Extraction of NNLO PDFs from Lattice QCD Calculations

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Based On ZYL, Ma, Qiu, 2006.12370

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Outline

- Introduction to Lattice Calculations of PDFs
- Determination of Renormalization Scheme
- Calculation of NNLO Matching Coefficients

PDFs and Lattice QCD

- PDFs encode important nonperturbative information of strong interactions, and were usually extracted from experimental data.

$$d\sigma_{hh'}(Q^2,S)=\sum_{i,j} f_{i/h}(x,\mu^2)\otimes f_{j/h'}(x',\mu^2)$$
$$\otimes\, d\hat{\sigma}_{ij}(x,x',\mu^2,Q^2,S)+O(\Lambda^2_{\mathrm{QCD}}/Q^2)$$

- Lattice QCD is the main nonperturbative approach to solve QCD.

- By definition, PDFs depend on time, so PDFs cannot be calculated by LQCD directly.

Quasi-PDFs and Pseudo-PDFs

- A new idea: use LQCD calculable functions to calculate PDFs

Ji, 1305.1539, 1404,6680

$$f_{q/h}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- p_+} \langle h(p)|\overline{\psi}_q(\xi_-)\gamma_+ \mathcal{P}\left\{e^{-ig_s\int_0^{\xi_-} d\eta_- A_+^{(f)}(\eta_-)}\right\}\psi_q(0)|h(p)\rangle$$

$$\tilde{F}_{q/h}(x,p_z^2,\mu^2) = \int \frac{d\xi_z}{4\pi} e^{ix\xi_z p_z} \langle h(p)|\overline{\psi}_q(\xi_z)\gamma_z \mathcal{P}\left\{e^{-ig_s\int_0^{\xi_z} d\eta_z A_z^{(f)}(\eta_z)}\right\}\psi_q(0)|h(p)\rangle$$

- Quasi-PDFs are factorizable into PDFs

$$\tilde{F}_{q_{ik}/h}\left(y,p_z^2,\mu^2\right) = \int_{-1}^{1}\frac{dx}{|x|} f_{q_{ik}/h}(x,\mu^2)\tilde{K}\left(y/x, x^2p_z^2,\mu^2\right) + O\left(\Lambda_{\rm QCD}^2/(yp_z)^2\right)$$

flavor non-singlet quark: $q_{ik} \equiv q_i - q_k$

- Pseudo-PDFs:

Radyushkin, 1705.01488

$$\hat{F}_{q/h}(x,\xi_z^2,\mu^2) = \int \frac{dp_z}{4\pi}\frac{\xi_z}{p_z} e^{ix\xi_z p_z} \langle h(p)|\overline{\psi}_q(\xi_z)\gamma_z \mathcal{P}\left\{e^{-ig_s\int_0^{\xi_z} d\eta_z A_z^{(f)}(\eta_z)}\right\}\psi_q(0)|h(p)\rangle$$

$$\hat{F}_{q_{ik}/h}\left(y,\xi_z^2,\mu^2\right) = \int_{-1}^{1}\frac{dx}{|x|} f_{q_{ik}/h}(x,\mu^2)\hat{K}\left(y/x,\xi_z^2,\mu^2\right) + O\left(\xi_z^2\Lambda_{\rm QCD}^2\right)$$

Lattice Cross Sections

Ma, Qiu, 1404.6860, 1709.03018

- Quark correlation functions:

$$\langle h(p)|\overline{\psi}_q(\xi_z)\gamma_z \mathcal{P}\left\{e^{-ig_s\int_0^{\xi_z} d\eta_z A_z^{(f)}(\eta_z)}\right\}\psi_q(0)|h(p)\rangle$$

$$\overset{q\to q_i - q_k}{=} \int_{-1}^{1}\frac{dx}{x} f_{q_{ik}/h}(x,\mu^2) K_z\left(xp_z\xi_z, \xi_z^2, \mu^2\right) + O\left(\xi_z^2\Lambda_{\text{QCD}}^2\right)$$

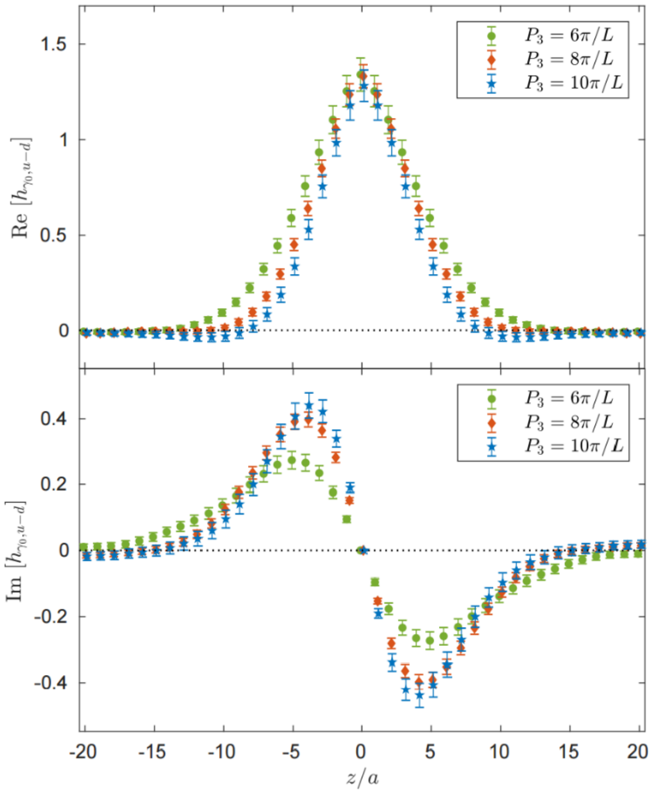
- Good LCSs:
- 1) Calculable in Euclidean-space lattice
- 2) renormalizable for UV divergence to ensure a reliable continue limit
- 3) factorizable to PDFs

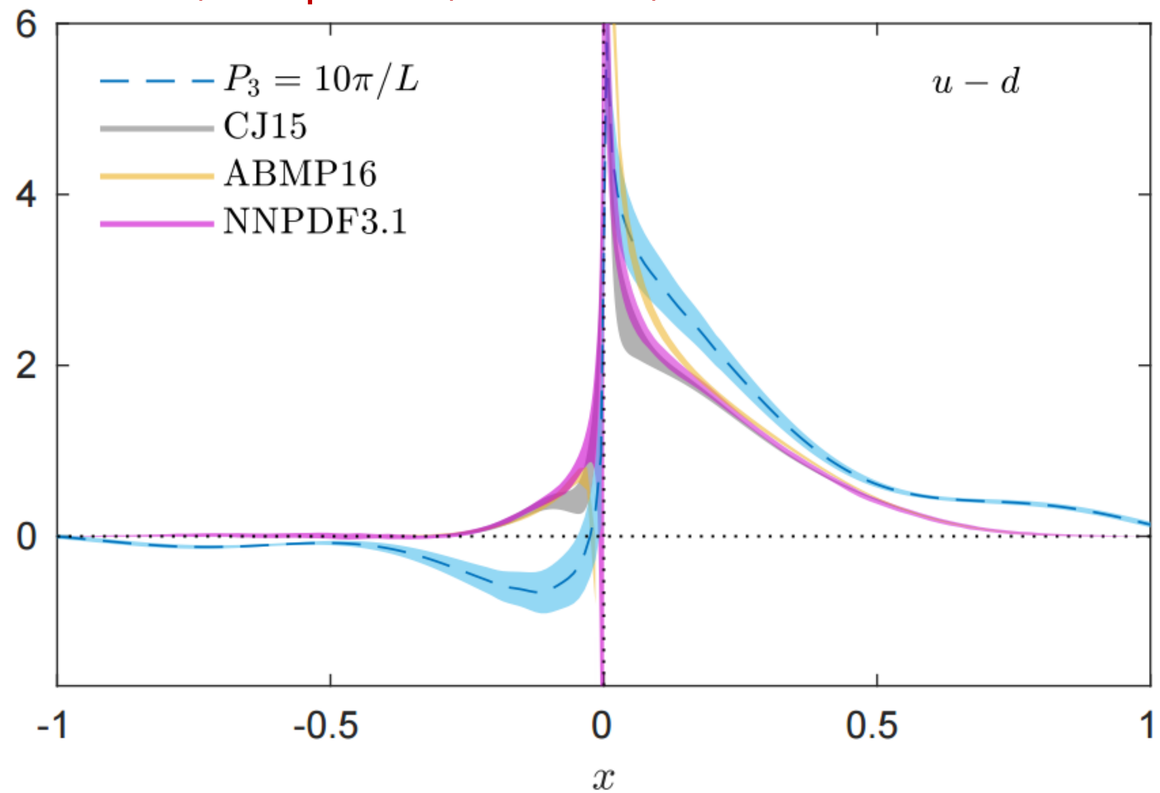
“quasi-PDFs”, “pseudo-PDFs” and “QCFs” are special cases of “good LCSs”

Lattice Results of QCFs

- Exploratory study

Alexandrou, Cichy, Constantinou, Hadjiyiannakou, Jansen, Scapellato, Steffens, 1902.00587





- Shape is similar to the experimental results

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Operator Renormalization

- Quark correlation operator:

$$\mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta) = \overline{\psi}_q(\xi)\,\gamma^\nu\,\Phi^{(f)}(\{\xi,0\})\,\psi_q(0)\Big|_{\mu^2,\delta}$$

- Multiplicatively renormalization: Ji, Zhang, Zhao, 1706.08962

Ishikawa, Ma, Qiu, Yoshida, 1707.03107 Green, Jansen, Steffens, 1707.07152

$$\mathcal{O}_q^{\nu,\mathrm{RS}}(\xi) = \mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta)/Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta)$$

- Non-perturbative renormalization schemes:

$$Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta) = \frac{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle}{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle^{(0)}}$$

an example: $|\mathrm{RS}\rangle = |h(p)\rangle_{p^2=-\mu_R^2}$ Stewart, Zhao, 1709.04933

a single-scale scheme: $|\mathrm{RS}\rangle = |\Omega\rangle$

Braun, Vladimirov, Zhang, 1810.00048

Factorization of Renormalized QCFs

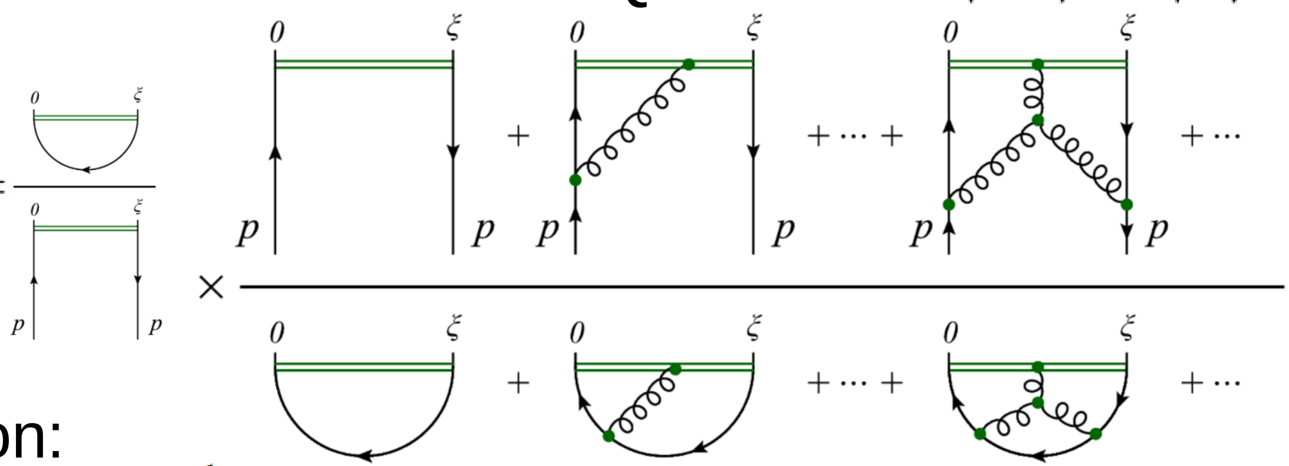
- Renormalized QCFs:

$$F^{\nu,\mathrm{RS}}_{q/h}(\omega,\xi^2) = \langle h(p)|\mathcal{O}^{\nu,\mathrm{RS}}_q(\xi)|h(p)\rangle \qquad \omega \equiv p\cdot\xi$$

  are independent of scale μ

- Perturbative calculations of QCFs with $|\mathrm{RS}\rangle = |\Omega\rangle$:

$$F^{\nu,\Omega,\mathrm{NNLO}}_{q_{ik}/q_i}(\omega,\xi^2\mu^2) =$$

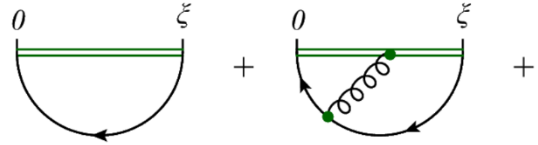


- Factorization:

$$F^{\nu,\mathrm{RS}}_{q_{ik}/h}(\omega,\xi^2) = \frac{1}{R^{\mathrm{RS}}}\int_{-1}^{1}\frac{dx}{x}\, f_{q_{ik}/h}(x,\mu^2)K^{\nu}(x\omega,\xi^2,\mu^2) + O(\xi^2\Lambda^2_{\mathrm{QCD}})$$

$$R^{\mathrm{RS}}(\xi^2,\mu^2) \equiv Z^{\mathrm{RS}}(\xi^2,\mu^2,\epsilon)/Z^{\overline{\mathrm{MS}}}(\xi^2,\mu^2,\epsilon)$$

Renormalization Factor

- Steps of calculating  :

- Fourier transformation. IBP reduction.
- Solve MIs by the method of difference equations.

- Obtain Z^{Ω} up to NNLO

$$Z_{\overline{\rm MS}} = 1 + \frac{\alpha_s S_\epsilon}{\pi\epsilon} C_F + \left(\frac{\alpha_s S_\epsilon}{\pi\epsilon}\right)^2 C_F \left\{ \left[\frac{C_F}{2} - \frac{13 C_A}{32} + \frac{n_f T_F}{8}\right] + \left[\left(-\frac{1}{8} + \frac{\pi^2}{12}\right) C_F + \left(\frac{25}{48} - \frac{\pi^2}{48}\right) C_A - \frac{n_f T_F}{6}\right]\epsilon \right\}$$

$$S_\epsilon \equiv (4\pi)^\epsilon / \Gamma(1-\epsilon)$$

$$R^{\Omega} = 1 + \frac{\alpha_s}{\pi} C_F \left(\frac{3}{4}L + 2 + \frac{\pi^2}{3}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left\{ \left[\frac{9}{32} C_F + \frac{11}{32} C_A - \frac{1}{8} n_f T_F\right] L^2 \right.$$
$$+ \left[\left(\frac{43}{32} + \frac{5\pi^2}{12}\right) C_F + \left(\frac{75}{32} + \frac{19\pi^2}{72}\right) C_A - \left(\frac{7}{8} + \frac{\pi^2}{9}\right) n_f T_F\right] L + \left[\left(\frac{153}{128} + \frac{13\pi^2}{12} - \frac{\zeta_3}{2} + \frac{\pi^4}{90}\right) C_F \right.$$
$$\left.\left. + \left(\frac{6413}{1152} - \frac{5\pi^2}{432} - \frac{13\zeta_3}{2} - \frac{\pi^4}{90}\right) C_A - \left(\frac{589}{288} - \frac{\pi^2}{27} - 2\zeta_3\right) n_f T_F\right] \right\},$$

$$L \equiv \ln(-\xi^2\mu^2/4) + 2\gamma_E$$

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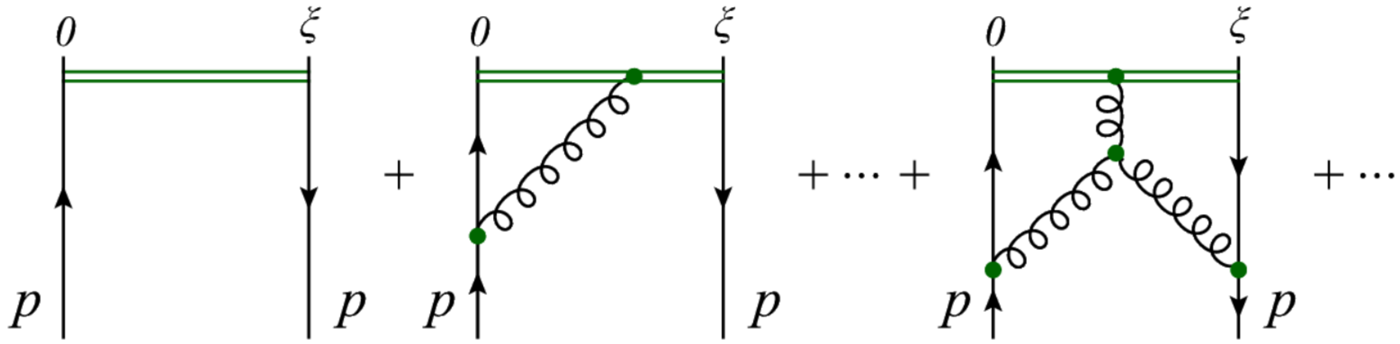
Partonic Matrix Elements

- Diagrams for calculating $F^{\nu,\overline{\mathrm{MS}}}_{q_{ik}/q_i}$:



- Steps of calculations:
- Fourier transformation. IBP reduction.
- Solve MIs by the method of differential equations. Solve boundary conditions with the help of vacuum MIs.
- Obtain Taylor series of ω . Assume an ansatz and obtain analytical expressions.

$\omega \equiv p \cdot \xi$

Matching Coefficients

- Structure of matching coefficients of QCFs:

$$K^\nu(x\omega,\xi^2,\mu^2) \equiv xp^\nu A(x\omega,\xi^2,\mu^2) + x\omega \frac{\xi^\nu}{-\xi^2} B(x\omega,\xi^2,\mu^2)$$

$$iA(\omega,\xi^2,\mu^2) = 2e^{i\omega} + \frac{\alpha_s}{\pi}\sum_{i=0}^{1} L^i C_F A_{i1}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \sum_{i=0}^{2} L^i C_F \left[C_F A_{i1}^{(2)} + C_A A_{i2}^{(2)} + n_f T_F A_{i3}^{(2)}\right]$$

$$L \equiv \ln(-\xi^2\mu^2/4) + 2\gamma_E$$

$$A_{ij}^{(m)} = a_{ij1}^{(m)}\, e^{i\omega} + \int_0^1 dz\, \frac{a_{ij2}^{(m)}(z)}{1-z}\left(e^{iz\omega} - e^{i\omega}\right) + \int_{-1}^0 dz\, \frac{a_{ij3}^{(m)}(z)}{1-z}\left(e^{iz\omega} - e^{i\omega}\right)$$

- Ansatz: polynomials time harmonic polylogarithms

examples:

$$a_{011}^{(2)} = -4\zeta_3 + \frac{\pi^2}{9} + \frac{223}{192}$$

$$a_{212}^{(2)} = -(1+z^2)H(1;z) + \cdots \qquad a_{213}^{(2)} = 0$$

$$a_{112}^{(2)} = (3-z^2)H(1,0;z) + \cdots \qquad a_{113}^{(2)} = -2(1+z^2)H(-1,0;-z) + \cdots$$

$$a_{012}^{(2)} = -(3-z^2)H(1,1,0;z) + \cdots \qquad a_{013}^{(2)} = -2(5+z^2)H(-1,-1,0;-z) + \cdots$$

harmonic polylogarithms:

$$H(1;z) = -\ln(1-z)\,,\; H(1,0;z) = -\ln(1-z)\ln(z) - Li_2(z)\,,\; H(1,1,0;z) = \zeta_3 + \frac{\pi^2}{6}\ln(1-z) - Li_3(1-z)\,,\;\cdots$$

Transformed Matching Coefficients

- Pseudo-PDFs:

  structure: $$\hat{K}(y,\xi^2\mu^2)=\hat{k}_1(\xi^2\mu^2)\delta(1-y)+\begin{cases}0 & 1<y\\ \left[\hat{k}_2(y,\xi^2\mu^2)\right]_+ & 0<y<1\\ \left[\hat{k}_3(y,\xi^2\mu^2)\right]_+ & -1\le y<0\\ 0 & y<-1\end{cases}$$

  when $a_{ij2}^{(m)}=H(n;z)$, $\hat{K}(y,\xi^2\mu^2)=\frac{\alpha_s^m}{\pi^m}C_j\begin{cases}0 & 1<y\\ \left[L^i\frac{H(n;y)}{1-y}\right]_+ & 0<y<1\\ 0 & y<0\end{cases}$

- Quasi-PDFs:

  structure: $$\tilde{K}(y,\mu^2/p_z^2)=\tilde{k}_1(\mu^2/p_z^2)\delta(1-y)+\begin{cases}\left[\tilde{k}_2(y,\mu^2/p_z^2)\right]_+ & 1<y\\ \left[\tilde{k}_3(y,\mu^2/p_z^2)\right]_+ & 0<y<1\\ \left[\tilde{k}_4(y,\mu^2/p_z^2)\right]_+ & -1\le y<0\\ -\left[\tilde{k}_2(y,\mu^2/p_z^2)\right]_+ & y<-1\end{cases}$$

  when $a_{0j2}^{(m)}=H(n;z)$, equals to psedo-PDFs

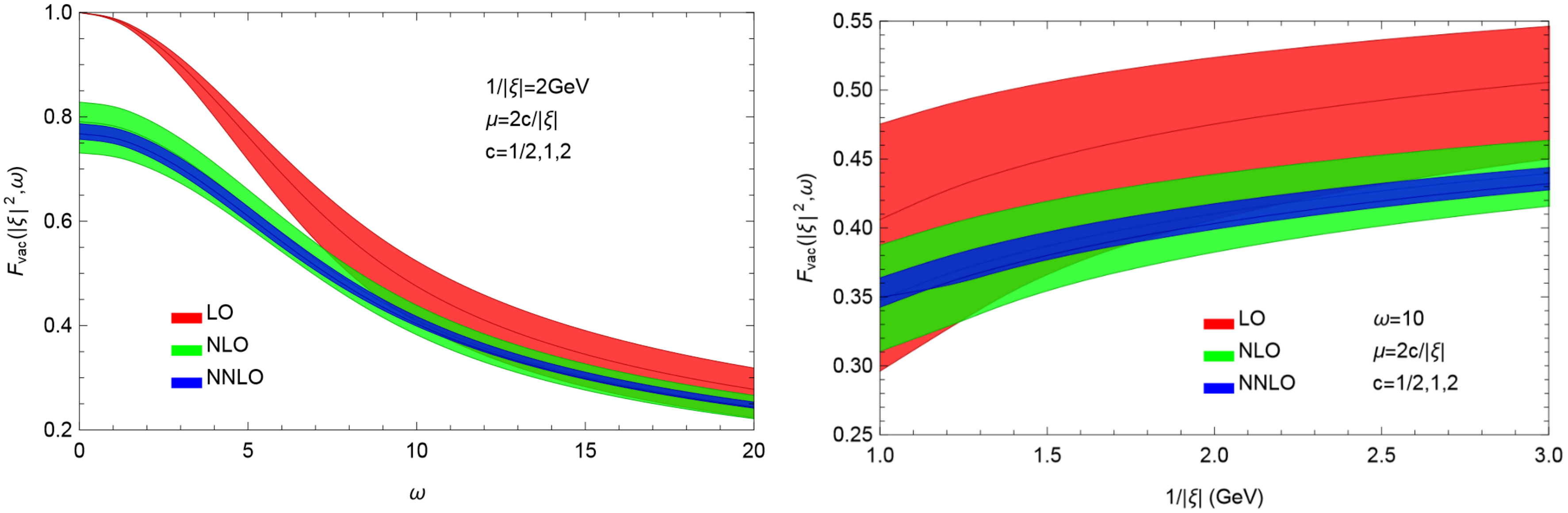
  for $i\neq 0$, for example, when $a_{112}^{(2)}=H(1,0;z)$

$$\tilde{K}(y,\mu^2/p_z^2)=\frac{\alpha_s^2}{\pi^2}C_F^2\begin{cases}\left[\frac{H(0,0,1;1/y)}{1-y}+\frac{H(1,0,1;1/y)}{1-y}\right]_+ & 1<y\\ \left[\ln(\frac{\mu^2}{4p_z^2})\frac{H(1,0;y)}{1-y}+\frac{2\zeta_3}{1-y}-\frac{3H(1,0,0;y)}{1-y}+\frac{H(1,0,1;y)}{1-y}+\frac{4H(1,1,0;y)}{1-y}\right]_+ & 0<y<1\\ \left[\frac{2\zeta_3}{1-y}-\frac{\pi^2}{6}\frac{H(-1;-y)}{1-y}+\frac{H(-1,0,-1;-y)}{1-y}-\frac{H(-1,0,0;-y)}{1-y}\right]_+ & y<0\end{cases}$$

Cross checked with Chen, Wang, Zhu, 2006.14825

Numerical Results

- Using PDFs from experimental data and matching coefficients, predict the lattice results of QCFs



- NNLO results reduce the uncertainty from the ambiguity of scale choice

Summary

- Obtain complete and analytic NNLO flavor non-singlet matching coefficients for QCFs.

- The method can be used for gluon cases and other LCSs.

- The method can be improved for higher order calculations. The evolution of PDFs beyond NNLO may be obtained from high order LCSs.

Thank you!