北京师范大学
BEIJING NORMAL UNIVERSITY

Threshold resummation for CGC hadron production

Based on [H.Y.Liu,Kang,Liu, Phys. Rev. D.2020(rapid communication)]

Liu Hao-yu

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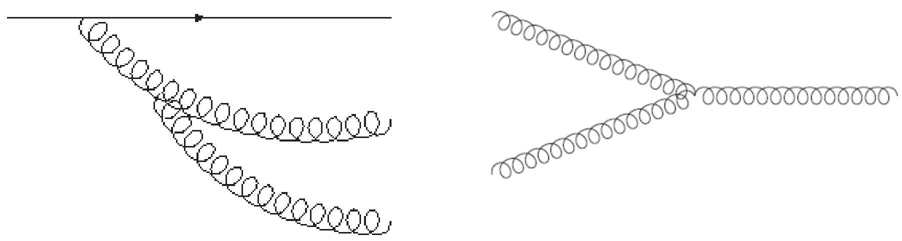
Outline

- CGC effective theory
- pA->hX and negative cross section problem
- Threshold resummation

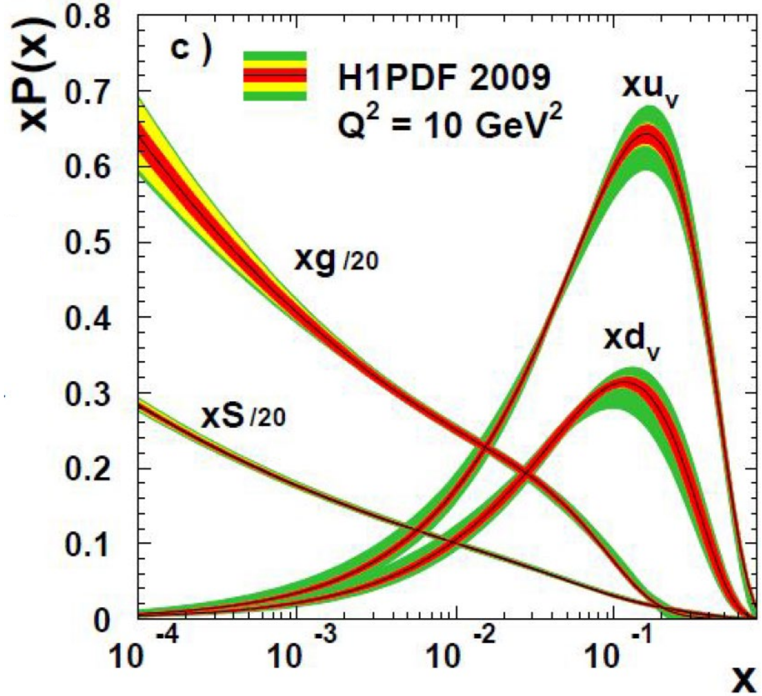
Gluon Saturation

Gluon dominate parton Distribution
for small Bjorken x



splitting recombination

saturation

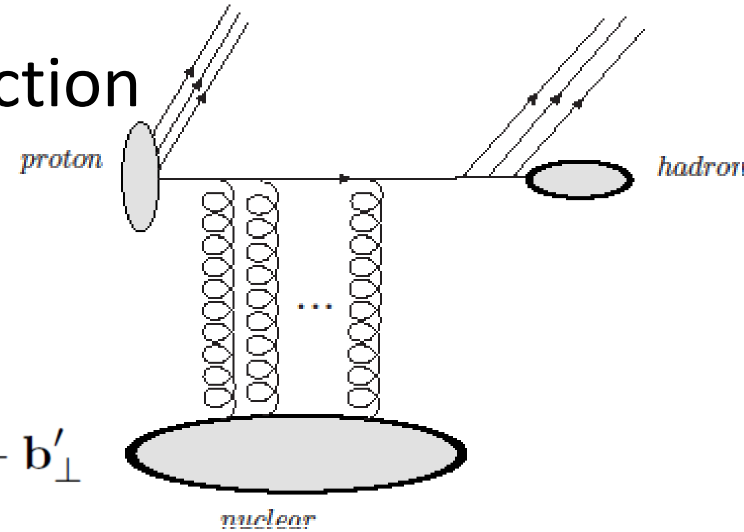


[Gelis, Iancu, Jalilian-Marian, Venugopalan, Ann.Rev.Nucl.Part.Sci,2010]

CGC effective theory (Color Glass Condensate)
is one of the most suitable theory for saturation

Saturation in forward pA collision

Leading order result of hadron production



$$\frac{d\sigma}{dy_h d^2 p_{h,\perp}} = \sum_{i,j=g,q} \frac{1}{4\pi^2} \int \frac{d\xi}{\xi^2} x_p f_{i/P}(x_p,\mu) D_{h/j}(\xi,\mu)$$

$$\times \int d^2 b_\perp d^2 b'_\perp \, e^{ip'_\perp \cdot r_\perp} \left\langle \langle \mathcal{M}_0(b'_\perp) | \mathcal{M}_0(b_\perp) \rangle \right\rangle_\nu$$

where $x_p = \frac{p_{h,\perp} e^{y_h}}{\xi\sqrt{s}}$ $x_A = \frac{p_{h,\perp} e^{-y_h}}{\xi\sqrt{s}}$ $r_\perp = \mathbf{b}_\perp - \mathbf{b}'_\perp$

Hybrid factorization y_h is large $x_A \ll 1$

ν is the factorization scale similar to the μ in collinear factorization

$$\langle M_0(b'_\perp) | M_0(b_\perp) \rangle = \frac{1}{C} Tr[W(\mathbf{b}_\perp) W^\dagger(\mathbf{b}'_\perp)] = S(\mathbf{b}_\perp, \mathbf{b}'_\perp)$$ distribution

$C = N_c$ for quark and N_c^2 for gluon

$$W(\mathbf{x}_\perp) = \mathcal{P}exp\{ig \int_{-\infty}^{+\infty} dx^+ t^c A_c^-(x^+, \mathbf{x}_\perp)\}$$

t^c fundamental for quark

adjoint for gluon

Evolution equation

Dipole amplitude $S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{b}'_\perp) = \frac{1}{N_c}\langle Tr[W(\mathbf{b}_\perp)W^\dagger(\mathbf{b}'_\perp)]\rangle_{X_f}$

$X_f = \nu/p_A^-$ is the dimension less factorization scale

$$\frac{\partial S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{b}'_\perp)}{\partial \ln(1/X_f)} = -\frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2\mathbf{x}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)^2}{(\mathbf{b}_\perp - \mathbf{x}_\perp)^2(\mathbf{b}'_\perp - \mathbf{x}_\perp)^2}[S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{b}'_\perp) - S^{(2)}_{X_f}(\mathbf{b}_\perp, \mathbf{x}_\perp)S^{(2)}_{X_f}(\mathbf{b}'_\perp, \mathbf{x}_\perp)]$$

Balitsky-Kovchegov evolution equation

Dynamic scale Q_s Typical Transverse Momentum Semi-hard

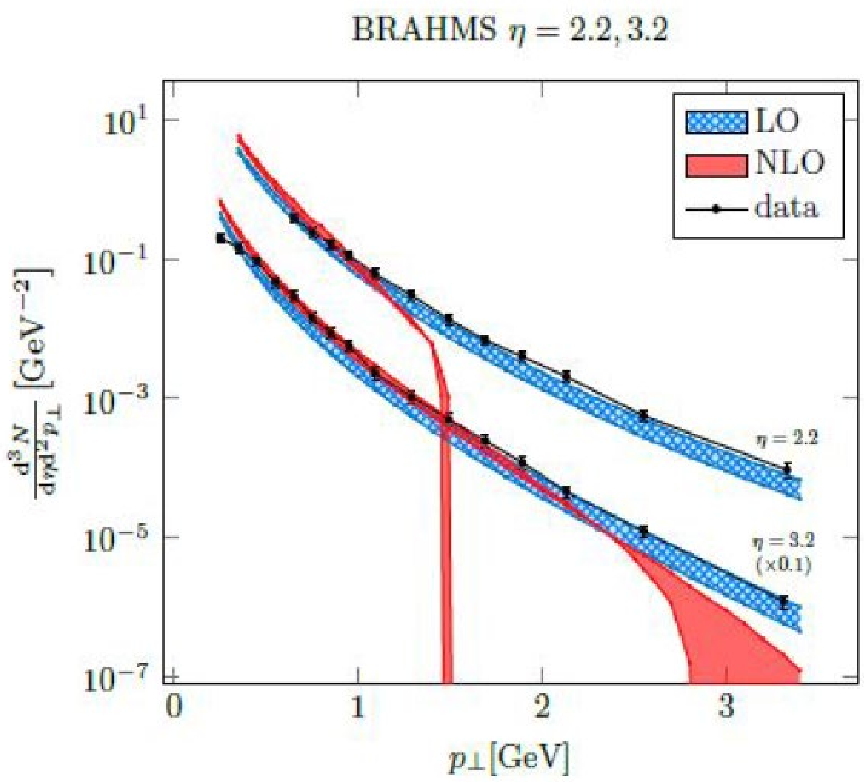
$\alpha_s(Q_s)$ is not very small, higher order calculation is necessary

Negative cross section problem

First NLO calculation for CGC factorization theory

Demonstrate CGC factorization for this process

Cross section becomes Negative when $p_{h,\perp} > Q_s$



[Chirilli,Xiao,Yuan, Phys.Rev.Lett.2012]
[Stasto et al,Phys.Rev.Lett,2014]

Further efforts to resolve this problem

Put in the kinematic constraint

Broaden the region where the cross section is positive

Still becomes negative for medium and large $p_{h,\perp}$ region

[Watanabe, Xiao, Yuan *et al.*PRD, **2015**]

Positive defined result

But give up the power counting and factorization formalism

[Iancu,Mueller,Triantafyllopoulos. JHEP,**2016**]
[Duclou, Lappi, Zhu, Phys. Rev. D **2017**]

Strict factorization framework

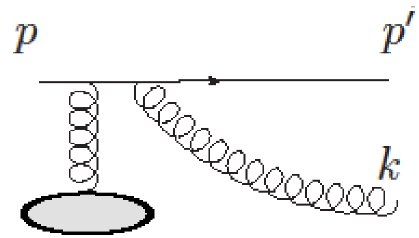
Kinematic constraint terms arise automatically

Further broaden the positive region

Still becomes negative for some region

[H.Y.Liu,Ma,Chao, Phys. Rev. D.2019(rapid communication)]

Dominate terms for large $p_{h,\perp}$



$$
\frac{d^2\hat{\sigma}^{(1)}}{dz d^2 p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)
$$

$$
-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{dx_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{{r'_\perp}^2{r''_\perp}^2}\right.
$$

$$
\left.+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{{r'_\perp}^2{r''_\perp}^2}\right]_+\right\}W_{aa'}(x_\perp)+\ldots
$$

$\mathbf{T}_i^a$ Catani color operator

$\mathbf{T}_i^2 = C_F$ or N_c

$\frac{\bar{n}\cdot p'}{\bar{n}\cdot p}=z \qquad \frac{\bar{n}\cdot k}{\bar{n}\cdot p}=1-z$

$\tilde{P}_{i\to i}(z)$ Splitting function

When $p_{h,\perp}$ is large, especially in forward region

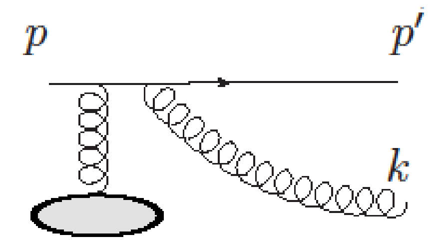
where y_h is large, $x_p = p_{h,\perp}e^{y_h}/\xi\sqrt{s} \to 1 \quad x_p < z < 1$

When $z\to 1$, $\tilde{P}_{i\to i}(z)\to\frac{2}{(1-z)_+}$

Threshold region

Threshold contribution

Dominate terms for large pT



$$\frac{\mathrm{d}^2\hat{\sigma}^{(1)}}{\mathrm{d}z\mathrm{d}^2p'_\perp} \propto -\frac{\alpha_s}{2\pi}\mathbf{T}_i^2 P_{i\to i}(z)\ln\frac{r_\perp^2\mu^2}{c_0^2}\left(1+\frac{1}{z^2}e^{i\frac{1-z}{z}p'_\perp\cdot r_\perp}\right)$$

$$-\frac{\alpha_s}{\pi}\mathbf{T}_i^a\mathbf{T}_j^{a'}\int\frac{\mathrm{d}x_\perp}{\pi}\left\{\frac{1}{z}\tilde{P}_{i\to i}(z)\,e^{i\frac{1-z}{z}p'_\perp\cdot r'_\perp}\frac{r'_\perp\cdot r''_\perp}{{r'_\perp}^2{r''_\perp}^2}\right.$$

$$\left.+\delta(1-z)\ln\frac{X_f}{X_A}\left[\frac{r_\perp^2}{{r'_\perp}^2{r''_\perp}^2}\right]_+\right\}W_{aa'}(x_\perp)+\ldots$$

$$\frac{\bar{n}\cdot p'}{\bar{n}\cdot p}=z \qquad \frac{\bar{n}\cdot k}{\bar{n}\cdot p}=1-z$$

$$\tilde{P}_{i\to i}(z)\to\frac{2}{(1-z)_+}$$

$$x_p=p_{h,\perp}e^{y_h}/\xi\sqrt{s}\to 1$$

$$x_p<z<1$$

The threshold contribution is proportional to $\dfrac{f(x_p/z)-f(x_p)}{1-z}$

Because the PDF decreases rapidly when x_p is large, $f(x_p/z)\ll f(x_p)$ even when z is not far from 1

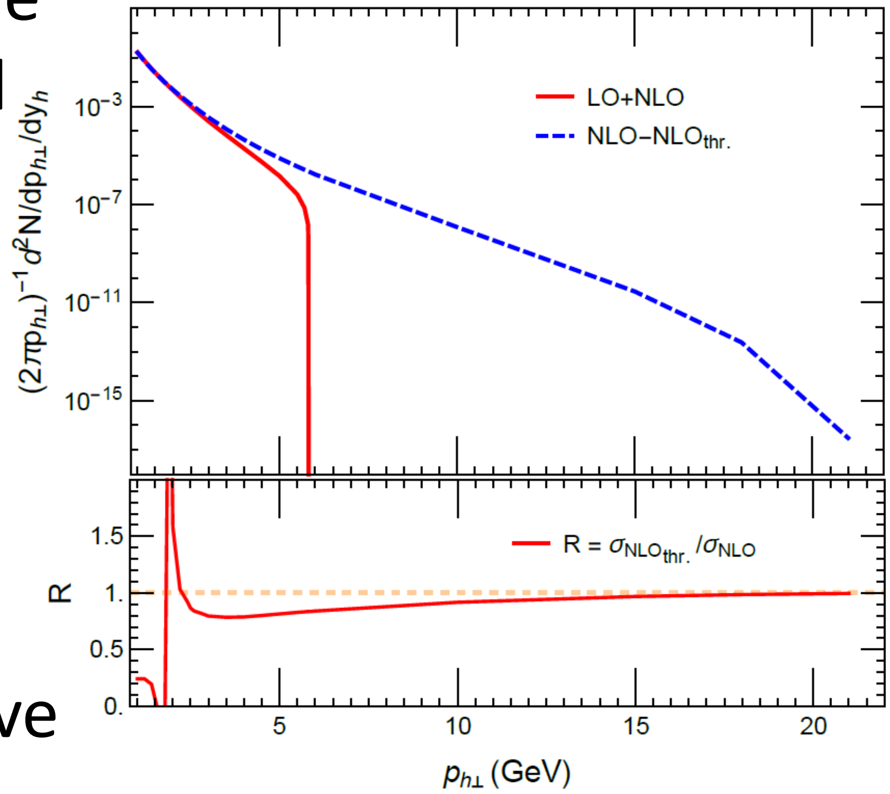
$\dfrac{f(x_p/z)-f(x_p)}{1-z}\to-\dfrac{f(x_p)}{1-z}$ and becomes a large log and is negative

Evaluate the threshold contribution

The cross section remains positive after threshold part is subtracted

The threshold part dominate the NLO part at large $p_{h,\perp}$

The numerical study indicates that the threshold contribution is the main reason for the negative cross section problem



$$1-\alpha_s L \rightarrow e^{-\alpha_s L}$$

Resummation of large log may solve negative cross section problem

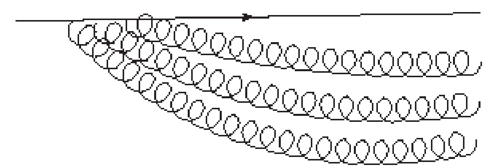
The factorization formula

We firstly reexamine the factorization formula by power counting

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y_h \mathrm{d}^2 p_{h\perp}} = \sum_{i,j=g,q} \frac{1}{4\pi^2} \int \frac{\mathrm{d}\xi}{\xi^2} \frac{\mathrm{d}x}{x} z x f_{i/P}(x,\mu) D_{h/j}(\xi,\mu)$$

$$\times \int \mathrm{d}^2 b_\perp \mathrm{d}^2 b'_\perp \, e^{i p'_\perp \cdot r_\perp} \left\langle \langle \mathcal{M}_0(b'_\perp) | \, \mathcal{J}(z,\mu,\nu,b_\perp,b'_\perp) \mathcal{S}(\mu,\nu,b_\perp,b'_\perp) | \mathcal{M}_0(b_\perp) \rangle \right\rangle$$

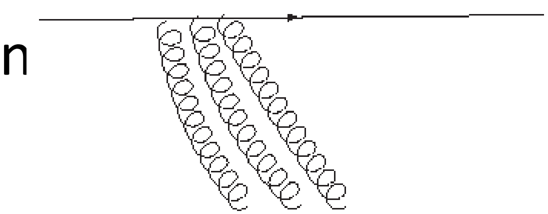
$\mathcal{J}$ Jet function Contribution from Collinear radiation



Gluon in forward direction with momentum

$$\sqrt{s}(1,\lambda^2,\lambda) \qquad \lambda \sim p_{h,\perp}/\sqrt{s} \ll 1$$

$\mathcal{S}$ Soft function Contribution from soft radiation



Gluon in central direction with momentum

$$\sqrt{s}(\lambda,\lambda,\lambda)$$

The soft function is responsible for the kinematic constraint missed in previous calculation

$\nu \propto X_f p_A^-$ is the scale to sperate forward and central momentum

Large log and evolution

For the threshold region $z \to 1$ $\quad \bar{n}\cdot k = \bar{n}\cdot p(1-z) \sim p'_\perp$

real emitted gluon $(\bar{n}\cdot k, n\cdot k, k_\perp) \sim \sqrt{s}(\lambda,\lambda,\lambda)$ soft

$\mathcal{J}$ Contains only virtual correction contribution

$\mathcal{S}$ Contains real correction contribution

$\mathcal{J}$ and $\mathcal{S}$ can be calculated perturbatively

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \ldots$$

$$S^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_s}\right) + \alpha_s D_s(\nu_s) + \ldots$$

We reproduce the full fixed order results with $z \to 1$

$D_s(\nu_s)$ contains $\ln(\nu_s/\bar{n}\cdot p)$, $\ln(\nu_s/p'_\perp)$ and $\dfrac{1}{(1-z)_+}$

$\nu_J = \bar{n}\cdot p \qquad \nu_s \sim p'_\perp \qquad p'_\perp \ll \bar{n}\cdot p$

So the evolution equation is $\nu \dfrac{d}{d\nu}\mathcal{F}(\nu) = \gamma_{\mathcal{F}}\mathcal{F}(\nu)$ $\quad \mathcal{F} = \mathcal{J}$ or $\mathcal{S}$

Leading log result

$$J^{(1)} \propto \alpha_s \ln\left(\frac{\nu}{\nu_J}\right) + \alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \dots$$

$$J^{(0)} + J^{(1)} \propto (1+\alpha_s \ln\left(\frac{\nu}{\nu_J}\right))(1+\alpha_s \ln\left(\frac{\nu_J}{\bar{n}\cdot p}\right) + \dots)$$

All order

All order

Evolution kernel $U_{\mathcal{F}}(\nu,\nu_{\mathcal{F}})$

$\mathcal{F}(\nu_{\mathcal{F}})$ Initial condition

$$\mathcal{F}(\nu) = U_{\mathcal{F}}(\nu,\nu_{\mathcal{F}})\,\mathcal{F}(\nu_{\mathcal{F}})$$

Leading log result

$$\boldsymbol{U}_J \boldsymbol{U}_S = \exp\left[-\frac{\alpha_s}{\pi}\int \frac{\mathrm{d}x_\perp}{\pi}\left(\ln\frac{\nu_S}{\nu_J} I_{BK,r} + \ln\frac{X_f}{X_A} I_{BK}\right)\mathbf{T}_i^a \mathbf{T}_j^{a'} W_{aa'}(x_\perp)\right]$$

$$d\sigma \propto \mathbf{U}_J \mathbf{U}_S \bigotimes S^{(2)}$$

$$\frac{d\sigma}{dX_f} = 0$$

X_f can be arbitrary value

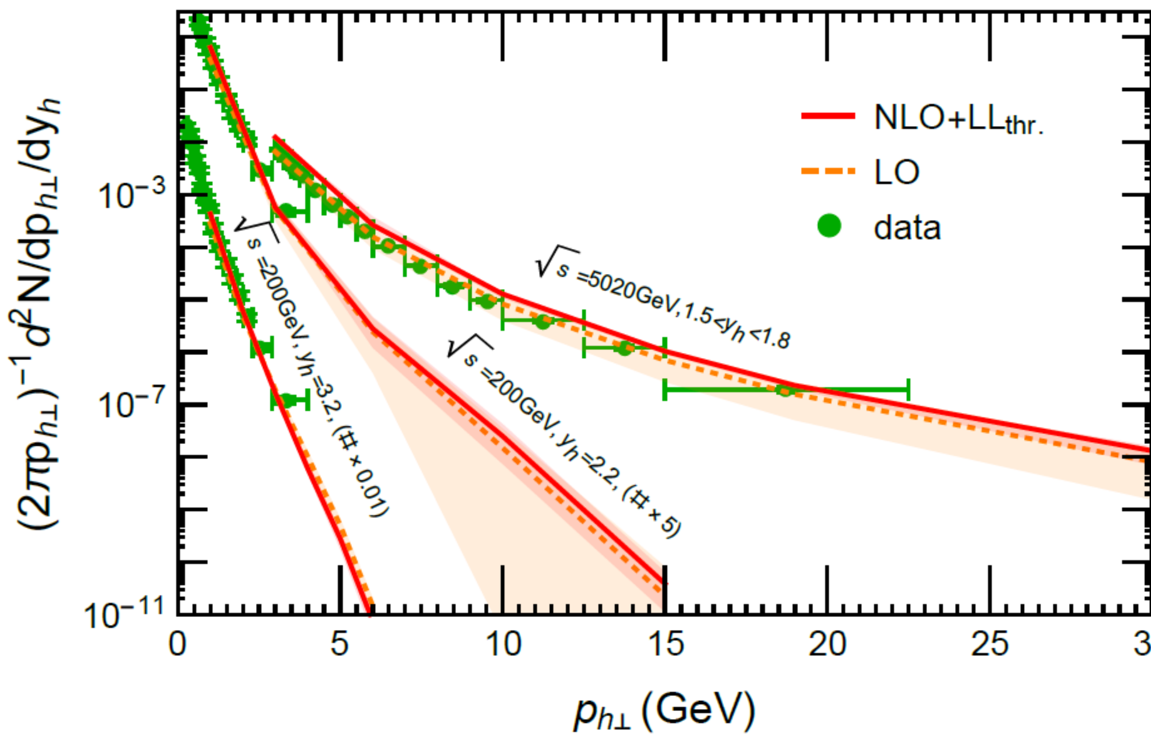
The exponential part is complicated, for simplicity, we take X_f to make $\boldsymbol{U}_J \boldsymbol{U}_S$ be around 1

Phemenology application

NLO+LL resummed result

Positive for the whole range of $p_{h,\perp}$

Consistent with the experiment data



Data from [G. Aad et al. [ATLAS Collaboration], Phys. Lett. B **2016**]

[I. Arsene et al. [BRAHMS Collaboration], PRL. **2004**]

Summary

- The threshold contribution is the main reason of the negative cross section problem
- The threshold log can be resumed by our factorization formalism
- The numerical NLO+LL result stays positive and is consistent with the experimental data
- Our method can be used to calculate other high order result by CGC factorization

Thank You!