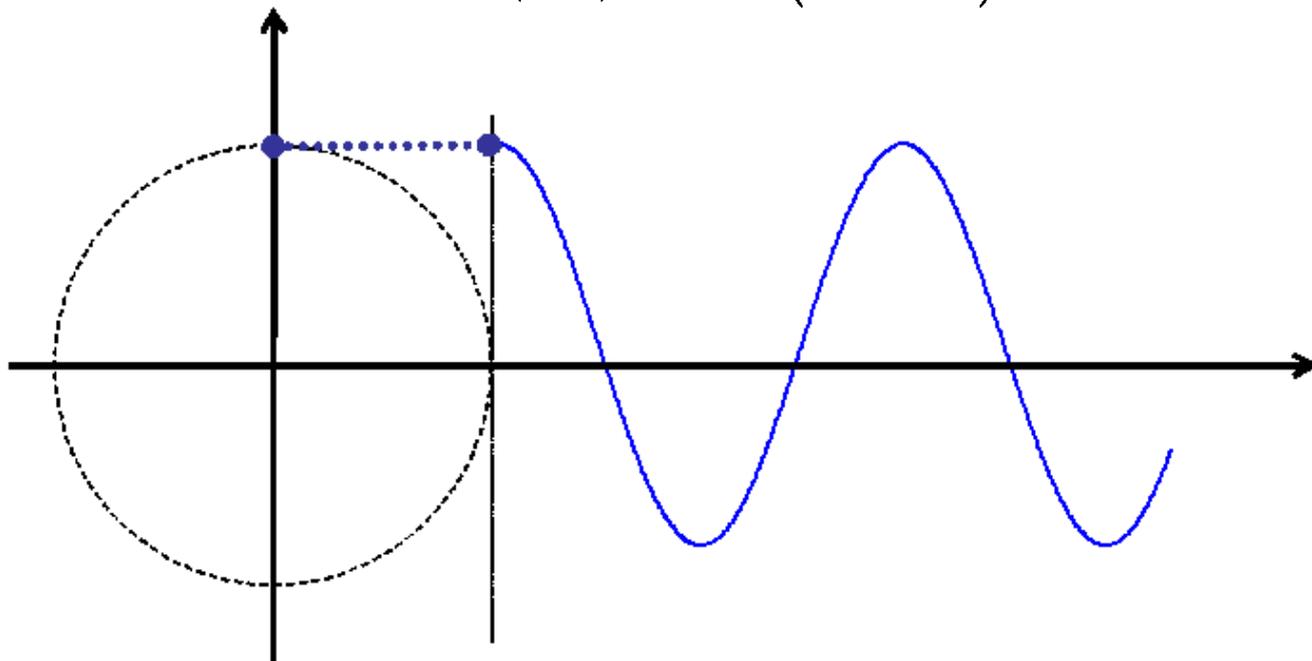
简谐波为定态波的基元成分，其标量形式：

$$U(P, t) = A(P) \cos(\omega t - \varphi(P))$$

振幅稳定，频率单一

○ 波函数的复数表示

$$U(t) = A \cos(\omega t) = \operatorname{Re}(Ae^{\pm i\omega t})$$



数学上复数的实部和虚部分别对应cos函数和sin函数：

$$\tilde{U} = Ae^{i\alpha} = A(\cos \alpha + i \sin \alpha)$$

$$\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}, \quad \sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}$$

$$U(P, t) = A(P) \cos(\omega t - \phi(P)) = \operatorname{Re}(\tilde{U}(P, t))$$



$$\tilde{U}(P, t) = A(P) e^{\pm i(\omega t - \phi(P))}$$



$$\tilde{U}(P, t) = A(P) e^{-i(\omega t - \phi(P))}$$

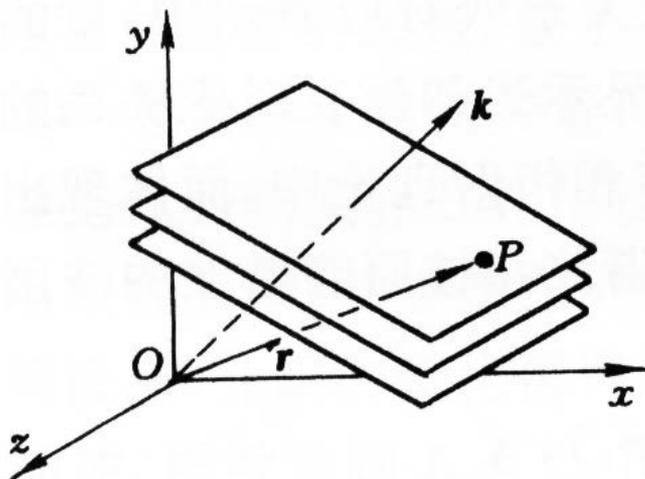
复数的模对应振幅，复数的复角对应相位，我们取-号， $\phi(P)$ 正号表示相位落后。

几种典型波的复数描述

平面简谐波：

$$U(\vec{r}, t) = A \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = A e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t} = A e^{i(k_x x + k_y y + k_z z)} e^{-i\omega t}; \quad (\text{设: } \phi_0 = 0)$$



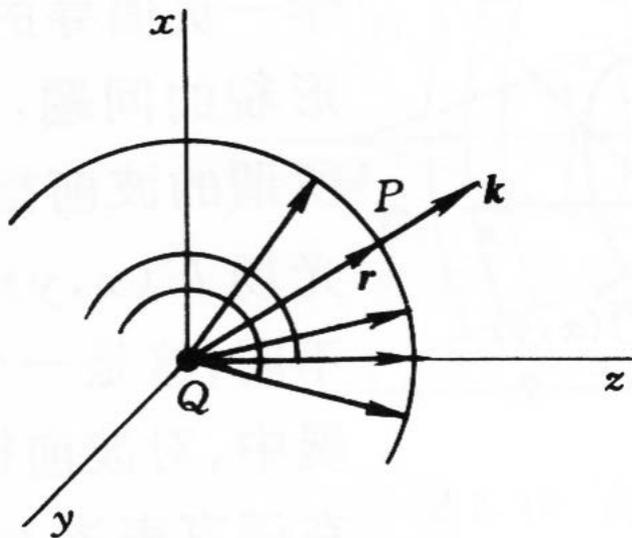
特征：振幅为空间和时间的常数，相因子为线性。

球面简谐波

$$U(\vec{r}, t) = \frac{A}{r} \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = \frac{A}{r} e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t}$$

$$= \frac{A}{\sqrt{x^2 + y^2 + z^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2 + z^2}} e^{-i\omega t}; \quad \text{设: } \phi_0 = 0$$



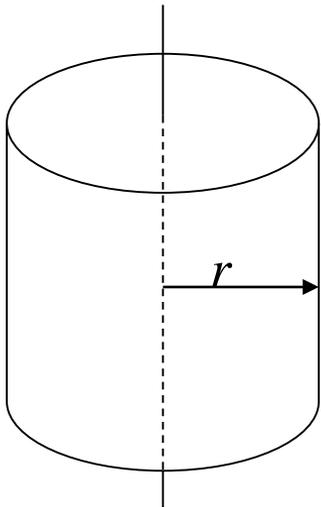
特征：振幅为空间函数，相因子为非线性。

柱面简谐波

$$U(\vec{r}, t) = \frac{A}{\sqrt{r}} \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = \frac{A}{\sqrt{r}} e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t}$$

$$= \frac{A}{\sqrt[4]{x^2 + y^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2}} e^{-i\omega t}; \quad (\text{设: } \phi_0 = 0)$$



特征：振幅为空间函数，相因子为非线性。

平面简谐波：
$$U(\vec{r}, t) = A \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = A e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t} = A e^{i(k_x x + k_y y + k_z z)} e^{-i\omega t}; \quad (\text{设 } \phi_0 = 0)$$

球面简谐波：
$$U(\vec{r}, t) = \frac{A}{r} \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = \frac{A}{r} e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t} = \frac{A}{\sqrt{x^2 + y^2 + z^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2 + z^2}} e^{-i\omega t}; (\text{设 } \phi_0 = 0)$$

柱面简谐波：
$$U(\vec{r}, t) = \frac{A}{\sqrt{r}} \cos(\omega t - \vec{k} \cdot \vec{r} - \phi_0)$$

$$\tilde{U}(\vec{r}, t) = \frac{A}{\sqrt{r}} e^{i\vec{k} \cdot \vec{r}} e^{-i\omega t} = \frac{A}{\sqrt{x^2 + y^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2}} e^{-i\omega t}; \quad (\text{设 } \phi_0 = 0)$$

复振幅概念： 定态波，时间频率单一，我们关心振幅的空间分布 $A(P)$ 和相位空间分布 $\varphi(P)$ ，引进复振幅概念：

$$\tilde{U}(P) = A(P) e^{i\varphi(P)}$$

三种典型光波的复振幅：

平面简谐波：

$$\tilde{U}(P) = Ae^{i\vec{k}\cdot\vec{r}} = Ae^{i(k_x x + k_y y + k_z z)}$$

球面简谐波：

$$\tilde{U}(P) = \frac{A}{r} e^{i\vec{k}\cdot\vec{r}} = \frac{A}{\sqrt{x^2 + y^2 + z^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2 + z^2}}$$

柱面简谐波：

$$\tilde{U}(P) = \frac{A}{\sqrt{r}} e^{i\vec{k}\cdot\vec{r}} = \frac{A}{\sqrt[4]{x^2 + y^2}} e^{i(\vec{e}_k \cdot \vec{e}_r) k \sqrt{x^2 + y^2}}$$

♣ 平面波复振幅及其特点

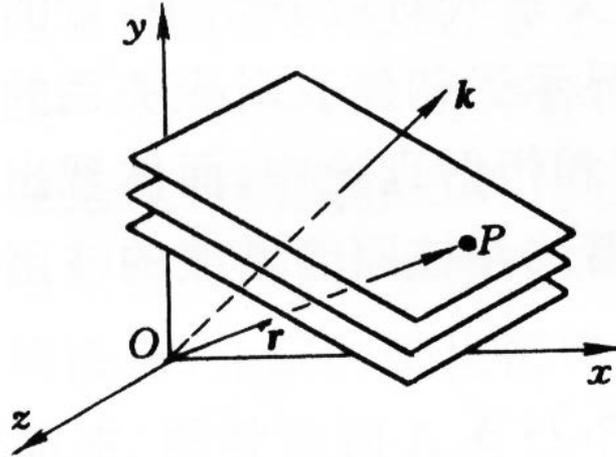


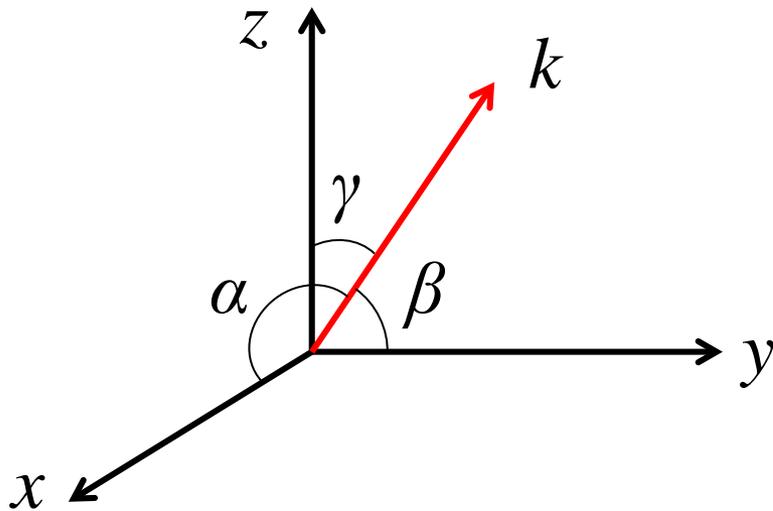
图 2.6 平面波特征矢量——波矢 k

$$\tilde{U}(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}} = Ae^{i(k_x\cdot x + k_y\cdot y + k_z\cdot z)} = Ae^{ik(\cos\alpha\cdot x + \cos\beta\cdot y + \cos\gamma\cdot z)}$$

平面波复振幅的特点：
1、振幅为常数
2、线性相因子

线性相因子系数：
 (k_x, k_y, k_z) 或 $(\cos \alpha, \cos \beta, \cos \gamma)$ } \Leftrightarrow 传播方向

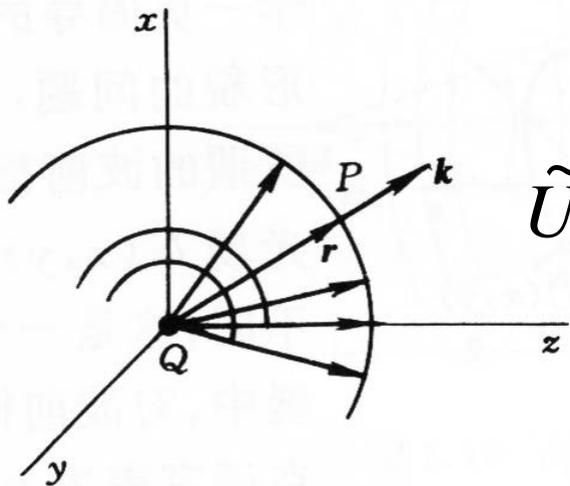
$$\sqrt{k_x^2 + k_y^2 + k_z^2} = k = \frac{2\pi}{\lambda}$$



$$\begin{cases} k_x = k \cos \alpha \\ k_y = k \cos \beta \\ k_z = k \cos \gamma \end{cases}$$

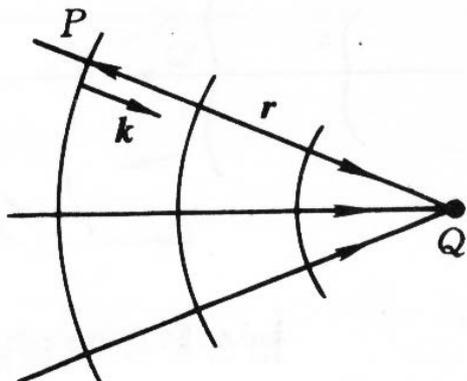
♣ 球面波复振幅及其特点

(1) 发散球面波



$$\tilde{U}(P) = \frac{A}{r} e^{ikr} = \frac{A}{\sqrt{x^2 + y^2 + z^2}} e^{ik\sqrt{x^2 + y^2 + z^2}}$$

(2) 会聚球面波



$$\tilde{U}(P) = \frac{A}{r} e^{-ikr} = \frac{A}{\sqrt{x^2 + y^2 + z^2}} e^{-ik\sqrt{x^2 + y^2 + z^2}}$$

(3) 轴外点源

$Q(x_0, y_0, z_0)$ 为点源，场点 $P(x, y, z)$

$$\tilde{U}(P) = \frac{A}{r} e^{\pm ikr},$$

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

+ \rightarrow 发散，**-** \rightarrow 会聚，聚散中心 (x_0, y_0, z_0)

♣ 光强和复振幅的关系

光强空间分布： $I(P) = \underline{\underline{\tilde{U}(P) \cdot \tilde{U}^*(P)}}} = A^2(P)$

这里 $\tilde{U}^*(P)$ 是 $\tilde{U}(P)$ 的复共轭：

$$\tilde{U}(P) = A(P)e^{i\phi(P)}$$

$$\tilde{U}^*(P) = A(P)e^{-i\phi(P)}$$

第三节、波前函数

♠ 波前和波前光学概述

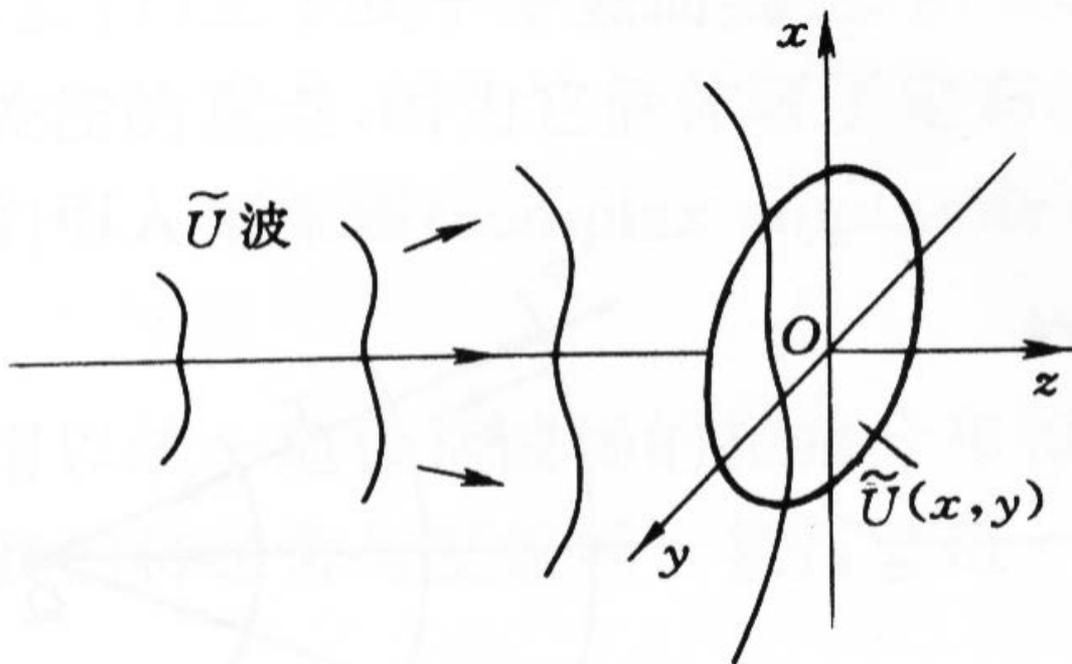


图 2.8 广义波前 $U(x, y)$

“波前”一词指波场中任一曲面，更多地指一个平面，如记录介质、感光底片、接收屏幕等所在的平面的复振幅分布 $U(x, y)$ 。

波前是分析波动光学行为的主要工具，包括：

1. 波前的描述和识别
2. 波前的叠加和干涉
3. 波前的变换和分析
4. 波前的记录和再现

♠ 平面和球面波前函数以及其共轭波前

(1) 一列平面波 $k_{1x} = k \sin \theta$, $k_{1y} = 0$, $k_{1z} = k \cos \theta$
波 U_1 :

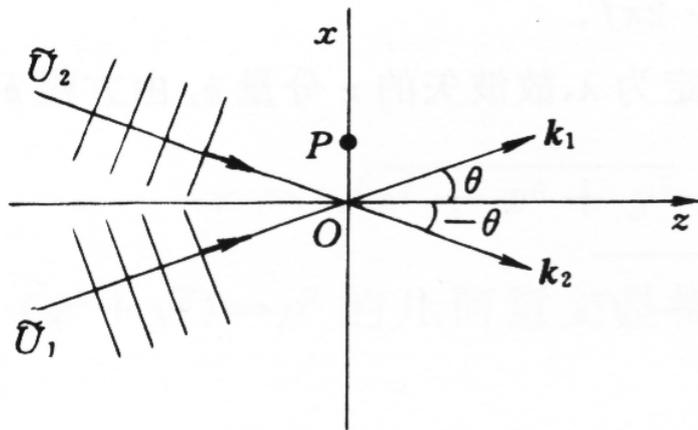


图 2.9 平面波及其共轭波前

$$\tilde{U}(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}} = Ae^{i(k_x\cdot x + k_y\cdot y + k_z\cdot z)} = Ae^{ik(\sin\theta\cdot x + \cos\theta\cdot z)}$$

在 $z=0$ 的平面上的波前函数:

$$\tilde{U}_1(x, y) = Ae^{ik \sin \theta \cdot x}$$

其共轭波前:

$$\tilde{U}_2(x, y) = \tilde{U}_1^*(x, y) = Ae^{-ik \sin \theta \cdot x} = Ae^{i(-k) \sin \theta \cdot x} = Ae^{ik \sin(-\theta) \cdot x}$$

(2) 一球面波, 点光源 $Q(0,0,-R)$

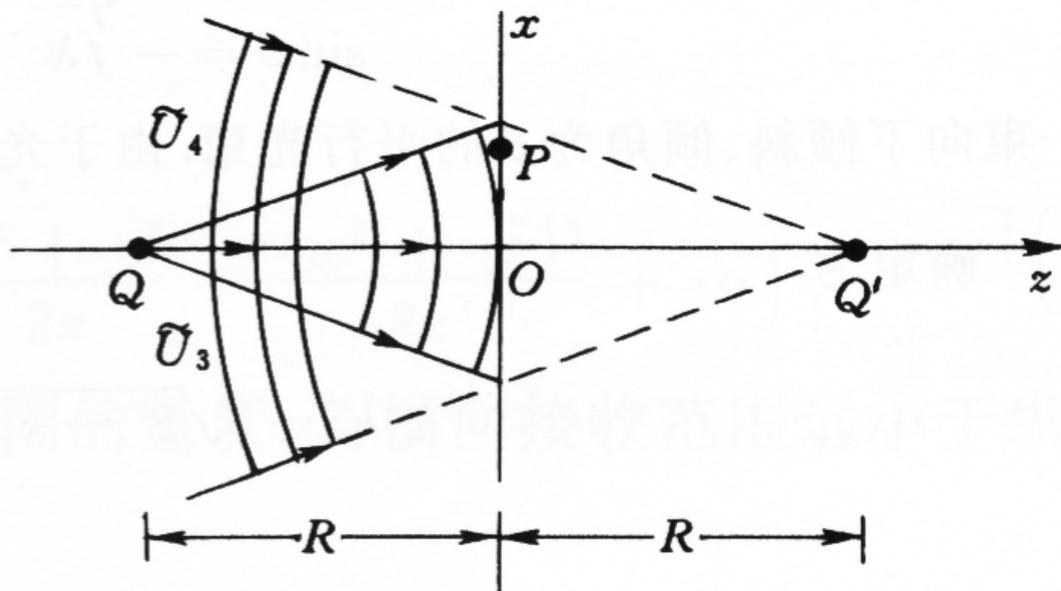


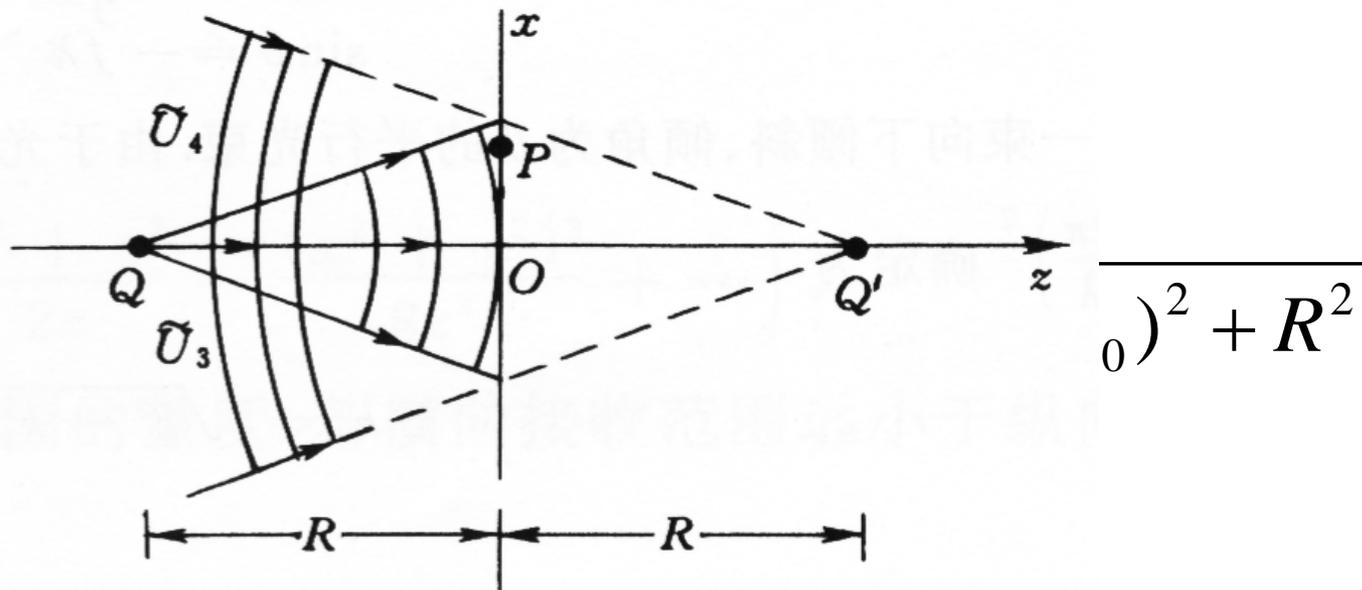
图 2.10 球面波及其共轭波前

$$\tilde{U}(P) = \frac{A}{r} e^{ikr}, \quad r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

$$x_0 = y_0 = 0, \quad z_0 = -R$$

$z=0$ 平面的波

$$\tilde{U}_3(x, y) =$$



$$\frac{1}{(x-x_0)^2 + R^2}$$

图 2.10 球面波及其共轭波前

其共轭波前:

$$\tilde{U}_4(x, y) = \tilde{U}_3^*(x, y) = \frac{A}{r} e^{-ikr},$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + R^2}$$

波的类型和特性 \Leftrightarrow 波前函数

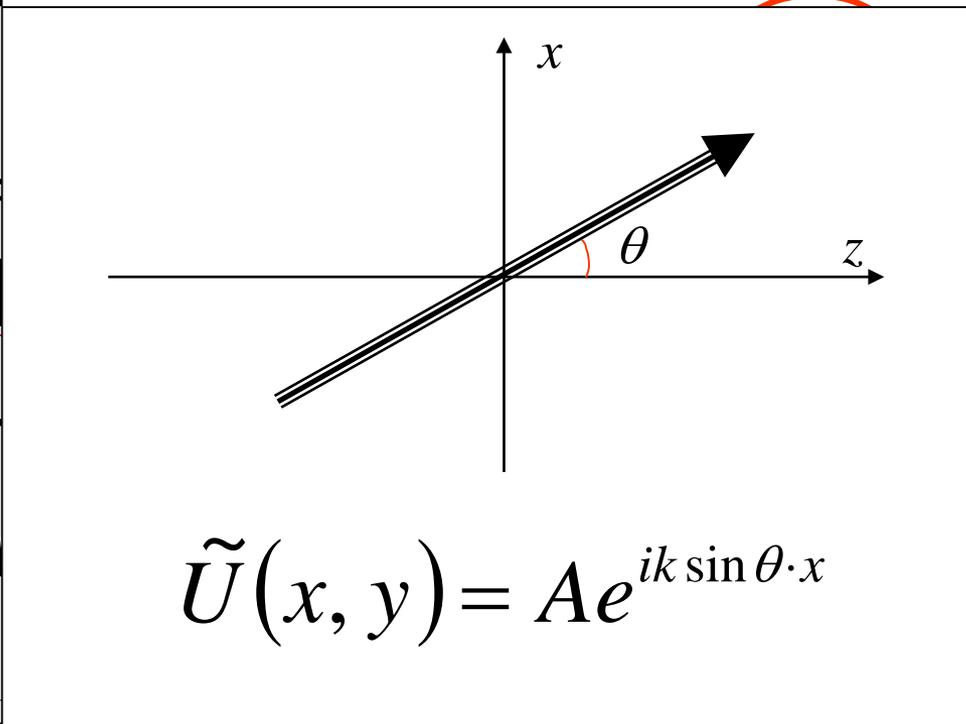
例题： 已知一列波长为 λ 的波，在 (x, y) 接收面上的波前函数为：

其中常数 f 和 λ 的
类型和特

线形因子-

$$\tilde{U}(x, y) = A e^{ik \sin \theta \cdot x}$$

所以： $\tilde{U}(x, y)$



$$\tilde{U}(x, y) = A e^{ik \sin \theta \cdot x}$$

前函数联系的波

无关，因此 $k_y=0$

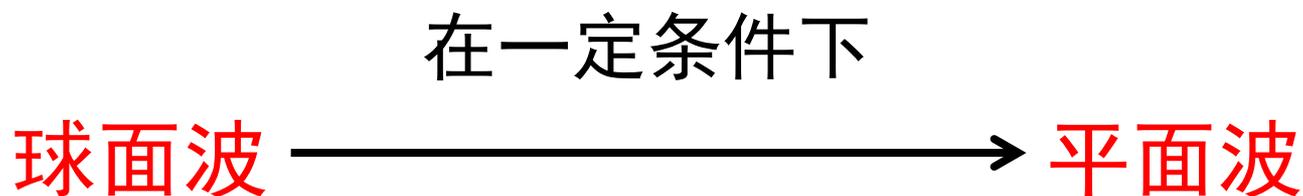
$$\sin \theta = -f \lambda \quad \text{或} \quad k_x = -2\pi f$$

$$k_z = \sqrt{k^2 - k_x^2} = 2\pi \sqrt{\frac{1}{\lambda^2} - f^2}$$

第四节、傍轴条件和远场条件

● 球面波和平面波的理论地位

平面简谐波或球面简谐波可以看成为复杂波场的基元成分，所有复杂波场都可以分解成一系列球面波或平面波的叠加。



● 傍轴条件或振幅条件—— $z_0^2 \gg \rho^2$

点源 $O(x_0, y_0)$ ，球面波到 (x, y) 面的波前函数：

$$\tilde{U}(x, y) = \frac{A}{r} e^{ikr},$$

其中：

$$r = \sqrt{x^2 + y^2 + z_0^2} = z_0 \sqrt{1 + \frac{x^2 + y^2}{z_0^2}} = z_0 \left(1 + \frac{x^2 + y^2}{2z_0^2} - \frac{(x^2 + y^2)^2}{8z_0^4} + \dots \right)$$

$\rho^2 = (x^2 + y^2)$ 横向接收范围的尺度。

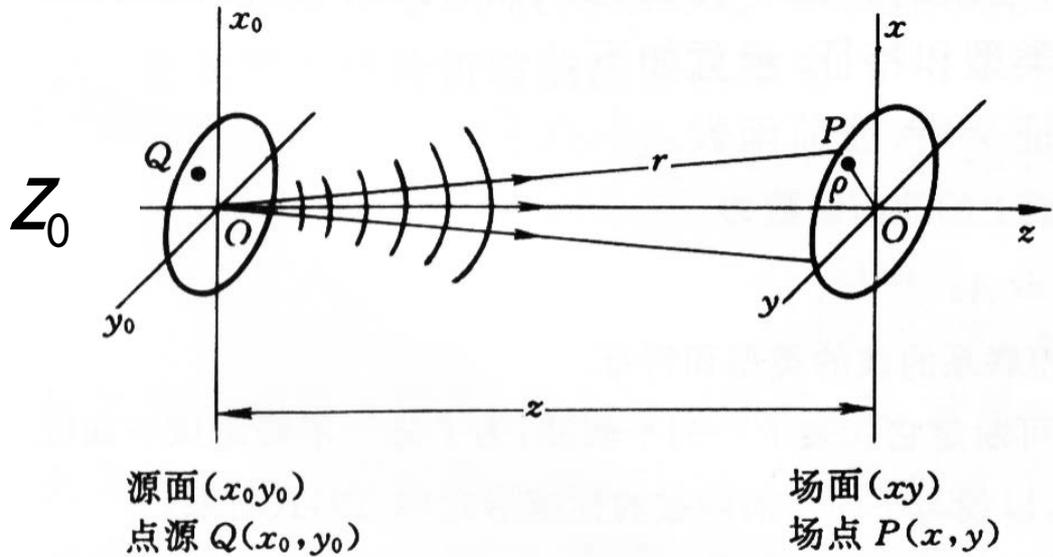


图 2.11 考察球面波向平面波的转化

在 $z_0^2 \gg \rho^2$ 下, 波前函数中的振幅可以近似为:

$$\frac{A}{r} \approx \frac{A}{z_0}$$

但是相因子中的二次项不能轻易舍弃, 保留到二次项:

$$\tilde{U}(x, y) \approx \frac{A}{z_0} e^{ik \frac{x^2 + y^2}{2z_0}} \cdot e^{ikz_0}$$

特点: 常数振幅, 非线性因子。

傍轴条件:

$$z_0^2 \gg \rho^2$$

- 远场条件或相位条件—— $z_0 \lambda \gg \rho^2$

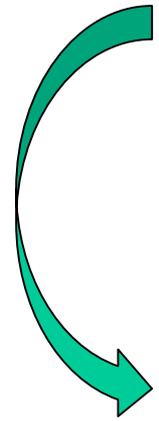
$$\tilde{U}(x, y) = \frac{A}{r} e^{ikr}$$

$$kr = k\sqrt{x^2 + y^2 + z_0^2} = \left(kz_0 + k \frac{x^2 + y^2}{2z_0} - k \frac{(x^2 + y^2)^2}{8z_0^3} + \dots \right)$$

相因子对物理状态的影响具有周期性，周期为 2π 。相因子中可以忽略的小量应该是远远小于 π 。

可以忽略二次项的条件：

$$k \frac{x^2 + y^2}{2z_0} = k \frac{\rho^2}{2z_0} = \frac{2\pi}{\lambda} \frac{\rho^2}{2z_0} \ll \pi$$



$z_0 \lambda \gg \rho^2$ 这是远场条件，或称相位条件。

相因子可以近似为：

$$e^{ikr} \approx e^{ikz_0}$$

振幅系数须保留二次项：

$$\tilde{U}(x, y) = \frac{A}{r} e^{ikz_0} \approx \frac{A}{z_0 + \frac{x^2 + y^2}{2z_0}} e^{ikz_0}$$

同时满足傍轴条件和远场条件：

$$U(x, y) = \frac{A}{z_0} e^{ikz_0}$$

为正入射平面波，这时，球面波转变成了平面波。

● 比较傍轴和远场条件

例题1、设波长 $\lambda \sim 500\text{nm}$, 横向距离 $\rho \sim 1\text{mm}$, 约定“ \gg ”为50倍, 推算傍轴条件下的纵向距离 Z_p 和远场条件的 Z_f .

$$Z_p \approx \sqrt{50}\rho = \sqrt{50} \times 1\text{mm} \approx 7\text{mm}$$

$$Z_f \approx 50 \frac{\rho^2}{\lambda} = 50\rho \left(\frac{\rho}{\lambda} \right) = 50 \times 1\text{mm} \times (2 \times 10^3) = 100\text{m}$$

此时: $Z_f > Z_p$, 因为 ρ/λ 带来的高倍率。

例题2、设波长 $\lambda \sim 1\text{m}$, 横向距离 $\rho \sim 10\text{cm}$, 约定“ \gg ”为50倍, 推算傍轴条件下的纵向距离 Z_p 和远场条件的 Z_f .

$$Z_p \approx \sqrt{50} \rho = \sqrt{50} \times 1\text{cm} \approx 70\text{cm}$$

$$Z_f \approx 50 \frac{\rho^2}{\lambda} = 50 \rho \left(\frac{\rho}{\lambda} \right) = 50 \times 10\text{cm} \times (0.1) = 50\text{cm}$$

此时: $Z_f > Z_p$, 因为 $\rho/\lambda < 1$ 。

结论：

1. 比较傍轴条件和远场条件谁对纵向距离要求更远，取决于 ρ/λ 。
2. 在可见光波段，通常 $\rho/\lambda \gg 1$ ，故 $Z_f \gg Z_p$

第五节、波前相因子分析法

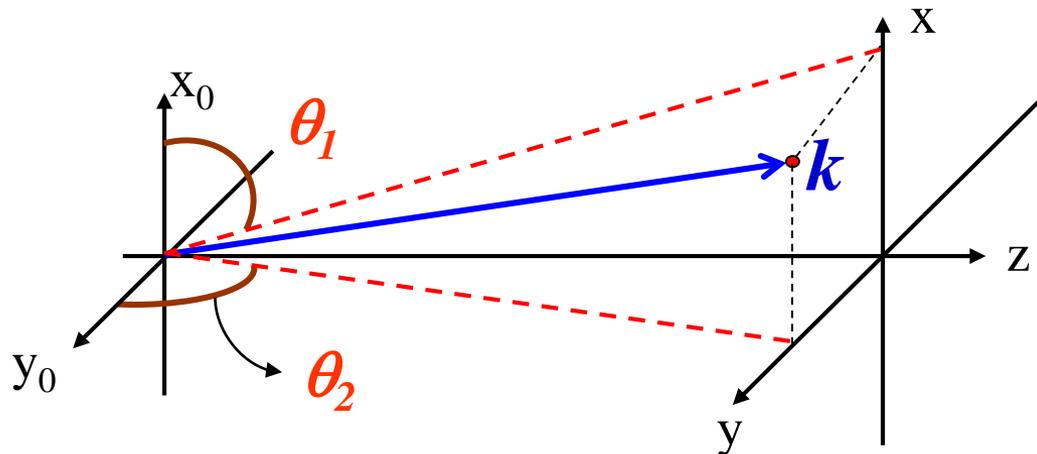
波的类型和特性 \Leftrightarrow 波前函数

波场的**主要特征**体现在波前函数的相因子中。如果能将一个复杂的波场中复振幅的相因子与平面波或球面波的相因子作一比较，使之联系起来，则复杂波场即可分解为一系列平面波或球面波成分，从而使我们比较容易从概念上去掌握它。所谓“**相因子判断法**”，简言之，即根据波前函数的相因子来判断波场的性质，分析波场的主要特征。为了使用相因子判断法，我们先要熟悉**两件事**：一是平面波和几种典型情况下球面波在波前上的相因子。

波的类型和特性 \Leftrightarrow 波前相因子

(1) 平面波

平面波的波前函数具有线性相因子，



θ_1 和 θ_2 是波矢 k 的两个方向余弦角。

$$k_x = k \cos \theta_1, \quad k_y = k \cos \theta_2$$

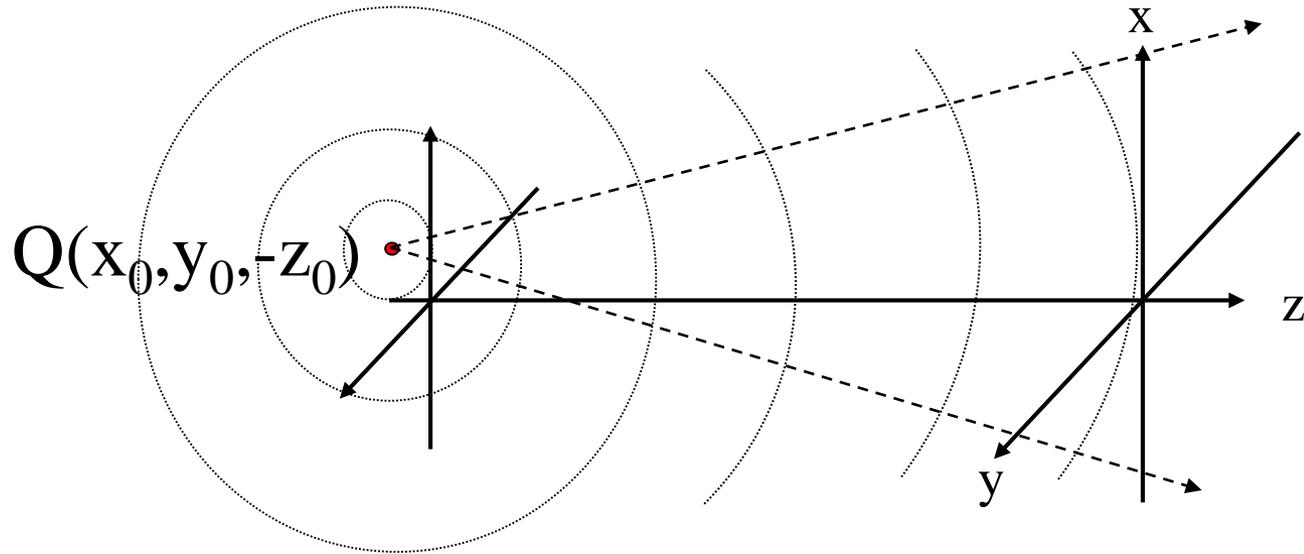
在 $z=0$ 平面的波前函数和**相因子**：

$$\tilde{U}(x, y) = A e^{ik(\cos \theta_1 \cdot x + \cos \theta_2 \cdot y)} \propto e^{ik(\cos \theta_1 \cdot x + \cos \theta_2 \cdot y)}$$

线性相因子系数 $(\cos \theta_1, \cos \theta_2)$ 与平面波的传播方向一一对应。

(2) 球面波

I、发散球面波



$$\tilde{U}(x, y) = \frac{A}{r} e^{ikr}, \quad r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + z_0^2}$$

在傍轴条件下：

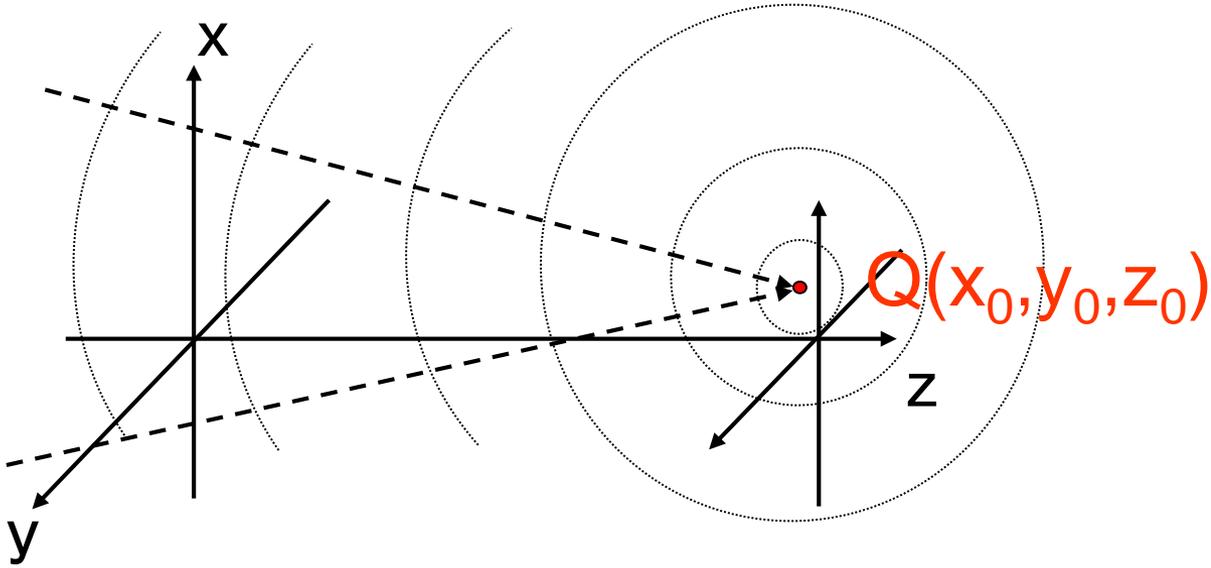
$$\tilde{U}(x, y) \approx \frac{A}{z_0} e^{ik \frac{(x-x_0)^2 + (y-y_0)^2}{2z_0}} \cdot e^{ikz_0}$$

$$\propto e^{ik \frac{x^2 + y^2}{2z_0}} \cdot e^{ik \frac{x_0^2 + y_0^2}{2z_0}} \cdot e^{-ik \frac{xx_0 + yy_0}{z_0}} \cdot e^{ikz_0}$$

$$\propto e^{ik \frac{x^2 + y^2}{2z_0}} \cdot e^{-ik \frac{xx_0 + yy_0}{z_0}}$$

发散中心为 **Q** $(x_0, y_0, -z_0)$

II、汇聚球面波



$$\tilde{U}(x, y) = \frac{A}{r} e^{-ikr}, \quad r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + z_0^2}$$

在傍轴条件下:

$$\tilde{U}(x, y) \approx \frac{A}{z_0} e^{-ik \frac{(x-x_0)^2 + (y-y_0)^2}{2z_0}} \cdot e^{-ikz_0}$$

$$\propto e^{-ik \frac{x^2 + y^2}{2z_0}} \cdot e^{-ik \frac{x_0^2 + y_0^2}{2z_0}} \cdot e^{ik \frac{xx_0 + yy_0}{z_0}} \cdot e^{ikz_0}$$

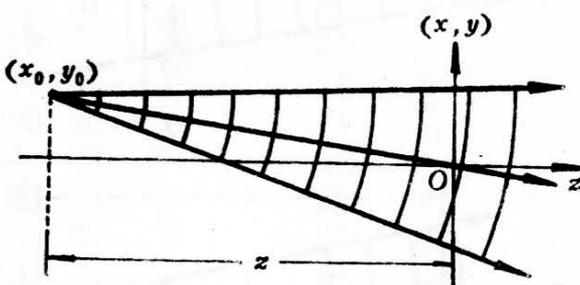
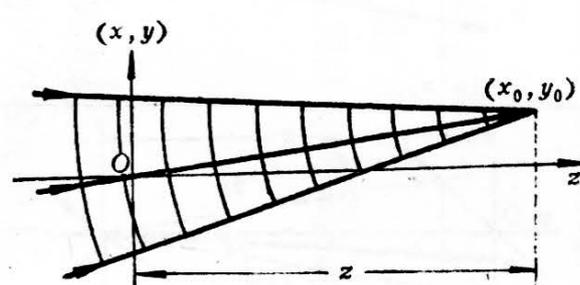
$$\propto e^{-ik \frac{x^2 + y^2}{2z_0}} \cdot e^{ik \frac{xx_0 + yy_0}{z_0}}$$

汇聚中心为 **Q** (x_0, y_0, z_0)

总结:

表 V-1 平面波和球面波在波前上的相因子

| 波的类型 | 特征 | 相因子 | 图 解 |
|-----------|--|--|-----|
| (1) 平面波 | 传播方向 (θ_1, θ_2) 当 $\theta_1 = \theta_2 = 0$ 时 | $\begin{cases} \exp[ik(\sin\theta_1 x + \sin\theta_2 y)] \\ 1 \end{cases}$ | |
| (2) 发散球面波 | 中心在轴上 坐标 $(0, 0, -z)$ | $\exp\left[ik \frac{x^2 + y^2}{2z}\right]$ | |
| (3) 会聚球面波 | 中心在轴上 坐标 $(0, 0, z)$ | $\exp\left[-ik \frac{x^2 + y^2}{2z}\right]$ | |

| 波的类型 | 特征 | 相因子 | 图解 |
|-----------|------------------------------|---|--|
| (4) 发散球面波 | 中心在轴外 坐标 $(x_0, y_0, -z)$ | $\exp\left[ik\left(\frac{x^2 + y^2}{2z} - \frac{xx_0 + yy_0}{z} \right) \right]$ |  |
| (5) 会聚球面波 | 中心在轴外 坐标 (x_0, y_0, z) | $\exp\left[-ik\left(\frac{x^2 + y^2}{2z} - \frac{xx_0 + yy_0}{z} \right) \right]$ |  |

例题：在 (x, y) 平面上波前函数的相因子为 $e^{-ik\left(4\frac{x^2+y^2}{D}\right)}$

k 为波矢，分析波场的类型和特征？

二次相因子 \rightarrow 傍轴球面波
无一次相因子 \rightarrow 中心在 z 轴上
相因子中的负号 \rightarrow 汇聚球面波

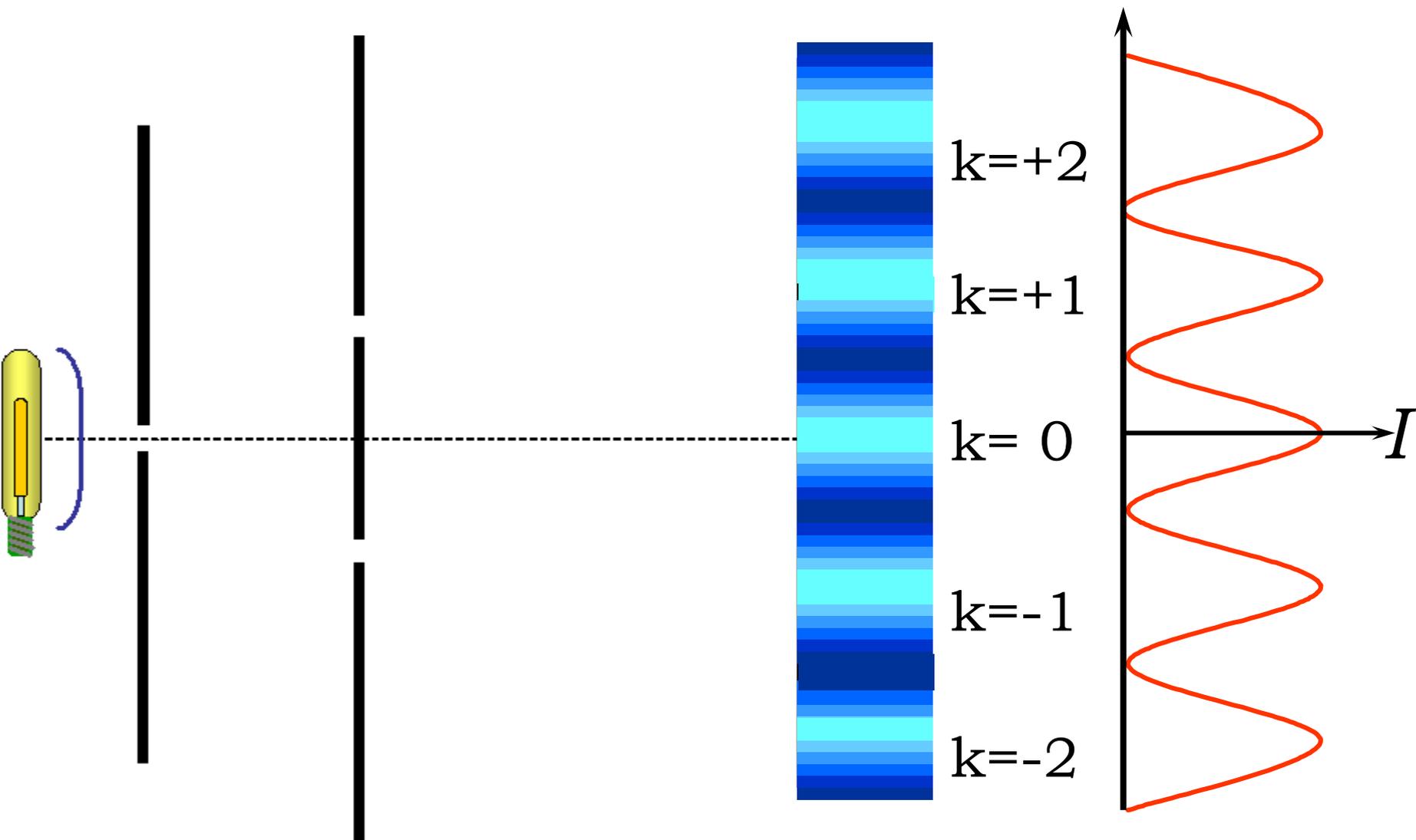
进一步确定中心位置，把相因子函数改写成标准形式：

$$e^{-ik\left(4\frac{x^2+y^2}{D}\right)} = e^{-ik\left(\frac{x^2+y^2}{2\left(\frac{D}{8}\right)}\right)}$$

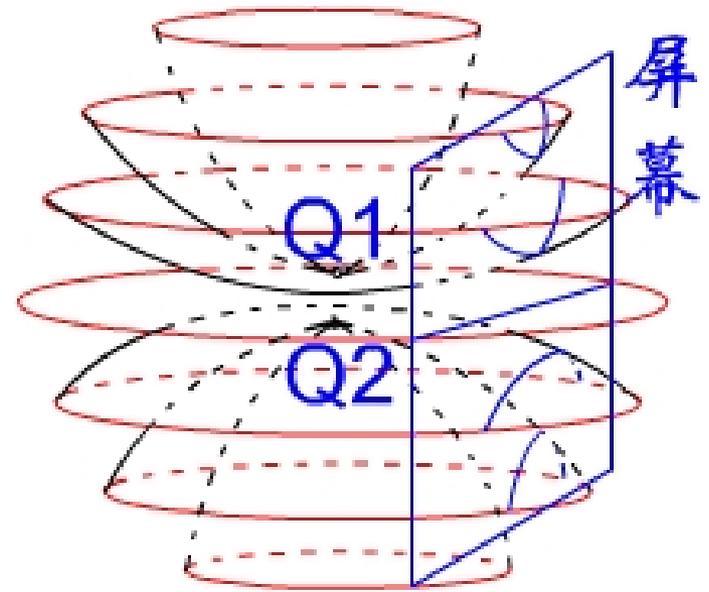
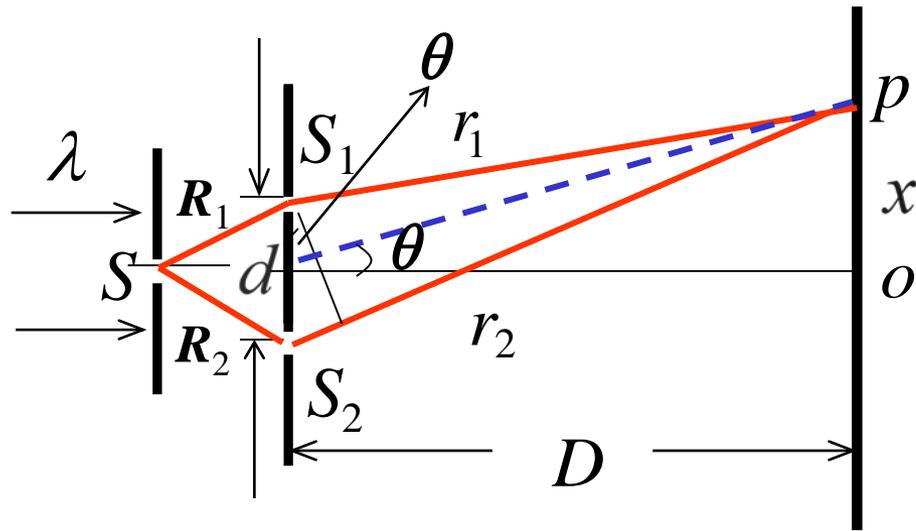
于是判断汇聚中心的位置为 $(0, 0, D/8)$

第六节、应用举例

实验装置



干涉条纹形状:



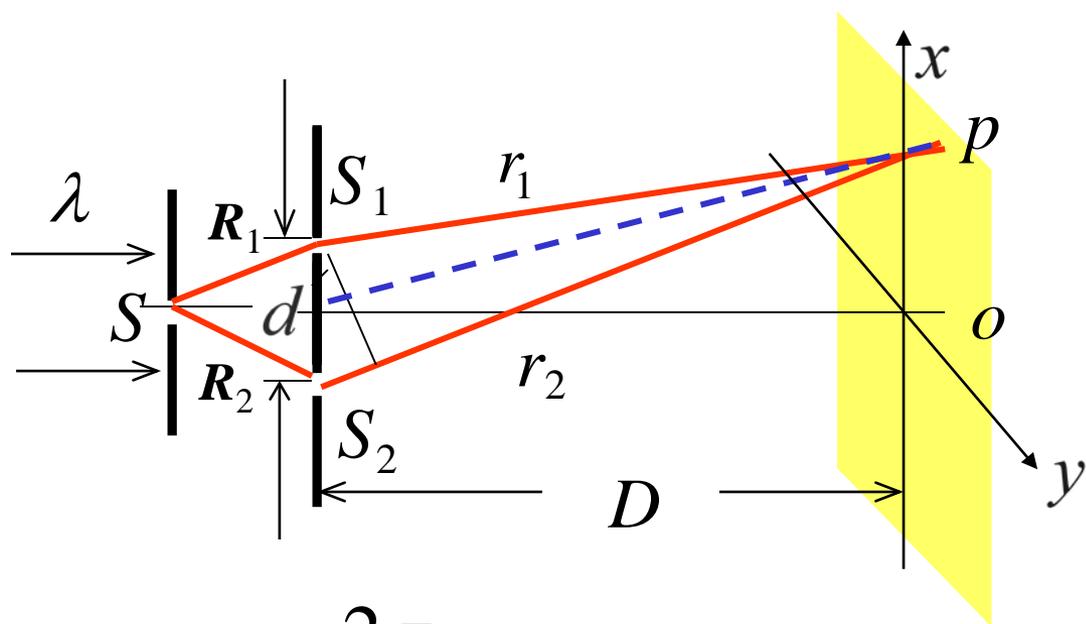
两球面波的干涉场

$$I(P) = I_1(P) + I_2(P) + 2\sqrt{I_1 I_2} \cos \delta(P)$$

$$\delta = \frac{2\pi}{\lambda} [(R_2 + r_2) - (R_1 + r_1)]$$

$r_2 - r_1$ 相同的点, 干涉情况相同。数学证明 $r_2 - r_1 = \text{常数}$ 的点的轨迹是以两点光源为焦点的回旋双曲面。

$$\delta(P) = ?$$



$$\delta = \frac{2\pi}{\lambda} [(R_2 + r_2) - (R_1 + r_1)] = \frac{2\pi}{\lambda} [r_2 - r_1] \text{ 对称装置, } R_1 = R_2$$

$$r_1 = \sqrt{D^2 + \left(x - \frac{1}{2}d\right)^2 + y^2}$$

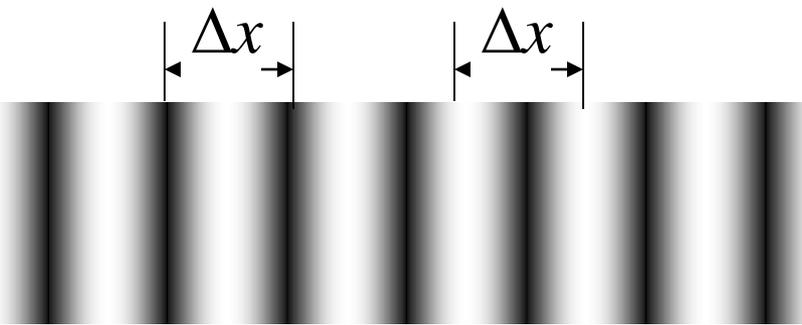
$$r_2 = \sqrt{D^2 + \left(x + \frac{1}{2}d\right)^2 + y^2}$$

$$d, x, y \ll D$$

$$r_2 - r_1 \approx \frac{d}{D} x$$

$$\delta = \frac{2\pi}{\lambda} \frac{d}{D} x$$

条纹间隙:



当 $\delta = \frac{2\pi d}{\lambda D} x = 2k\pi$, 振动相长, 为亮条纹。

亮条纹的位置: $x = \frac{D}{d} k\lambda$

当 $\delta = \frac{2\pi d}{\lambda D} x = (2k+1)\pi$, 振动相消, 为暗条纹。

暗条纹的位置: $x = \frac{D}{d} (k + \frac{1}{2})\lambda$

条纹间距 Δx 为两条相邻亮条纹或暗条纹之间的距离:

$$\Delta x = \frac{D}{d} \lambda$$

如果 $I_1=I_2=I_0$

$$I = 2I_0(1 + \cos \delta) = 4I_0 \cos^2\left(\frac{\delta}{2}\right) = 4I_0 \cos^2\left(\frac{\pi d}{\lambda D} x\right)$$

光波的叠加原理，观察屏上的波前函数为：

$$\tilde{U}(x, y) = \tilde{U}_1(x, y) + \tilde{U}_2(x, y) + \dots$$

观察屏上光强分布：

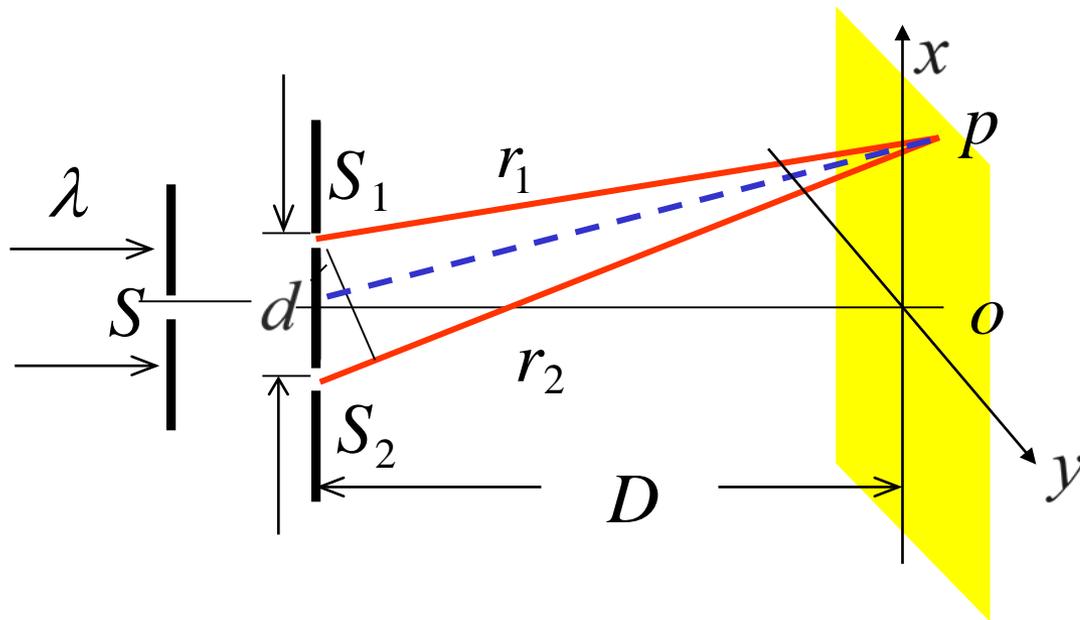
$$\begin{aligned} I(x, y) &= \tilde{U}(x, y) \cdot \tilde{U}^*(x, y) \\ &= \left(\tilde{U}_1(x, y) + \tilde{U}_2(x, y) + \dots \right) \cdot \left(\tilde{U}_1(x, y) + \tilde{U}_2(x, y) + \dots \right)^* \end{aligned}$$

几种常用几种波前函数：

$$\tilde{U}(x, y) = A e^{i(k_x \cdot x + k_y \cdot y)}$$

$$\tilde{U}(x, y) \approx \frac{A}{z_0} e^{ik \frac{x^2 + y^2}{2z_0}} \cdot e^{-ik \frac{xx_0 + yy_0}{z_0}}$$

$$\tilde{U}(x, y) \approx \frac{A}{z_0} e^{-ik \frac{x^2 + y^2}{2z_0}} \cdot e^{ik \frac{xx_0 + yy_0}{z_0}}$$



$$\tilde{U}_{1or2}(x, y) = \frac{A_{1or2}}{D} e^{ik \frac{x^2+y^2}{2z_0}} \cdot e^{-ik \frac{xx_0+yy_0}{z_0}} = A e^{ik \frac{x^2+y^2}{2z_0}} \cdot e^{-ik \frac{xx_0+yy_0}{z_0}}$$

$$\tilde{U}_1 : x_0 = \frac{d}{2}, y_0 = 0, z_0 = D \quad ; \quad \tilde{U}_2 : x_0 = -\frac{d}{2}, y_0 = 0, z_0 = D$$

叠加场波前函数： $\tilde{U} = \tilde{U}_1 + \tilde{U}_2$

$$\begin{aligned} I(x, y) &= (\tilde{U}_1 + \tilde{U}_2) \cdot (\tilde{U}_1 + \tilde{U}_2)^* \\ &= \tilde{U}_1 \cdot \tilde{U}_1^* + \tilde{U}_2 \cdot \tilde{U}_2^* + \tilde{U}_1 \cdot \tilde{U}_2^* + \tilde{U}_2 \cdot \tilde{U}_1^* \\ &= 2A^2 + 2A^2 \cos \left[k \frac{d}{D} x \right] \\ &= 2I_0 \left[1 + \cos \left(\frac{2\pi d}{\lambda D} x \right) \right] \end{aligned}$$

条纹间隙:

当 $\frac{2\pi d}{\lambda D} x = 2k\pi$, $I(x, y)$ 极大, 为亮条纹。

亮条纹的位置: $x = \frac{D}{d} k\lambda$

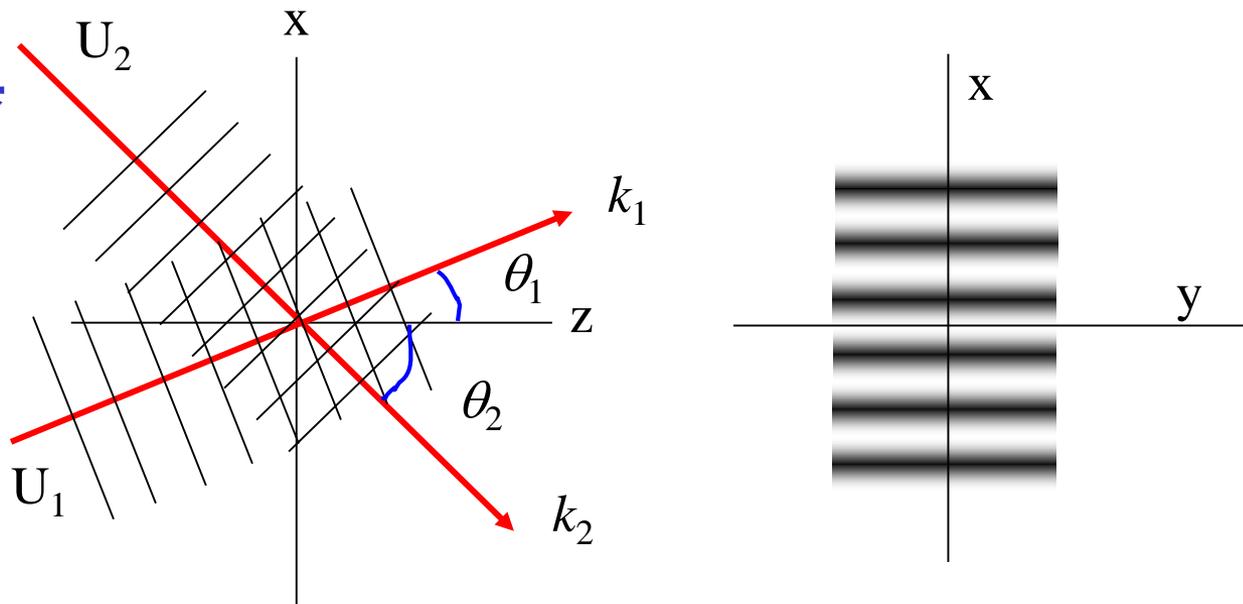
当 $\delta = \frac{2\pi d}{\lambda D} x = (2k + 1)\pi$, $I(x, y)$ 极小, 为暗条纹。

暗条纹的位置: $x = \frac{D}{d} (k + \frac{1}{2})\lambda$

条纹间距 Δx 为两条相邻亮条纹或暗条纹之间的距离,

$$\Delta x = \frac{D}{d} \lambda$$

例题：平行光干涉



$$\tilde{U}_1(x, y) = A_1 e^{i(k \sin \theta_1 \cdot x - \phi_{10})} \quad \tilde{U}_2(x, y) = A_2 e^{i(-k \sin \theta_2 \cdot x - \phi_{20})}$$

$$\begin{aligned} I(x, y) &= (\tilde{U}_1 + \tilde{U}_2) \cdot (\tilde{U}_1 + \tilde{U}_2)^* \\ &= \tilde{U}_1 \cdot \tilde{U}_1^* + \tilde{U}_2 \cdot \tilde{U}_2^* + \tilde{U}_1 \cdot \tilde{U}_2^* + \tilde{U}_2 \cdot \tilde{U}_1^* \\ &= \underline{A_1^2} + \underline{A_2^2} + 2A_1A_2 \cos \left[\underline{k(\sin \theta_1 + \sin \theta_2)x - (\phi_{10} - \phi_{20})} \right] \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \underline{\delta(x, y)} \end{aligned}$$

The diagram includes blue arrows pointing from the underlined A_1^2 and A_2^2 terms to I_1 and I_2 respectively. A red arrow points from the underlined phase term in the third line to the underlined $\delta(x, y)$ term in the final line.

$$\gamma = \frac{I_M - I_m}{I_M + I_m} \quad \longrightarrow \quad \gamma = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$$

$$I = I_0(1 + \gamma \cos \delta(x, y)), \quad I_0 = I_1 + I_2$$

条纹间距：

$$\Delta\delta = 2\pi, \text{ 即: } k(\sin\theta_1 + \sin\theta_2)\Delta x = 2\pi, \quad \left(k = \frac{2\pi}{\lambda}\right)$$

$$\Delta x = \frac{\lambda}{\sin\theta_1 + \sin\theta_2}$$

空间频率概念：

条纹间距的倒数被定义为空间频率，记着 f ，常用单位 mm^{-1}

$$f = \frac{1}{\Delta x}$$

两平行光干涉的空间频率：

$$f = \frac{\sin \theta_1 + \sin \theta_2}{\lambda}$$

干涉场强度空间分布可以用空间频率改写成：

$$I = I_0 \left(1 + \gamma \cos(2\pi f x + \phi_0) \right), \quad I_0 = I_1 + I_2$$

例题1、两束相干的平行光，传播方向角 $\theta_1=\pi/6$ ， $\theta_2=\pi/4$ ，光波长为633nm，求干涉条纹的间距和空间频率？

间距：

$$\Delta x = \frac{\lambda}{\sin \theta_1 + \sin \theta_2} = \frac{633nm}{\sin \frac{\pi}{6} + \sin \frac{\pi}{4}} \approx 0.53 \mu m$$

空间频率：

$$f = \frac{1}{\Delta x} = \frac{1}{0.53 \mu m} \approx 1896 mm^{-1}$$

例题2、想获得低频 $f=20\text{mm}^{-1}$ ，求两平行光的夹角，设光波长为 633nm 。

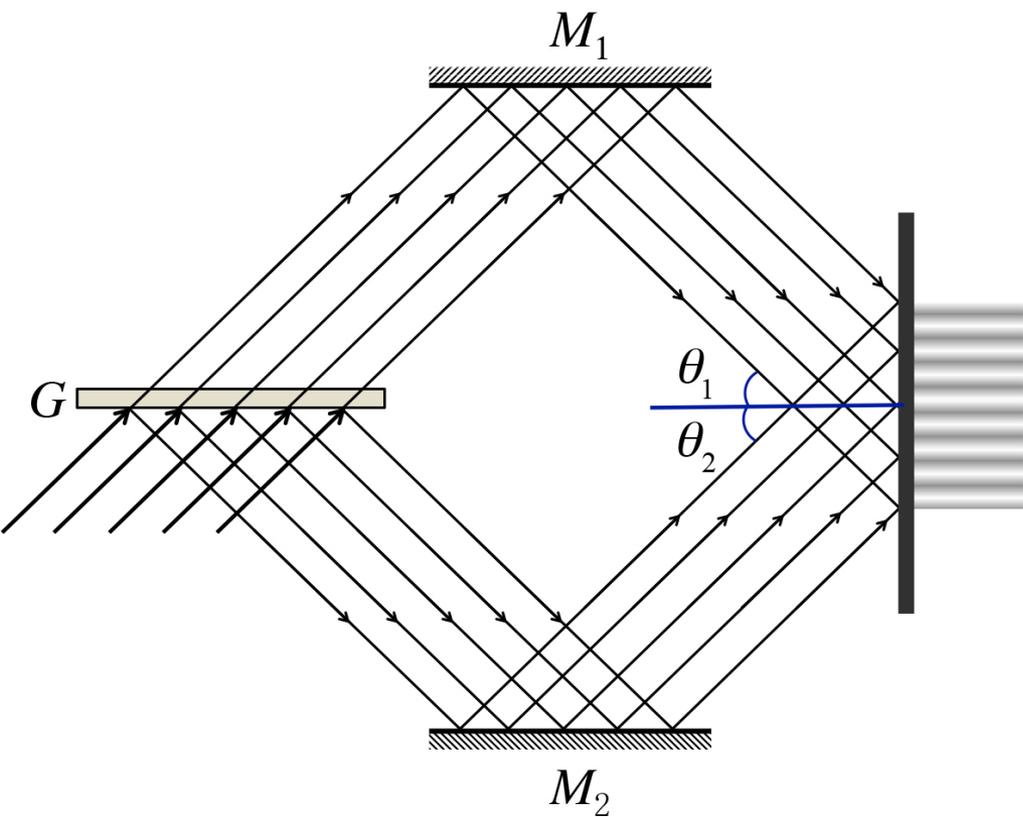
$$f = \frac{\sin \theta_1 + \sin \theta_2}{\lambda} \approx \frac{\theta_1 + \theta_2}{\lambda} = \frac{\Delta \theta}{\lambda}$$

于是：

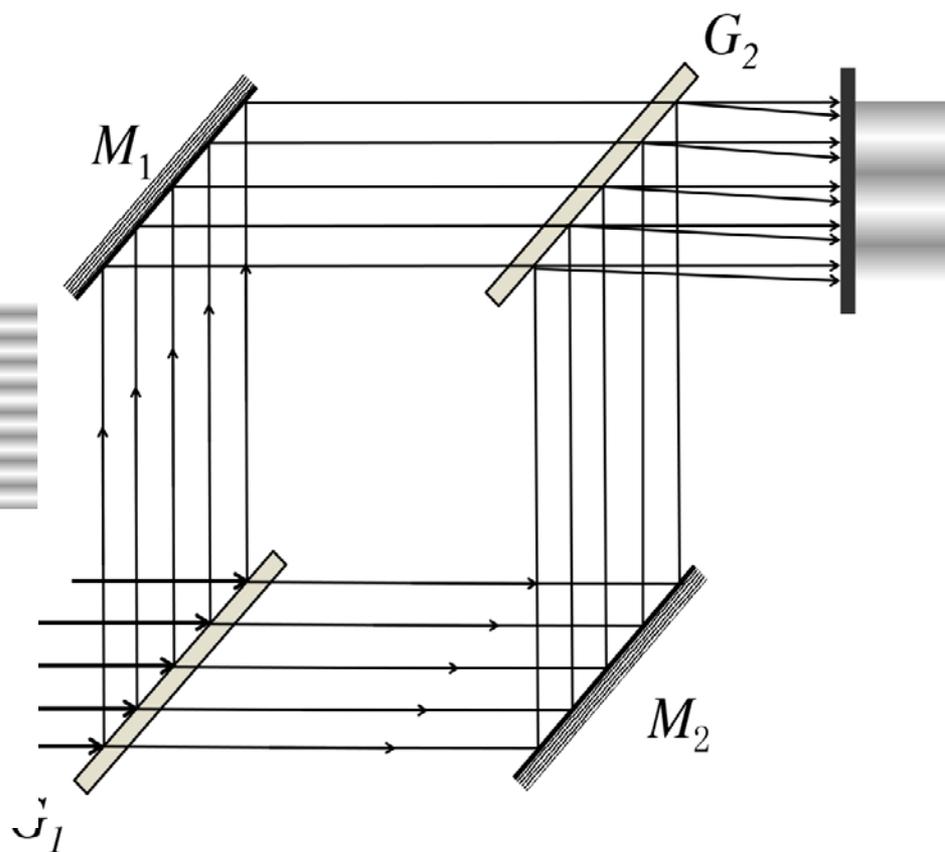
$$\Delta \theta \approx f \lambda = 20 \times 633 \frac{\text{nm}}{\text{mm}} \approx 0.013 \text{rad} \approx 45'$$

大夹角 \Leftrightarrow 高空间频率，小夹角 \Leftrightarrow 低空间频率

两种典型的光路获得空间高频和低频：



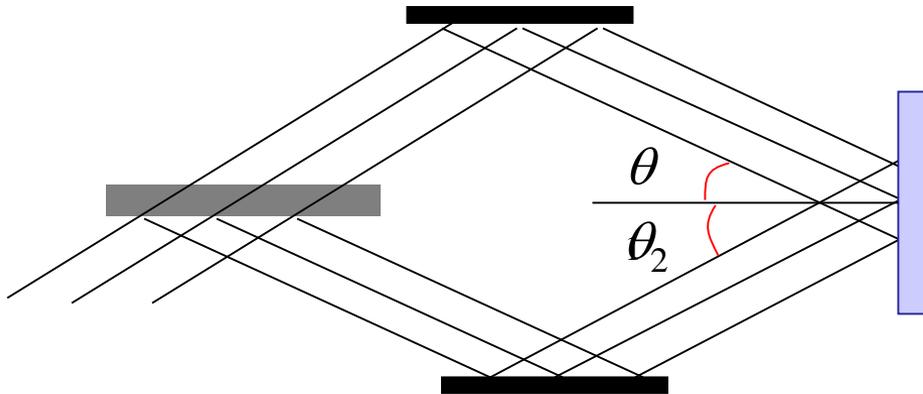
高频



低频

2、余弦光栅的衍射场

余弦光栅的制备：



$$I(x, y) = I_0 (1 + \gamma \cos(2\pi fx + \phi_0)) , \quad f = \frac{\sin \theta_1 + \sin \theta_2}{\lambda}$$

用干板记录，通过显影和定影。

干板底片的透过率函数为：

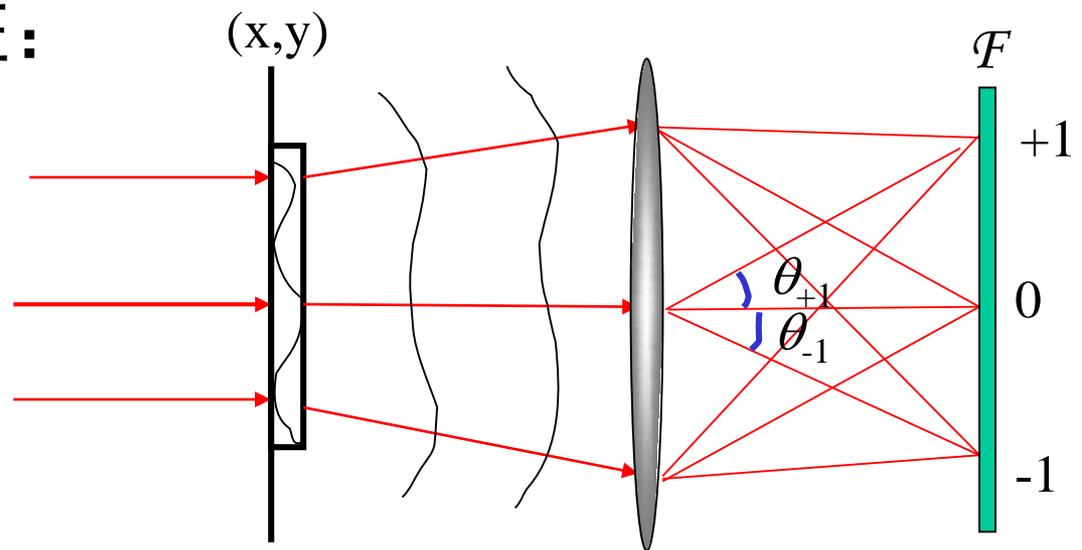
$t(x, y) \propto I(x, y)$ ，即：

$$t(x, y) = \alpha + \beta I(x, y) = t_0 + t_1 \cos(2\pi fx + \phi_0)$$

余弦光栅的衍射特征：

平面波正入射，其入射波前为：

$$\tilde{U}_1(x, y) = A_1$$



经过余弦光栅后的透射波前为：

$$\tilde{U}_2(x, y) = \tilde{t}(x, y)\tilde{U}_1(x, y) = A_1 [t_0 + t_1 \cos(2\pi fx + \phi_0)]$$

$$= A_1 \left[t_0 + t_1 \left(\frac{e^{i(2\pi fx + \phi_0)} + e^{-i(2\pi fx + \phi_0)}}{2} \right) \right]$$

$$= A_1 t_0 + \frac{1}{2} A_1 t_1 e^{i(2\pi fx + \phi_0)} + \frac{1}{2} A_1 t_1 e^{-i(2\pi fx + \phi_0)} = \tilde{U}_0 + \tilde{U}_{+1} + \tilde{U}_{-1}$$



所以衍射方向：

$$\tilde{U}_0 = A_1 t_0$$

0级为正出射的平面波，衍射角为0；

$$\tilde{U}_{+1} = \frac{1}{2} A_1 t_1 e^{i(2\pi f x + \phi_0)} = \frac{1}{2} A_1 t_1 e^{i \frac{2\pi}{\lambda} (f\lambda)x + i\phi_0} = \frac{1}{2} A_1 t_1 e^{ik \sin \theta_{+1} \cdot x + i\phi_0}$$

+1级 \tilde{U}_{+1} 向上斜出射的平面波，衍射角 θ_{+1} 满足： $\sin \theta_{+1} = f\lambda$

$$\tilde{U}_{-1} = \frac{1}{2} A_1 t_1 e^{-i(2\pi f x + \phi_0)} = \frac{1}{2} A_1 t_1 e^{-i \frac{2\pi}{\lambda} (f\lambda)x - i\phi_0} = \frac{1}{2} A_1 t_1 e^{ik \sin \theta_{-1} \cdot x - i\phi_0}$$

-1级 \tilde{U}_{-1} 向下斜出射的平面波，衍射角 θ_{-1} 满足： $\sin \theta_{-1} = -f\lambda$

最重要的特点：

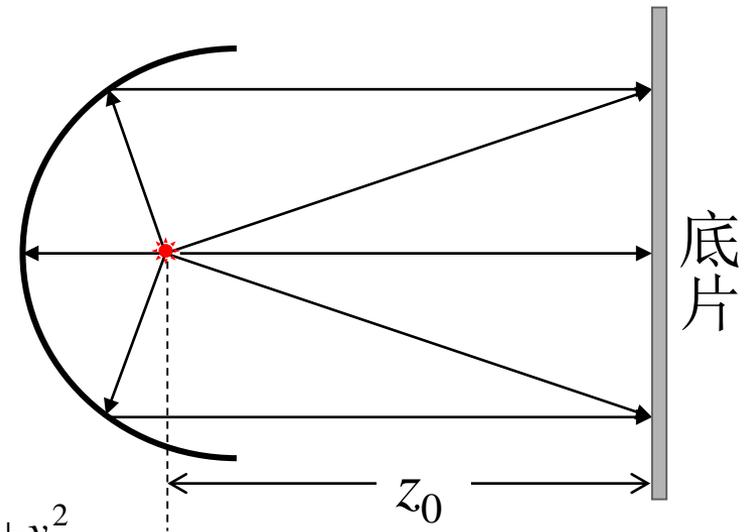
± 1 级衍射斑的方位角与余弦光栅的空间频率对应。

全息学发展

余弦型环状波带片的衍射

余弦型环状波带片的制备：

平面光和球面光干涉，



$$\tilde{U}_1(x, y) = A_1 e^{i\varphi_0}, \quad \tilde{U}_2(x, y) = A_2 e^{ik \frac{x^2 + y^2}{2z_0}}$$

$$\begin{aligned} I(x, y) &= \left(\tilde{U}_1(x, y) + \tilde{U}_2(x, y) \right) \cdot \left(\tilde{U}_1(x, y) + \tilde{U}_2(x, y) \right)^* \\ &= \left(A_1 e^{i\varphi_0} + A_2 e^{ik \frac{x^2 + y^2}{2z_0}} \right) \cdot \left(A_1 e^{-i\varphi_0} + A_2 e^{-ik \frac{x^2 + y^2}{2z_0}} \right) \\ &= A_1^2 + A_2^2 + 2A_1 A_2 \cos \left(k \frac{x^2 + y^2}{2z_0} - \varphi_0 \right) \end{aligned}$$

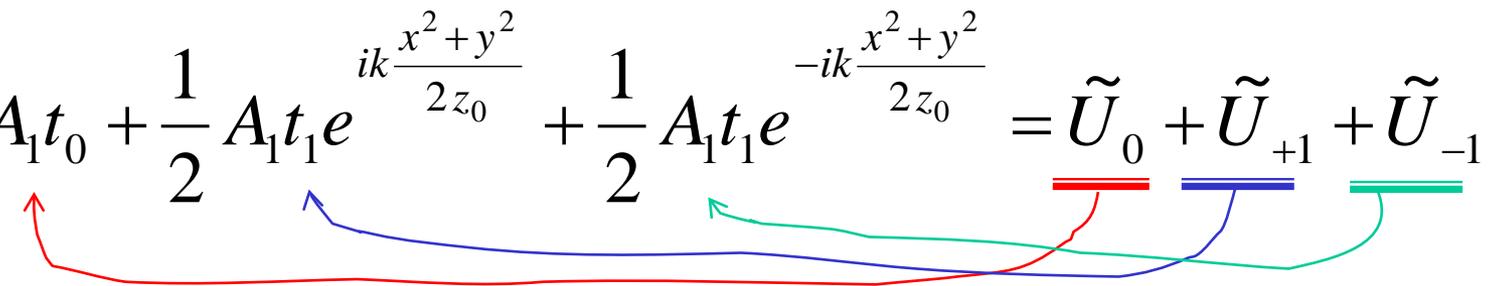
用干板记录，通过显影和定影，干板的透过率为：

$t(x, y) \propto I(x, y)$ 即：

$$t(x, y) = \alpha + \beta I(x, y) = \alpha + \beta \left(A_1^2 + A_2^2 + 2A_1A_2 \cos \left(k \frac{x^2 + y^2}{2z_0} \right) \right)$$
$$= t_0 + t_1 \cos \left(k \frac{x^2 + y^2}{2z_0} \right) \text{ 为振幅型。}$$

平面波正入射，其入射波前为： $\tilde{U}_1(x, y) = A_1$

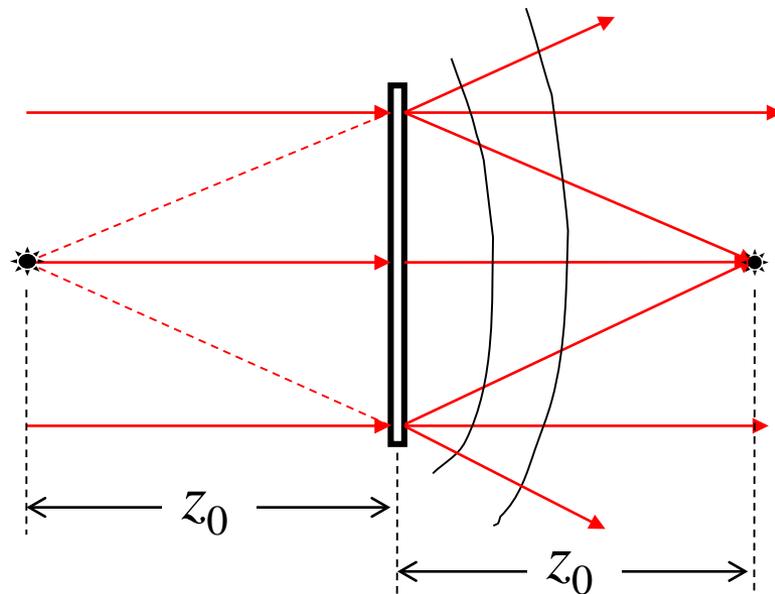
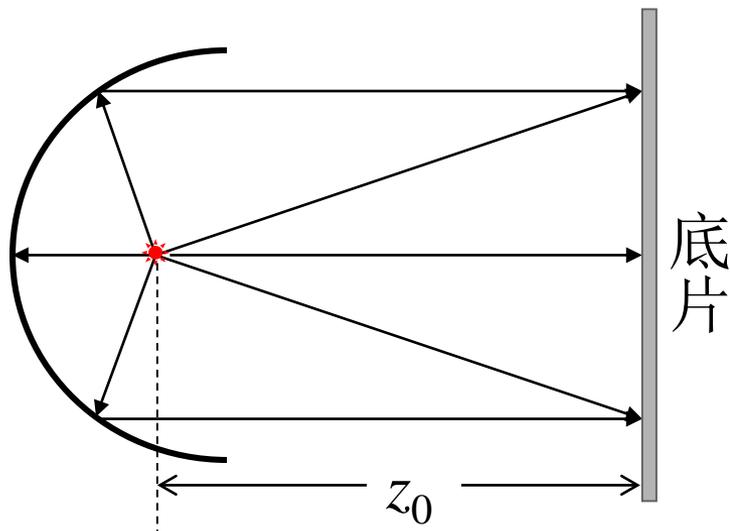
如果入射光和记录光的波长相同，则 $k_1 = k_2 = k$ 。经过环状余弦波带片后，透射波前为：

$$\begin{aligned}\tilde{U}_2(x, y) &= \tilde{t}(x, y)\tilde{U}_1(x, y) = A_1 \left[t_0 + t_1 \cos\left(k \frac{x^2 + y^2}{2z_0}\right) \right] \\ &= A_1 \left[t_0 + t_1 \left(\frac{e^{ik \frac{x^2 + y^2}{2z_0}} + e^{-ik \frac{x^2 + y^2}{2z_0}}}{2} \right) \right] \\ &= A_1 t_0 + \frac{1}{2} A_1 t_1 e^{ik \frac{x^2 + y^2}{2z_0}} + \frac{1}{2} A_1 t_1 e^{-ik \frac{x^2 + y^2}{2z_0}} = \tilde{U}_0 + \tilde{U}_{+1} + \tilde{U}_{-1}\end{aligned}$$


0级 $\tilde{U}_0 = A_1 t_0$ 为正出射的平面波

+1级 $\tilde{U}_{+1} = \frac{1}{2} A_1 t_1 e^{ik \frac{x^2+y^2}{2z_0}}$ 代表发散球面波，发散点： $(0,0,-z_0)$

-1级 $\tilde{U}_{-1} = \frac{1}{2} A_1 t_1 e^{-ik \frac{x^2+y^2}{2z_0}}$ 代表汇聚球面波，发散点： $(0,0,z_0)$



余弦型环状波带片的制备和衍射场

谢谢！