Recent Progress in Perturbative QCD

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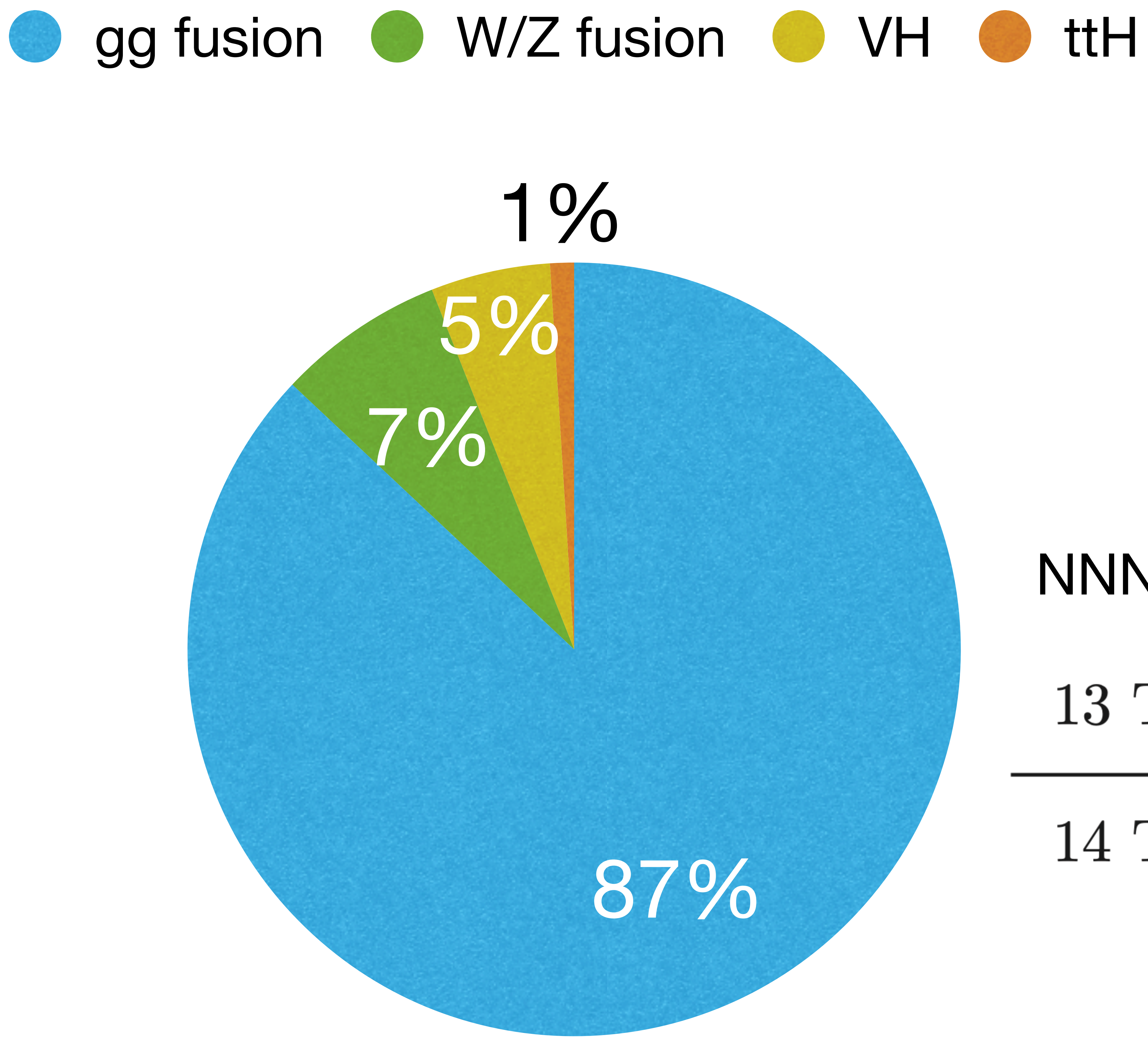
Outline

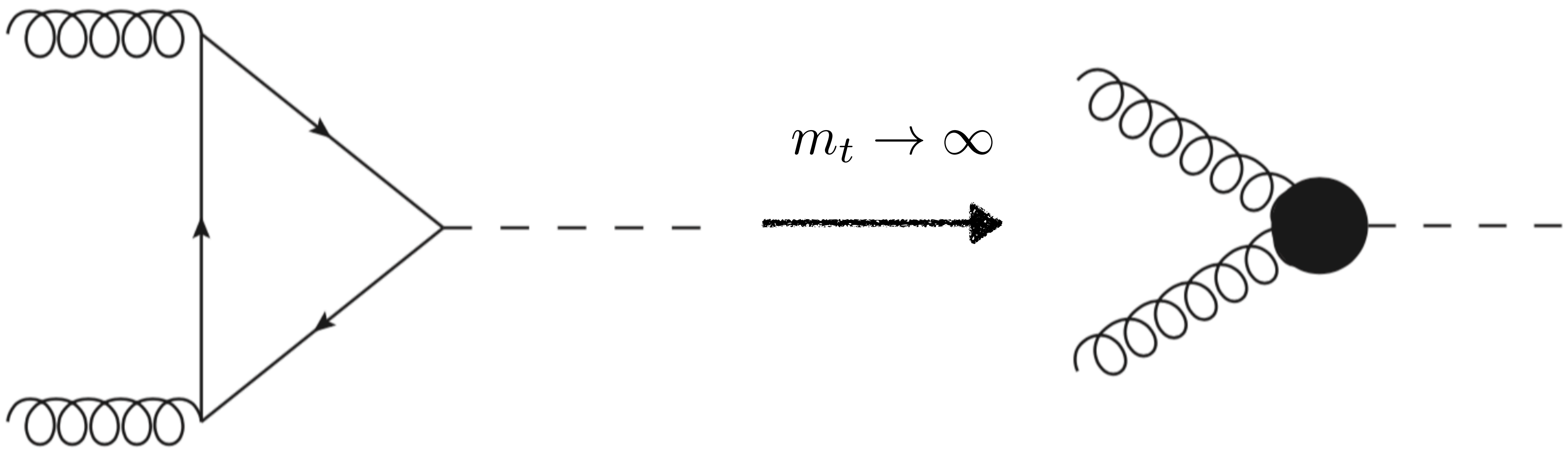
1. Progress in Higgs sector
2. Progress in Jet or photon production
3. Precision calculation of parton distribution functions
4. Summary

Single Higgs production



Expected exp. accuracy: 3%

Les Houches 2019, 2003.01700



NNNLO QCD prediciton in Heavy Top Limit Mistlberger 18'

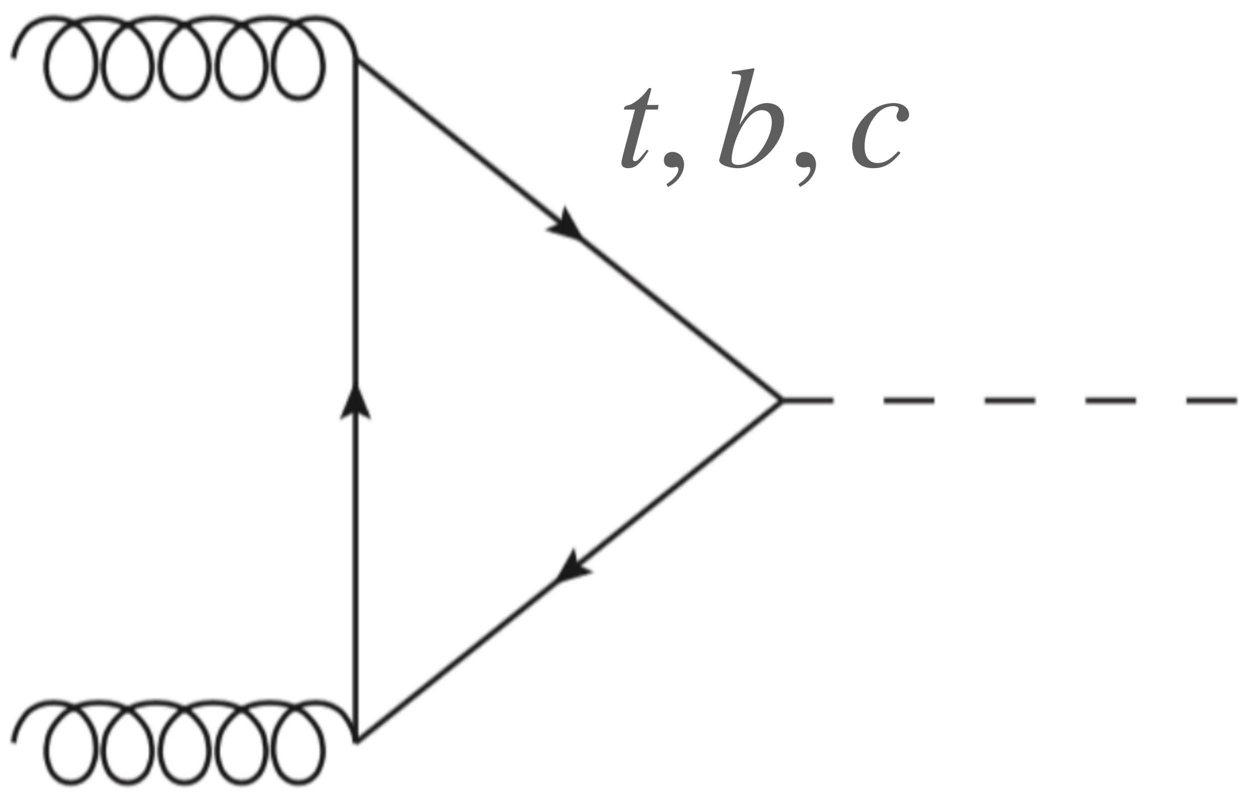
		Scale	PDF	α_s
13 TeV	48.68 pb	$^{+2.07\text{pb}}_{-3.16\text{pb}}\left(^{+4.26\%}_{-6.48\%}\right)$	$\pm 0.89\,\text{pb}\,(\pm 1.85\%)$	$^{+1.25\text{pb}}_{-1.26\text{pb}}\left(^{+2.59\%}_{-2.62\%}\right)$
14 TeV	54.80 pb	$^{+2.34\text{pb}}_{-3.54\text{pb}}\left(^{+4.28\%}_{-6.46\%}\right)$	$\pm 1.00\,\text{pb}\,(\pm 1.86\%)$	$^{+1.40\text{pb}}_{-1.42\text{pb}}\left(^{+2.60\%}_{-2.62\%}\right)$

Finite top quark mass effects are found to be less than 1%.

Harlander, Ozeren 09'; Pak, Rogal, Steinhauser 09'

Single Higgs production

Light quark in the loop:



	$\overline{\text{MS}}$	OS	$\delta\sigma^{sc}$
$\sigma^{LO}_{ex;t}$	16.00 pb	16.04 pb	0.25%
$\sigma^{LO}_{ex;t+b}$	14.94 pb	14.24 pb	-4.8%
$\sigma^{LO}_{ex;t+b+c}$	14.83 pb	13.81 pb	-6.9%
$\sigma^{NLO}_{ex;t}$	36.60 pb	36.63 pb	0.08%
$\sigma^{NLO}_{ex;t+b}$	34.96 pb	34.49 pb	-1.3%
$\sigma^{NLO}_{ex;t+b+c}$	34.77 pb	34.04 pb	-2.1%

$$\delta\sigma^{sc} = (\sigma^{\text{OS}}/\sigma^{\overline{\text{MS}}} - 1) \times 100\%$$

Anastasiou et al,16'

The light quark contributions are around -7% at LO and -5% at NLO in MSbar scheme.

Their effects become more significant in OS scheme, and can be reduced after including higher orders.

The large logarithms $\ln(m_H^2/m_q^2)$ have been evaluated to all orders, giving (-0.34, 0.08)pb. Anastasiou, Penin,20'

Three loop form factor has been obtained semi-numerically and analytically for leading color or massless quark loop .

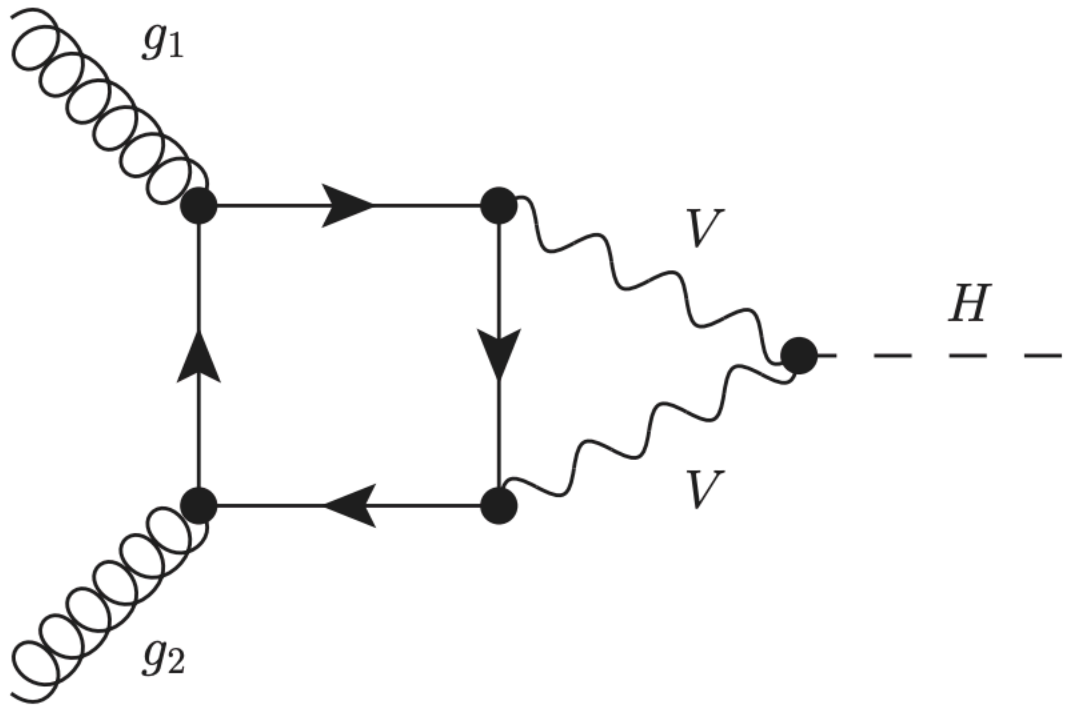
Czakon, Niggetiedt,20'

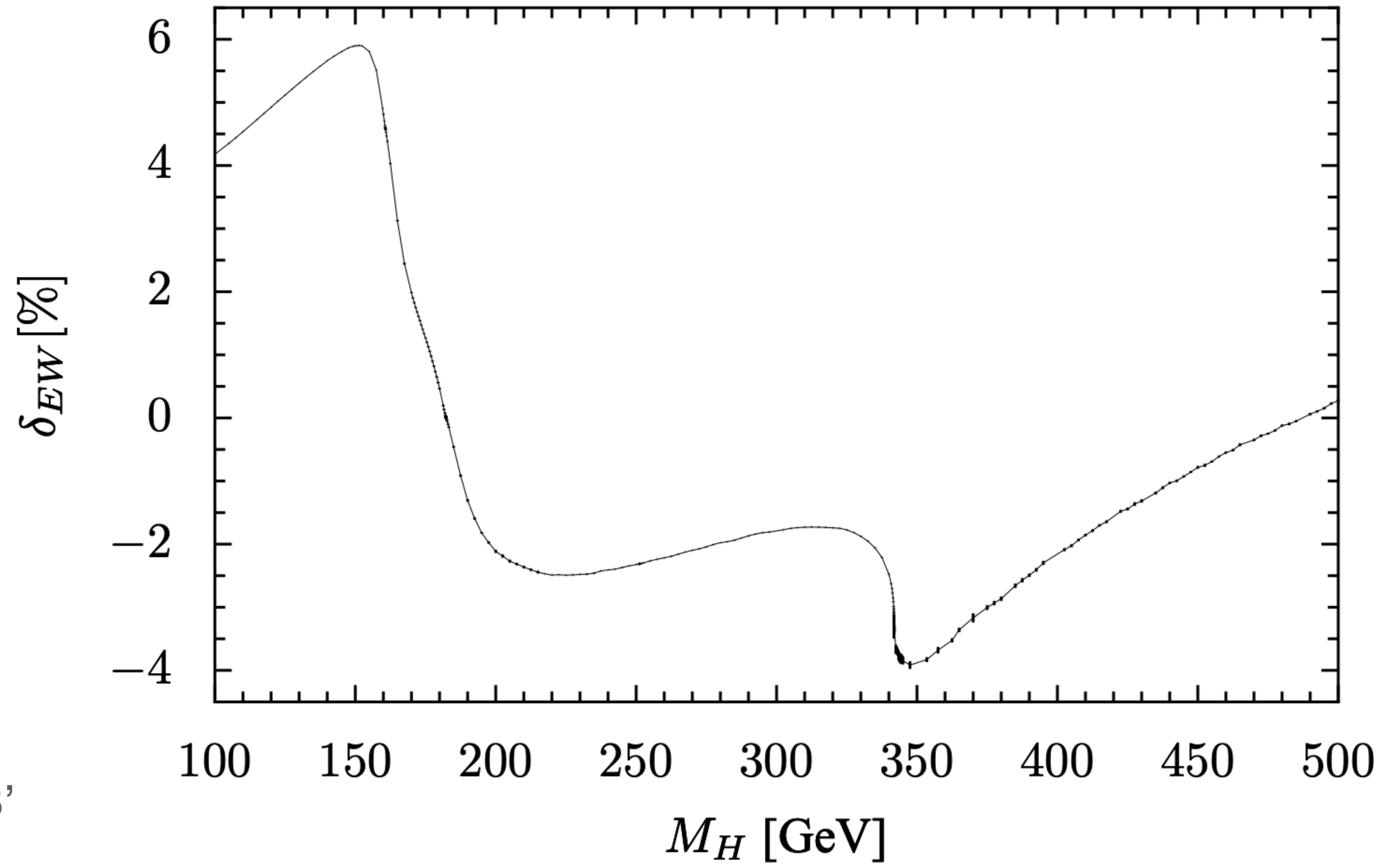
In the two-loop one real corrections, master integrals have been known. Frellesvig et al,19'

Harlander, Prausa, Usovitsch,19',20'

Single Higgs production

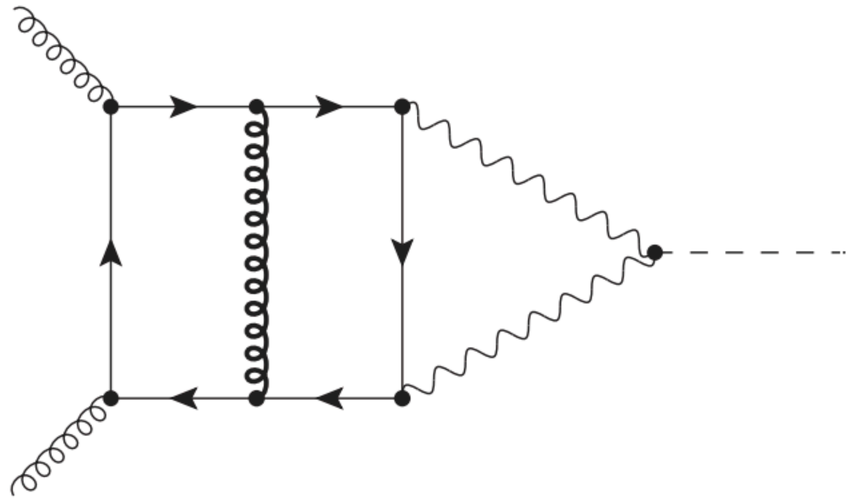
W/Z in the loop:





The EW correction is around +5%.

Actis et al,08'

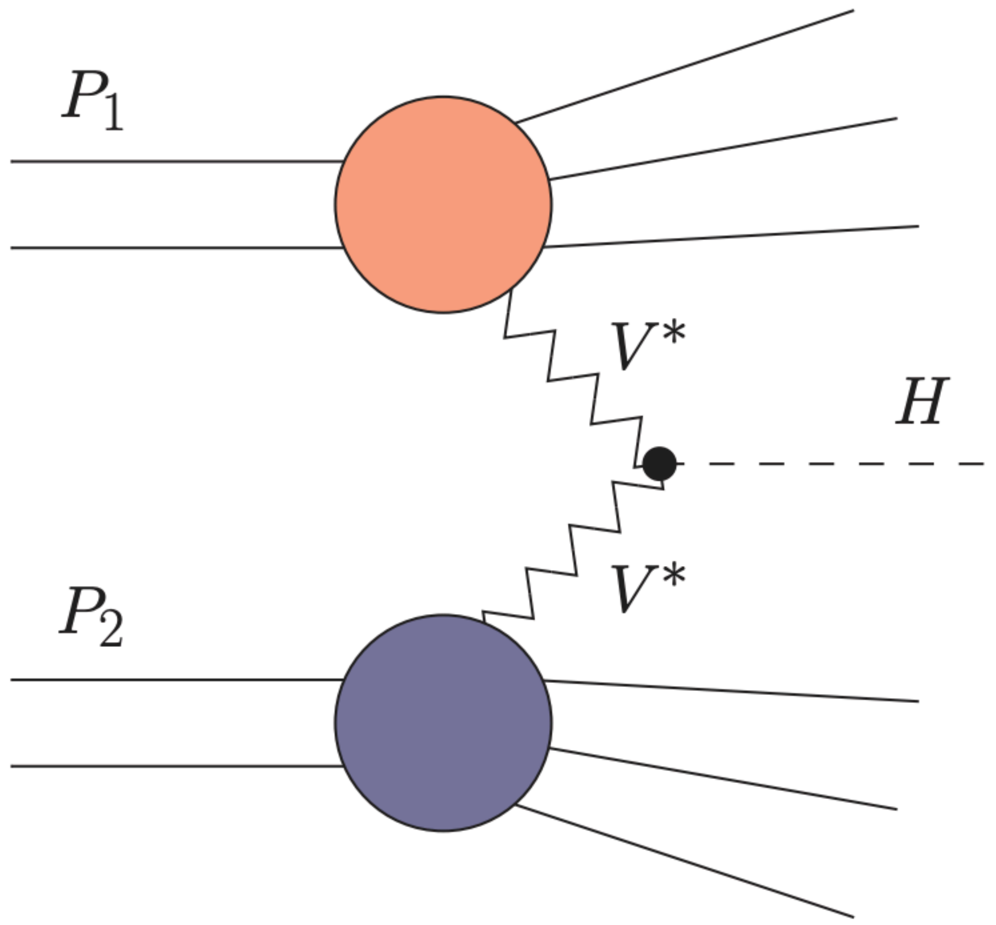


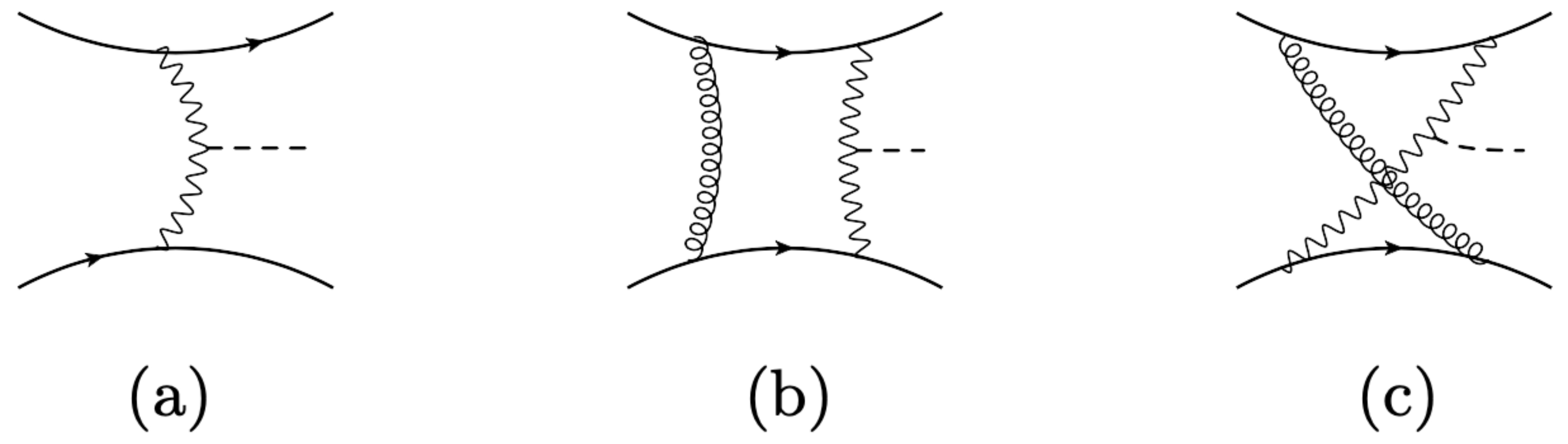
+ soft gluon approximation in real correction gives ~5% even at NLO.

Bonetti et al,18'

Full real and NLO corrections have been obtained recently. Bonetti et al,20'; Becchetti et al, 20'

VBF Higgs production





In the eikonal approximation Liu, Melnikov, Penin,19'

$$\Delta_{\rm NF} = \frac{\sigma_{\rm VBF}^{\rm NNLO,NF}}{\sigma_{\rm VBF}^{\rm LO}} \times 100\% = -0.39\%$$

NNNLO QCD prediciton in structure function

Dreyer, Karlberg,16'

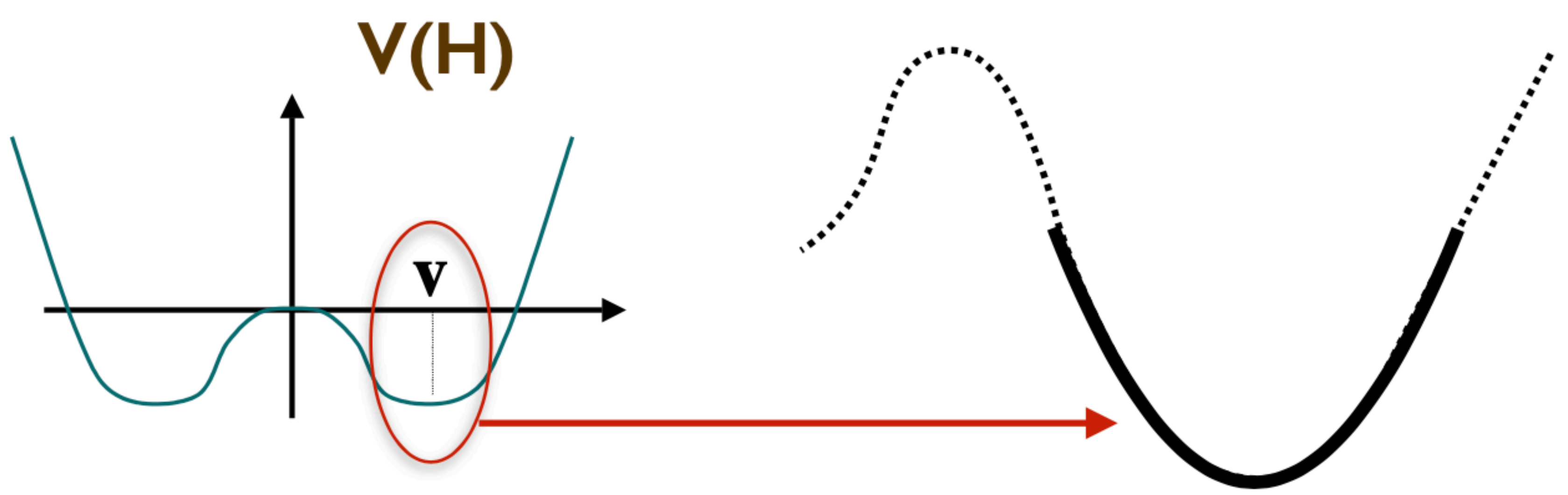
	$\sigma^{(13\ \text{TeV})}$ [pb]	$\sigma^{(14\ \text{TeV})}$ [pb]	$\sigma^{(100\ \text{TeV})}$ [pb]
LO	$4.099^{+0.051}_{-0.067}$	$4.647^{+0.037}_{-0.058}$	$77.17^{+6.45}_{-7.29}$
NLO	$3.970^{+0.025}_{-0.023}$	$4.497^{+0.032}_{-0.027}$	$73.90^{+1.73}_{-1.94}$
NNLO	$3.932^{+0.015}_{-0.010}$	$4.452^{+0.018}_{-0.012}$	$72.44^{+0.53}_{-0.40}$
N^3LO	$3.928^{+0.005}_{-0.001}$	$4.448^{+0.006}_{-0.001}$	$72.34^{+0.11}_{-0.02}$

The eikonal approximation has been checked.
With typical selection cuts, the non-factorizable NNLO corrections are small. For full inclusive phase space, they are of the same order as the NNLO factorisable ones.

Dreyer, Karlberg, Tancredi,20'

See Tao Liu's talk

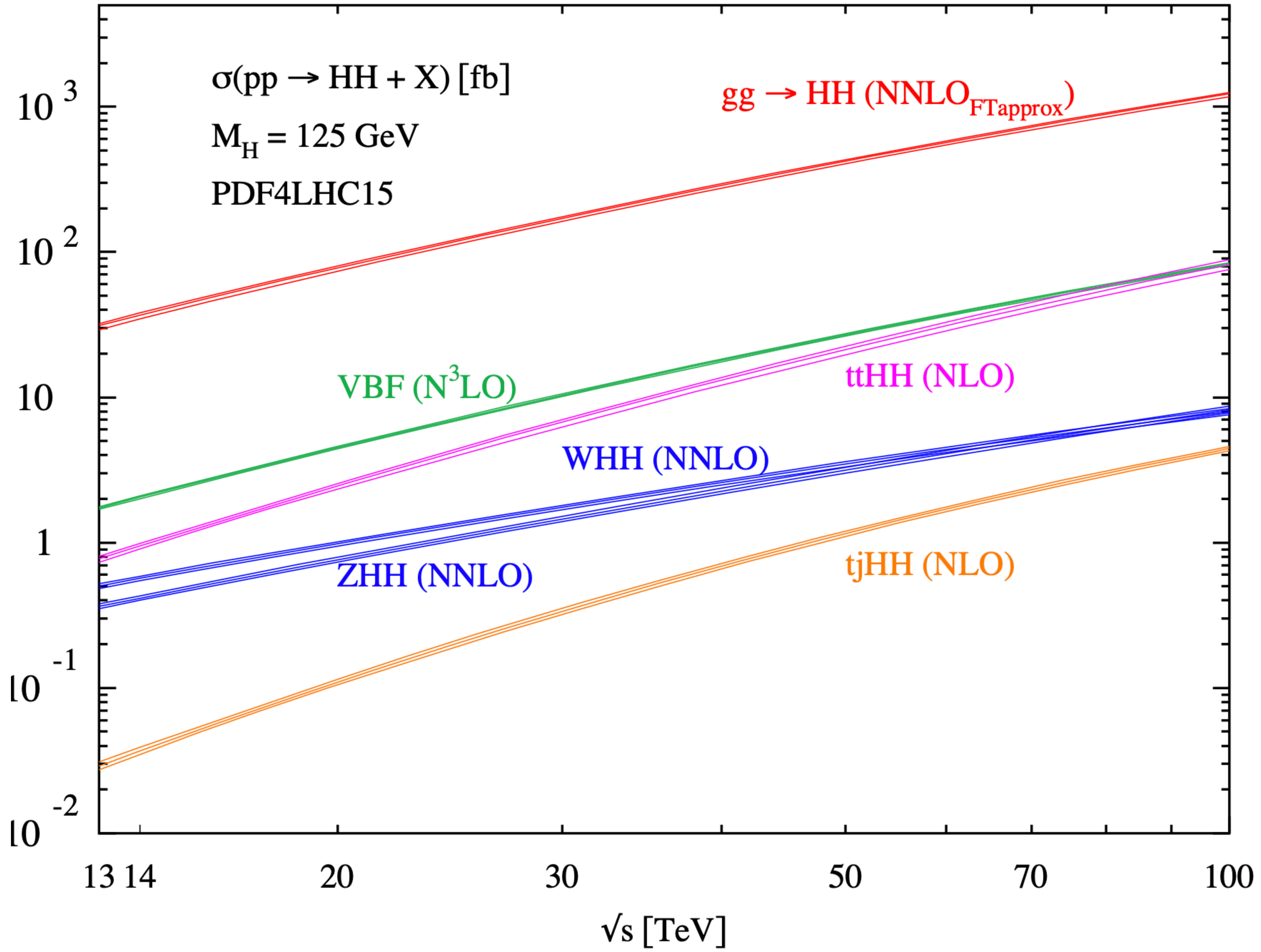
Double Higgs production



Higgs pair production is a process to access the trilinear Higgs Boson self-coupling.

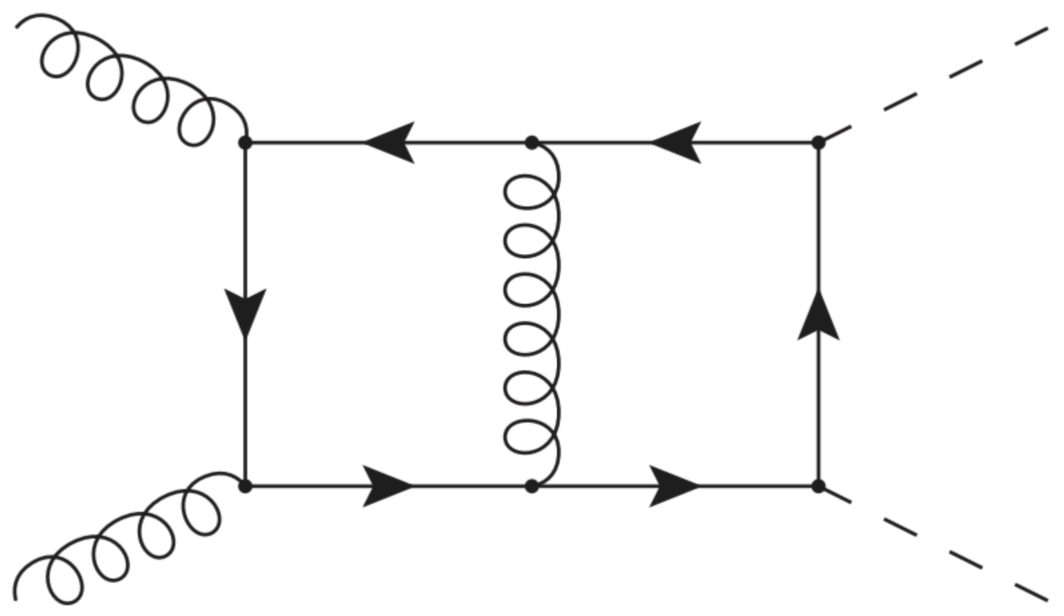
Expected accuracy:
~50% at HL-LHC,
~5% at FCC-hh @ 100 TeV

1902.00134
Mangano et al, 20'



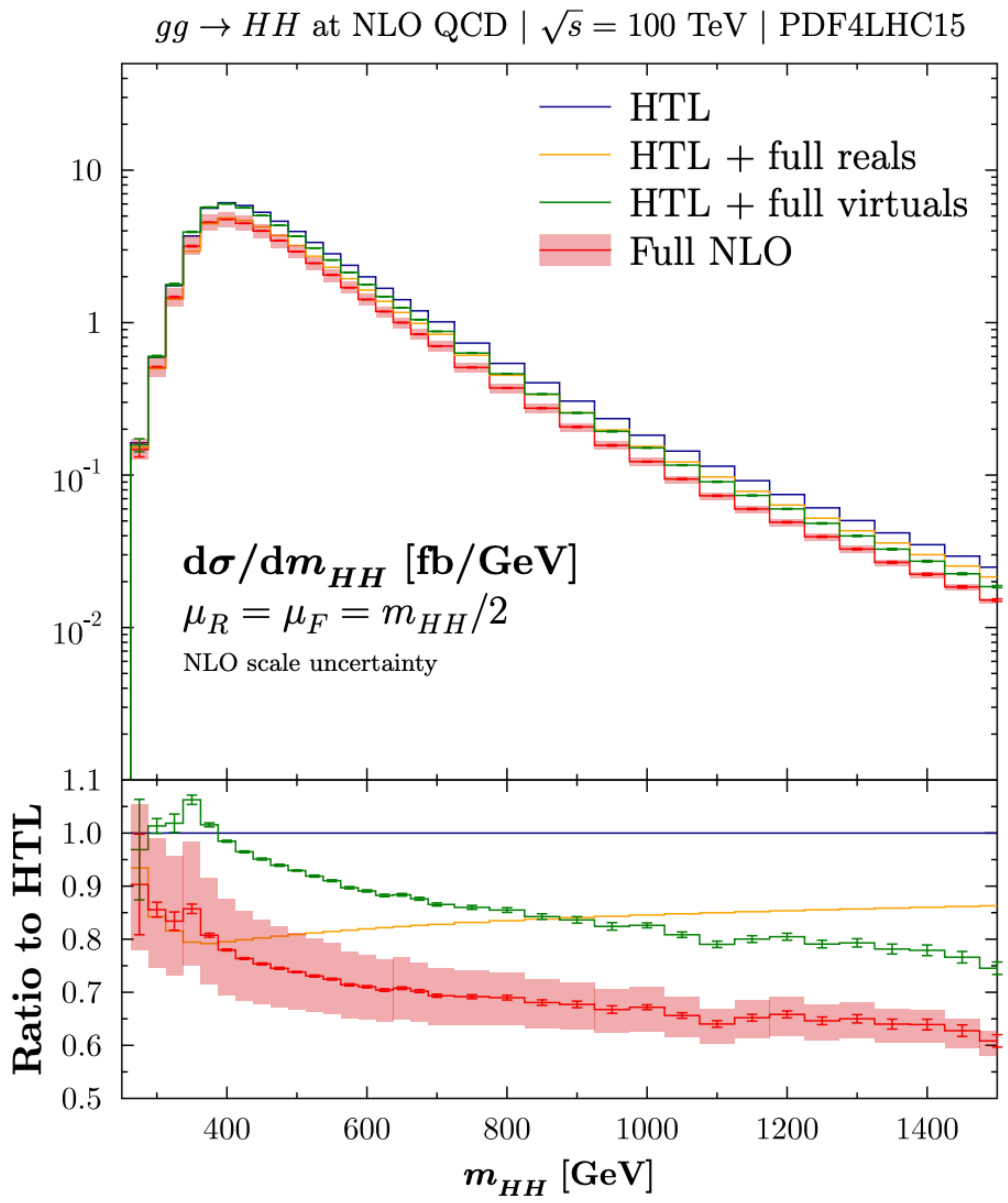
Handbook of LHC Higgs XS, 1610.07922

Double Higgs production



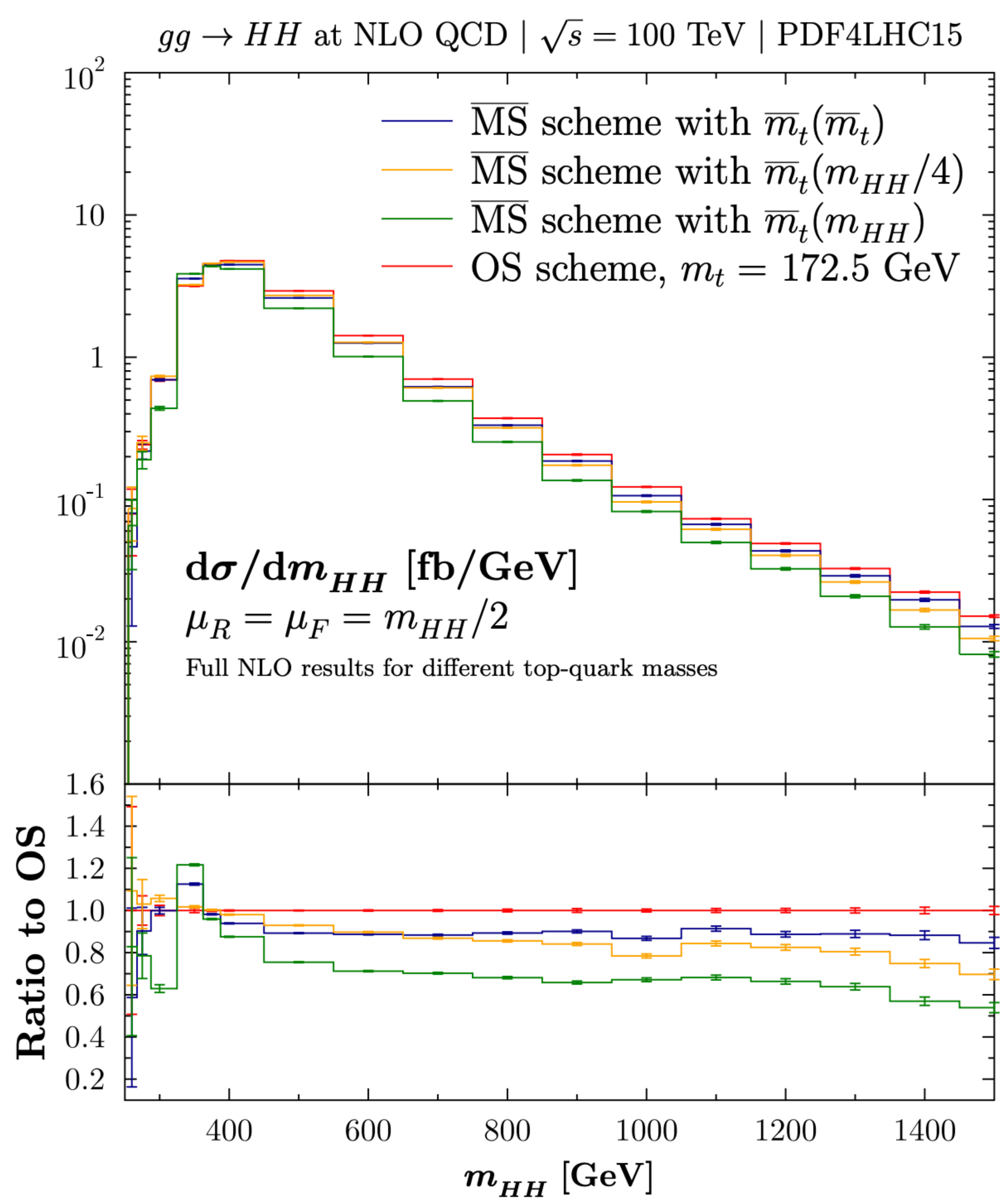
Full NLO QCD corrections:

1. Sector decomposition Borowka et al,16'
2. Contour deformation + Richardson extrapolation Baglio et al,18',20'



The HTL overshoots the full NLO by 10-40%.

Double Higgs production



Scale uncertainties:

$$\sqrt{s} = 13\text{ TeV}: \quad \sigma_{tot} = 27.73(7)^{+13.8\%}_{-12.8\%}\text{ fb},$$
$$\sqrt{s} = 14\text{ TeV}: \quad \sigma_{tot} = 32.81(7)^{+13.5\%}_{-12.5\%}\text{ fb},$$
$$\sqrt{s} = 27\text{ TeV}: \quad \sigma_{tot} = 127.0(2)^{+11.7\%}_{-10.7\%}\text{ fb},$$
$$\sqrt{s} = 100\text{ TeV}: \quad \sigma_{tot} = 1140(2)^{+10.7\%}_{-10.0\%}\text{ fb},$$

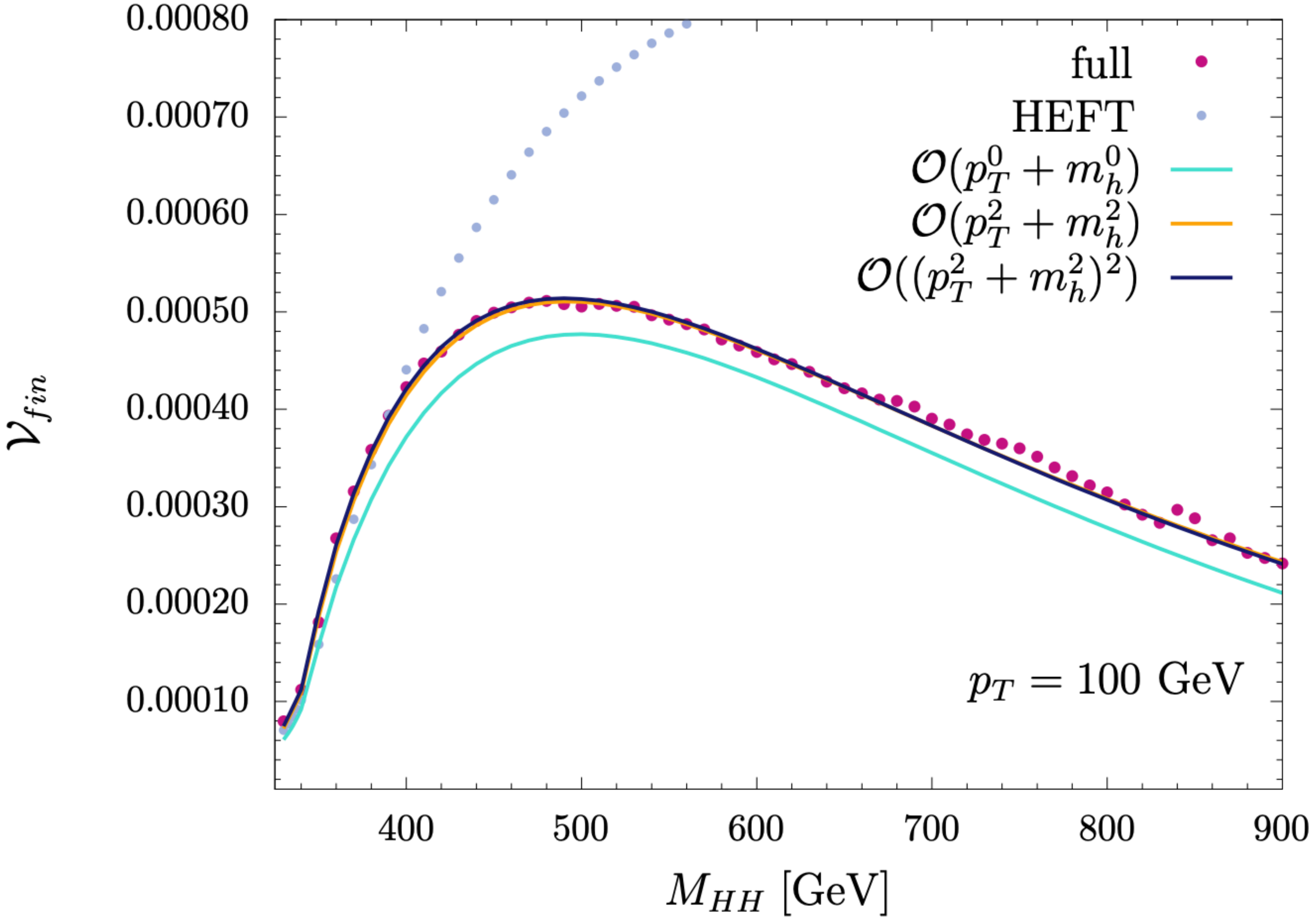
Top mass scheme uncertainties:

$$\sqrt{s} = 13\text{ TeV}: \quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%}\text{ fb},$$
$$\sqrt{s} = 14\text{ TeV}: \quad \sigma_{tot} = 32.81(7)^{+4\%}_{-18\%}\text{ fb},$$
$$\sqrt{s} = 27\text{ TeV}: \quad \sigma_{tot} = 127.0(2)^{+4\%}_{-18\%}\text{ fb},$$
$$\sqrt{s} = 100\text{ TeV}: \quad \sigma_{tot} = 1140(2)^{+3\%}_{-18\%}\text{ fb}$$

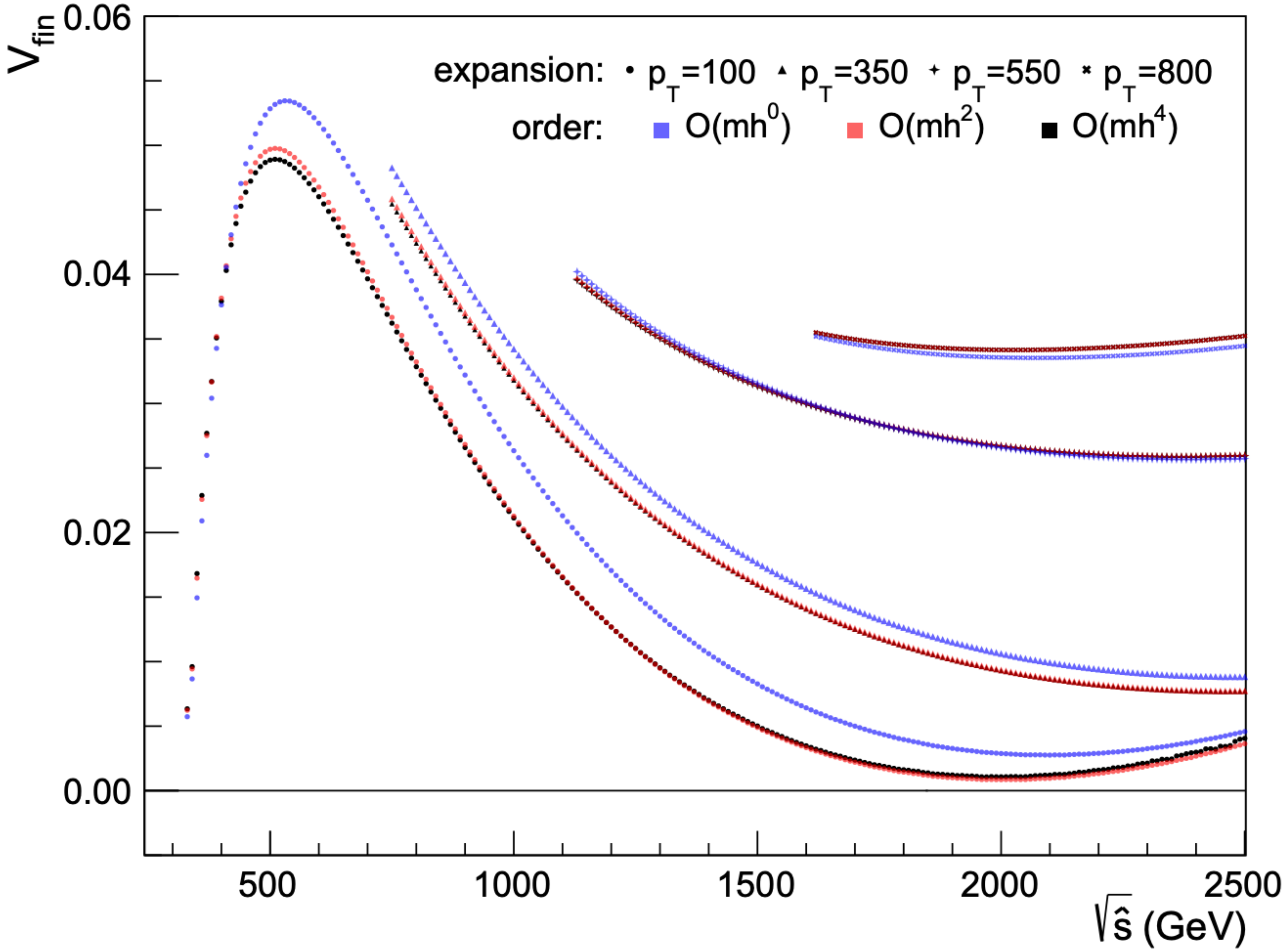
Baglio et al,18',20'

Double Higgs production

Analytical approach:



Bonciani et al,18’

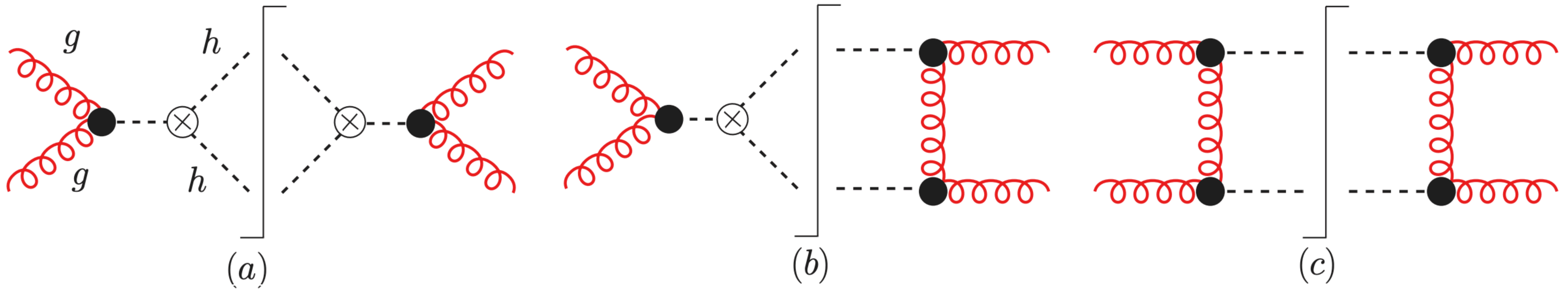


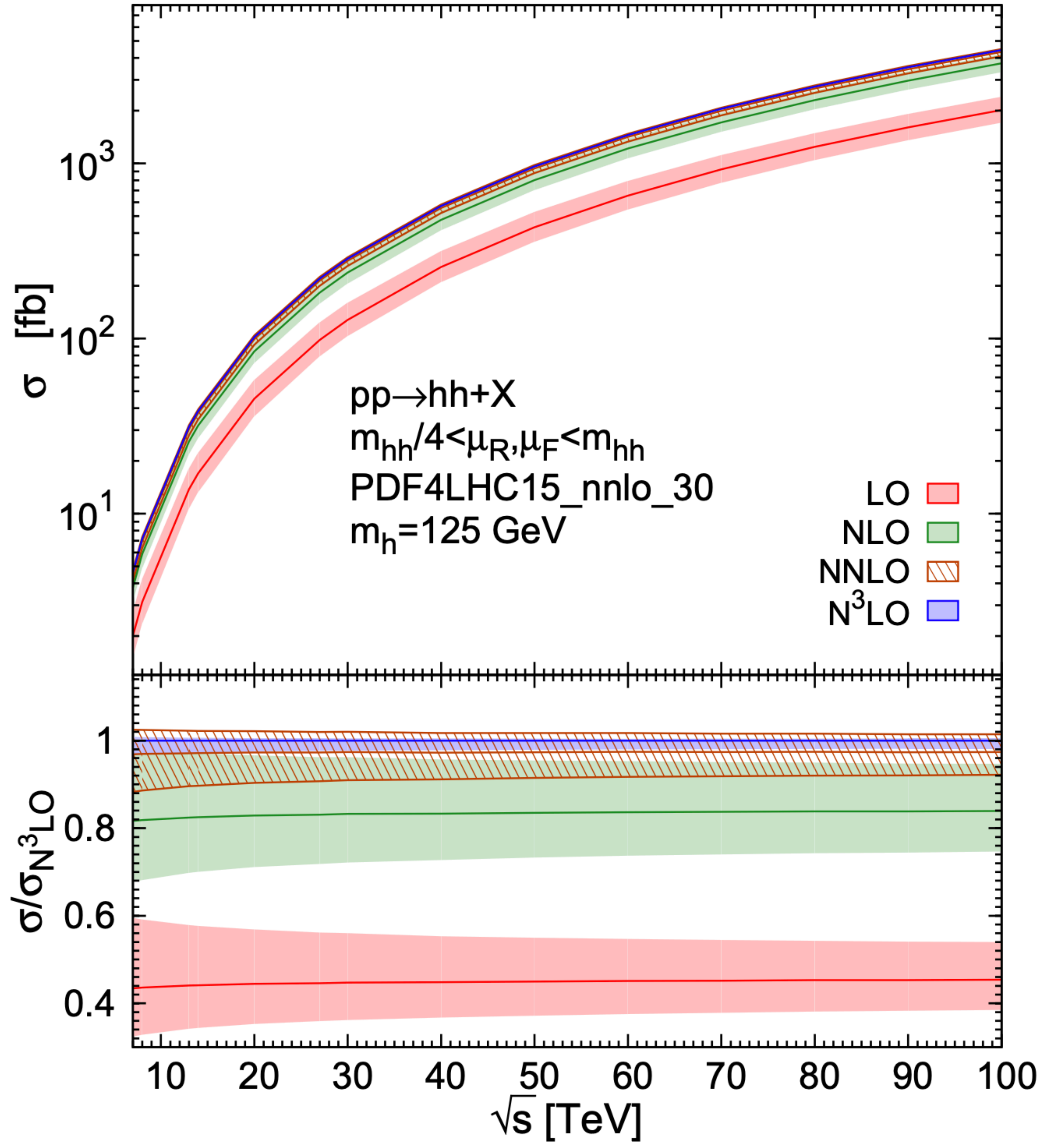
Xu, Yang 18’; Wang et al,20’

Double Higgs production

NNNLO QCD corrections in HTL

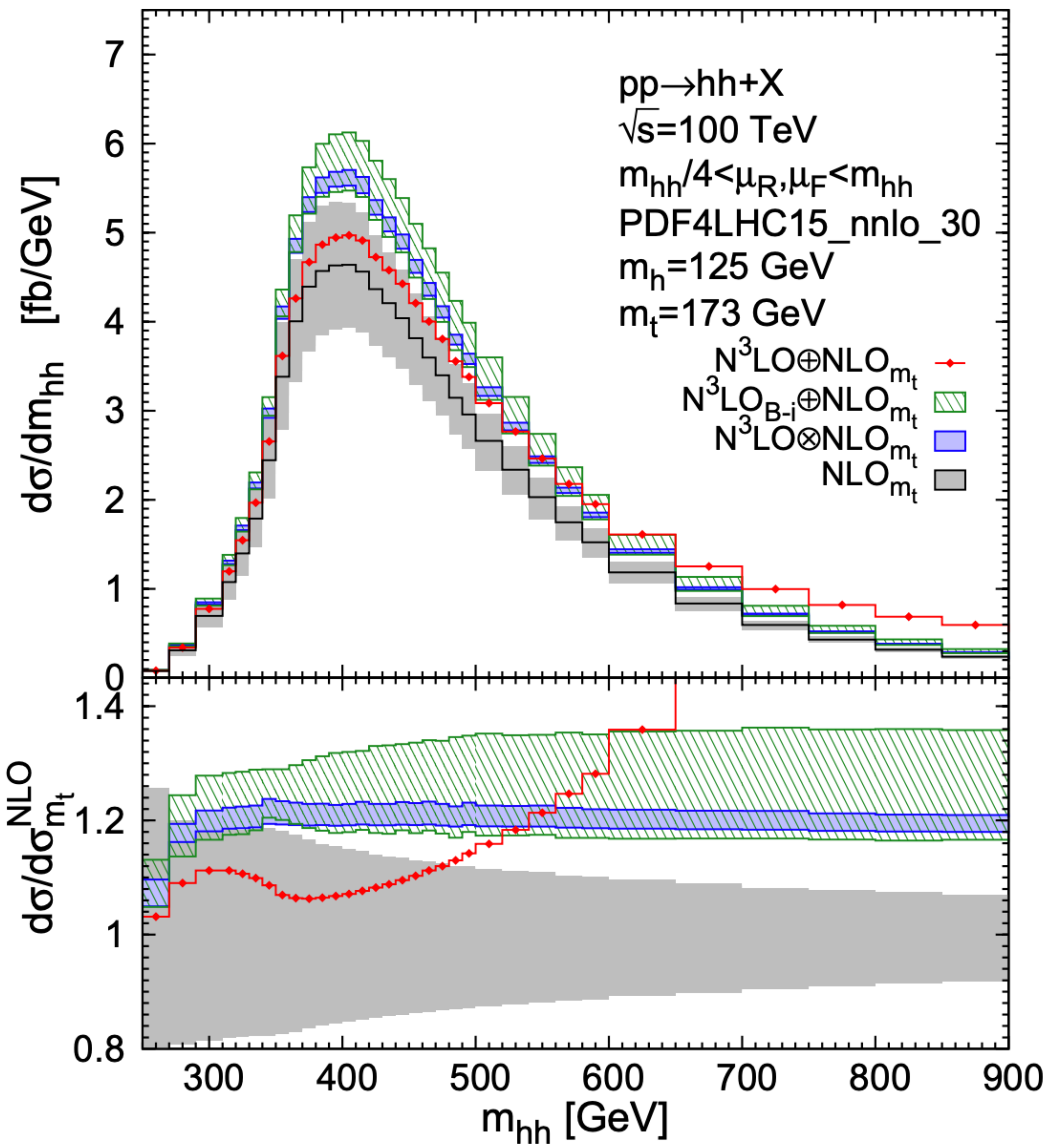
Chen, Li, Shao, JW,19',20





$$\frac{\sigma_{\mathrm{NNNLO}}}{\sigma_{\mathrm{NNLO}}} \sim 3\,\%$$

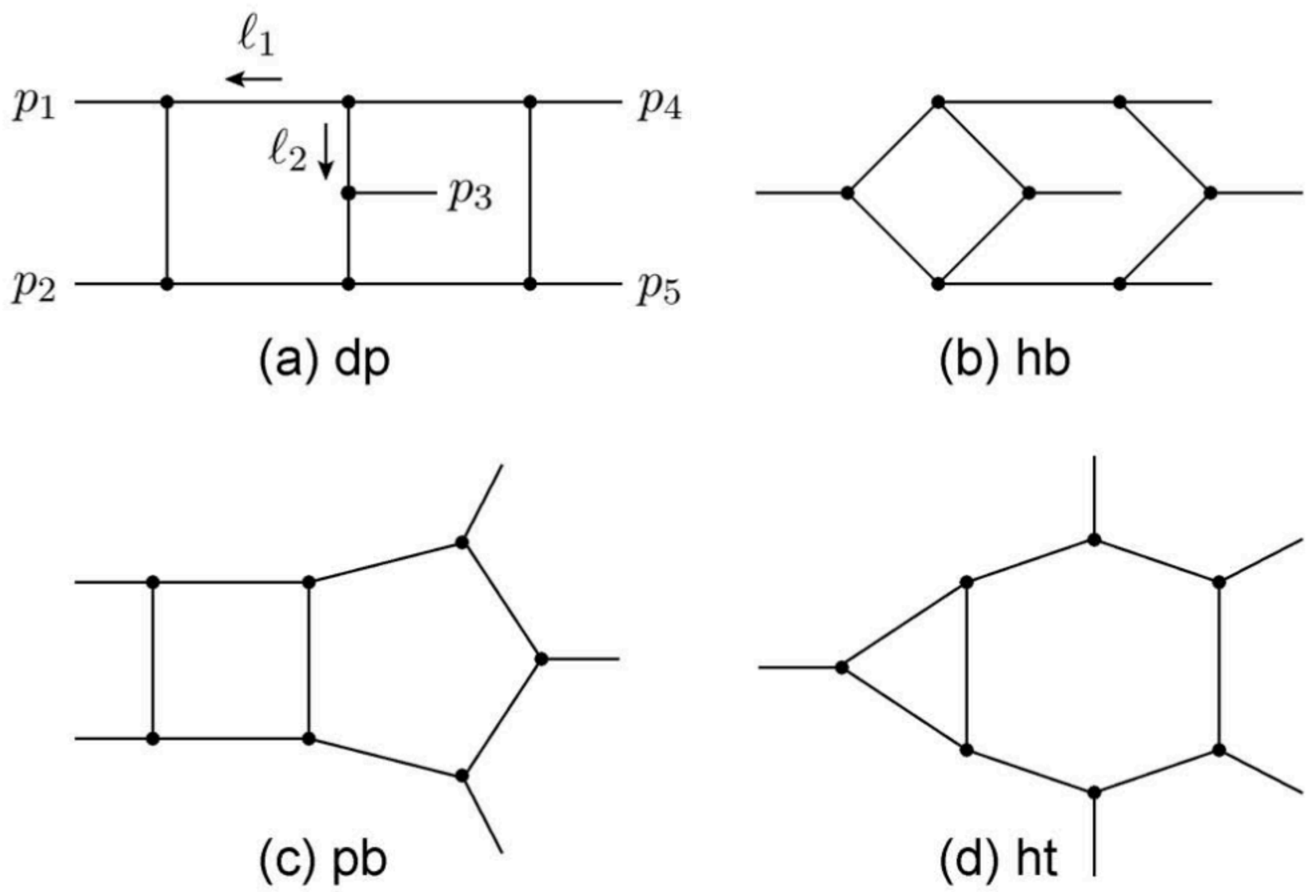
Scale unc.
NNLO: 7%
NNNLO: 3%



Improve the NLO by ~20%

Jet or photon production

2 to 3 process: Two loop virtual corrections



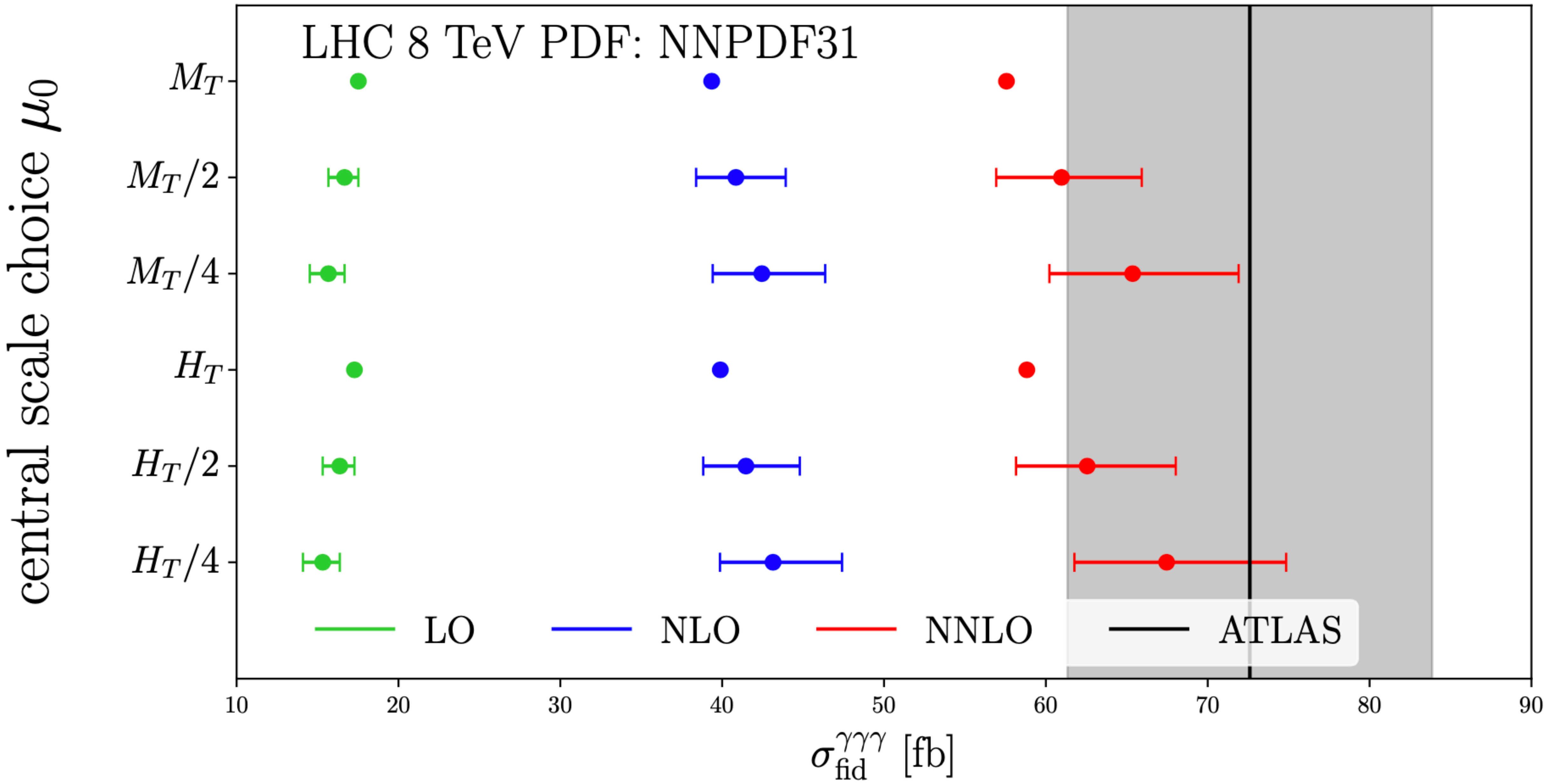
- Two- loop massless five-particle scattering amplitudes (all plus helicity or leading color) in the physical scattering region.

Abreu et al, 18'; Badger et al,19'; Laurentis Maitre 20'

- New methods have been developed for the reduction.

Guan, Liu, Ma 19'

top.	#int.	#MIs	t_{search} (h)	t_{solve} (s)	size(MB)
(a)	3914	108	112	0.17	66
(b)	3584	73	31	0.090	40
(c)	3458	61	56	0.075	31
(d)	2634	28	8	0.035	11



- Application in 3 photon production

Chawdhry, et al 19'; Kallweit et al 20'; Abreu et al 20'

- Extended to W+2 parton

Abreu et al, 20'

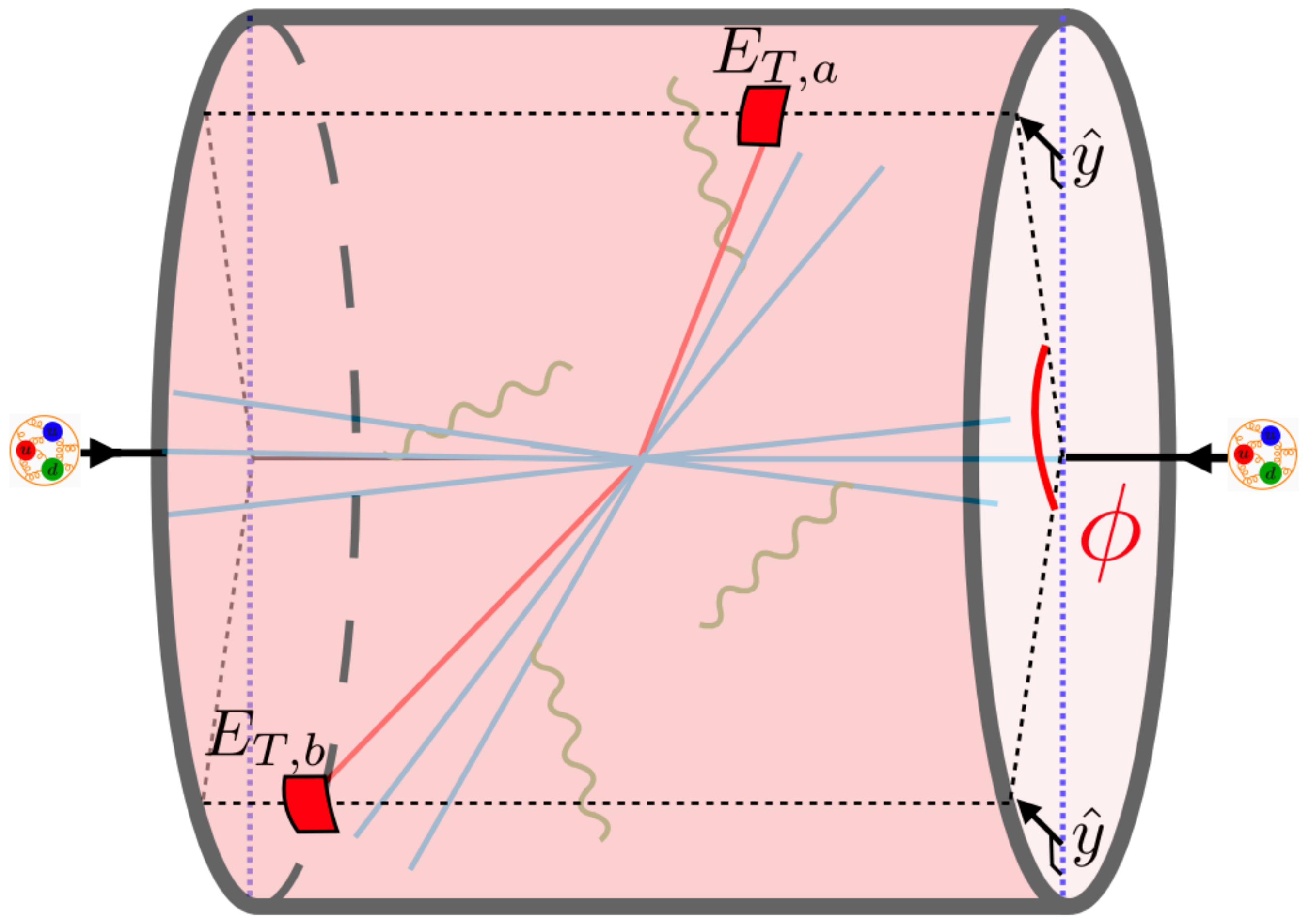
Jet or photon production

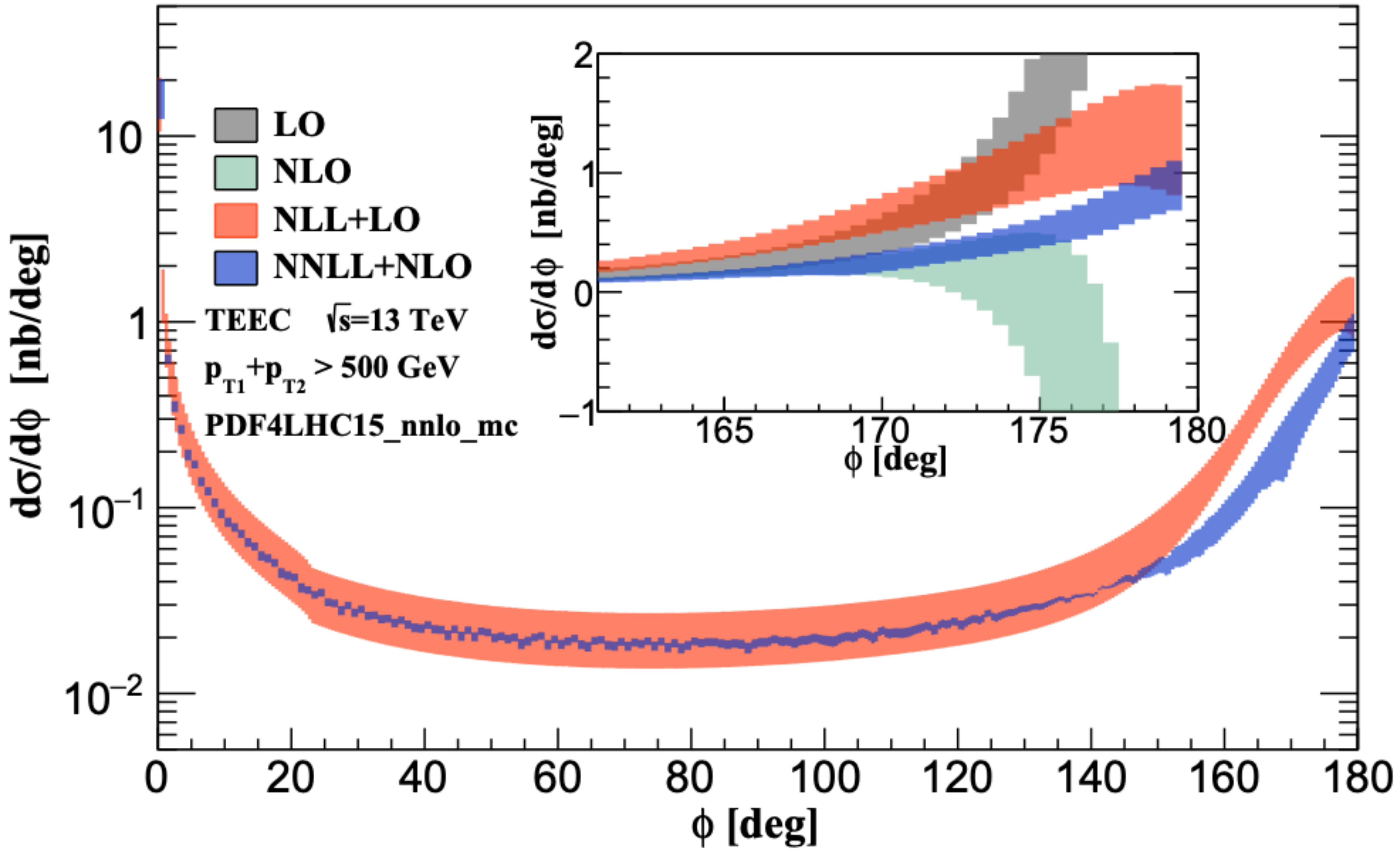
Transverse Energy-Energy Correlator

$$\frac{d\sigma^{(0)}}{d\tau} = \frac{p_T}{16\pi s^2(1+\delta_{f_3 f_4})\sqrt{\tau}} \sum_{\text{channels}} \frac{1}{N_{\text{init}}} \int \frac{dy_3 dy_4 dp_T^2}{\xi_1 \xi_2} \int_{-\infty}^{\infty} \frac{db}{2\pi} e^{-2ib\sqrt{\tau}p_T} \text{tr}\left[\mathbf{H}^{f_1 f_2 \to f_3 f_4}(p_T, y^*, \mu)\mathbf{S}(b, y^*, \mu, \nu)\right]$$
$$\cdot B_{f_1/N_1}(b,\, \xi_1,\, \mu,\, \nu)\, B_{f_2/N_2}(b,\, \xi_2,\, \mu,\, \nu) J_{f_3}(b, \mu, \nu)\, J_{f_4}(b, \mu, \nu)\,.$$

Hard function   Soft function

Beam function   Jet function





$$\text{TEEC} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

Extended to DIS   Li, Vitev, Zhu,20'

Gao, Li, Moult, Zhu,19'

Parton distribution function at NNNLO

In small qT region,

$$\frac{d\sigma}{d^2q_\perp} \sim \sigma_0 H(Q) \int d^2b_\perp e^{ib_\perp \cdot q_\perp} \mathcal{B} \otimes \mathcal{B}\mathcal{S}\,,$$

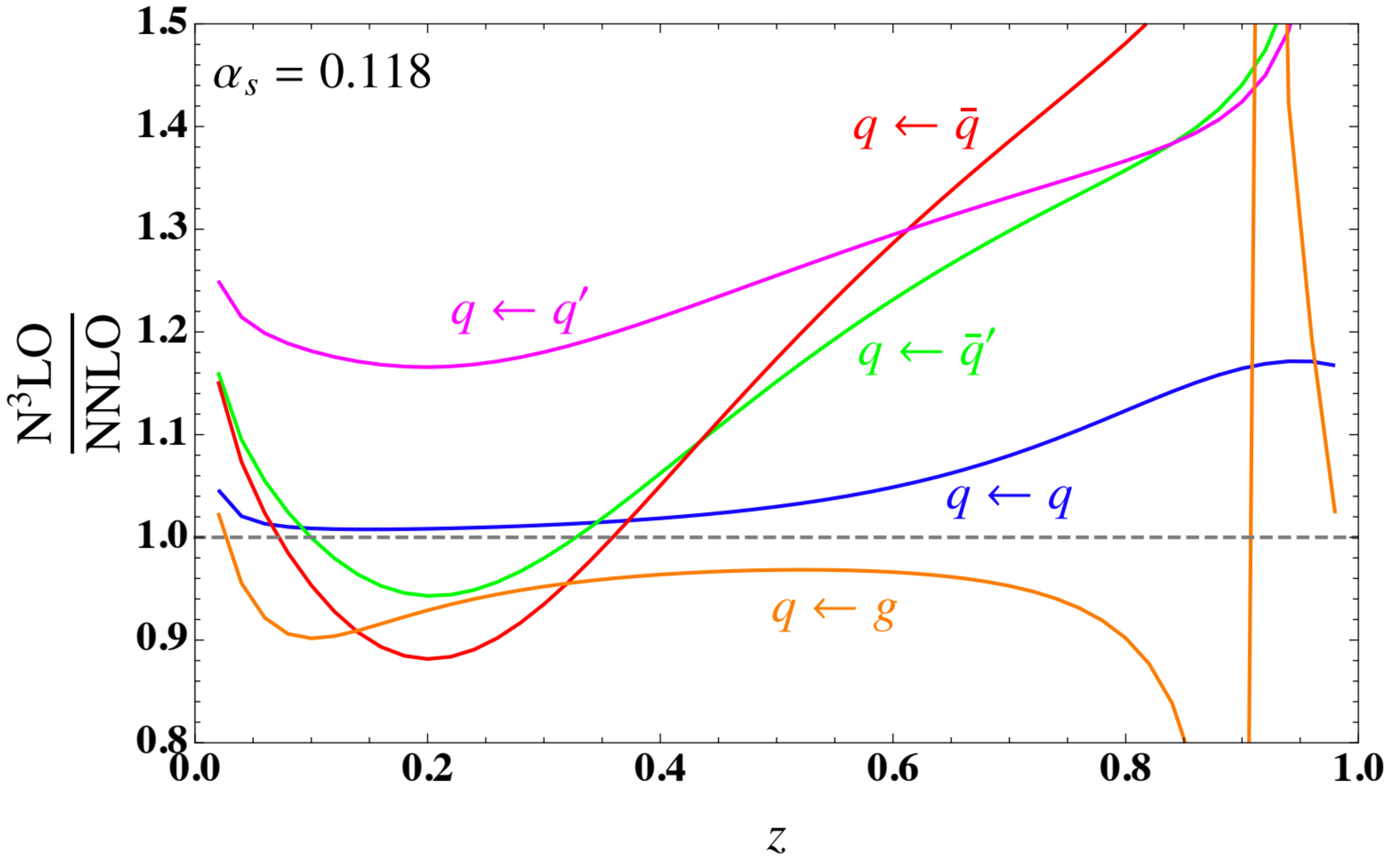
qT dep. soft function

qT dep. beam function

Transverse momentum dep. PDFs

$$f_{T,ij}(z, b_\perp, P^+, \mu) = \mathcal{I}_{ij}(z, b_\perp, P^+, \mu, \nu)\sqrt{\mathcal{S}(b_\perp, \mu, \nu)}$$

Matching coefficient of $\mathscr{B}$ to normal PDF



N-jettiness beam function at NNNLO

Ebert, Mistlberger, Vita 20'

Luo, Yang, Zhu, Zhu 19'

Slicing method for differential calculation at NNNLO: three loop jet function

Bruser, Liu, Stahlhofen 18'

quasi-Parton distribution function at NNLO

Light cone PDF:

$$f_{q/H}(x,\mu)=\int\frac{d\xi^-}{4\pi}\,e^{-ixp^+\xi^-}\langle p|\bar{q}(\xi^-)\gamma^+\exp\left(-ig\int_0^{\xi^-}d\eta^-A^+(\eta^-)\right)q(0)|p\rangle$$

Quasi PDF: Ji 13'

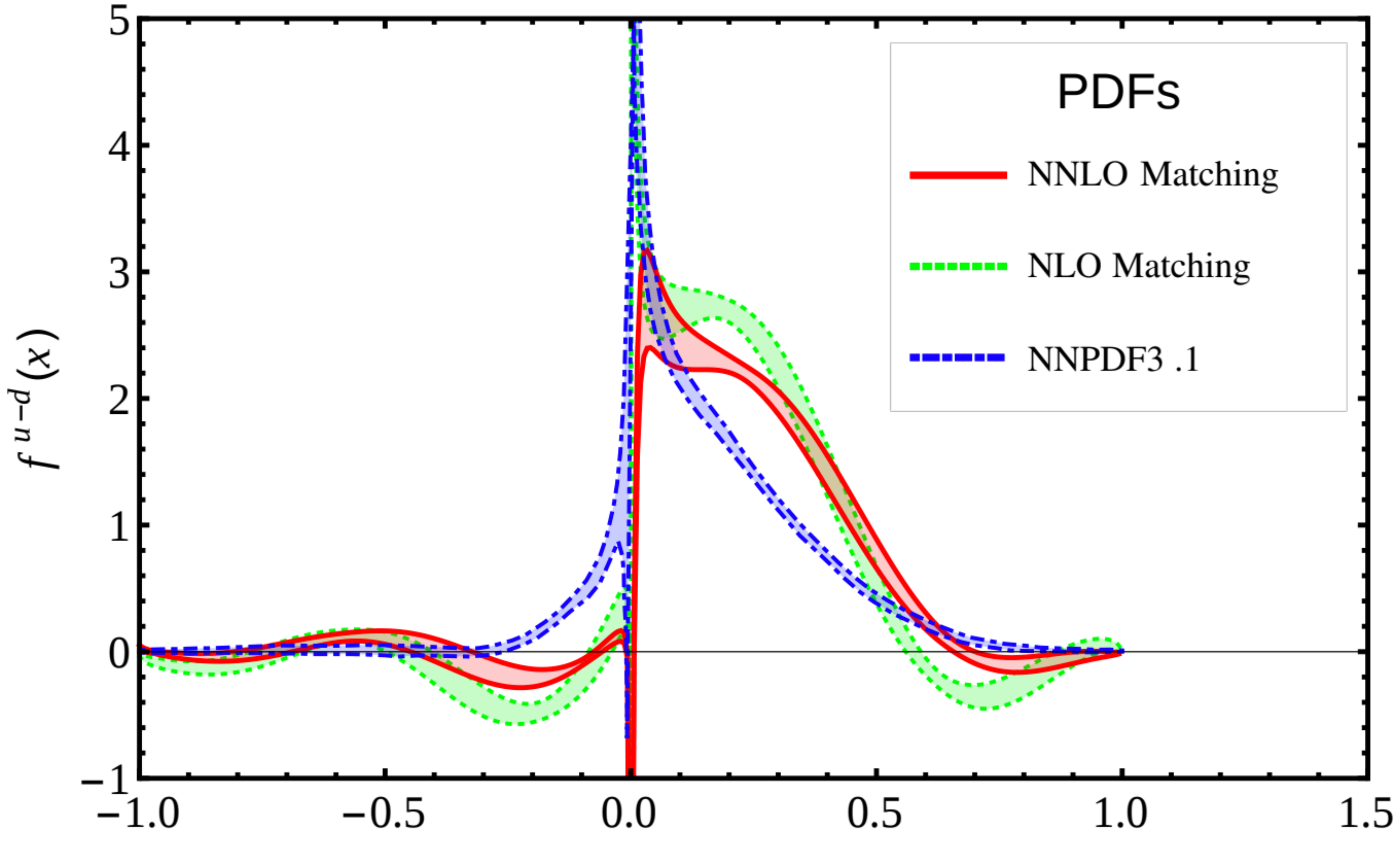
$$\tilde{f}_{q/H}(x,p^z)\;=\;N\int\frac{dz}{4\pi}e^{izxp^z}\langle p|\bar{q}(z)\Gamma\exp\left(-ig\int_0^z dz'A^z(z')\right)q(0)|p\rangle$$

Matching:

$$\tilde{f}_{q/H}(x,p^z)=\int_{-1}^{1}\frac{dy}{|y|}\left[C_{qq'}\left(\frac{x}{y},\frac{|y|p^z}{\mu}\right)f_{q'/H}(y,\mu)\right]$$

NNLO result: Chen, Wang, Zhu 20'; Li, Ma, Qiu 20'

See Zheng Yang Li 's talk



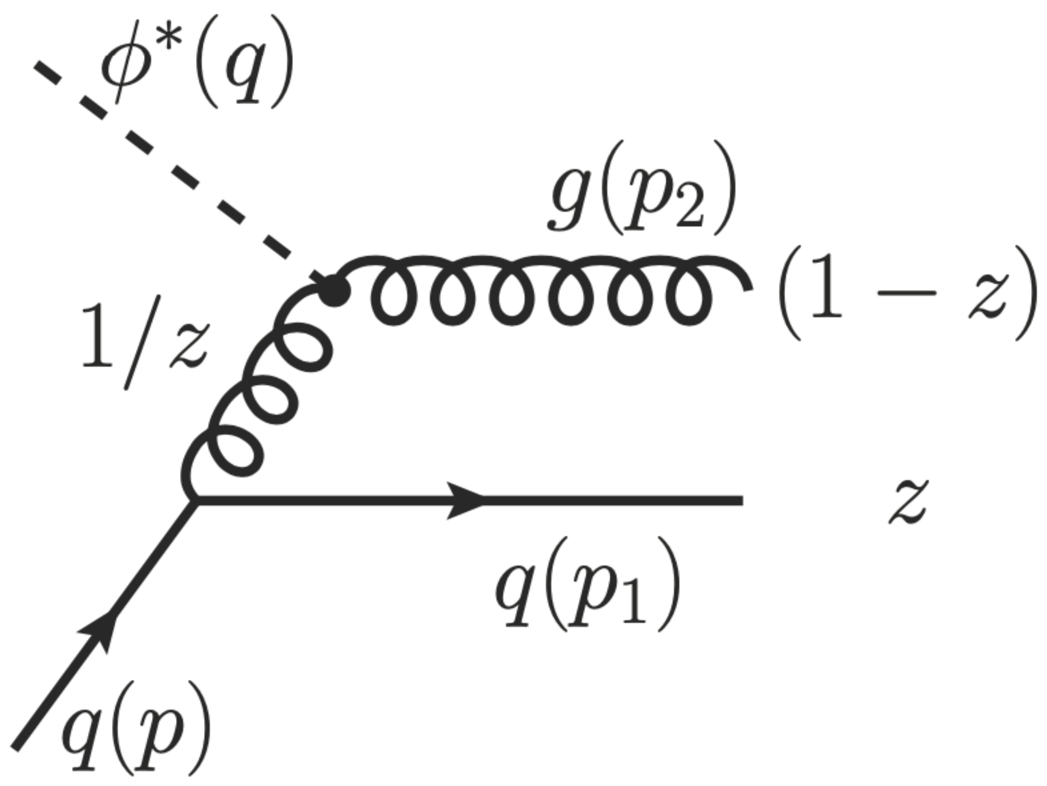
Interesting double logarithms in PDF

All order $q \to g$ DGLAP splitting function : Vogt 10'

$$P_{gq}^{\text{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N \qquad \mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n$$

Structure function:

$$\sum_i (W_{\phi,i}f_i)^{\text{NLP}} = W_{\phi,q}^{\text{NLP}} f_q^{\text{LP}} + W_{\phi,\bar{q}}^{\text{NLP}} f_{\bar{q}}^{\text{LP}} + W_{\phi,g}^{\text{NLP}} f_g^{\text{LP}} + W_{\phi,g}^{\text{LP}} f_g^{\text{NLP}}$$



Making use of consistency relation: Moult, Stewart, Zhu 19'

$$F_{\text{pole}}(w,a) = \sum_{k\geq 1}\frac{1}{w^k}\sum_{n\geq 0}\frac{B_n}{n!(n+k)!}a^{n+k}$$

Algebraic all order expression:

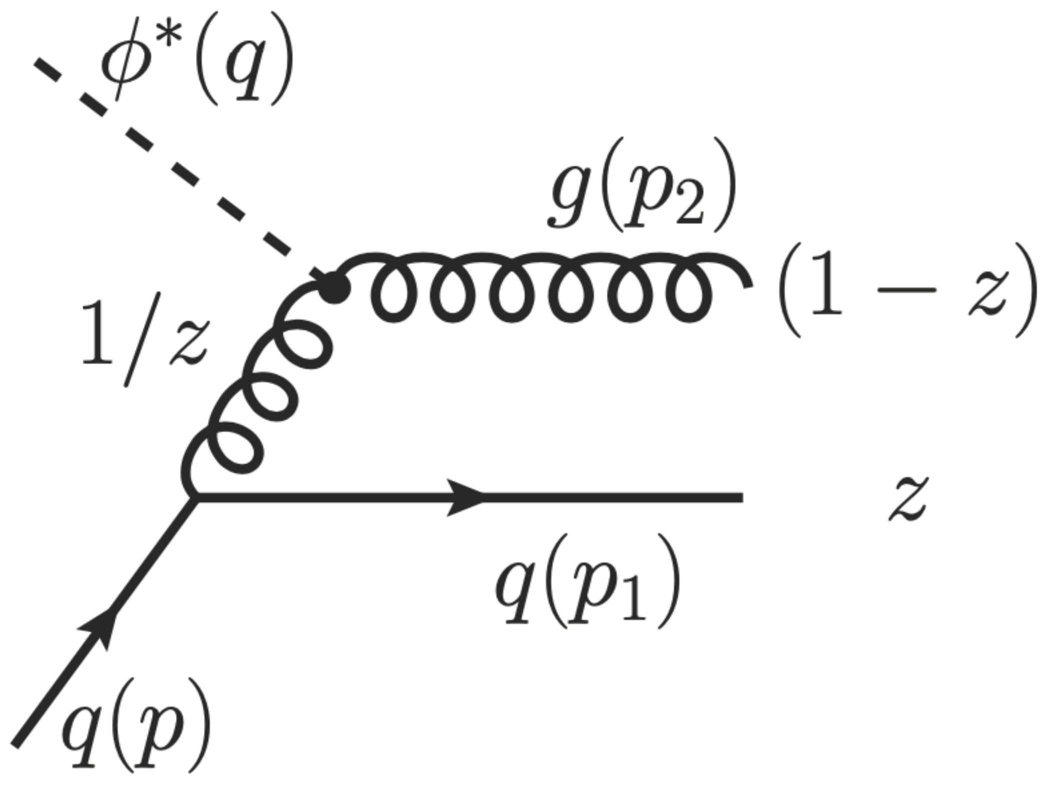
$$\gamma_{gq}^{\text{NLP,LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\left[F_{\text{pole}}(w,a) - w\frac{d}{da}F_{\text{pole}}(w,a)\right] = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a)$$

Beneke et al 20'

End point divergences at subleading power factorization and resummation

Interesting double logarithms in PDF

Description of the off-diagonal process in Effective theory: Beneke et al 20'



$$F_A^{\mu\nu} F_{\mu\nu}^A = C^{A0}(Q^2,\mu^2)\, J^{A0} \qquad J^{A0} = 2g^{\mu\nu} n_-\partial\mathcal{A}_{\perp\mu}^{A,z\text{-}\overline{hc}}\, n_+\partial\mathcal{A}_{\perp\nu}^{A,z\text{-}hc}$$

The left diagram can be represented by

$$C^{A0}\left(Q^2,\mu^2\right)\langle q(p_1) g(p_2)| \int d^d x\, T\left\{J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z\text{-}\overline{sc}}}(x)\right\}|q(p)\rangle$$

Second matching

$$\mathcal{L}^{(1)}_{\xi q_{z\text{-}\overline{sc}}}(x) = \bar{q}_{z\text{-}\overline{sc}}(x_-)\, W^\dagger_{z\text{-}hc} i\not{D}_{\perp,z\text{-}hc}\xi_{z\text{-}hc} + \text{h.c.}$$

$$\int d^d x\, T\left\{J^{A0}, \mathcal{L}^{(1)}_{\xi q_{z\text{-}\overline{sc}}}(x)\right\} = D^{B1}(zQ^2,\mu^2)\, J^{B1}$$

RG equation:

$$\frac{d}{d\ln\mu} C^{A0}\left(Q^2,\mu^2\right) D^{B1}\left(zQ^2,\mu^2\right) J^{B1} = 0 \qquad \frac{d}{d\ln\mu} C^{A0}(Q^2,\mu^2) = \Gamma_{A0}\, C^{A0}(Q^2,\mu^2) = \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\, C^{A0}(Q^2,\mu^2)$$

$$\left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\text{bare}} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F - C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]$$

Summary

- With the accumulating data at the LHC, the experimental errors become less and less, requiring high precise predictions from theory.
- New methods have been developed during the last ten years, e.g. differential equations, generalized unitarity, finite field, intersection numbers…. We understand the microworld more clearly. Meanwhile, there are problems to be solved.
- In the future, after scrutinizing the data with highest precise theory, we are looking forward to some new physics discovery.
- Sorry for many topics not covered in this talk.

Thanks a lot for your attention.