Fit to mass spectrum with various interference configurations

A Toy MC study

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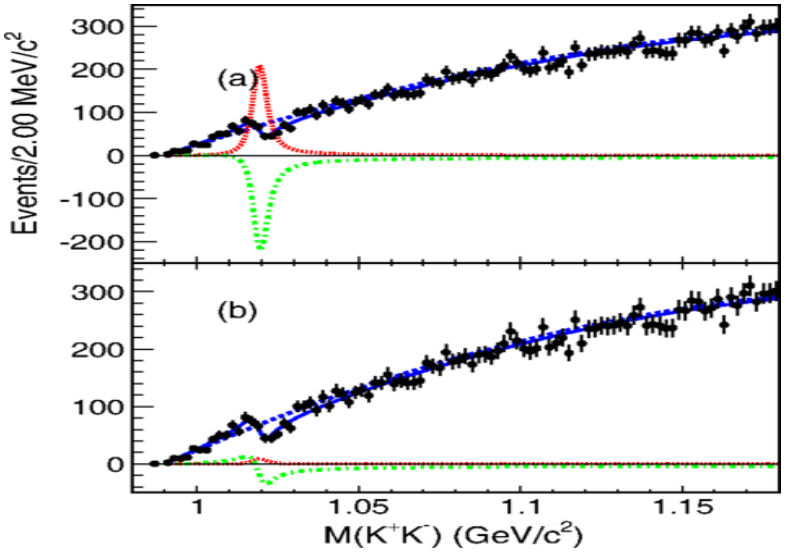
Introduction: interference effects

- Interference may change the observed mass spectrum significantly: asymmetry shape, dip etc.
- When there are such signs, a fit include interference may help us understand data

$J/\psi \to \phi\pi^0, \phi \to K^+K^-$



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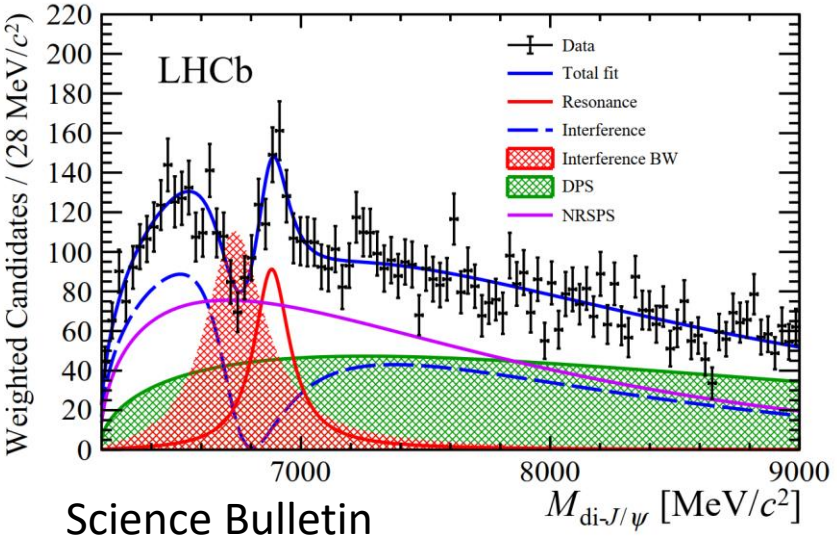


Science Bulletin

arXiv: 2006.16957

Pdf for interfering resonances

- Interfering resonances
  - Ideal case

$$\mathrm{pdf}(m) = |S_1(m) + f_{S_2} \cdot \exp(i\phi_{S_2}) \cdot S_2|^2$$

where S_i is Breit-Wigner function, f_{S_i} and ϕ_{S_i} are the coefficient and relative phase.

  - Real case
    - Include resolution

$$\mathrm{pdf}(m) = |S_1(m) + f_{S_2} \cdot \exp(i\phi_{S_2}) \cdot S_2|^2 \otimes R$$

R is the resolution function.

    - Include efficiency

$$\mathrm{pdf}(m) = |S_1(m) + f_{S_2} \cdot \exp(i\phi_{S_2}) \cdot S_2|^2 \otimes R \cdot \varepsilon(m)$$

$\varepsilon(m)$ is the efficiency variation.

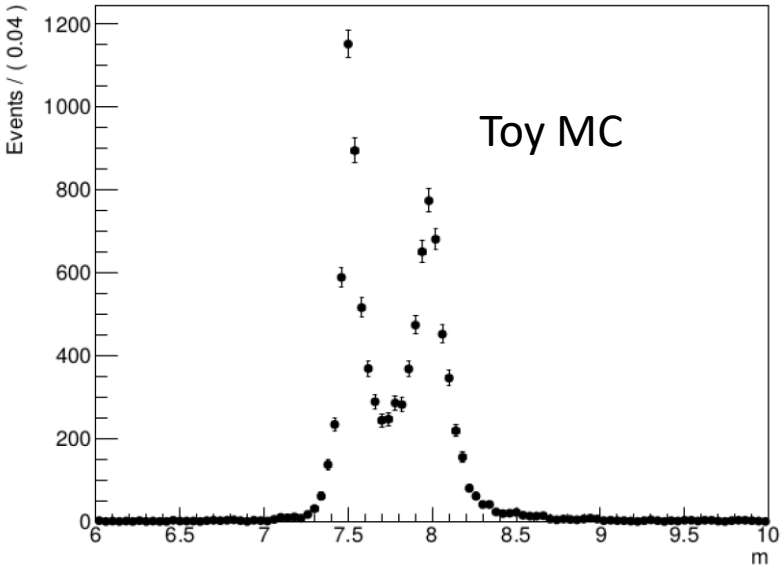
Toy MC of two interfering resonances

- Pdf for generation

$$\mathrm{pdf}(m) = |S_1(m) + f_{S_2} \cdot \exp(i\phi_{S_2}) \cdot S_2|^2$$

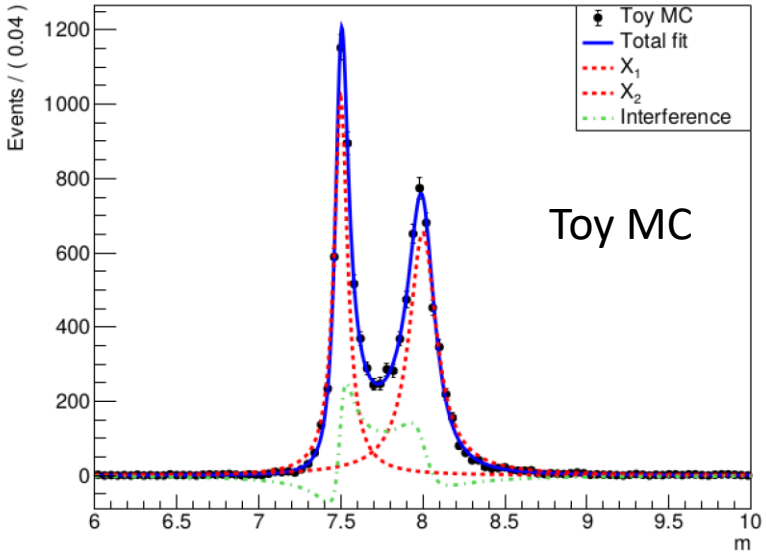
- Generation parameters and toy MC

	M (GeV)	Γ (GeV)	Coef	ϕ
X_1	7.5	0.1	1.0 (fix)	0.0 (fix)
X_2	8.0	0.2	0.8	2.5



A fit result to the toy MC

- Use the same pdf to fit the toy MC
- The Green dashed line is for the interference



- Fit is perfect, but work doesn't stop at here

Multiple solutions in fits with interference

- When interference is included in the fit, multiple solution is an avoidable result

➢ Two solutions for two Breit-Wigners

A mathematical review on the multiple-solution problem*

Int.J.Mod.Phys.A 26 (2011) 4511-4520

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The recent multiple-solution problem in extracting physics information from a fit to the experimental data in high energy physics is reviewed in a mathematical viewpoint.

➢ 2^N solutions for N + 1 Breit-Wigners

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General mathematical analysis on multiple solutions of interfering resonances combinations

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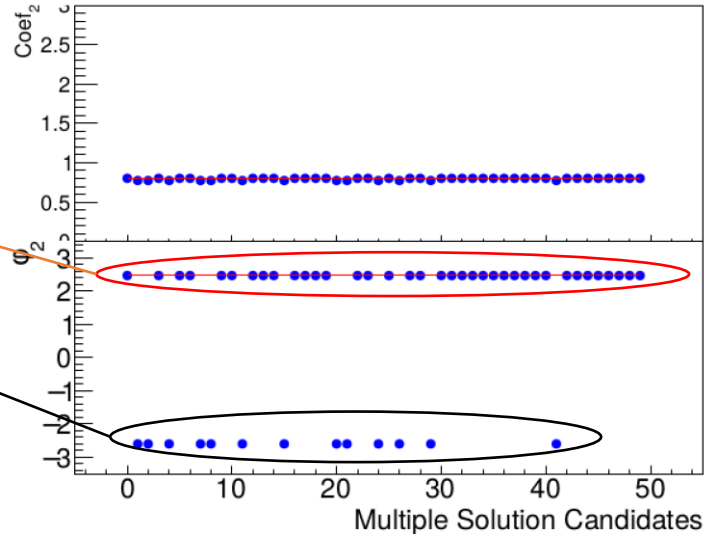
When fitting cross sections with several resonances or interfering background and resonances, one usually obtains multiple solutions of parameters with equal fitting quality. In the present work, we find the source of multiple solutions for a combination of several resonances or interfering background and resonances by analyzing the mathematical structure of the Breit-Wigner function. We find that there are

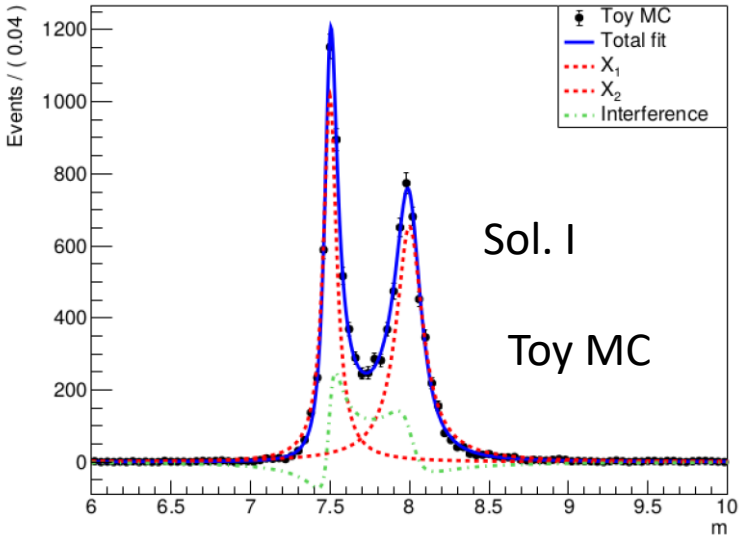
How to find all solutions

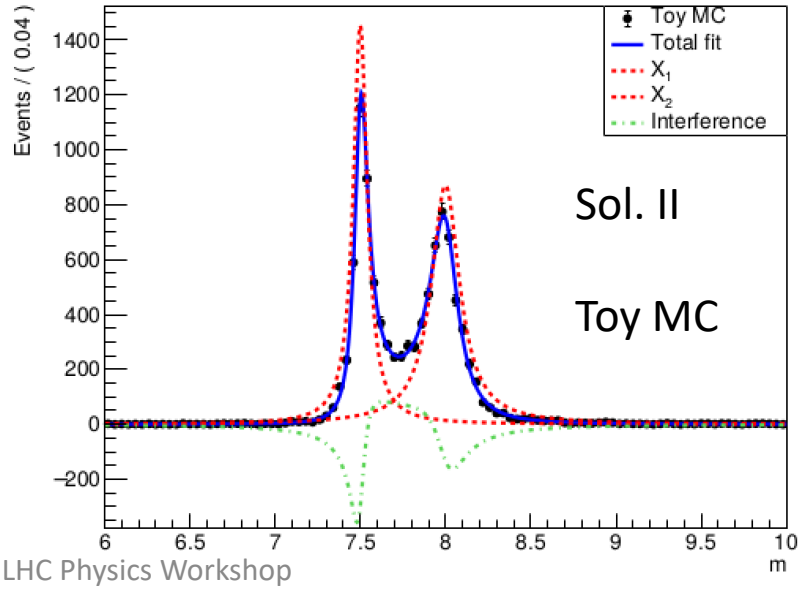
- Multiple solutions have same likelihood –logL and same fit quality. That's why we can not select a 'best' as the unique solution.
- Multiple solutions have different parameters (considering uncertainty), otherwise they are the same solution.
- To find all solutions:
  - Do many independent fits
  - Gather fits with minimum –logL ($\Delta(-\log L) < 0.5$)
  - Check the fit parameters, especially for the coefficient and relative phase

Double solution for two interfering resonances

- Solution I
- Solution II
- Different phase and interference term







Pdf for interfering resonance and NR

- Resonance and coherent non-resonant contribution
  - Replace S_2 with non-resonant (NR)

NR

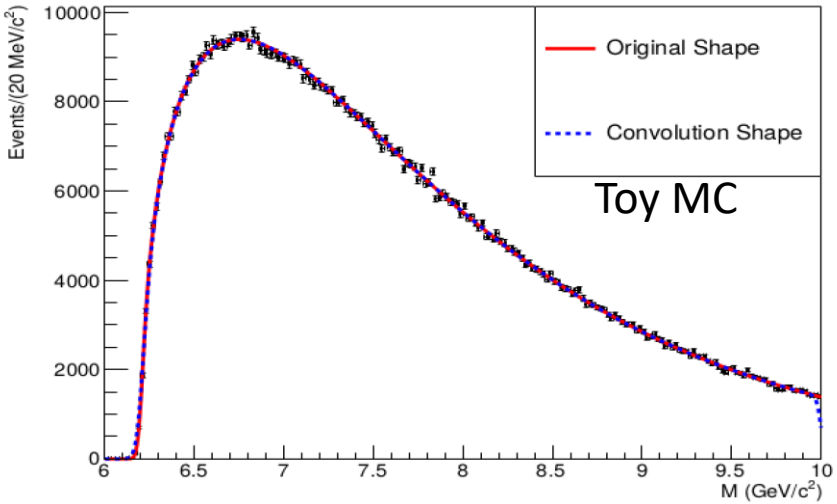
$$\mathrm{pdf}(m) = |S_1(m) + f_{S_2} \cdot \exp(i\phi_{S_2}) \cdot S_2|^2 \otimes R \cdot \varepsilon(m)$$

  - Since NR shape usually obtained at reconstruction level, it already includes resolution and efficiency effects in the shape
    - Remove efficiency: NR/ε
    - Slowly varied shape is not sensitive to the resolution: no special treatment for NR in the coherent sum
  - The final pdf

$$\mathrm{pdf}(m) = \left|S_1(m) + f_{NR} \cdot \exp(i\phi_{NR}) \cdot \sqrt{NR/\varepsilon(m)}\right|^2 \otimes R \cdot \varepsilon(m)$$

Resolution on slowly changed shape

- Slowly changed shape is not sensitive to the resolution
- Red line is for the original shape (include resolution effects)
- Blue dashed line is for the convolution of original shape and a Gaussian
- The shapes for convolution once and twice are same: put the shape obtained at reconstruction level in the coherent sum is a practical approximation if we do not know the true shape.



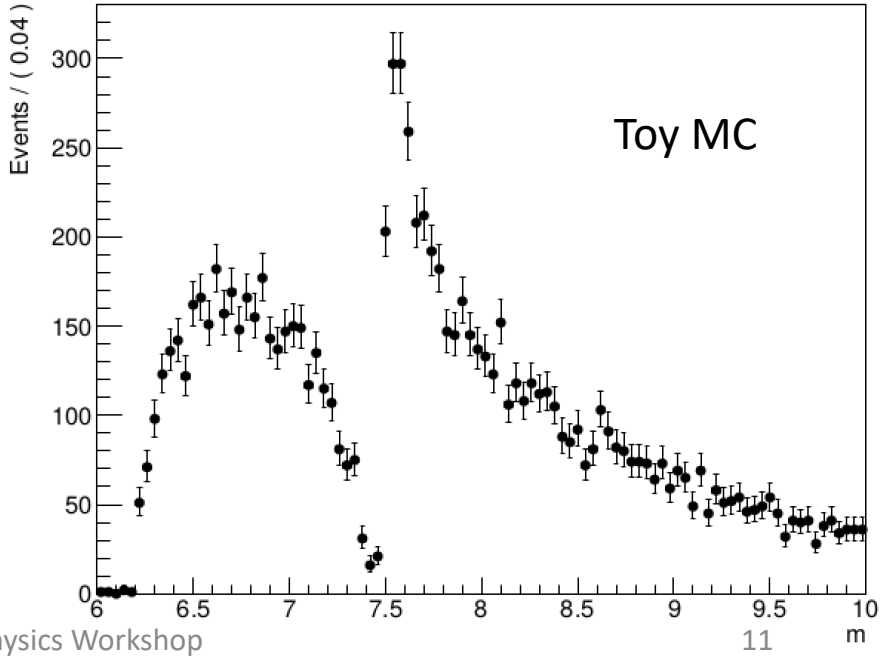
Toy MC of resonance and interfering NR

- Pdf for generation

$$\mathrm{pdf}(m) = \left| S_1(m) + f_{NR} \cdot \exp(i\phi_{NR}) \cdot \sqrt{NR} \right|^2$$

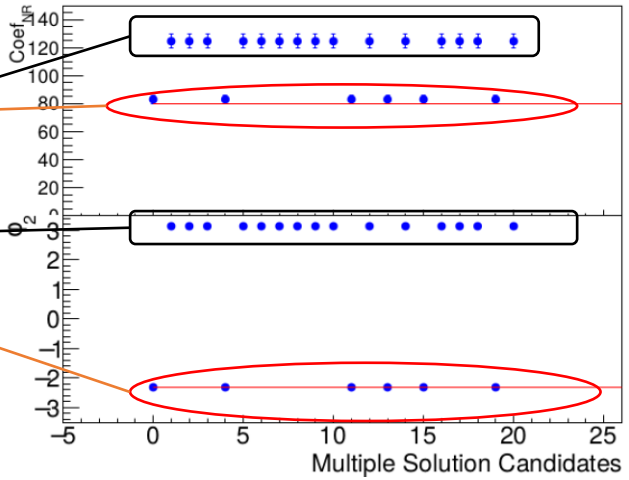
- Generation parameters and toy MC

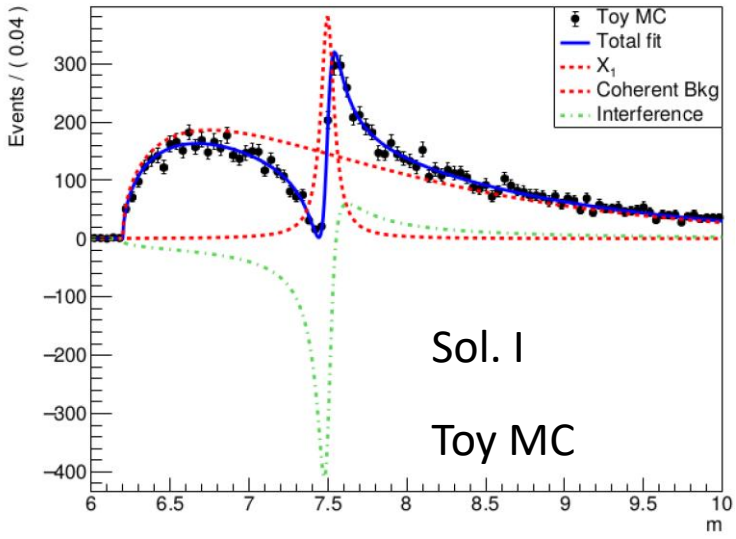
	M (GeV)	Γ (GeV)	Coef	ϕ
X_1	7.5	0.1	1.0 (fix)	0.0 (fix)
NR	–	–	80	-2.3

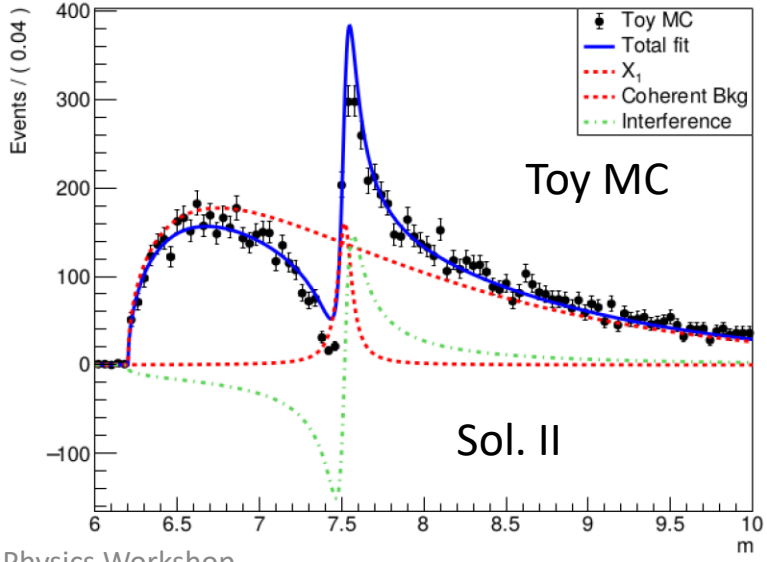


Resonance and interfering NR

- Solution I
- Solution II
- Different phase, coefficient, and interference term







Multiple interfering resonances and NR

- It could happen that resonances and NR interfere with each other
- Combine the first two type pdf

$$\mathrm{pdf}(m) = \left| \sum_i S_i(m) \cdot \exp(i\phi_i) + f_{NR} \cdot \exp(i\phi_{NR}) \cdot \sqrt{NR/\varepsilon(m)} \right|^2 \otimes R \cdot \varepsilon(m)$$

- Incoherent term can be added to these pdf if necessary

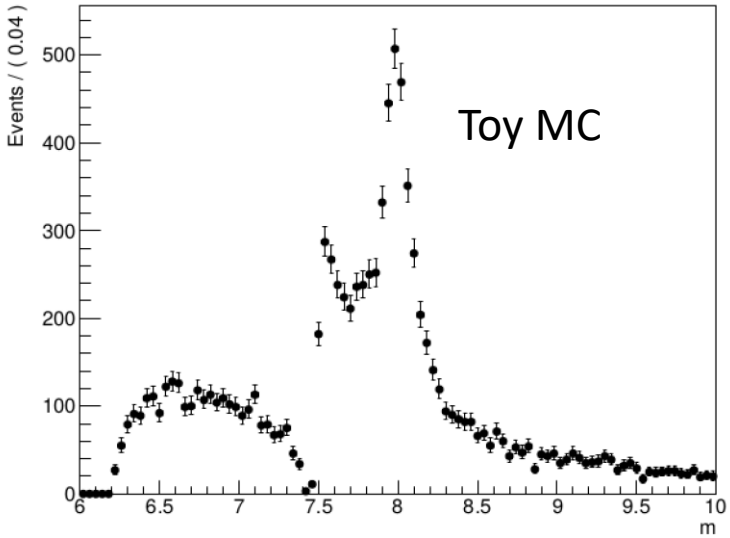
Toy MC for two resonance and interfering NR

- Pdf for generation

$$\mathrm{pdf}(m) = \left| S_1(m) + f_{S_2} \cdot \exp(i\phi_2) \cdot S_2 + f_{NR} \cdot \exp(i\phi_{NR}) \cdot \sqrt{NR} \right|^2$$

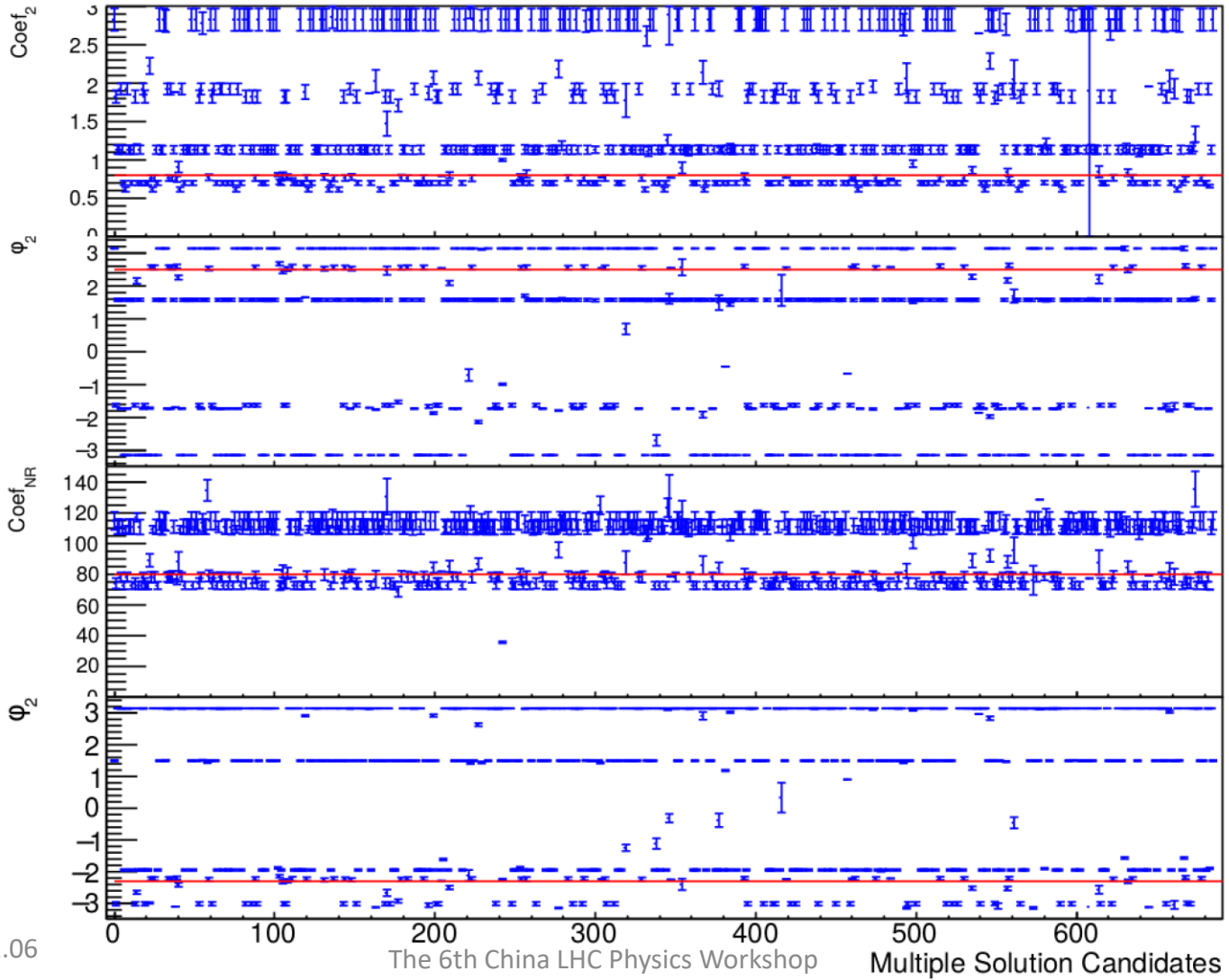
- Generation parameters and toy MC

	M (GeV)	Γ (GeV)	Coef	ϕ
X_1	7.5	0.1	1.0 (fix)	0.0 (fix)
X_2	8.0	0.2	0.8	2.5
NR	–	–	80	-2.3



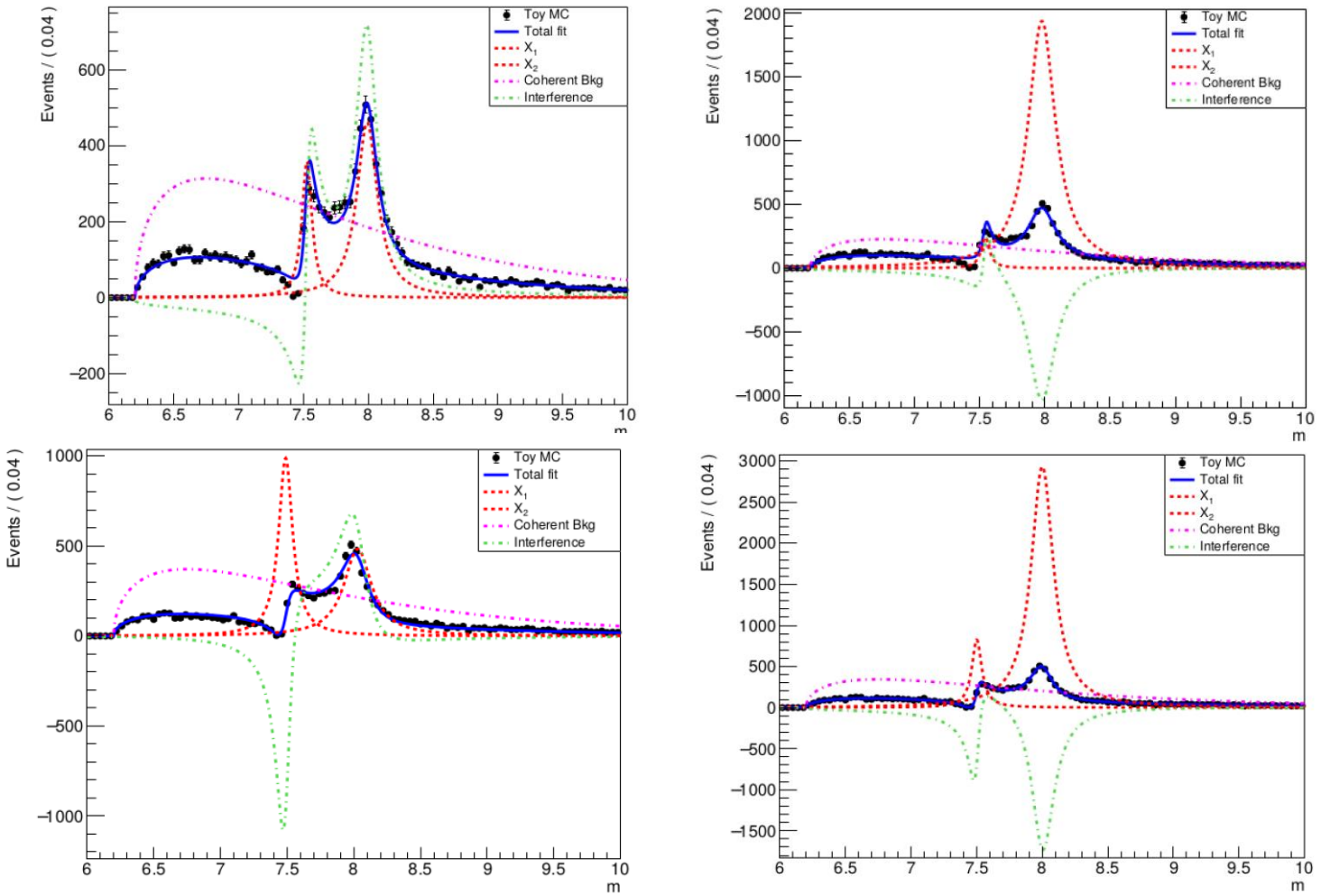
Two resonances and interfering NR

- More than two solutions
- And becomes more difficult to find all solutions



Two resonances and interfering NR

- Fit results for different interference



Summary

- Interference may significantly change invariant mass distribution.

- We present a toy MC study for various interference configurations in mass spectrum fit:

  Pdf functions, multiple solutions, and projections

Thanks