第六届中国LHC物理研讨会（CLHCP2020），2020/11/06

Leading penguin amplitude at NNLO in QCD Factorization

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Outline

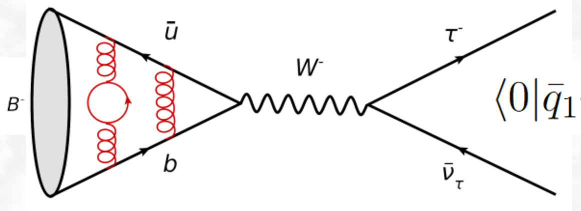
- ☐ Introduction
- ☐ Theoretical framework
- ☐ Leading penguin amplitudes at NNLO
- ☐ Summary

G. Bell, M. Beneke, T. Huber and Xin-Qiang Li, *"Two-loop current-current operator contribution to the non-leptonic QCD penguin amplitude,"* Phys. Lett. B **750** (2015) 348-355 [arXiv:1507.03700 [hep-ph]].

G. Bell, M. Beneke, T. Huber and Xin-Qiang Li, *"Two-loop non-leptonic penguin amplitude in QCD factorization,"* JHEP **04** (2020) 055 [arXiv:2002.03262 [hep-ph]].

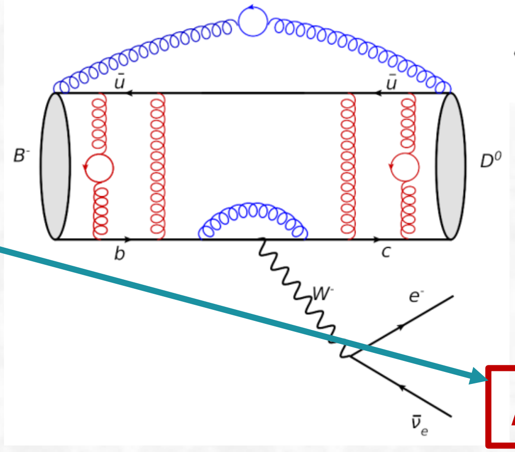
Classification of B decays

- **Purely leptonic decays:** decay constant



$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = i p_\mu f_P$$

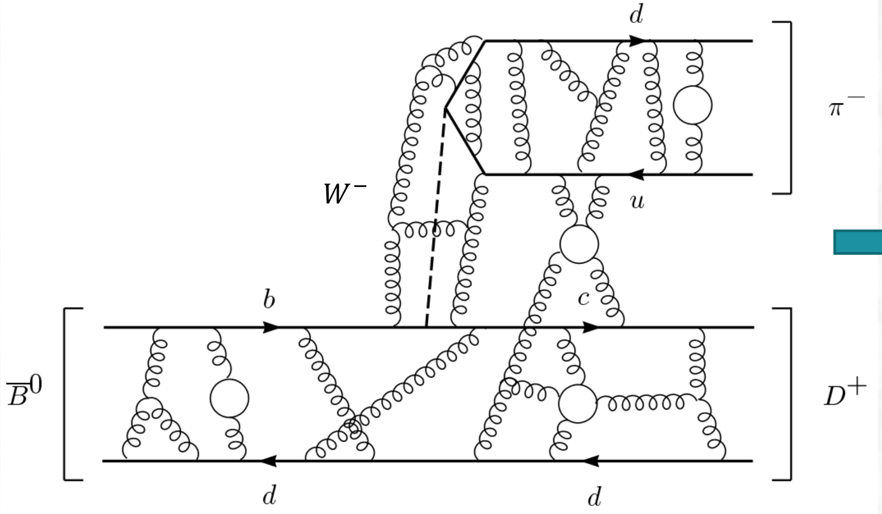
- **Semi-leptonic decays:** transition form factors



$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu + [f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

Lattice QCD or LCSR et al.

- **Hadronic decays:** hadronic matrix elements



multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

How to deal with hadronic decays?

Why hadronic B decays

http://ckmfitter.in2p3.fr/; frequentist

http://utfit.org/UTfit; Bayesian

- **Test the (C)KM mechanism of quark flavor mixings and CP violation;** [J. Zupan, 1903.05062]

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\mathcal{O}(\lambda^3) \qquad \mathcal{O}(\lambda^3) \qquad \mathcal{O}(\lambda^3)$$

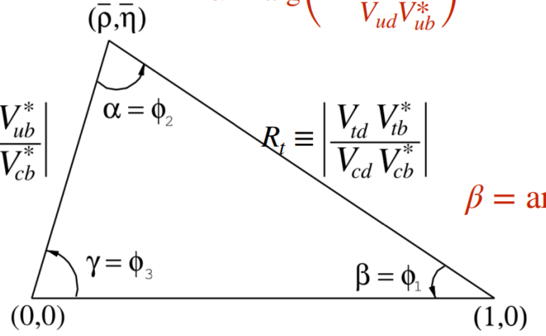
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

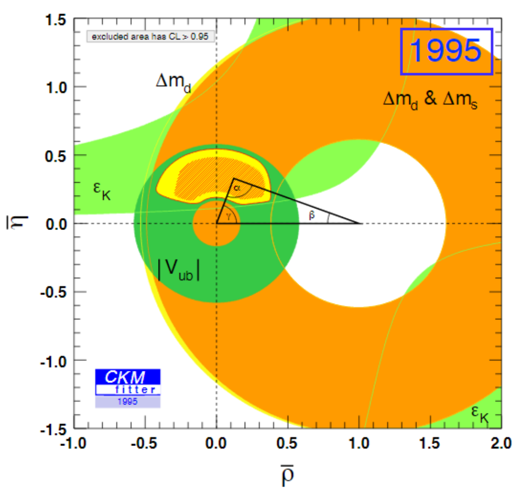
$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

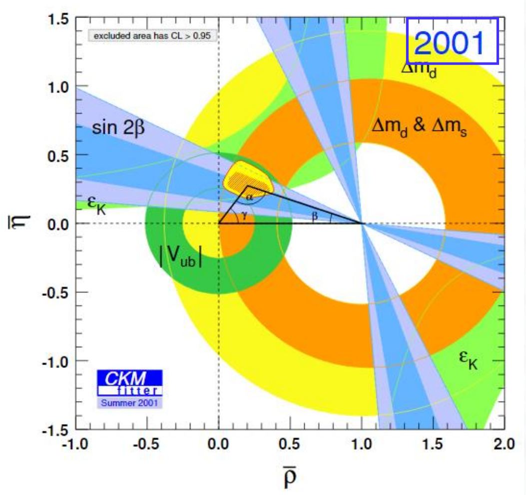
$$R_u \equiv \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| \qquad R_t \equiv \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right|$$

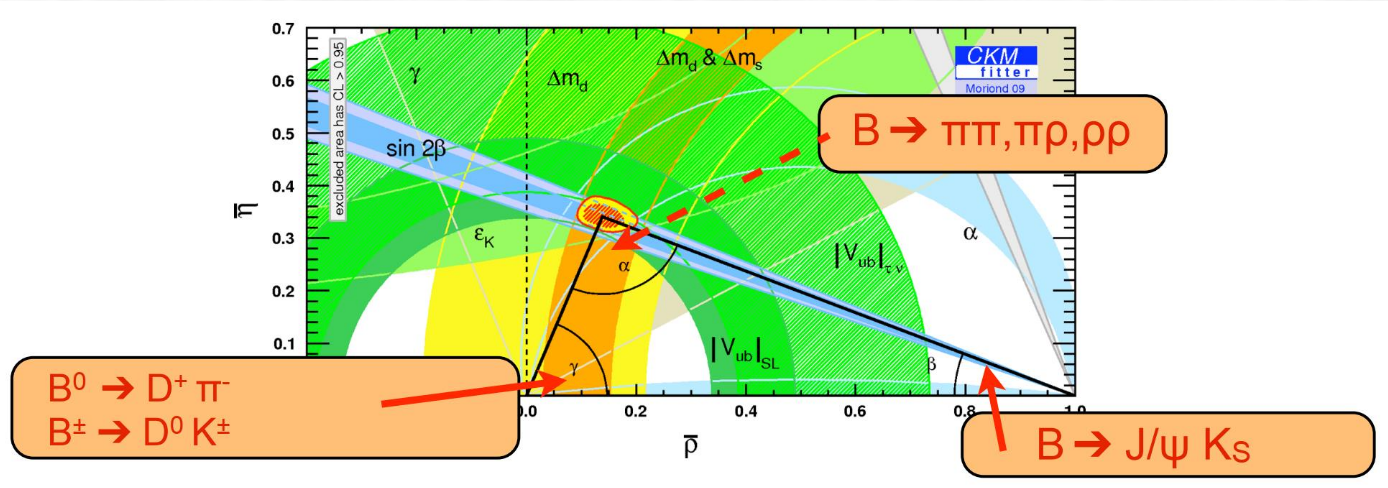
$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

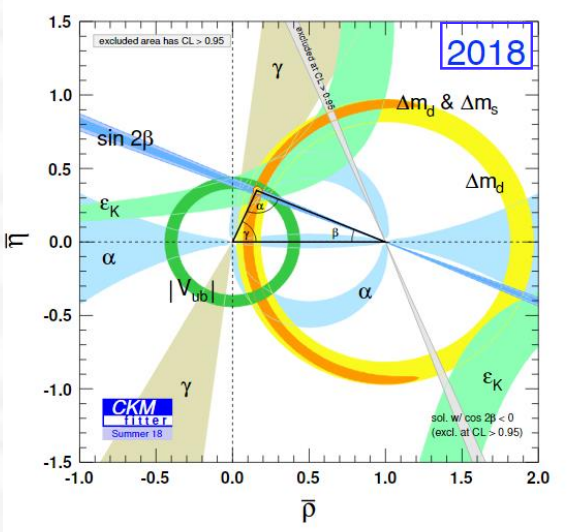
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$



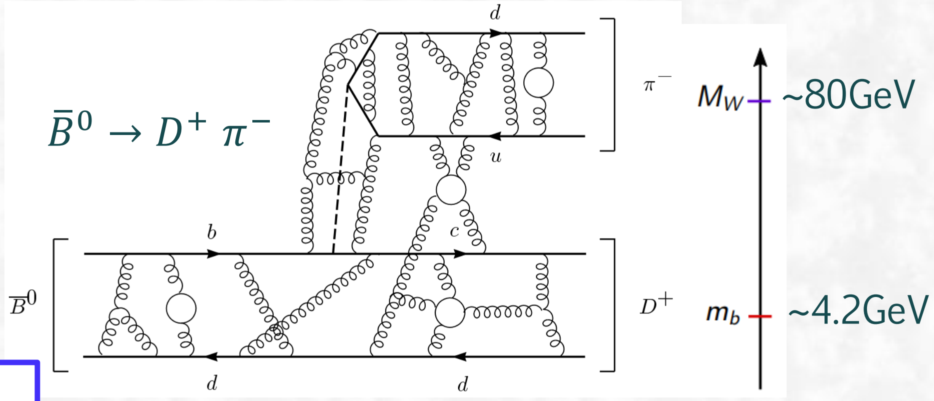








Why hadronic B decays



- **Understand various aspects of strong interactions;**
- **Problem:** multiple scales spoil PT convergence due to large logs;

$$P(M_W, m_b) = 1 + \alpha_s \left(\# \ln \frac{M_W}{m_b} + * \right) + \alpha_s^2 \left(\# \ln^2 \frac{M_W}{m_b} + * \right) + \ldots$$

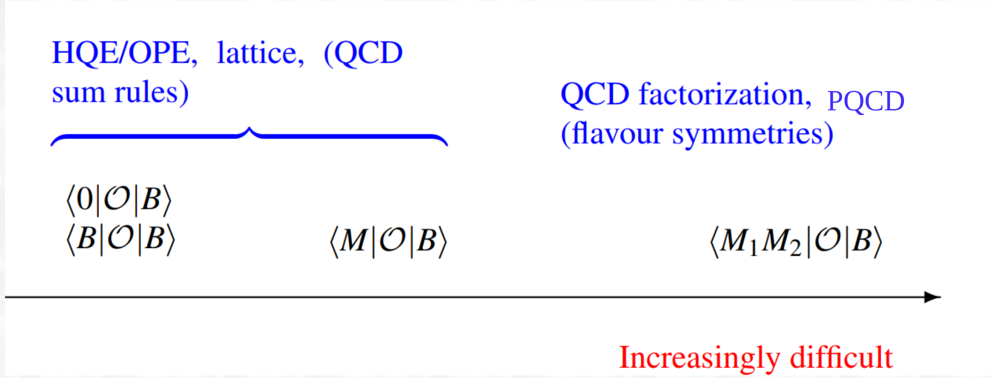
- **Solution:** perturbative series should be re-organized, and all $(\alpha_s \ln \frac{m_W}{m_b})^n$ should be re-summed!

$$P(M_W, m_b) = \underbrace{C(M_W, \mu_{\text{high}}) U(\mu_{\text{high}}, \mu_{\text{low}})}_{C_{\text{RGimproved}}(M_W, \mu_{\text{low}})} D(m_b, \mu_{\text{low}})$$

- **Final result based on RG-improved PT:**

$U(\mu_{\text{high}}, \mu_{\text{low}})$ is generally an exponential, and hence re-sums large logs $(\alpha_s \ln \frac{\mu_{\text{high}}}{\mu_{\text{low}}})^n$!

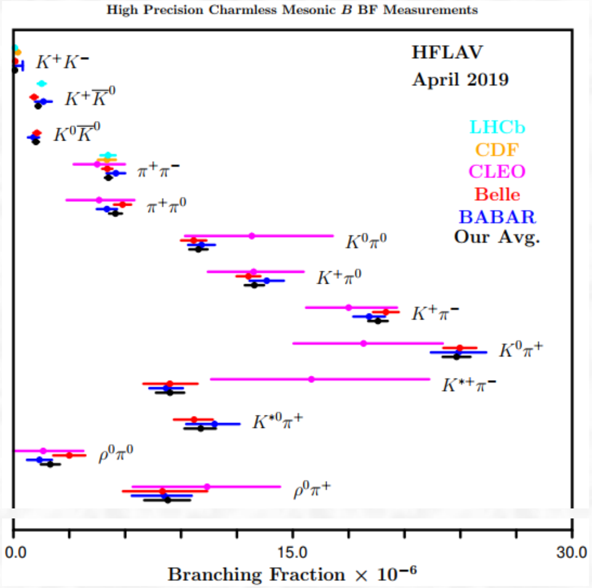
- **Hadronic matrix elements:**

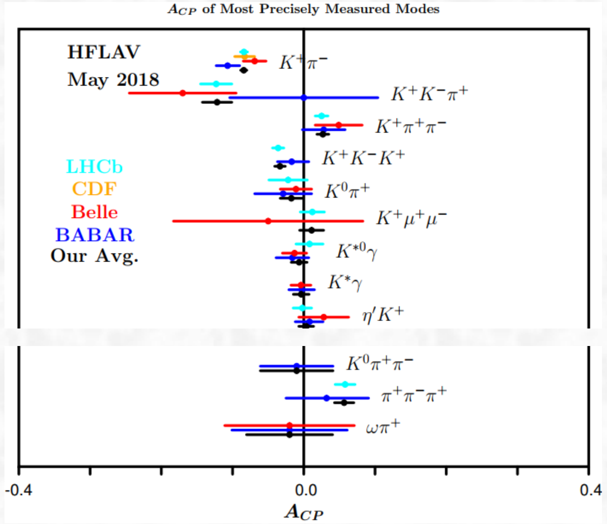


Precision era of B physics

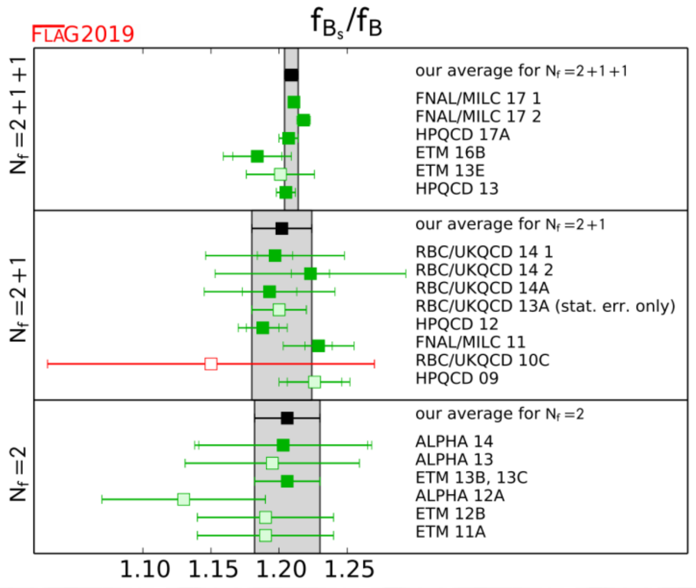
- ☐ **More data and more precise from B-dedicated exps.!**

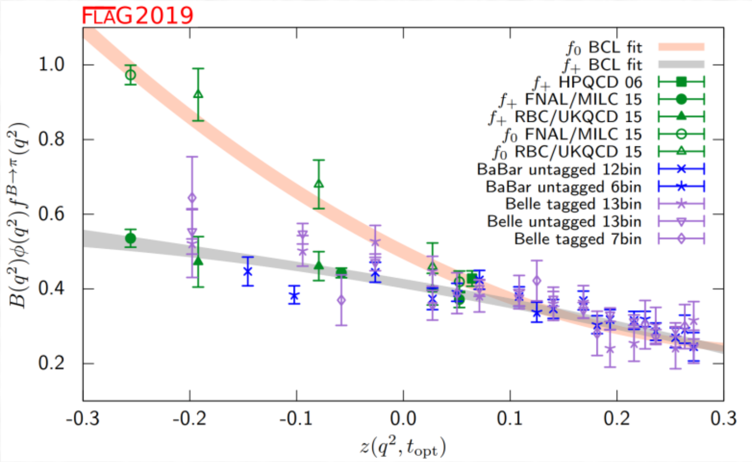
BaBar, Belle, LHCb, and Belle II, …





https://hflav.web.cern.ch/





- ☐ **More precise results for non-pert. hadronic parameters from LQCD & LCSR !**

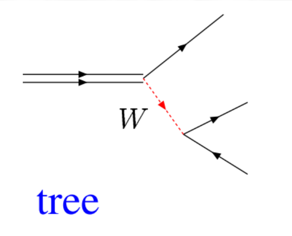
exciting

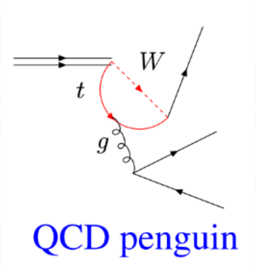
http://flag.unibe.ch/2019/

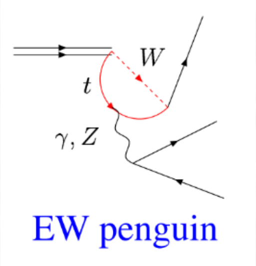
We are entering an era of precision flavor physics!

Effective Hamiltonian for B decays

- **The starting point $\mathcal{H}_{\text{eff}} = -\mathcal{L}_{\text{eff}}$:** obtained by integrating out heavy d.o.f. $(m_{W,Z,t} \gg m_b)$;







[BBL '96; CMM '98]

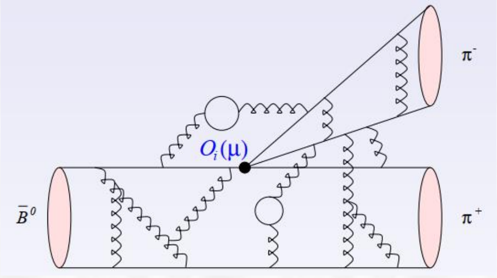
$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \Big(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i=\text{pen}} C_i \mathcal{O}_{i,\text{pen}} \Big)$$

- **Decay amplitude for a given decay:**

$$\mathcal{A}(\bar{B} \to f) = \sum_i \left[\lambda_{\text{CKM}} \times C \times \langle f | \mathcal{O} | \bar{B} \rangle_{\text{QCD+QED}} \right]_i$$

- **Wilson coefficients C_i:** all physics above m_b; perturbative calculable, NNLL program complete; [Gorbahn,Haisch '04; Czakon,Haisch,Misiak '06]

- **Hadronic matrix elements $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:**



$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \approx \langle M_1 | J_i^a | \bar{B} \rangle \langle M_2 | J_i^b | 0 \rangle$$

- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, $\cdots$
  [Keum, Li, Sanda, Lü, Yang '00;
  Beneke, Buchalla, Neubert, Sachrajda, '00;
  Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]

- Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, $\cdots$
  [Zeppenfeld, '81;
  London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

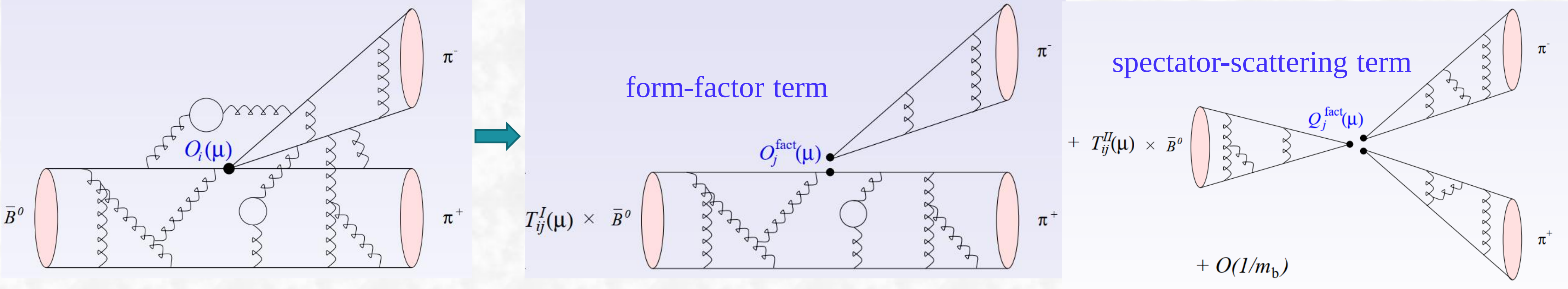
QCD factorization

[BBNS, '99-'03]

- **QCDF for $\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle$**: systematic framework to all orders in α_s, but limited by $1/m_b$ corrections.

$$\langle M_1 M_2|Q_i|\bar{B}\rangle = F^{BM_1}(0)\int_0^1 du\, T_i^I(u)\Phi_{M_2}(u) + \int_0^\infty d\omega \int_0^1 dudv\, T_i^{II}(\omega,u,v)\,\Phi_B(\omega)\Phi_{M_1}(v)\Phi_{M_2}(u)$$

- At LO in α_s and $1/m_b$, reproduces the naïve factorization result.
- Higher-order pert. corrections in α_s could be calculated systematically.
- Factorization generally broken at higher-order power in $1/m_b$.

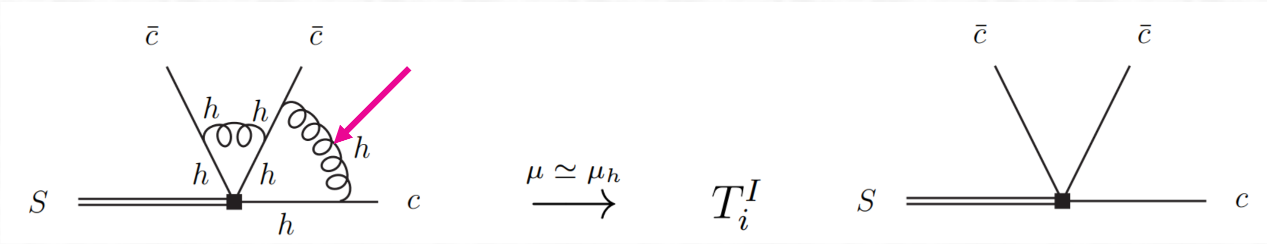


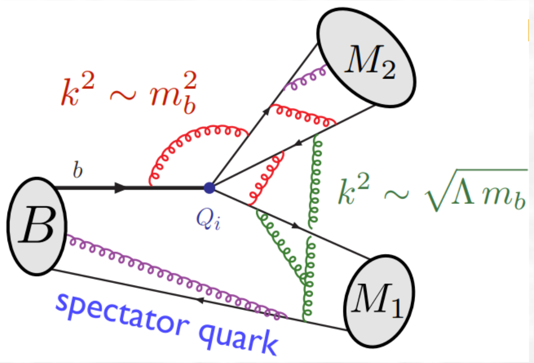
reduces $\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle$ to simpler $\langle M|j_\mu|\bar{B}\rangle$ (form factors), $\langle 0|j_\mu|\bar{B}\rangle$, $\langle M|j_\mu|0\rangle$ (decay constants & light-cone distribution amplitudes), all can be obtained from exp. data, lattice-QCD, or LCSR.

Soft-collinear factorization from SCET

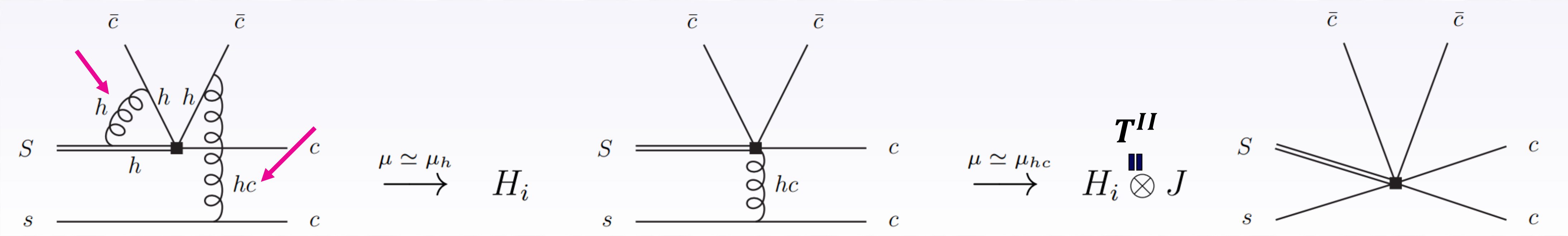
- **SCET:** a suitable framework for studying factorization and re-summation for processes involving light energetic particles; [Bauer etal. '00; Beneke etal. '02; Becher, Broggio, Ferroglia '14]

- **For hard kernel T^I:** one-step matching, QCD → $\mathrm{SCET_I(hc, c, s)}$!





- **For hard kernel T^{II}:** two-step matching, QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!



- **SCET result exactly the same as QCDF, but more apparent & efficient;** [Beneke, 1501.07374]

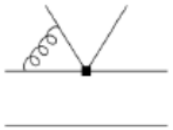
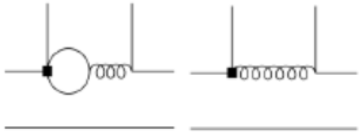
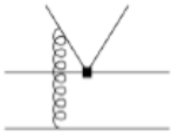
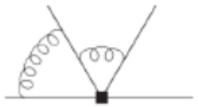
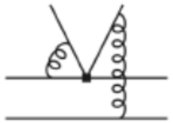
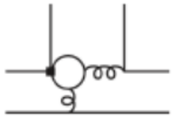
Status of the calculation of T^I and T^{II}

- **For each Q_i insertion, both tree & penguin topologies, and contribute to both T^I & T^{II}.**

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

vertex corrections: $T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$

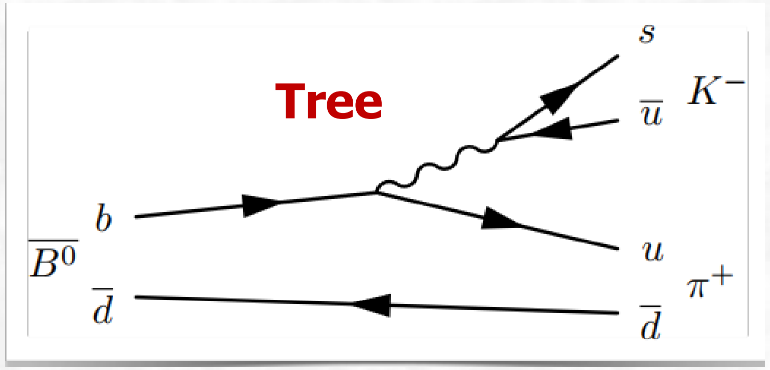
spectator scattering: $T^{II} = \mathcal{O}(\alpha_s) + \cdots$

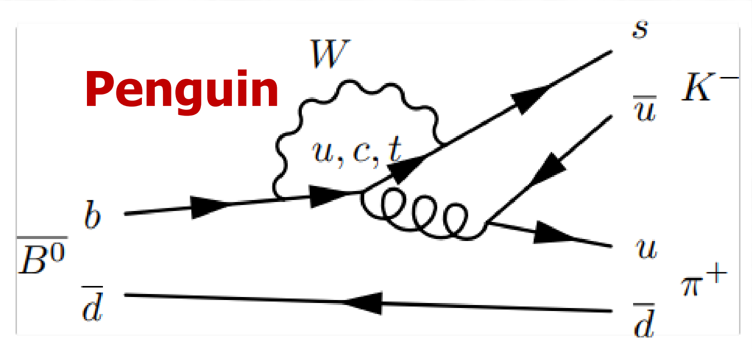
	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11, Bell Beneke, Huber, Li '15 Bell, Beneke, Huber, Li '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

- **NNLO calculation for hard kernels now complete!**
- **Soft-collinear fact. at 2-loop proved at leading power!**

QCD penguin amplitudes

- **Hadronic decays mediated by $b \to sq\bar{q}$ transitions:**





$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4)$

$\lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$

→ Penguin-dominated!

- **Interference between tree and penguin: main source of direct CPV;**

$$\left[\frac{P^c}{T}\right]_{\pi\pi}, \quad \left[\frac{T}{P^c}\right]_{\pi K} \quad \rightarrow \quad C_{\pi^+\pi^-} = -0.32 \pm 0.04, \quad \mathcal{A}_{\bar{B}^0\to K^-\pi^+} = -0.084 \pm 0.004$$

Why NNLO QCD penguin amplitudes

- **At LP in QCDF, strong phases generated only via hard loops with virtuality m_b^2 (not $m_b\Lambda$) ;**

$$A_{\rm CP} = [c \times \alpha_s]_{\rm NLO} + \mathcal{O}(\alpha_s^2, \Lambda/m_b)$$

NNLO is only NLO for direct CPV, and large effects still possible.

- **To predict accurately direct CPV, we need calculate both tree and penguin to NNLO;**

- **Driven by the exp. data;**

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\overline{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\overline{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\rm EW}^c\big],$$
$$\mathcal{A}_{\overline{B}^0\to\pi^+K^-} = A_{\pi\overline{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}
$B^+ \to \pi^+ K^0$	23.79 ± 0.75	-0.017 ± 0.016	
$B^+ \to \pi^0 K^+$	12.94 ± 0.52	0.040 ± 0.021	
$B_d^0 \to \pi^- K^+$	19.57 ± 0.53	-0.082 ± 0.006	
$B_d^0 \to \pi^0 K^0$	9.93 ± 0.49	-0.01 ± 0.10	0.57 ± 0.17

$$\Delta A_{CP} = A_{CP}(\pi^0K^-) - A_{CP}(\pi^+K^-) = (12.2 \pm 2.2)\%$$

differs from 0 by 5.5σ

ΔA_{CP} puzzle

How about the situation @ NNLO?

Penguin topologies & various insertions

- **Effective Hamiltonian including penguin operators:** [BBL '96; CMM '98]

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\,(\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\,(\bar{D}_L \gamma_\mu p_L),$$

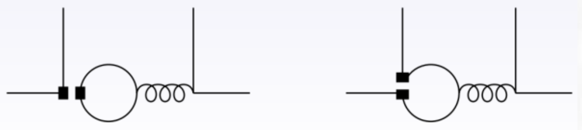
current-current operators

$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q}\gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q}\gamma_\mu \gamma_\nu \gamma_\rho T^A q).$$

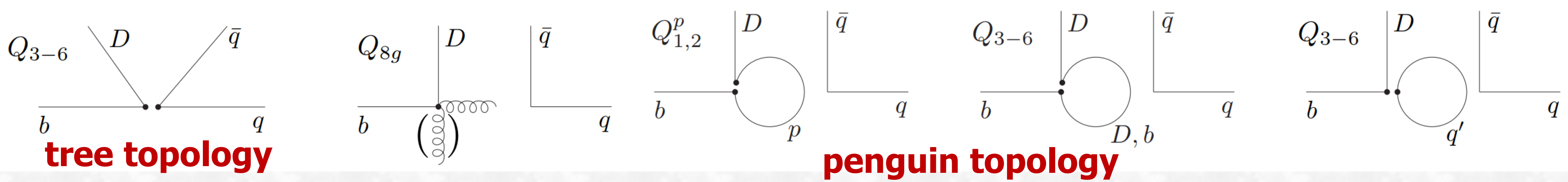
QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5) G^{\mu\nu} b,$$

chromo-magnetic dipole operators



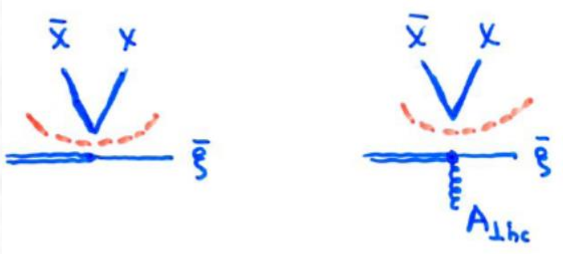
- **Various operator insertions but all featured by wrong insertion:**



(i) Dirac structure of Q_i, (ii) color structure of Q_i, (iii) types of contraction, and (iv) quark mass in the fermion loop;

Matching procedure

- **Hard kernels $T^{I,II}$ by QCD → SCET$_{\mathrm{I}}$ matching;** [see Beneke, Jager '06]



$$Q = \int d\hat{t}\, \tilde{T}^{\mathrm{I}}(\hat{t}) O^{\mathrm{I}}(t) + \int d\hat{t} d\hat{s}\, \tilde{H}^{\mathrm{II}}(\hat{t},\hat{s}) O^{\mathrm{II}}(t,s)$$

$$O^{\mathrm{I}}_{L,R}(t) = \Big[(\bar{\chi} W_{c2})(tn_-)\frac{\not{n}_-}{2}(1\mp\gamma_5)(W^\dagger_{c2}\chi)\Big]\Big[\tilde{C}^{(A0)}_{f_+}(\bar{\xi} W_{c1})\not{n}_+(1-\gamma_5)h_v - \frac{1}{m_b}\int d\hat{s}\, \tilde{C}^{(B1)}_{f_+}(\hat{s})(\bar{\xi} W_{c1})\not{n}_+[W^\dagger_{c1} i\not{D}_{\perp c1} W_{c1}](sn_+)(1+\gamma_5)h_v\Big],$$

$$O^{\mathrm{II}}_{L,R}(t,s) = \frac{1}{m_b}\Big[(\bar{\chi} W_{c2})(tn_-)\frac{\not{n}_-}{2}(1\mp\gamma_5)(W^\dagger_{c2}\chi)\Big] \times \Big[(\bar{\xi} W_{c1})\frac{\not{n}_+}{2}[W^\dagger_{c1} i\not{D}_{\perp c1} W_{c1}](sn_+)(1+\gamma_5)h_v\Big],$$

- **QCD matched onto SCET$_{\mathrm{I}}$:**

$$\begin{aligned}\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\sum_{p=u,c} V^*_{pD} V_{pb}\Big(&\delta_{pu}\{T^{\mathrm{I}}_1 * O^{\mathrm{I}}_L([\bar{q}_s u][\bar{u} D]) + H^{\mathrm{II}}_1 * O^{\mathrm{II}}_L([\bar{q}_s u][\bar{u} D]) \\ &+ T^{\mathrm{I}}_2 * O^{\mathrm{I}}_L([\bar{q}_s D][\bar{u} u]) + H^{\mathrm{II}}_2 * O^{\mathrm{II}}_L([\bar{q}_s D][\bar{u} u])\} \\ &+ \sum_{k=L,R}\Big\{T^{\mathrm{I},p}_{3k} * \sum_q O^{\mathrm{I}}_k([\bar{q}_s D][\bar{q} q]) + H^{\mathrm{II},p}_{3k} * \sum_q O^{\mathrm{II}}_k([\bar{q}_s D][\bar{q} q]) \\ &+ T^{\mathrm{I},p}_{3k,\mathrm{EW}} * \sum_q \frac{3}{2} e_q O^{\mathrm{I}}_k([\bar{q}_s D][\bar{q} q]) + H^{\mathrm{II},p}_{3k,\mathrm{EW}} * \sum_q \frac{3}{2} e_q O^{\mathrm{II}}_k([\bar{q}_s D][\bar{q} q])\Big\} \\ &+ \sum_{k=L,R}\Big\{T^{\mathrm{I},p}_{4k} * \sum_q O^{\mathrm{I}}_k([\bar{q}_s q][\bar{q} D]) + H^{\mathrm{II},p}_{4k} * \sum_q O^{\mathrm{II}}_k([\bar{q}_s q][\bar{q} D]) \\ &+ T^{\mathrm{I},p}_{4k,\mathrm{EW}} * \sum_q \frac{3}{2} e_q O^{\mathrm{I}}_k([\bar{q}_s q][\bar{q} D]) + H^{\mathrm{II},p}_{4k,\mathrm{EW}} * \sum_q \frac{3}{2} e_q O^{\mathrm{II}}_k([\bar{q}_s q][\bar{q} D])\Big\}\Big),\end{aligned}$$

- **Matrix elements of SCET$_{\mathrm{I}}$ operators:**

$$\langle P|(\bar{\chi} W_{c2})(tn_-)\frac{\not{n}_-}{2}(1\pm\gamma_5)(W^\dagger_{c2}\chi)|0\rangle = \mp\frac{i f_P m_B}{2}\int_0^1 du\, e^{iu\hat{t}}\phi_P(u),$$

$$\langle P|(\bar{\xi} W_{c1})\frac{\not{n}_+}{2}[W^\dagger_{c1} i\not{D}_{\perp c} W_{c1}](sn_+)(1+\gamma_5)h_v|\bar{B}\rangle = -m_b m_B\int_0^1 d\tau\, e^{i\tau\hat{s}}\,\Xi_P(\tau)$$

- **SCET$_{\mathrm{I}}$ matched onto SCET$_{\mathrm{II}}$:**

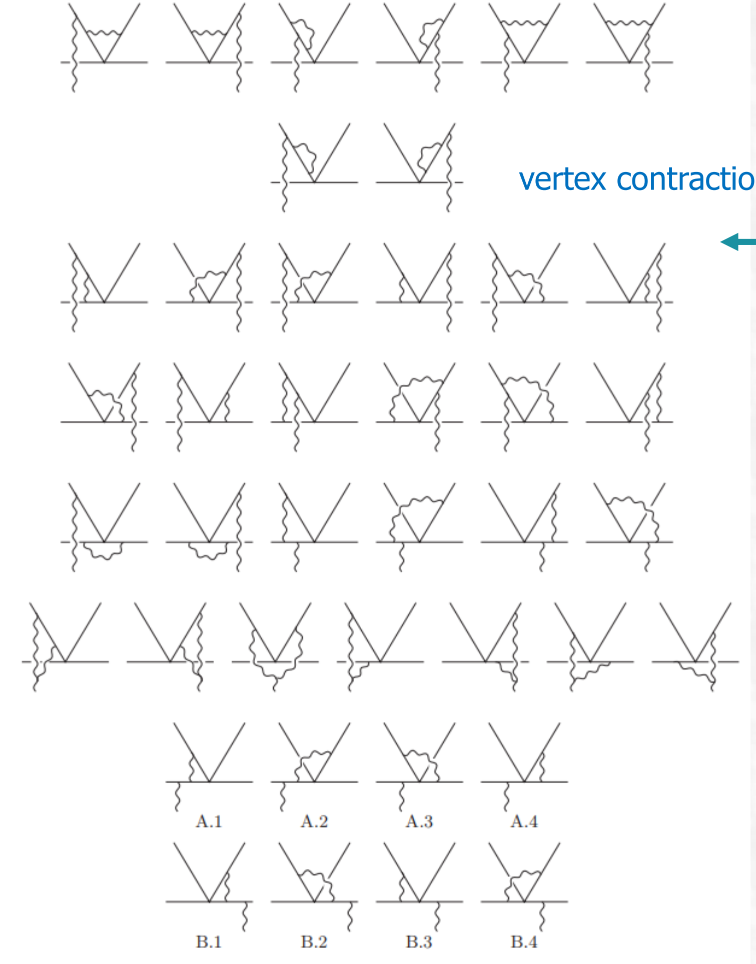
SCET$_{\mathrm{I}}$ form factor

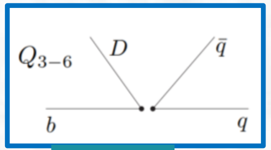
$$\hat{\Xi}(\tau) = \frac{m_B}{4m_b}\int_0^\infty \frac{d\omega}{\omega}\hat{f}_B\phi_{B+}(\omega)\int_0^1 dw\, f_{M_1}\phi_{M_1}(w) J(\tau; w, \omega)$$

Hard kernel T^{II} at NNLO

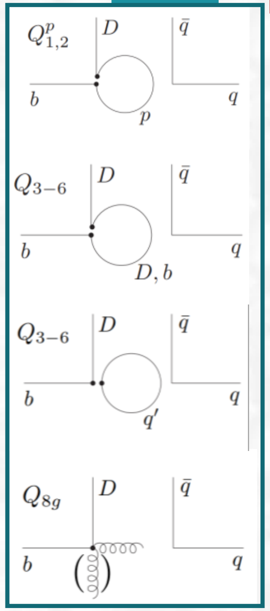
$$T_i^{\rm II} \sim H_i \star J \sim \left(1 + h_i\,\alpha_s + \mathcal{O}(\alpha_s^2)\right)\,\left(j^{(0)}\alpha_s + j^{(1)}\alpha_s^2 + \mathcal{O}(\alpha_s^3)\right)$$

- **Final results for hard kernels $T^{I,II}$:**
- **Hard function $H^{II} = H^{II}_V + H^{II}_P$;** [Beneke, Jager '06]

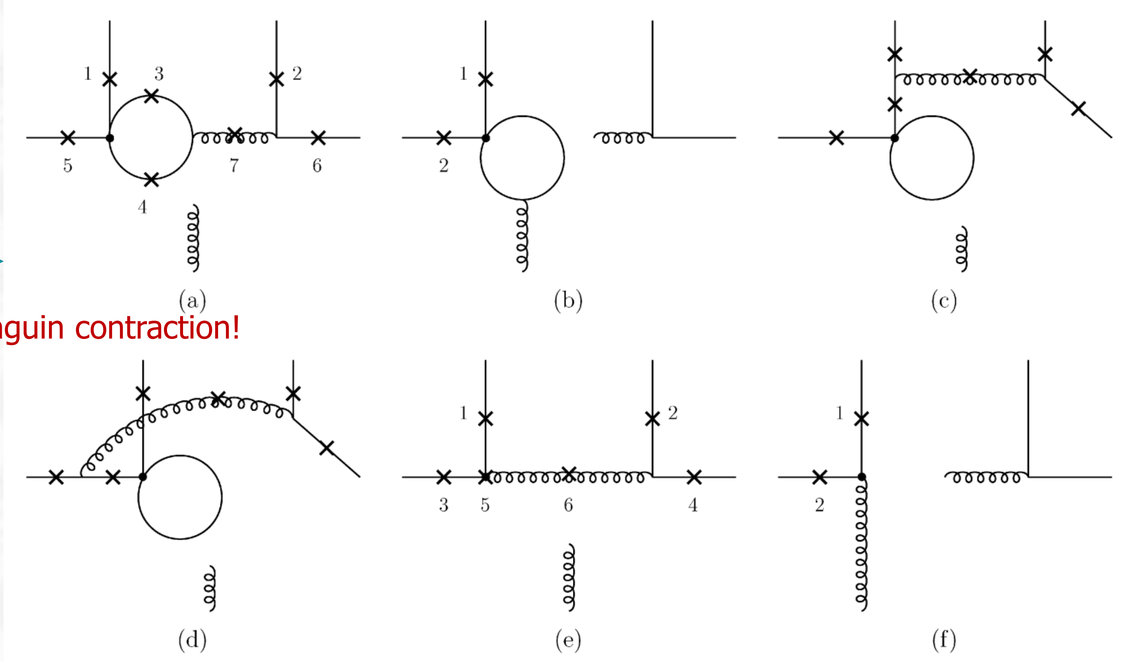




vertex contraction!



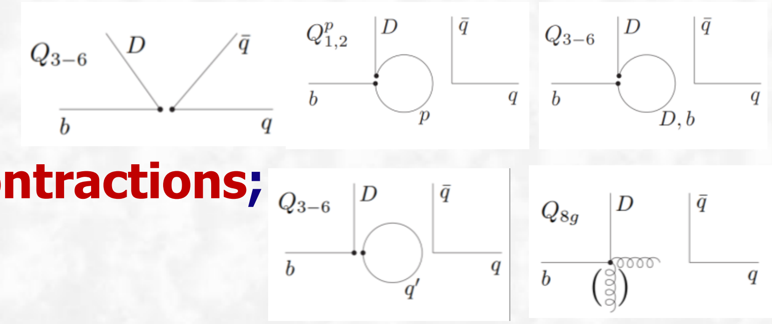
penguin contraction!



- **Hard kernel T^{II} at NNLO;** [Beneke, Jager '06]

$$\overbrace{H^{\rm II}(\mu_{\rm h}) * U_{\|}(\mu_{\rm h},\mu_{\rm hc}) * J(\mu_{\rm hc})}^{T^{\rm II}}$$

Hard kernel T^I at NNLO



□ **QCD → SCET$_\mathrm{I}$ matching calculation:**

$$\langle Q_i \rangle = \sum_a \widetilde{H}_{ia} \langle \widetilde{O}_a \rangle$$

□ **Note: different contractions;**

□ **Complete SCET operator basis:**

$$O_1 = \sum_{q=u,d,s} \left[\bar{\chi}_D \frac{\not{n}_-}{2}(1-\gamma_5)\chi_q\right] \left[\bar{\xi}_q \not{n}_+(1-\gamma_5)h_v\right],$$

the only physical operator and factorizes into FF*LCDA.

$$\tilde{O}_n = \sum_{q=u,d,s} \left[\bar{\xi}_q \gamma_\perp^\alpha \gamma_\perp^{\mu_1} \gamma_\perp^{\mu_2} \cdots \gamma_\perp^{\mu_{2n-2}} \chi_q\right] \left[\bar{\chi}_q (1+\gamma_5) \gamma_{\perp\alpha} \gamma_{\perp\mu_{2n-2}} \gamma_{\perp\mu_{2n-3}} \cdots \gamma_{\perp\mu_1} h_v\right],$$

n now up to 4, with 7 gamma matrices

$\tilde{O}_1 - O_1/2$ is another evanescent operator

□ **On-shell matrix elements at NNLO:**

QCD side

$$\begin{aligned}\langle Q_i \rangle = \Big\{ & \widetilde{A}^{(0)}_{ia} + \frac{\alpha_s}{4\pi}\left[\widetilde{A}^{(1)}_{ia} + Z^{(1)}_{\text{ext}} \widetilde{A}^{(0)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja}\right] \\ & + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\widetilde{A}^{(2)}_{ia} + Z^{(1)}_{ij} \widetilde{A}^{(1)}_{ja} + Z^{(2)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\text{ext}} \widetilde{A}^{(1)}_{ia} + Z^{(2)}_{\text{ext}} \widetilde{A}^{(0)}_{ia}\right. \\ & \left. + Z^{(1)}_{\text{ext}} Z^{(1)}_{ij} \widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\alpha} \widetilde{A}^{(1)}_{ia} + (-i)\,\delta m^{(1)}\, \widetilde{A}'^{(1)}_{ia}\right] + \mathcal{O}(\alpha_s^3) \Big\} \langle \widetilde{O}_a \rangle^{(0)}\end{aligned}$$

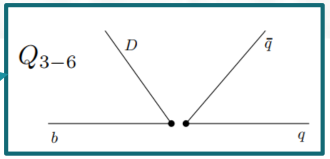
□ **On-shell matrix elements at NNLO:**

SCET side

$$\langle \widetilde{O}_a \rangle = \left\{ \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \widetilde{Y}^{(1)}_{ab} + \left(\frac{\hat{\alpha}_s}{4\pi}\right)^2 \left[\widetilde{M}^{(2)}_{ab} + \widetilde{Y}^{(2)}_{ab}\right] + \mathcal{O}(\hat{\alpha}_s^3) \right\} \langle \widetilde{O}_b \rangle^{(0)}$$

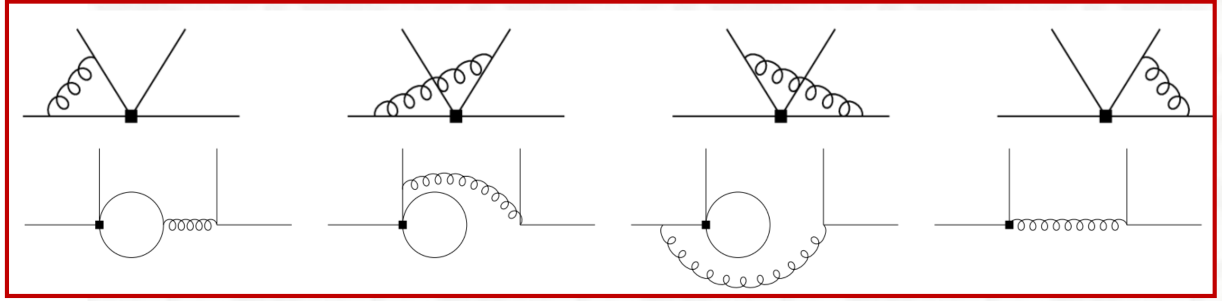
T^I at NNLO

$$\frac{1}{2}\,\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)}\,,$$



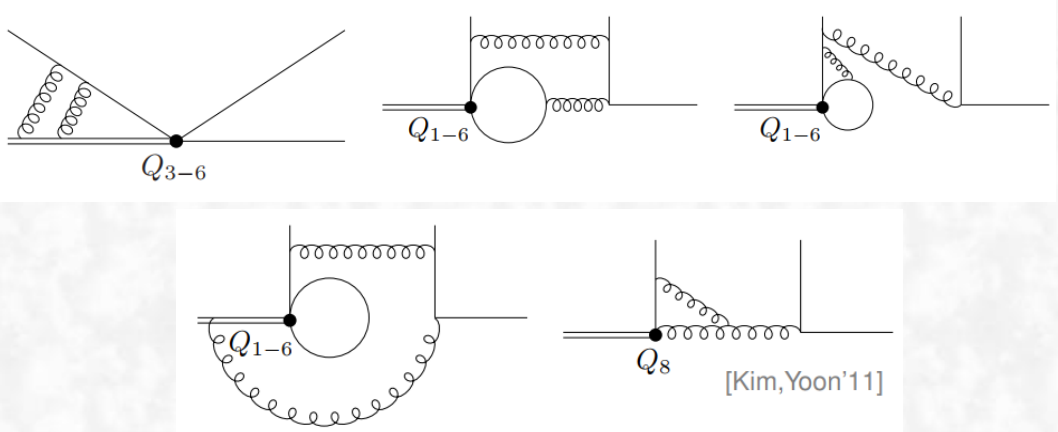
$$\frac{1}{2}\,\widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\text{f}} - A_{31}^{(1)\text{f}}\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{\sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(1)}}_{\mathcal{O}(\epsilon)}$$

- **Master formula for T^I:**



$$\begin{aligned}
\frac{1}{2}\,\widetilde{T}_i^{(2)} = {} & \widetilde{A}_{i1}^{(2)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\text{nf}} \\
& + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\text{nf}} + Z_{\text{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\text{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\,\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\text{f}} - A_{31}^{(2)\text{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\text{f}} - A_{31}^{\prime(1)\text{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& + (Z_{\alpha}^{(1)} + Z_{\text{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\text{f}} - A_{31}^{(1)\text{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}\,.
\end{aligned}$$

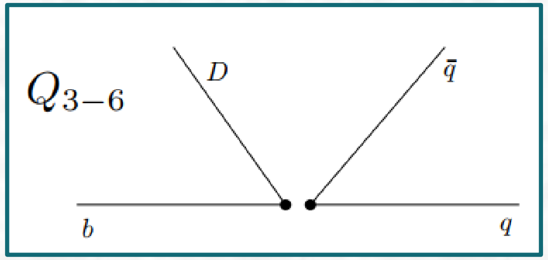
about 100 Feynman diagrams



[Kim,Yoon'11]

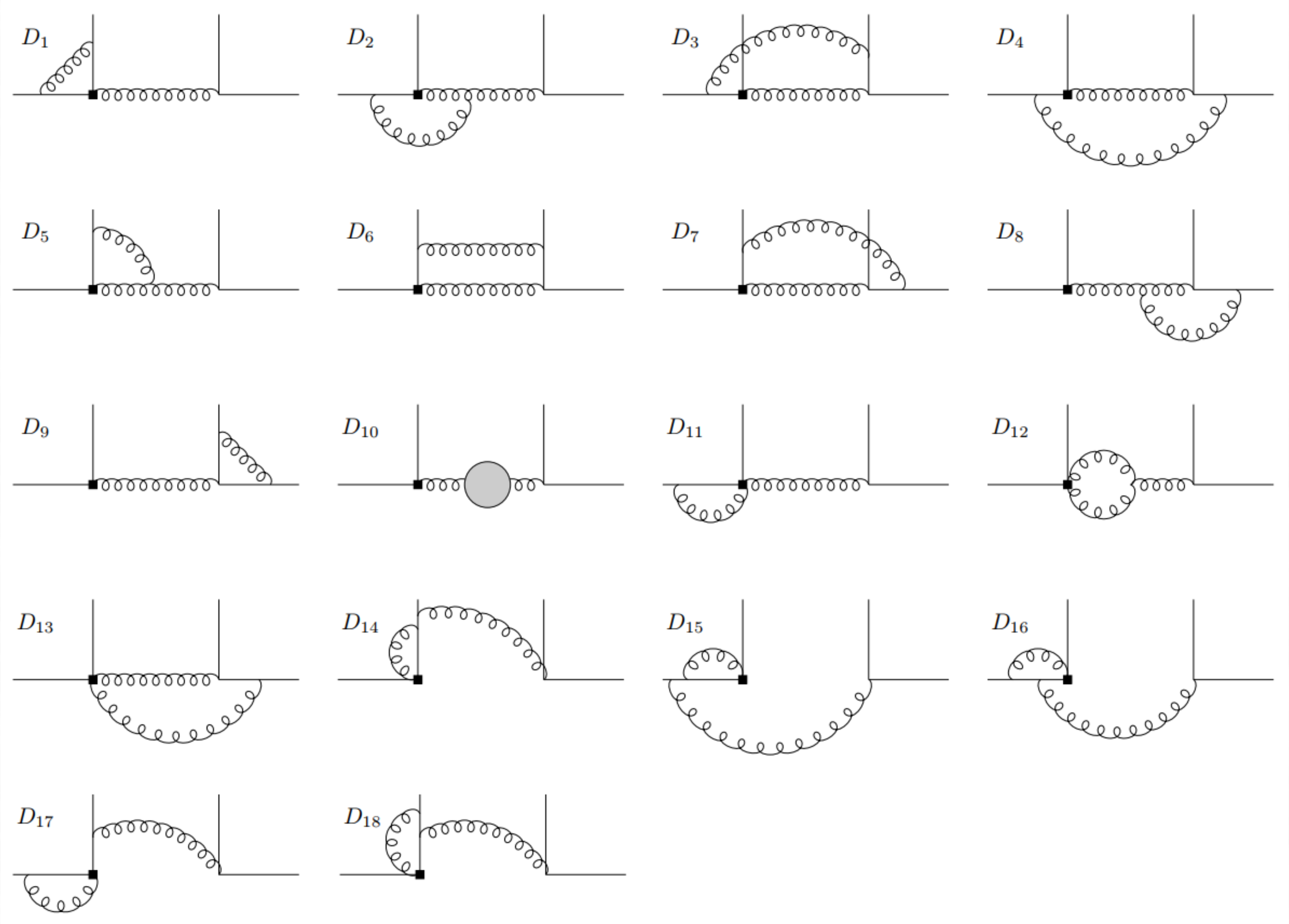
On the QCD side

- For $\widetilde{A}_{i1}^{(2)\mathrm{nf}}$, need consider Q_{3-6} insertion into **tree topology** (66 diagrams);



- Similar to color-suppressed tree amplitude, but with more complicated Dirac structures!

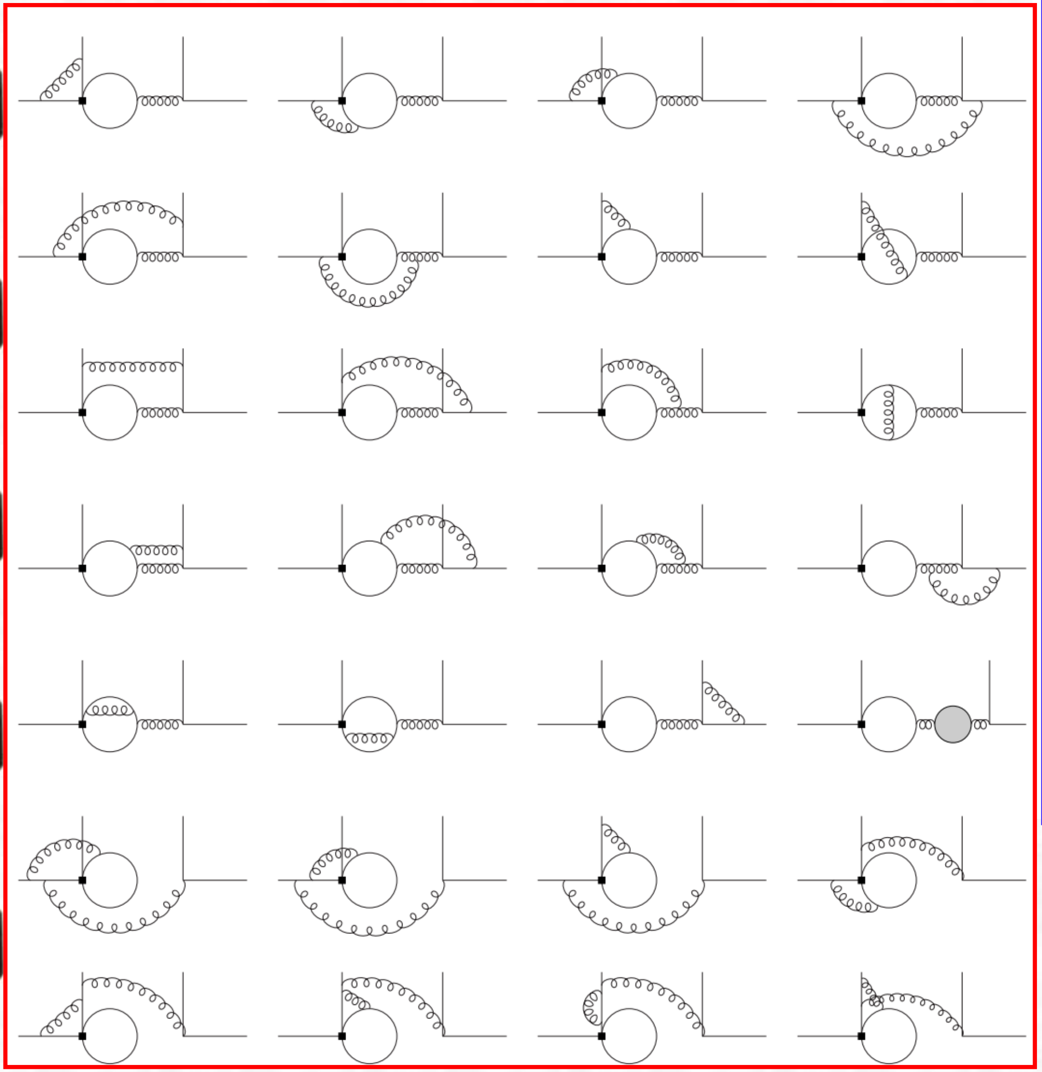
- 1-loop diagrams with Q_{8g} insertion; [Kim, Yoon '11]

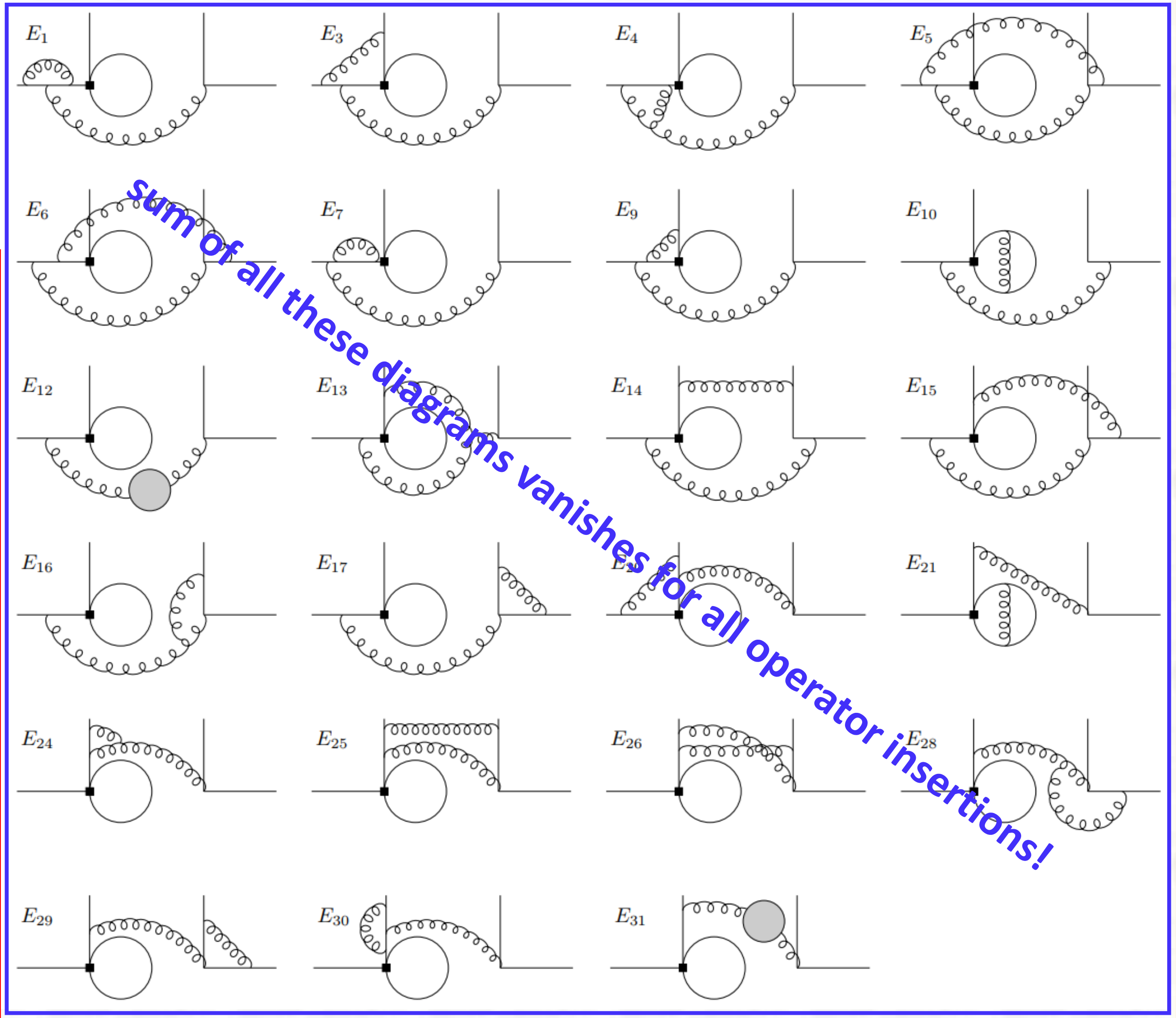


- Relatively simpler since just 1-loop!

On the QCD side

- $Q_{12,3-6}$ insertion into penguin topology;





(i) fermion loop with either $m = 0, m = m_c$ or $m = m_b$.

(ii) genuine 2-loop two-scale problem: $\bar{u},\ z_c = m_c^2/m_b^2$.

(iii) threshold at $\bar{u} = 4z_c$, need properly consider imaginary.

Techniques for 2-loop 2-scale diagrams

- Dimensional regularisation with $D = 4 - 2\epsilon$ regulates UV and IR. Poles up to $1/\epsilon^4$.
- Passarino-Veltman reduction of tensor integrals to scalar integrals *[Passarino, Veltman'79]*

thousands of scalar integrals

- Reduction of scalar integrals to a small set of master integrals
  - Integration-by-parts and Lorentz-invariance identities *[Tkachov'81; Chetyrkin, Tkachov'81; Gehrmann, Remiddi'99]*
  - System of equations solved by Laporta algorithm *[Laporta'01;Anastasiou, Lazopoulos'04;Smirnov'08]*

Get totally additional 36 MIs!

- Use differential equations in canonical form [Henn'13]

$$d\,\vec{M}(\epsilon, x_n) \;=\; \epsilon\;\; d\tilde{A}(x_n)\;\; \vec{M}(\epsilon, x_n)$$

- Found canonical basis for all masters, including boundary conditions [Bell,TH'14]
  - First example of canonical basis in case of 2 different internal masses
  - Analytic solution in terms of iterated integrals (GPLs) over alphabet

$$\left\{0\,,\ \pm 1\,,\ \pm 3\,,\ \pm i\sqrt{3}\,,\ \pm r\,,\ \pm\frac{r^2+1}{2}\,,\ \pm(1+2\sqrt{z_c})\,,\ \pm(1-2\sqrt{z_c})\right\}$$

All MIs computed analytically in terms of iterated integrals over generalized weight functions!

Final results for a_4^p

◻ Leading penguin amplitude a_4^p up to NNLO;

$$
\begin{aligned}
a_4^p = {} & \frac{C_3}{N_c} + \frac{C_F}{N_c} C_4 + \frac{16 C_5}{N_c} + \frac{16 C_F}{N_c} C_6 \\
& + \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \Bigg[\left(C_3 - \frac{C_4}{2N_c} + 16 C_5 - \frac{8 C_6}{N_c} \right) (-6L + I_t^{(1)}) \\
& \quad + \left(C_{8g} + C_3 - \frac{C_4}{2\,N_c} + 20 C_5 - \frac{10\,C_6}{N_c} \right) (-2)\, I_{8g}^{(1)} \\
& \quad + \left(C_2 - \frac{C_1}{2N_c} \right) \left(-\frac{2}{3} L + \frac{2}{3} - \delta_{pu}\, I_0^{(1)} - \delta_{pc}\, I_c^{(1)} \right) \\
& \quad + \left(C_3 - \frac{C_4}{2N_c} + 16 C_5 - \frac{8 C_6}{N_c} \right) \left(-\frac{4}{3} L + \frac{4}{3} - I_0^{(1)} - I_b^{(1)} \right) \\
& \quad + \left(C_4 + 10 C_6 \right) \left(-\frac{2}{3} n_f\, L - n_0\, I_0^{(1)} - I_c^{(1)} - I_b^{(1)} \right) \\
& \quad - N_c C_4 + \frac{16}{3} C_5 - 4 \left(10 N_c + \frac{2}{3N_c} - n_f \right) C_6 \Bigg] \\
& + \left(\frac{\alpha_s}{4\pi} \right)^2 \Bigg[C_1 \left(\delta_{pu}\, I_{1u}^{(2)} + \delta_{pc}\, I_{1c}^{(2)} \right) + C_2 \left(\delta_{pu}\, I_{2u}^{(2)} + \delta_{pc}\, I_{2c}^{(2)} \right) \\
& \quad + \sum_{i=3}^{6} C_i\, I_i^{(2)} + C_{8g}\, I_{8g}^{(2)} \Bigg] .
\end{aligned}
$$

◻ Convolution with light-meson LCDA;

$$
\int_0^1 du\, \widetilde{T}_i(u)\, \phi_M(u) = \int_r^{+i\infty} ds\, \frac{2s(r^2-1)}{(1-s^2)^2}\, \widetilde{T}_i(u(s))\, \phi_M(u(s))
$$

- Expand LCDA of light meson in Gegenbauer polynomials

$$
\phi_M(u) = 6u\bar{u} \left[1 + \sum_{n=1}^{\infty} a_n^M\, C_n^{(3/2)}(2u-1) \right]
$$

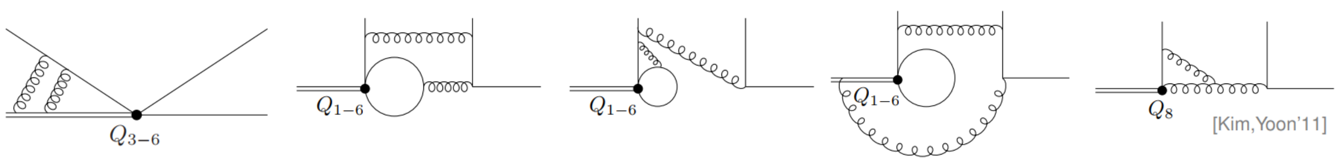
- Re-write in terms of $s = \sqrt{1 - 4z_c/\bar{u}}$ and $r = \sqrt{1 - 4z_c}$
- Integrate from $s = r \ldots + i\infty$

◆ all terms that involve powers of $L = \ln \mu^2/m_b^2$ completely analytically.

◆ in L^0 pieces a few terms are obtained only as an interpolation in z_c.

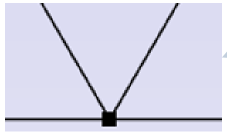
◆ for a_4^u completely analytic, for a_4^c a few terms as an interpolation in $z_c = m_c^2/m_b^2$.

Final results for a_4^p



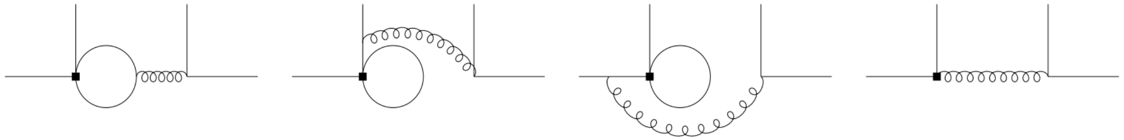
□ Final numerical results:

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1} - [0.32 + 0.71i]_{P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{P_2,\,Q_{3-6,8}}$$



$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14 + 0.12i]_{HV} - [0.01 - 0.05i]_{HP} + [0.07]_{tw3}\right\}$$

$$= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,,$$



$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1} - [0.77 + 0.50i]_{P_2,\,Q_{1,2}} + [0.33 + 0.38i]_{P_2,\,Q_{3-6,8}}$$

$$+ \left[\frac{r_{sp}}{0.434}\right]\left\{[0.13]_{LO} + [0.14 + 0.12i]_{HV} + [0.01 + 0.03i]_{HP} + [0.07]_{tw3}\right\}$$

$$= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,.$$

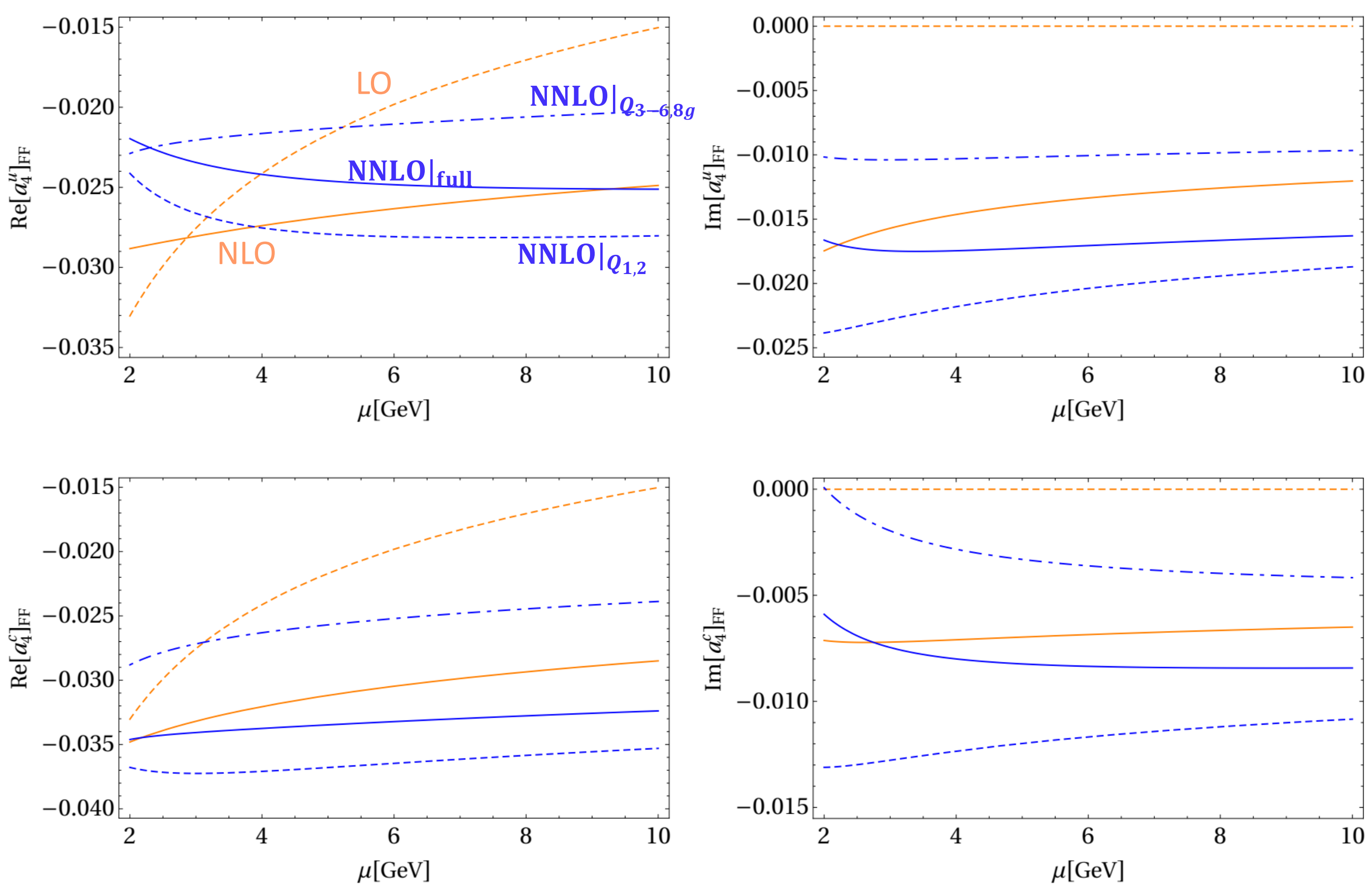
$$T^{II} = (H_V^{II} + H_P^{II}) * J$$

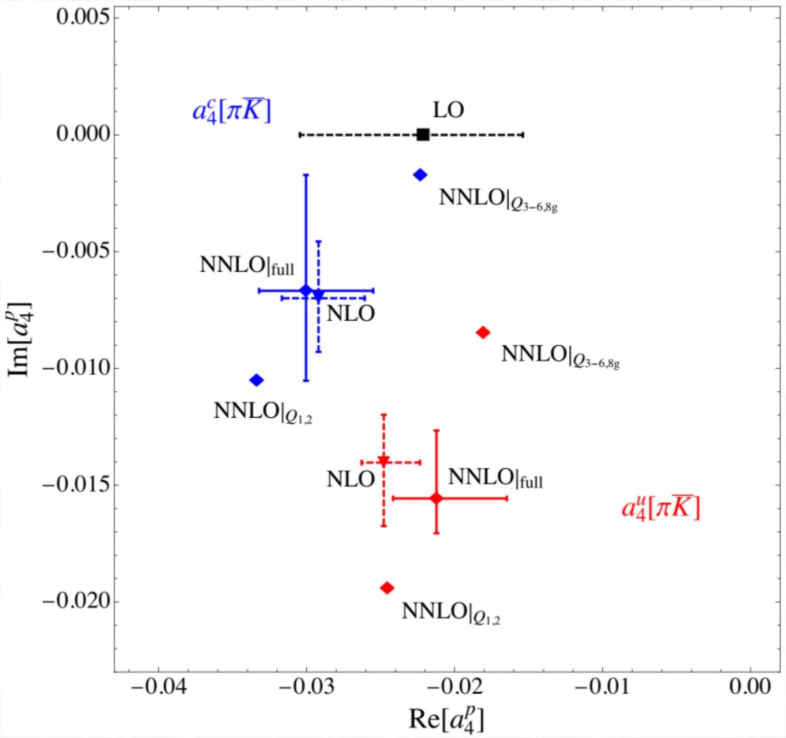
- spectator-scattering has only a small effect.

- **NNLO real part constitutes a (10 - 15)% correction relative to LO.**
- **NNLO imaginary part represents a -27% correction for a_4^u and reaches -54% for a_4^c.**
- **Strong cancellation between NNLO correction from $Q_{1,2}^p$ and from $Q_{3-6,8g}$ observed!**

Scale dependence of a_4^p

- strong cancellation between $Q_{1,2}^p$ and; $Q_{3-6,8g}$;

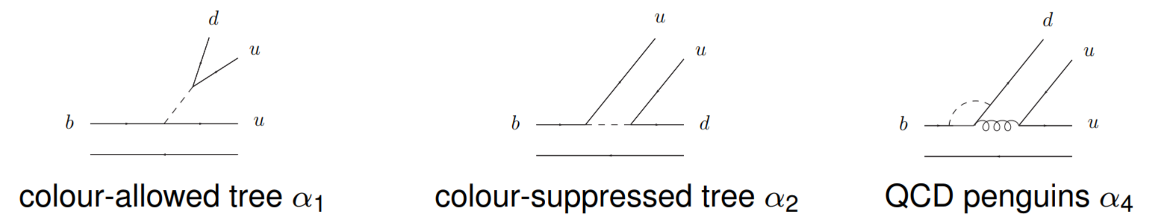




Scale dependence negligible, especially for μ > 4 GeV.

- Scale dependence of a_4^p: only form-factor term;

Summary



- **NNLO corrections to color-allowed, color-suppressed tree & leading penguin amplitudes now complete.**
- **Individual contributions sizeable, but tend to cancel each other among them, and thus NNLO shift in amplitudes is rather small.**
- **Updated phen. analyses including these NNLO corrections and progress from other sides are work in progress.**

Thank You for your attention!

For questions, email me: xqli@mail.ccnu.edu.cn