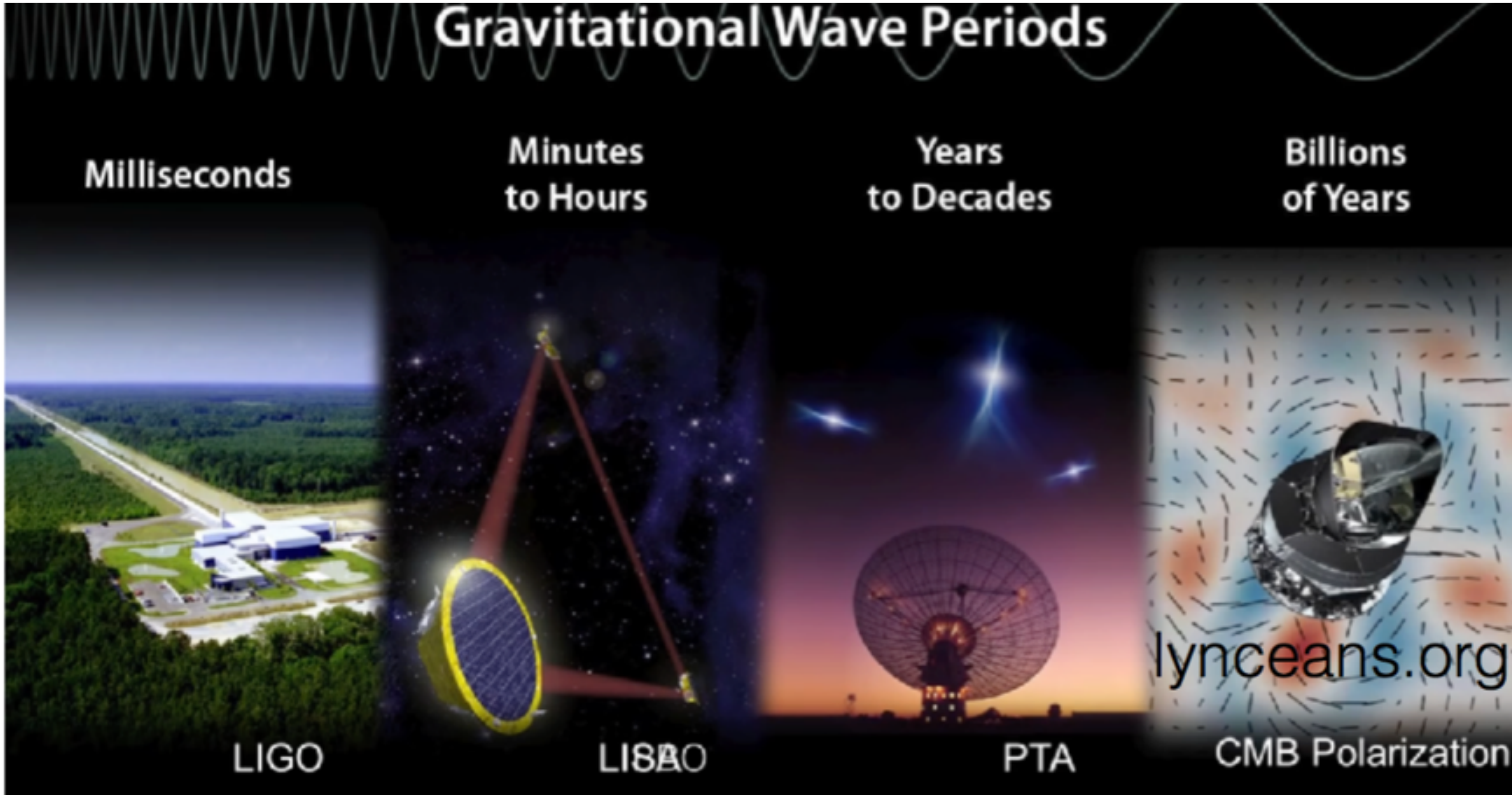
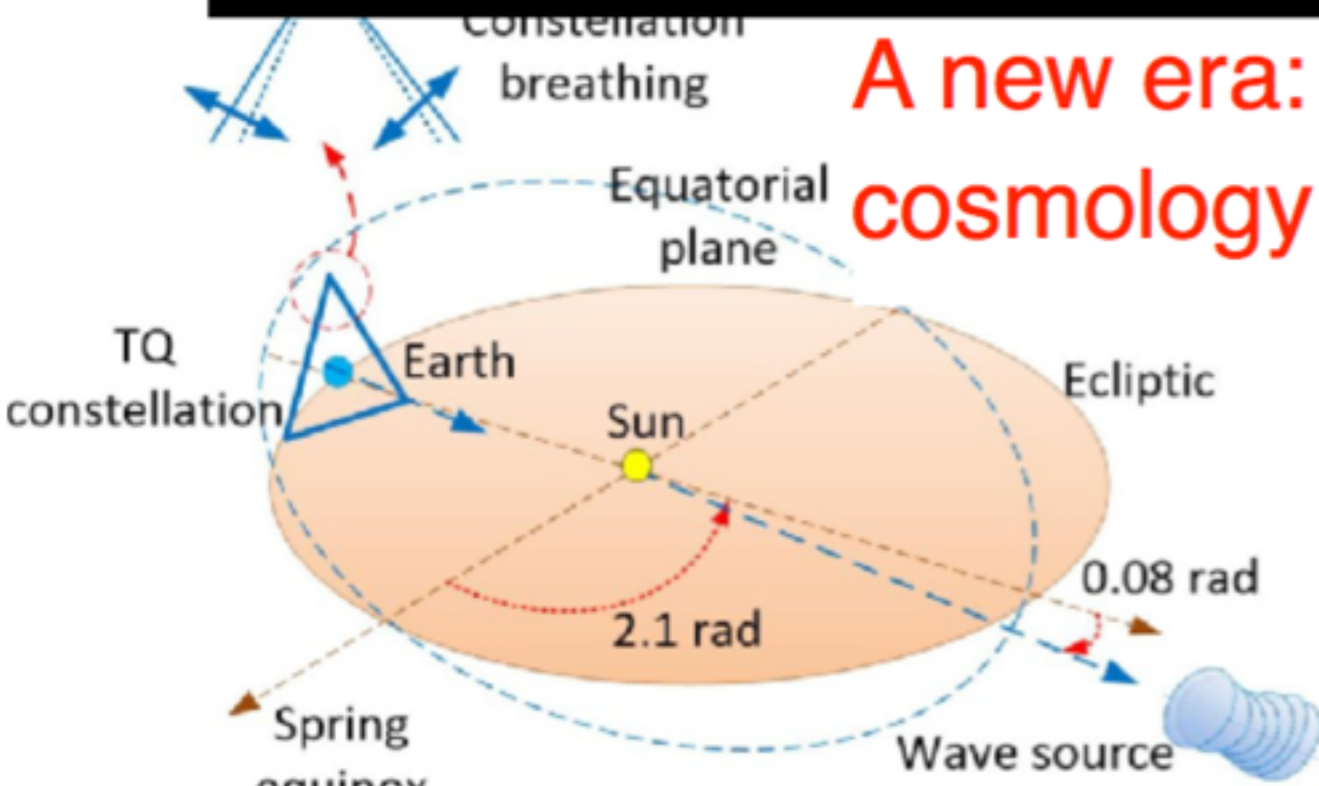
SGWB search for new physics

Ligong Bian (边立功)
Chongqing University (重庆大学)
lgbycl@cqu.edu.cn

CLHCP2020

Based on work with Rong-Gen Cai, Huai-Ke Guo, Jing Liu, Yongcheng Wu, Jing Yang, Ruiyu Zhou, Phys.Rev.D 101 (2020) 3, 035011, JHEP 04 (2020) 071,2006.13872,2009.13893.





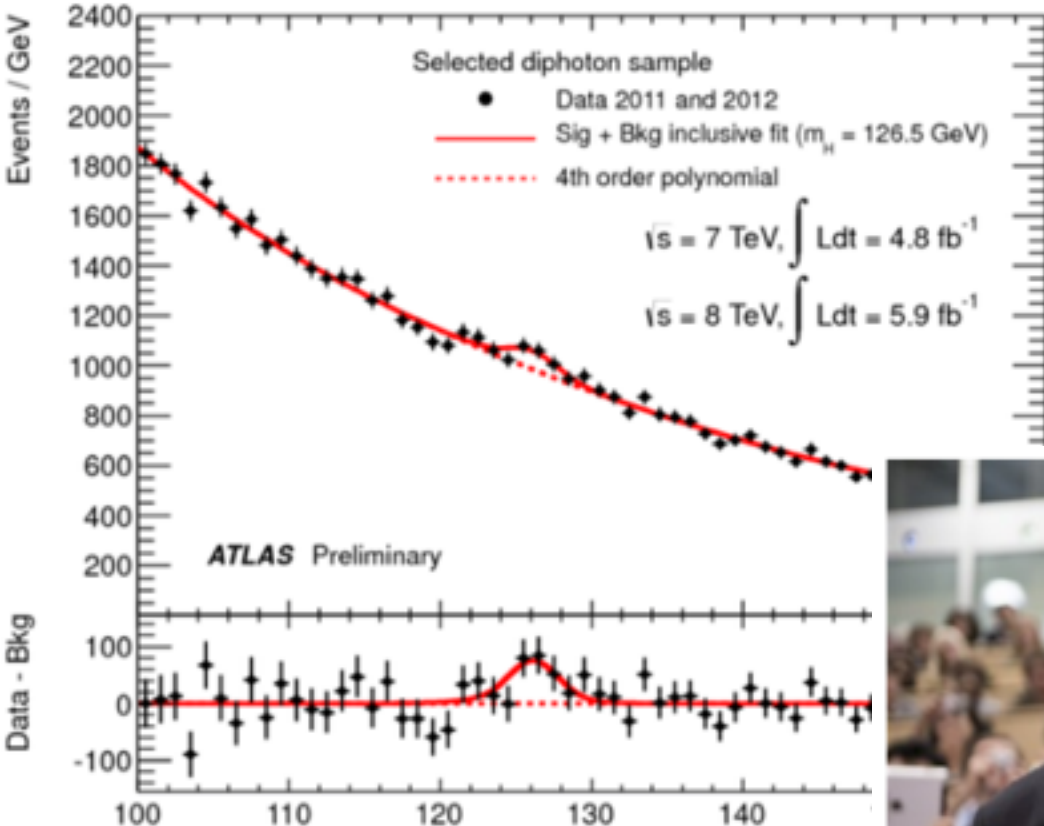
A new era: GWs as a new tool for probing cosmology and high energy physics

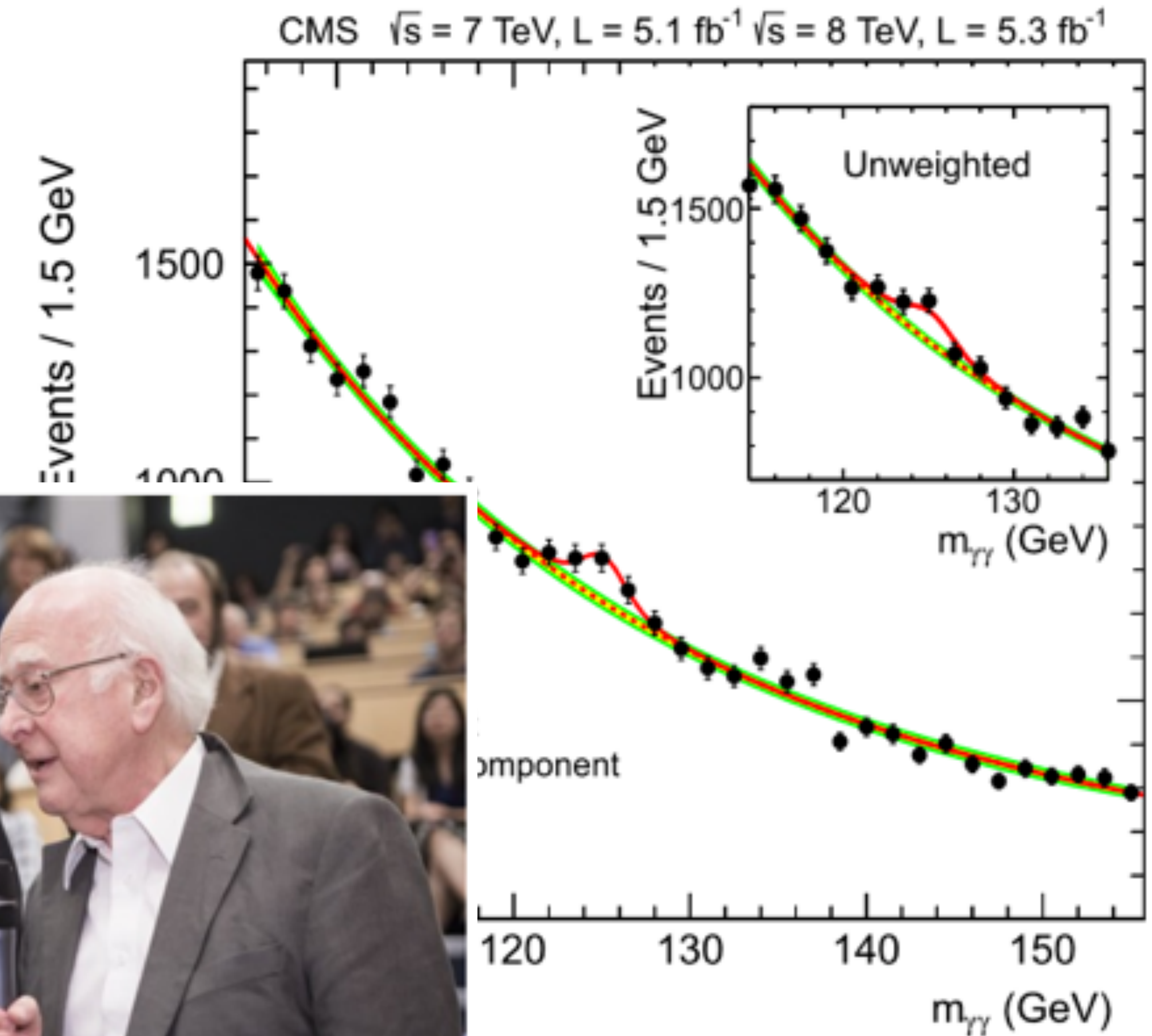
Current: LIGO,PTA,…

Future GW experiments:
LISA, Taiji, TianQin, BBO, DECIGO, ET, CE,...

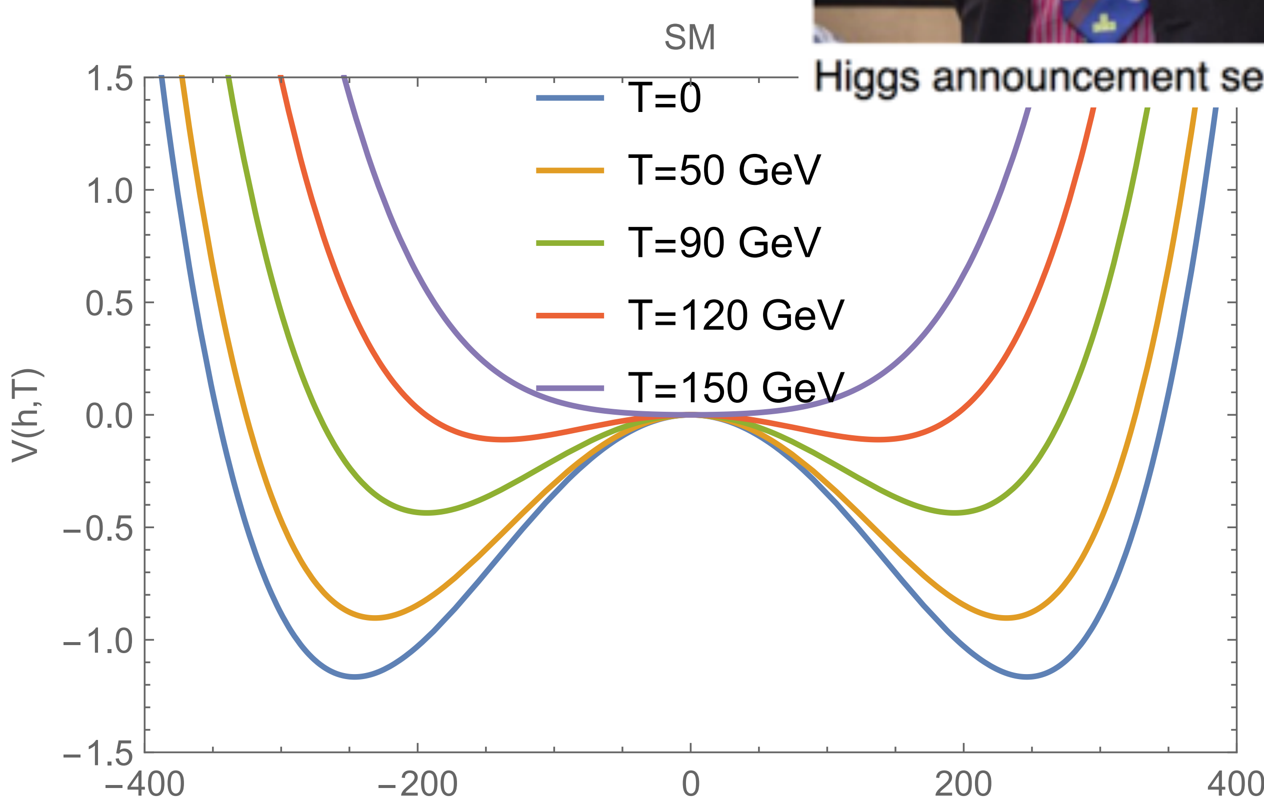
Contents

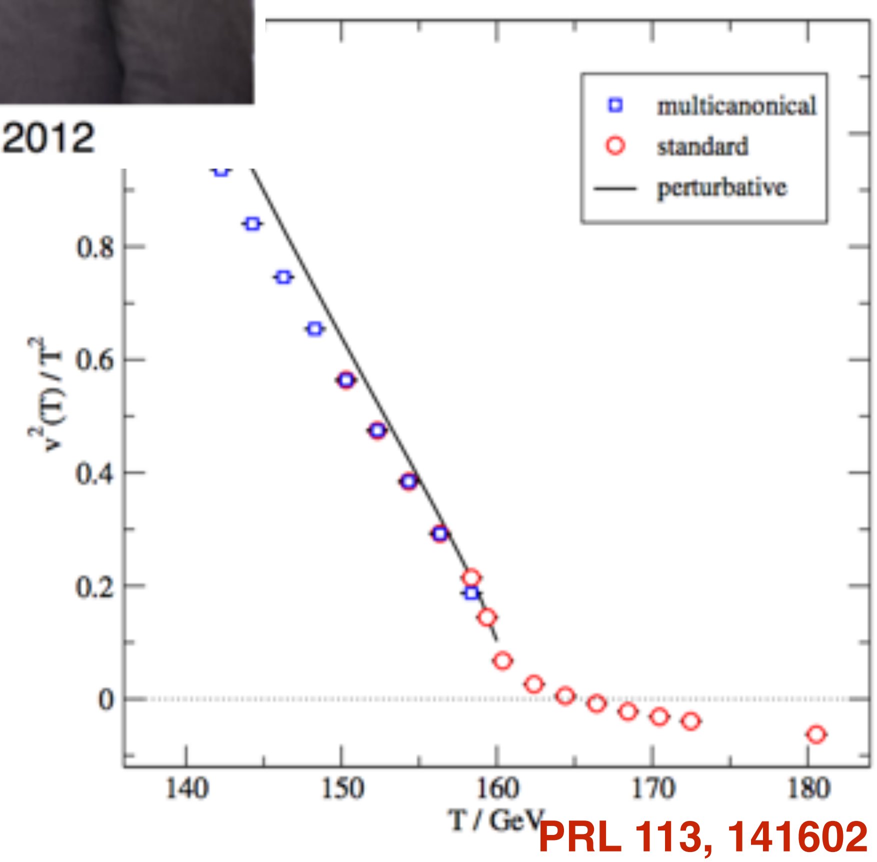
Implication of 125 GeV Higgs






Higgs announcement seminar on 4 July 2012

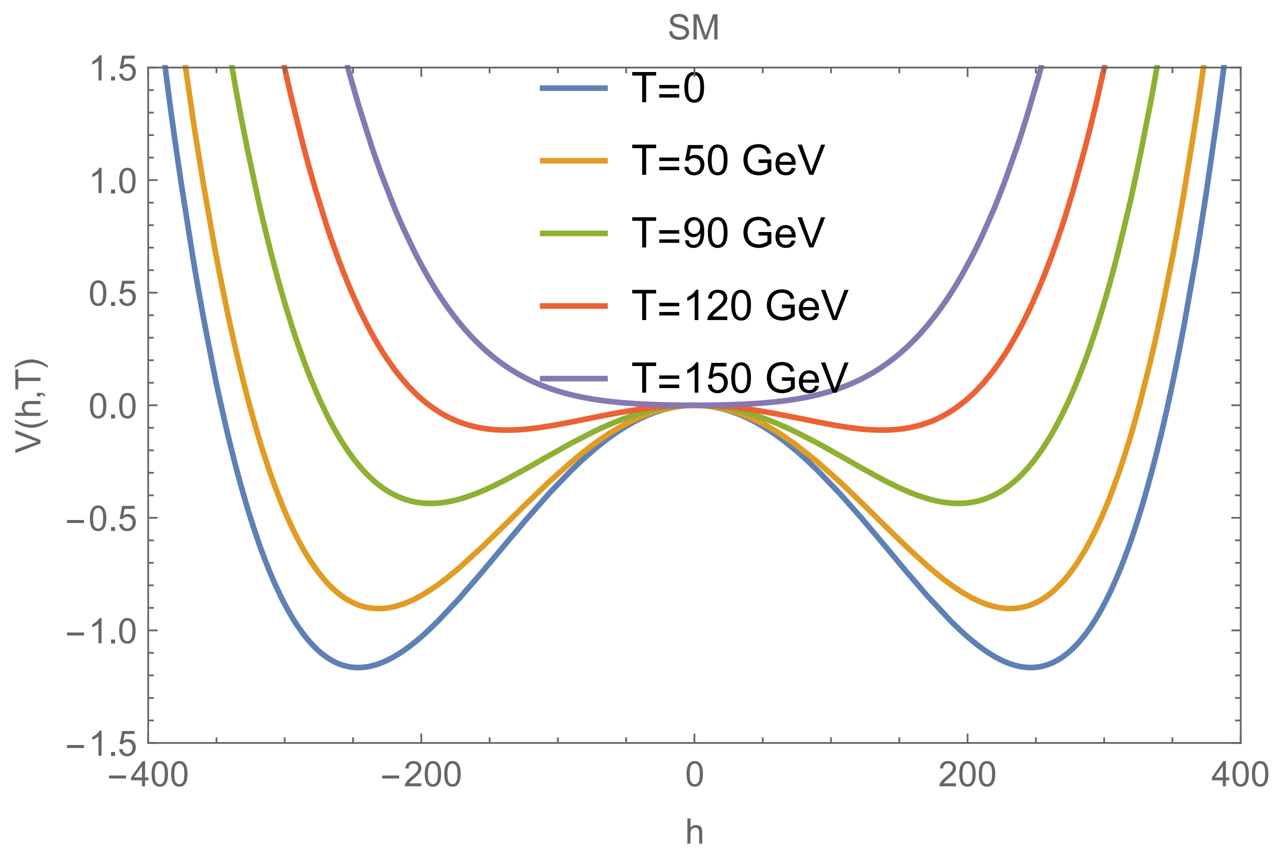


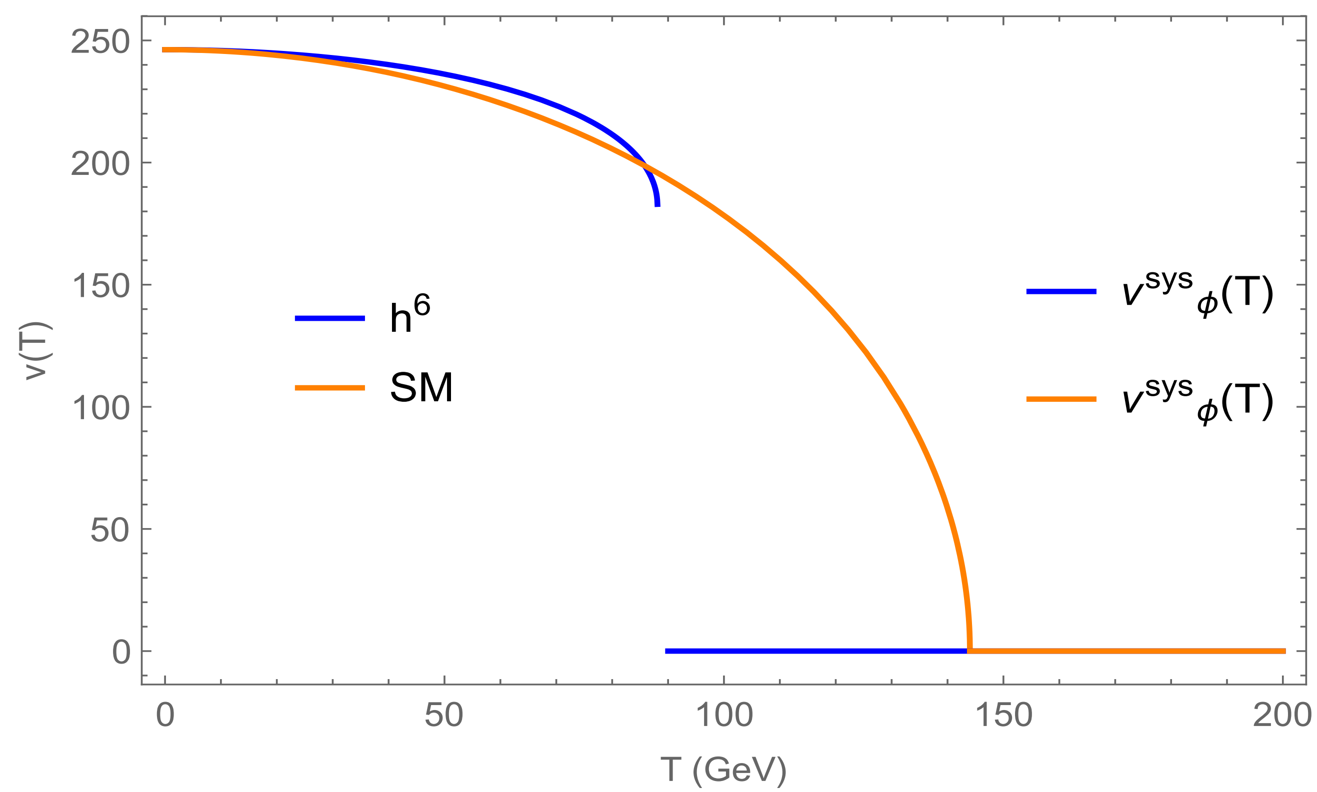


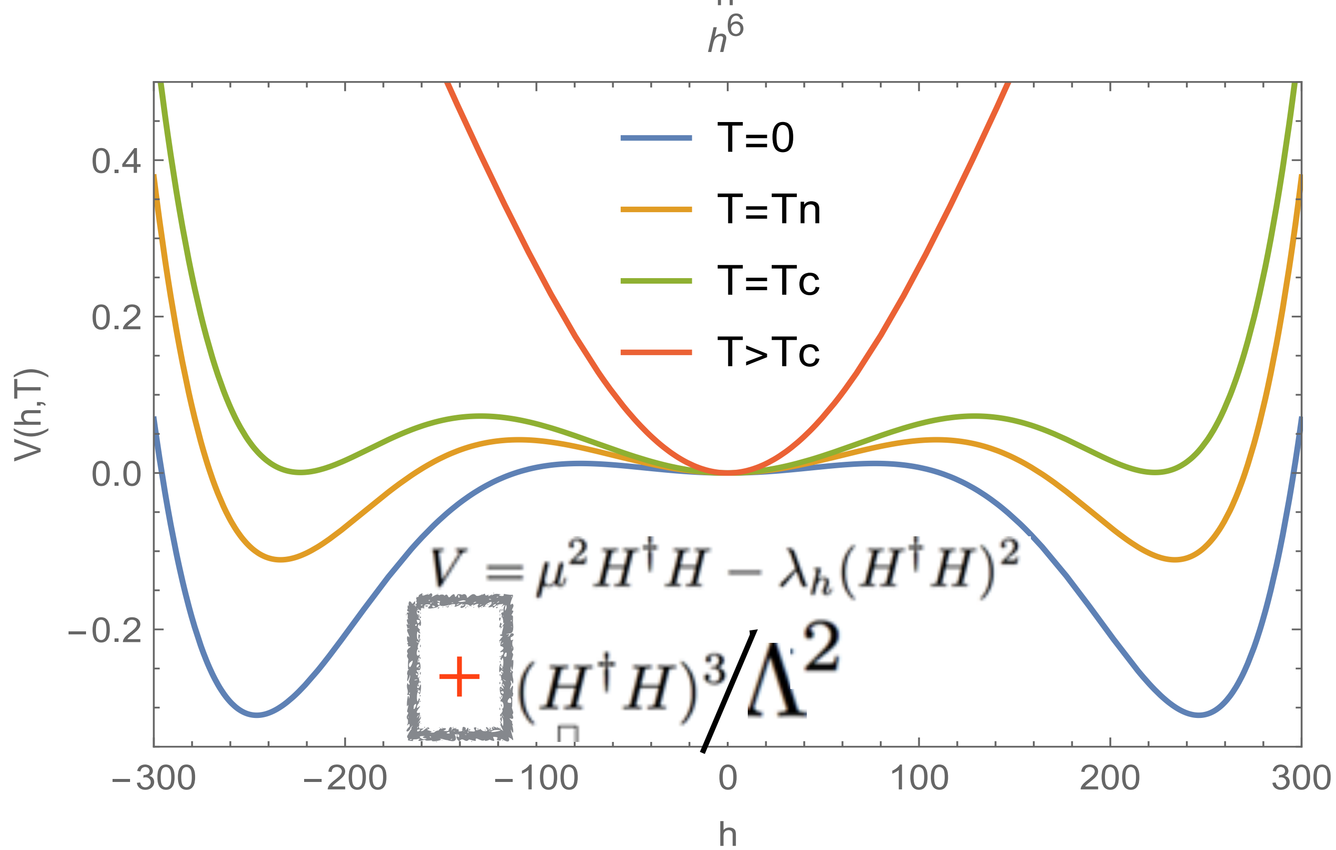
PRL 113, 141602 (2014)

Higgs Potential Shape??? EFT or ???

First or second order



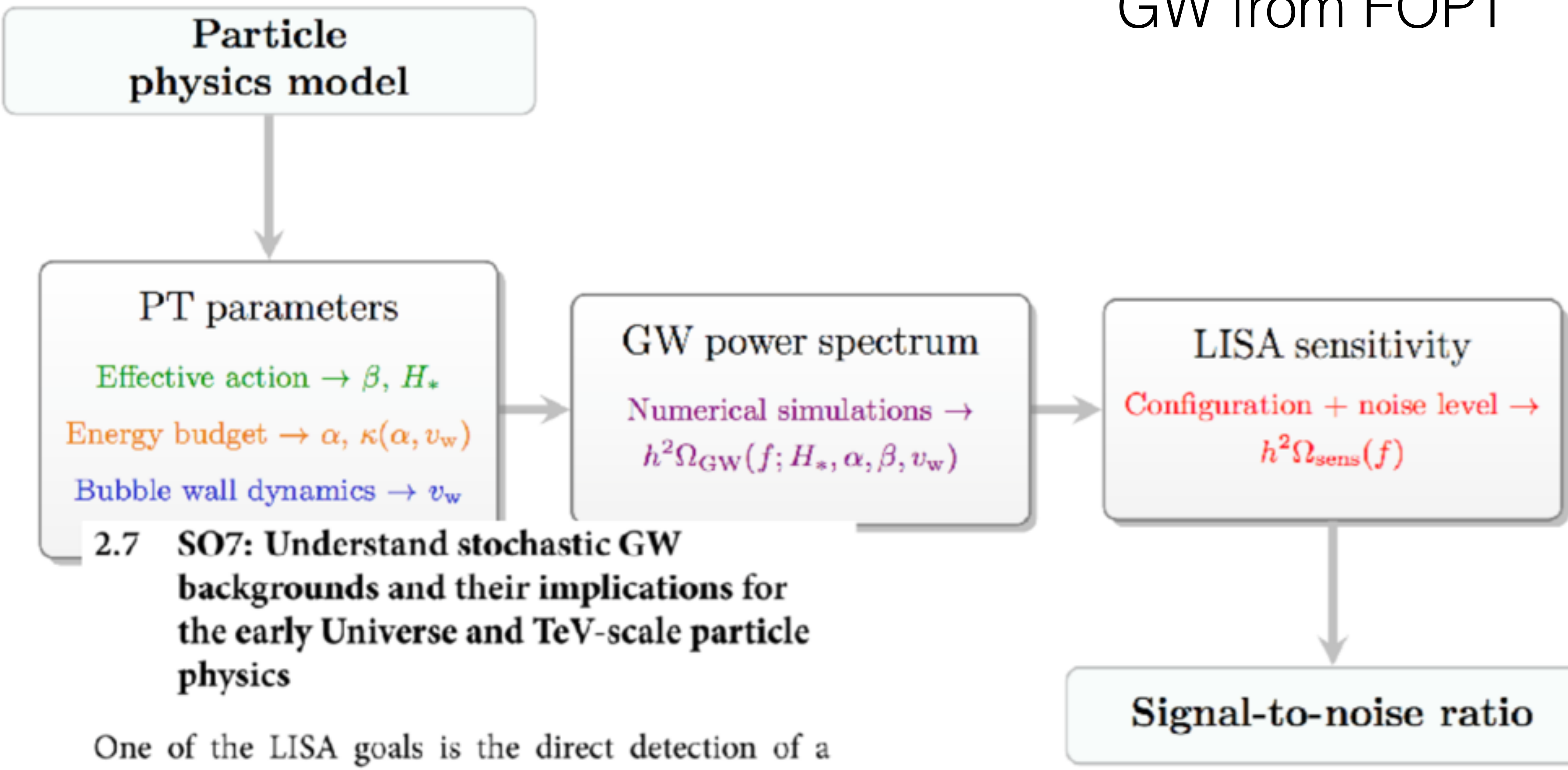




Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner (2015)
F.P. Huang, Gu, Yin, Yu, Zhang (2015) F.P. Huang, Wan, Wang, Cai, Zhang (2016) Cao, F.P. Huang, Xie, & Zhang (2017)

LHC say the quantum fluctuation (quadratic oscillation) around h=v with mh=126 GeV, not sensitive to the specifically potential shape

GW from FOPT



2.7 SO7: Understand stochastic GW backgrounds and their implications for the early Universe and TeV-scale particle physics

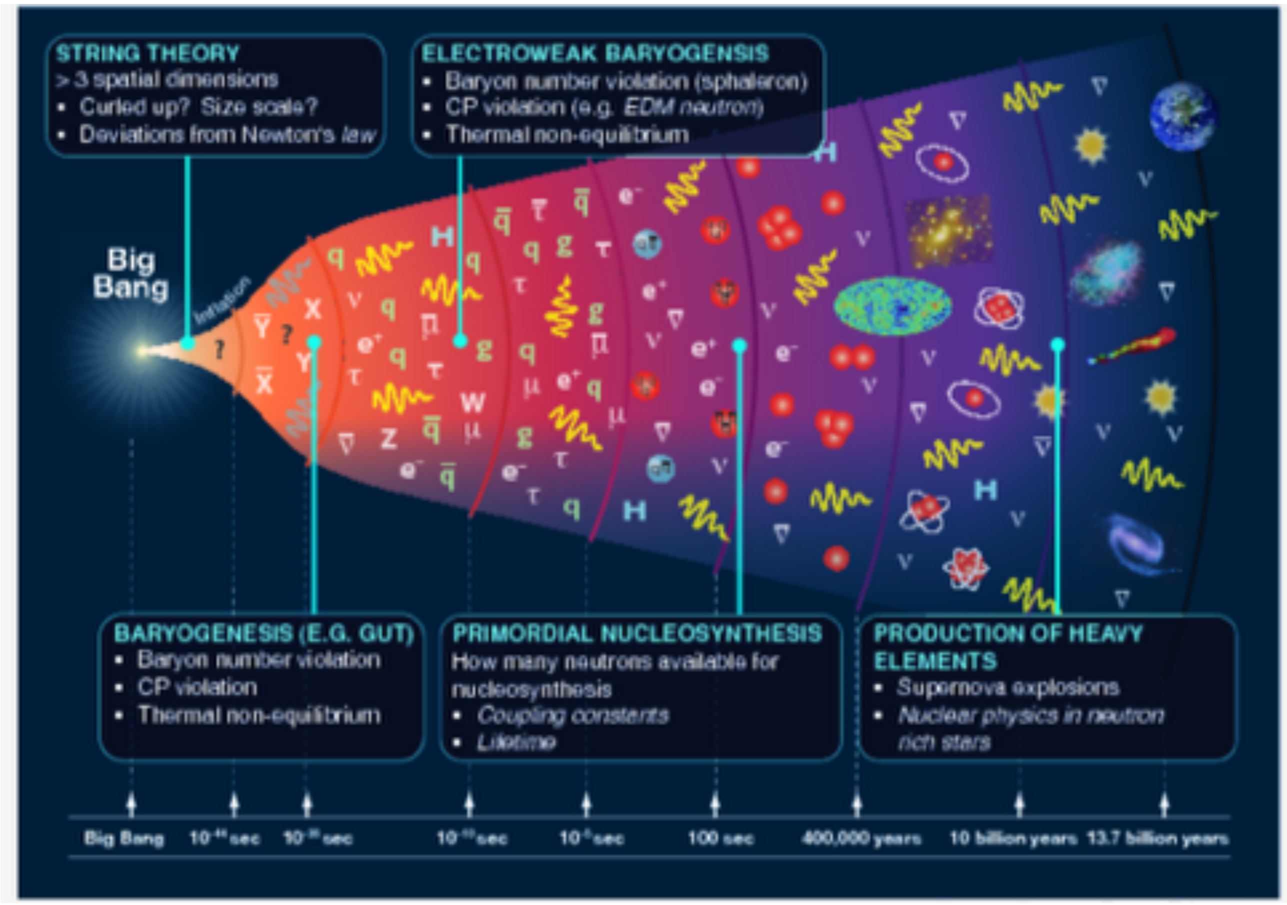
One of the LISA goals is the direct detection of a stochastic GW background of cosmological origin (like for example the one produced by a first-order phase transition around the TeV scale) and stochastic foregrounds. Probing a stochastic GW background of cosmological origin provides information on new physics in the early Universe. The shape of the signal gives an indication of its origin, while an upper limit allows to constrain models of the early Universe and particle physics beyond the standard model.

1702.00786

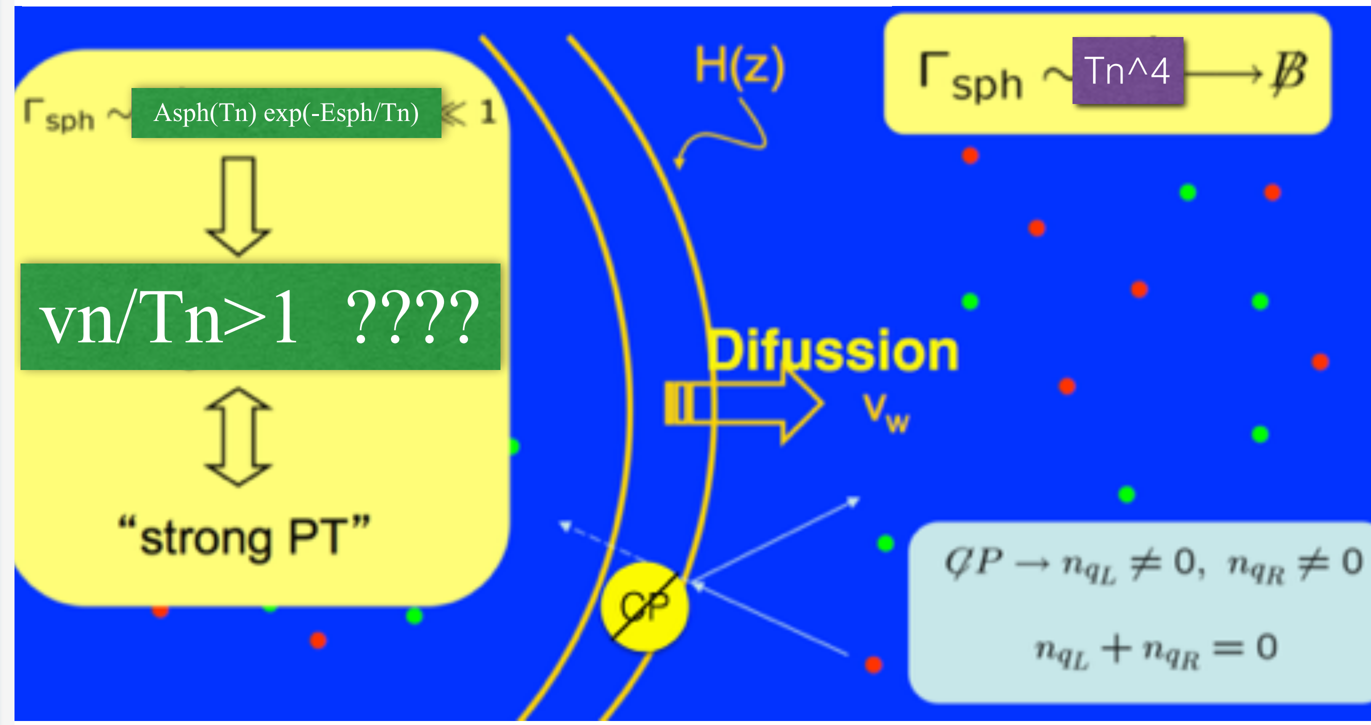
$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{Sens}}(f)}\right]^2}$$

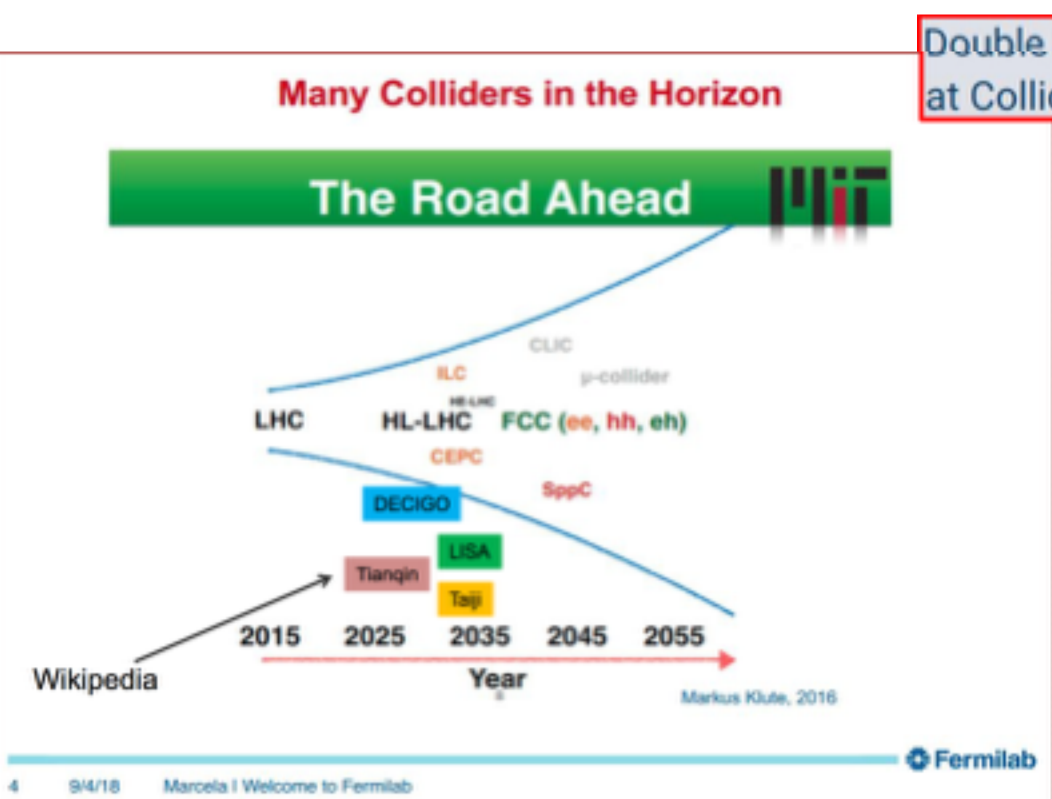
JCAP03(2020)024

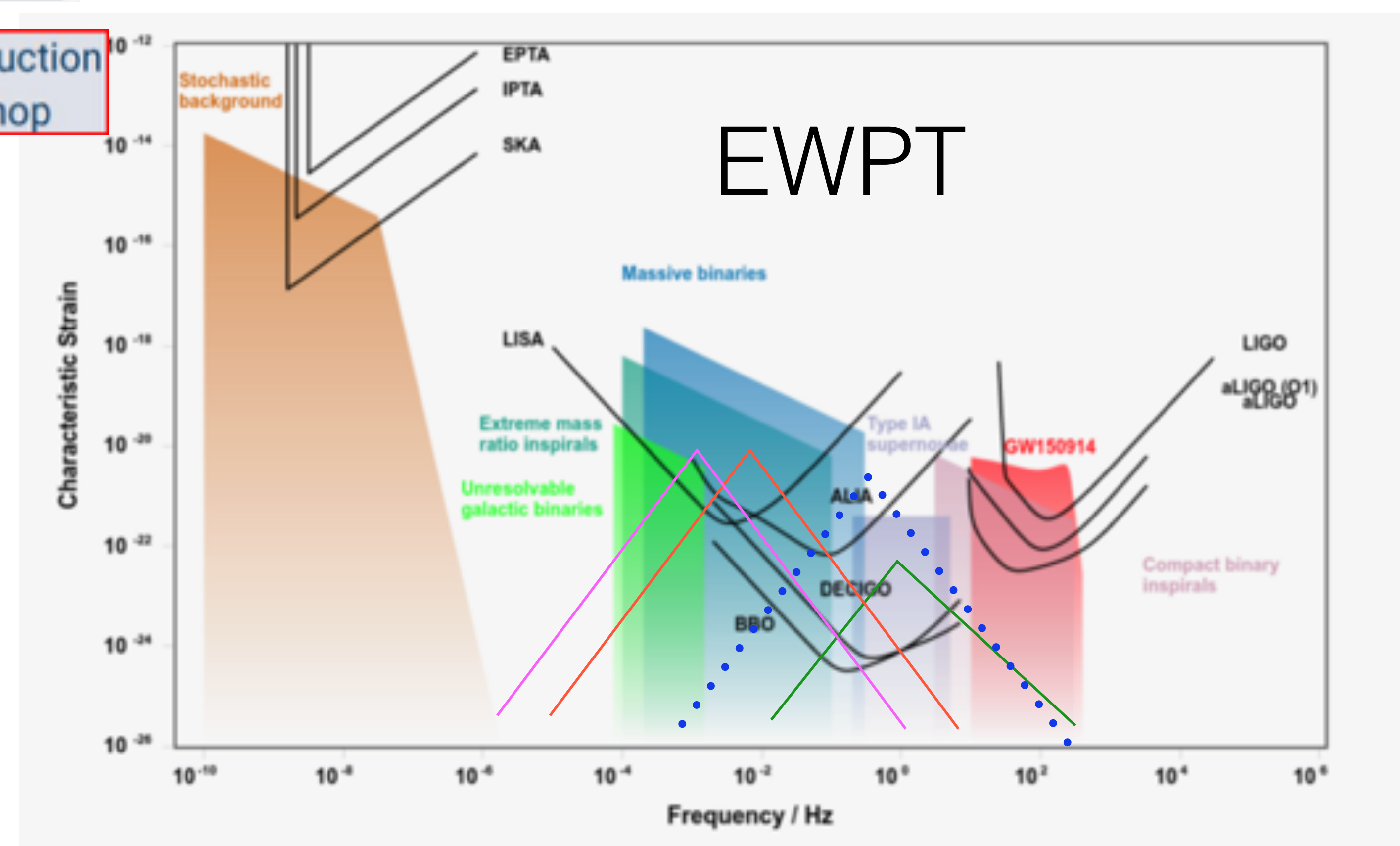
Why SFOEWPT



$$\frac{n_b}{s} \approx (0.7-0.9)\times 10^{-10} \neq 0$$



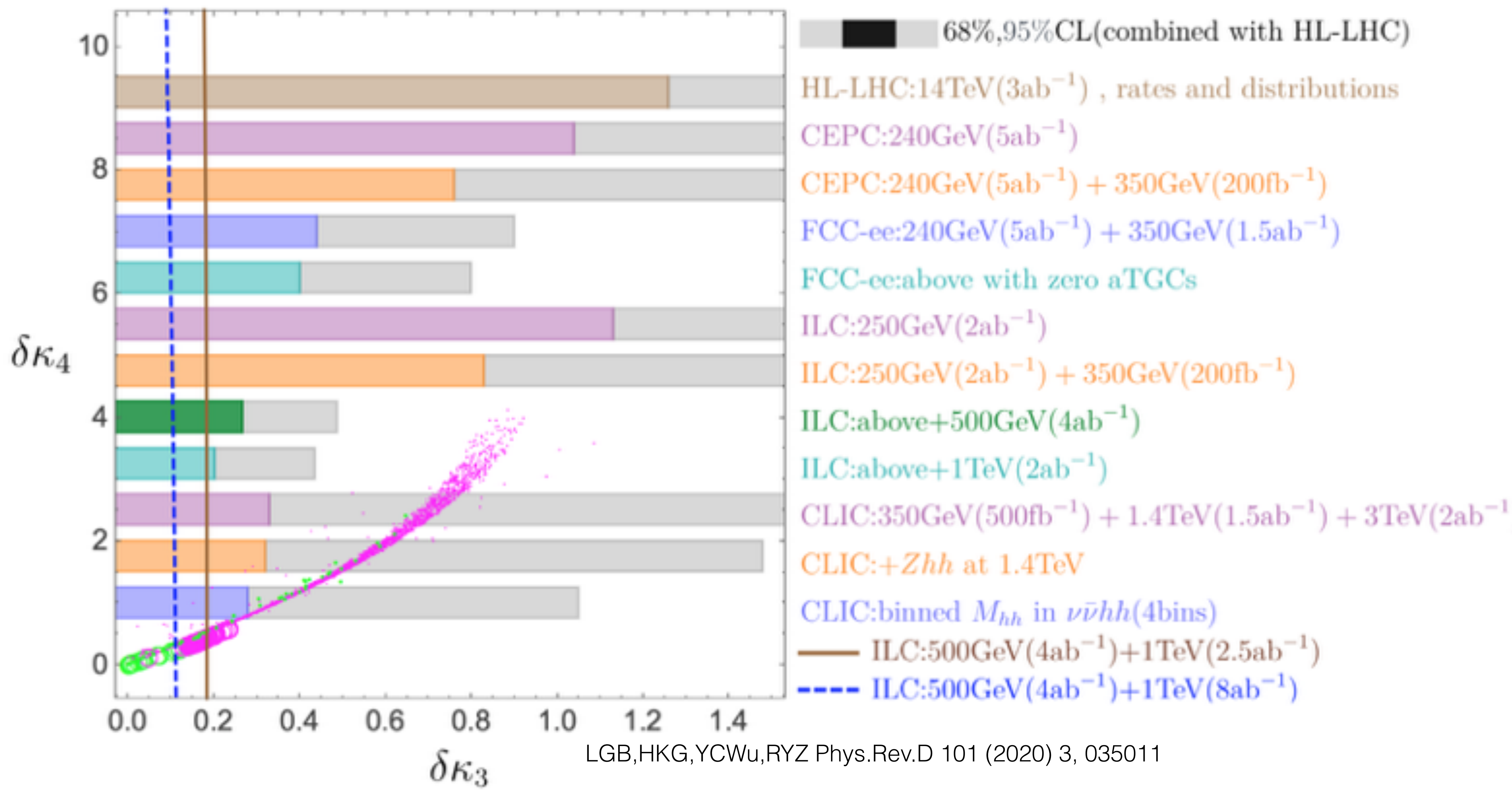




C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

Collider & GW complementary search

SNR > 10 points for two-step and one-step SFOEWPT



LGB,HKG,YCWu,RYZ Phys.Rev.D 101 (2020) 3, 035011

Circles and the dotted points for the GM and xSM scenarios

$$\delta\kappa_3^{\mathrm{xSM}} = \alpha_H^2\left[-\frac{3}{2}+\frac{2m_H^2-2b_3v_s-4b_4v_s^2}{m_h^2}\right]+\mathcal{O}(\alpha_H^3),$$

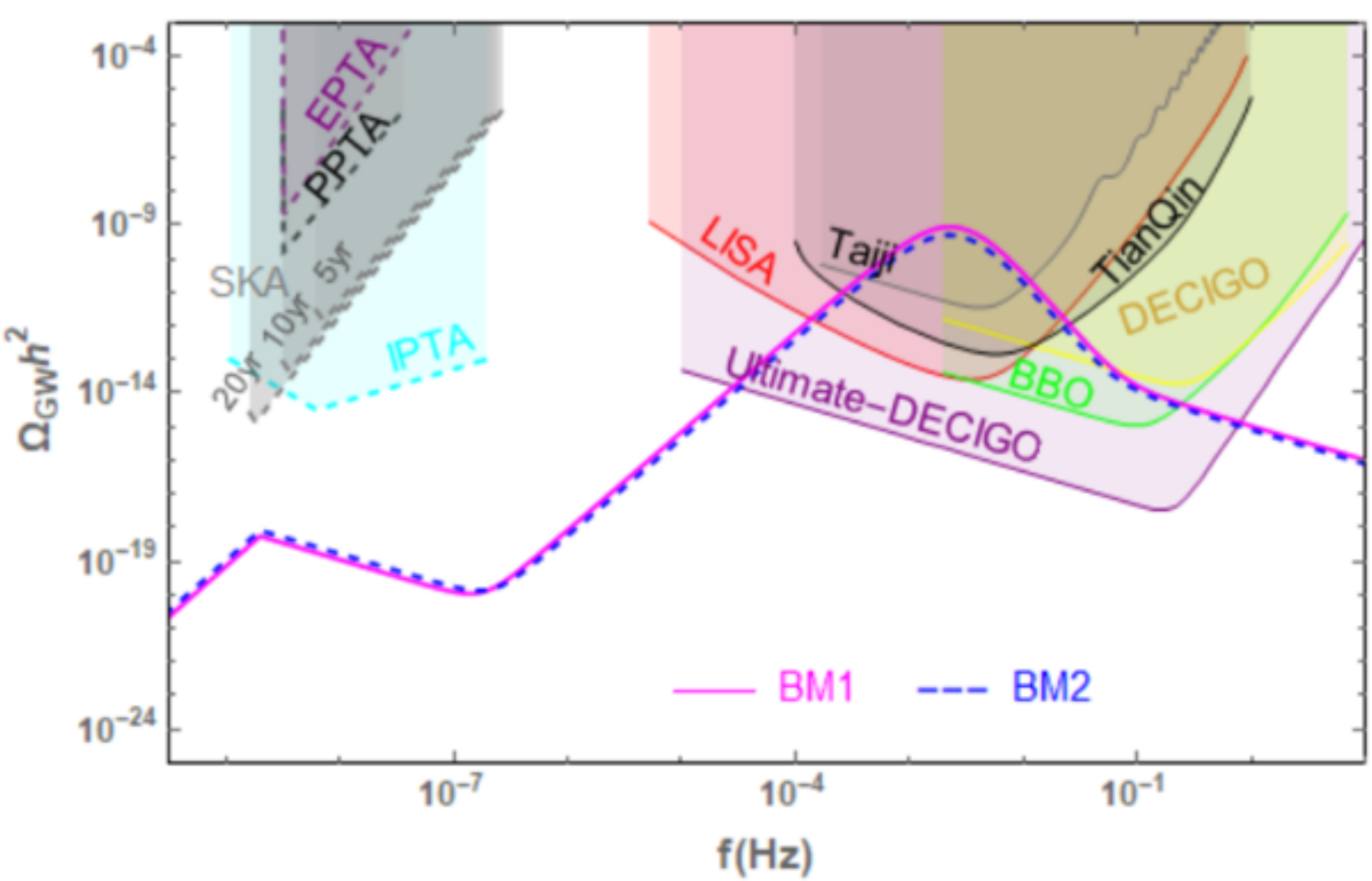
$$\delta\kappa_4^{\mathrm{xSM}} = \alpha_H^2\left[-3+\frac{5m_H^2-4b_3v_s-8b_4v_s^2}{m_h^2}\right]+\mathcal{O}(\alpha_H^3).$$

$$\delta\kappa_3^{GM} = -\alpha_H\frac{\sqrt{3}\mu_1 v}{2m_h^2}+\frac{\alpha_H v^2(4\alpha_H-\sqrt{6}\theta_H)(2\lambda_4+\lambda_5}{2m_h^2} - \frac{(3\alpha_H^2+\theta_H^2)}{2}+\mathcal{O}(\alpha_H^3,\theta_H^3),$$

$$\delta\kappa_4^{GM} = -2\alpha_H^2\left(1-\frac{2(2\lambda_4+\lambda_5)v^2}{m_h^2}\right)+\mathcal{O}(\alpha_H^3).$$

DW: SSB of discrete symmetry   Low scale

	m_χ (GeV)	m_{h_2} (GeV)	v_s (GeV)	θ	T_n (GeV)	β/H_n	α
BM_1	625.08	361.31	184.10	0.30	50.16	219.62	1.02
BM_2	814.81	370.24	243.05	0.13	69.46	152.41	0.66

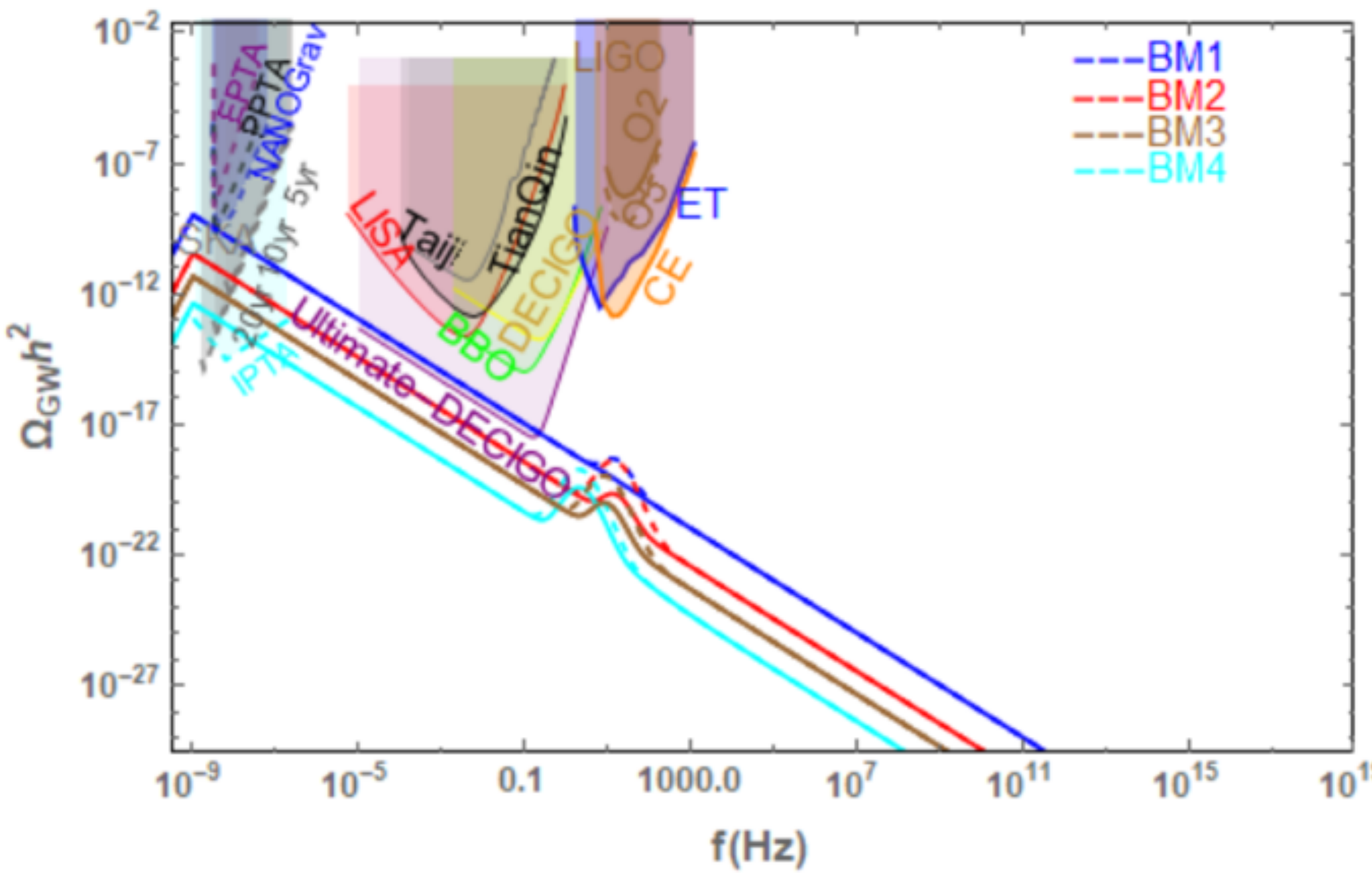


RYZ, JY, LGB,JHEP 04 (2020) 071

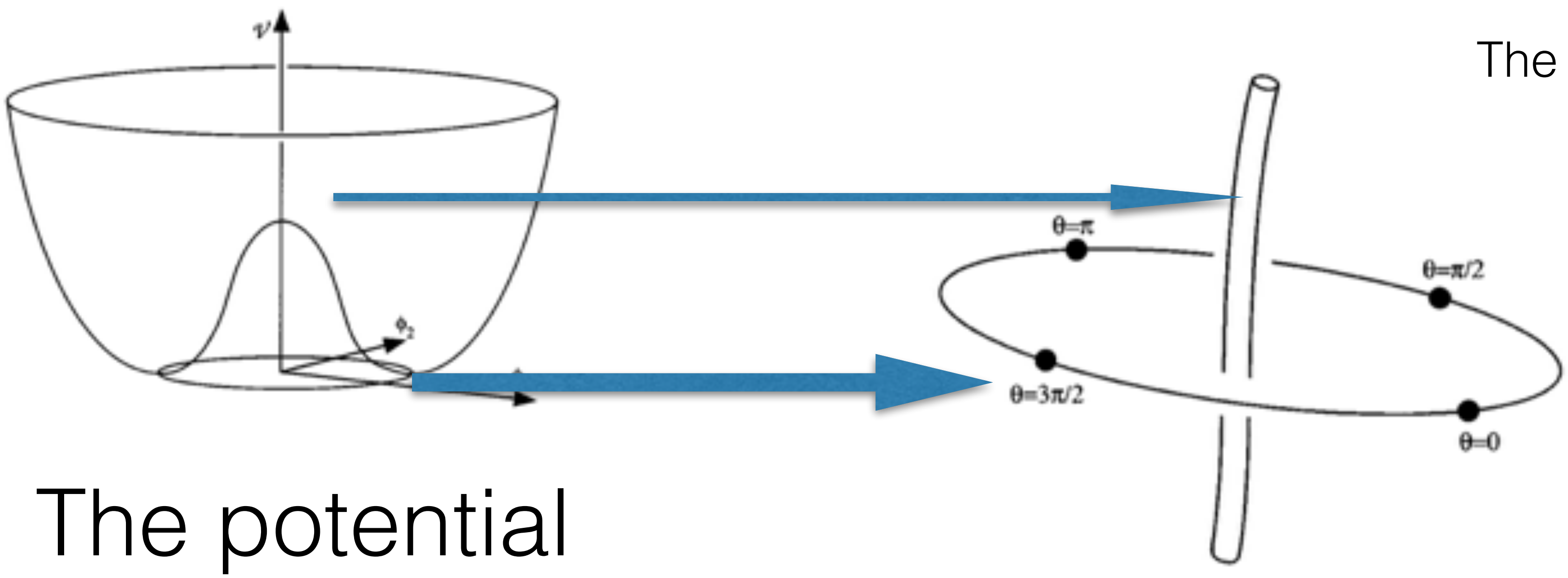
DW: SSB of discrete symmetry

High scale

Benchmark points	λ_s	m_s	m_χ (GeV)	v_n (GeV)	T_n (GeV)	α	β/H_n
BM_1	1.517	14991.4	40278.8	14878.4	12235.2	0.005	14310.7
BM_2	1.266	7889.67	21655.6	8859.84	6291.07	0.006	6991.27
BM_3	0.354	3843.19	9959.31	7686.83	4856.62	0.004	5428.57
BM_4	10.92	7294.71	21557.2	3054.29	2618.39	0.004	2474.3



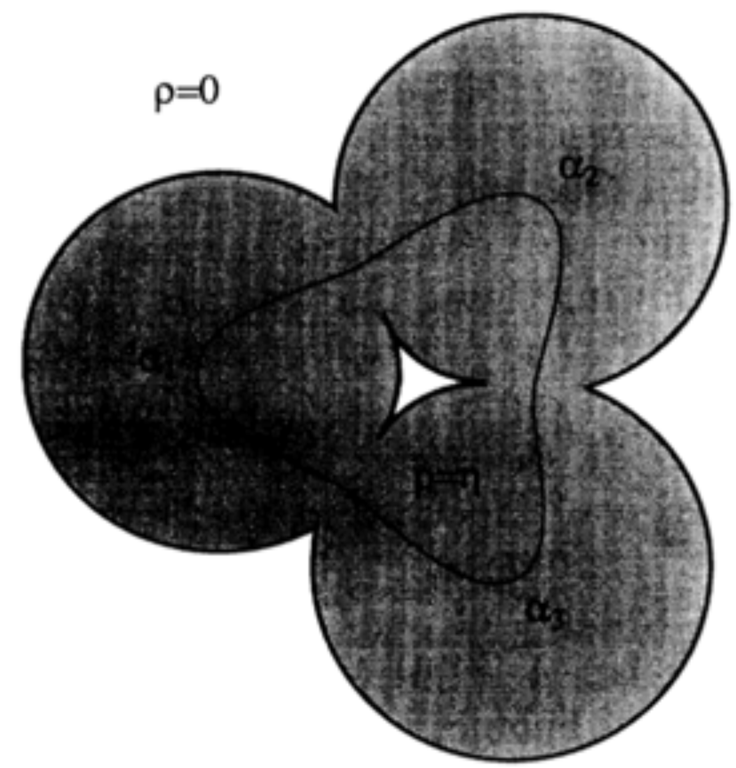
CS: SSB of U(1) symmetry   CS formation from first-order PT



The one-dimension topological defects:
cosmic string

T. W. B. Kibble

The potential



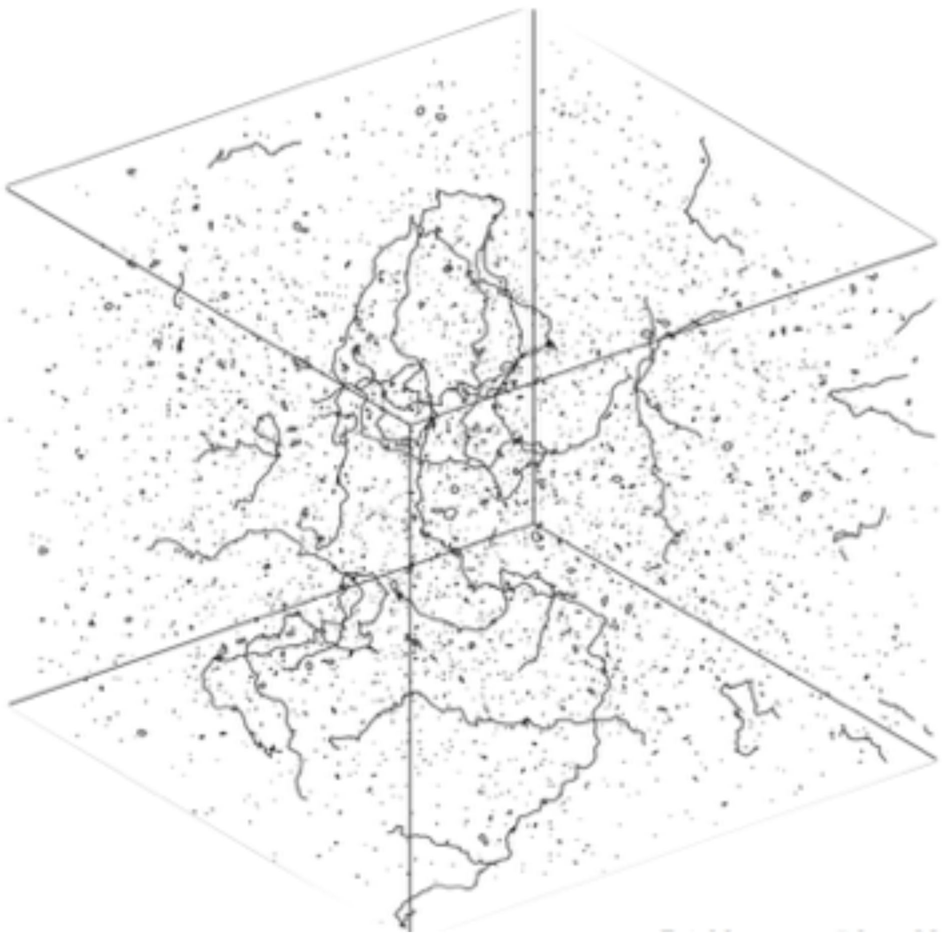
SFOPT with bubble nucleation, three bubbles meet along a line, it may happen that the net phase change around it is ±2π; in that case a string is trapped. Roughly speaking, we expect this to happen along one in four edges, so in this case the initial scale ξ_{str} of the string network is essentially the typical bubble diameter.

Aust. J. Phys., 1997, 50, 697–722

FIG. 1. Three bubbles of the broken symmetry phase ($\rho=\eta$) colliding. If the phase change of the scalar field around the loop γ is $\pm 2\pi$, a string (or antistring) is formed. If the phases α_i are ordered, then the requirement for a string is $\alpha_1+\pi<\alpha_3<\alpha_2+\pi$.

Mark Hindmarsh et al.   Phys.Rev.D 49 (1994) 1944-1950

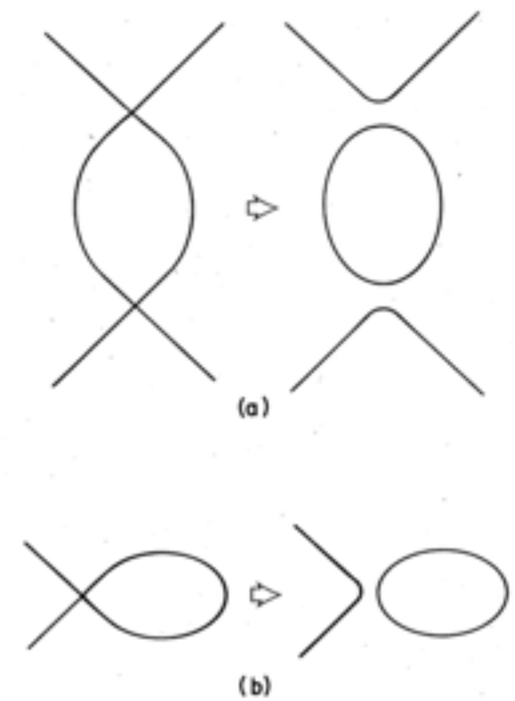
CS: SSB of U(1) symmetry



Allen&Shellard
PRL 64,119 (1990)

CS loops and GWs emission

Loop formation



Closed-loop formation by intercommuting strings.

Cosmic string loop formation. A loop forms (a) when two strings

interacts in 2 separate points or (b) when a string crosses itself.

Tanmay Vachaspati, Alexander Vilenkin   Phys.Rev.D 30 (1984) 2036

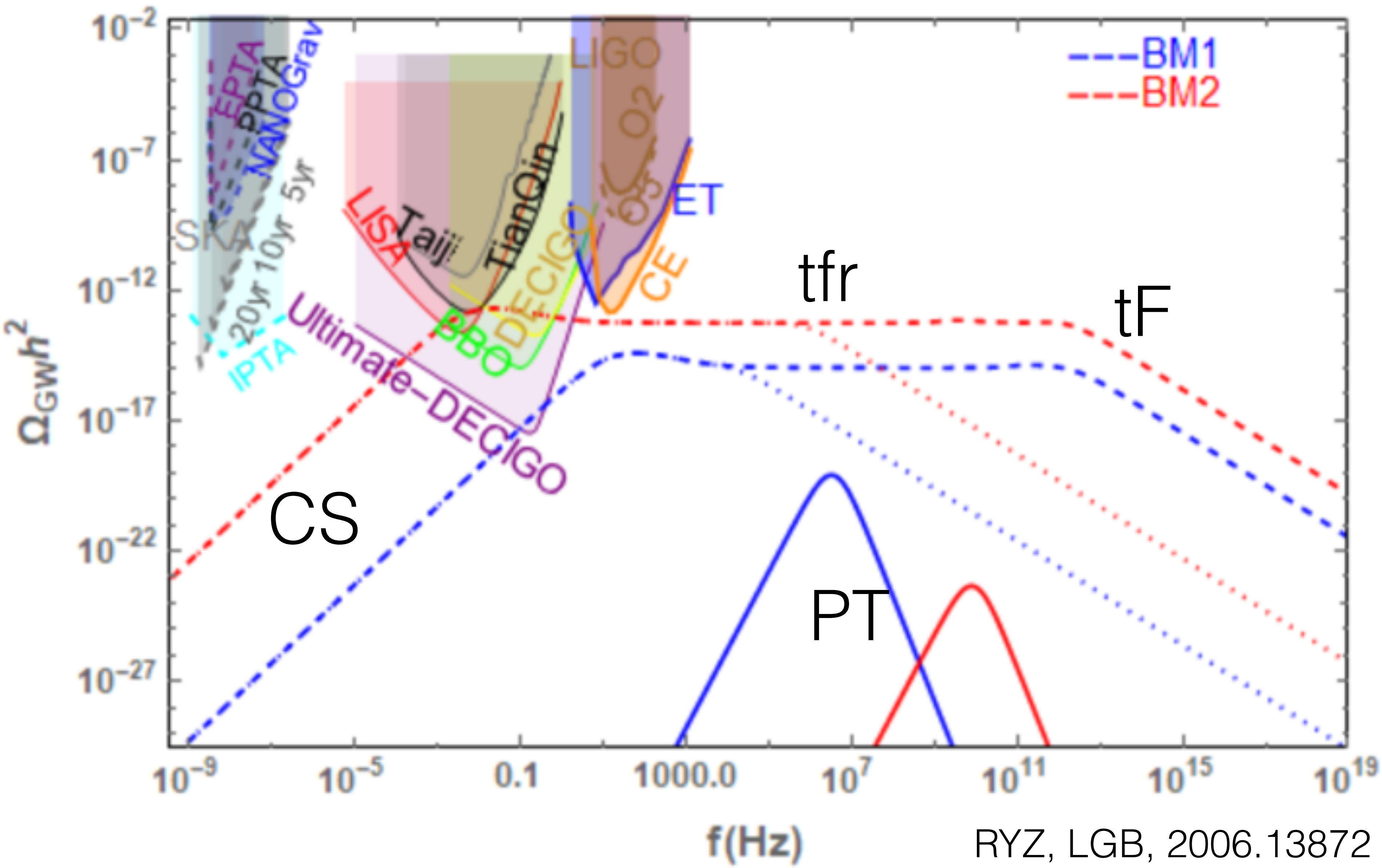
GW emission

Cusps: a pointed and highly Lorentz-boosted region which appear few times per oscillation period

CS: SSB of U(1) symmetry

GWs from CS after FOPT

Nambu-Goto cosmic strings characterized solely by the string tension μ



RYZ, LGB, 2006.13872

CMB temperature fluctuation: $G\mu \leq 1.1\times10^{-7}$

EPTA $G\mu \leq 8 \times 10^{-10}$

NANOGrav $G\mu \leq 5.3 \times 10^{-11}$

3.1. *Models of time-correlated processes*

The principal results of this paper are referred to a fiducial power-law spectrum of characteristic GW strain

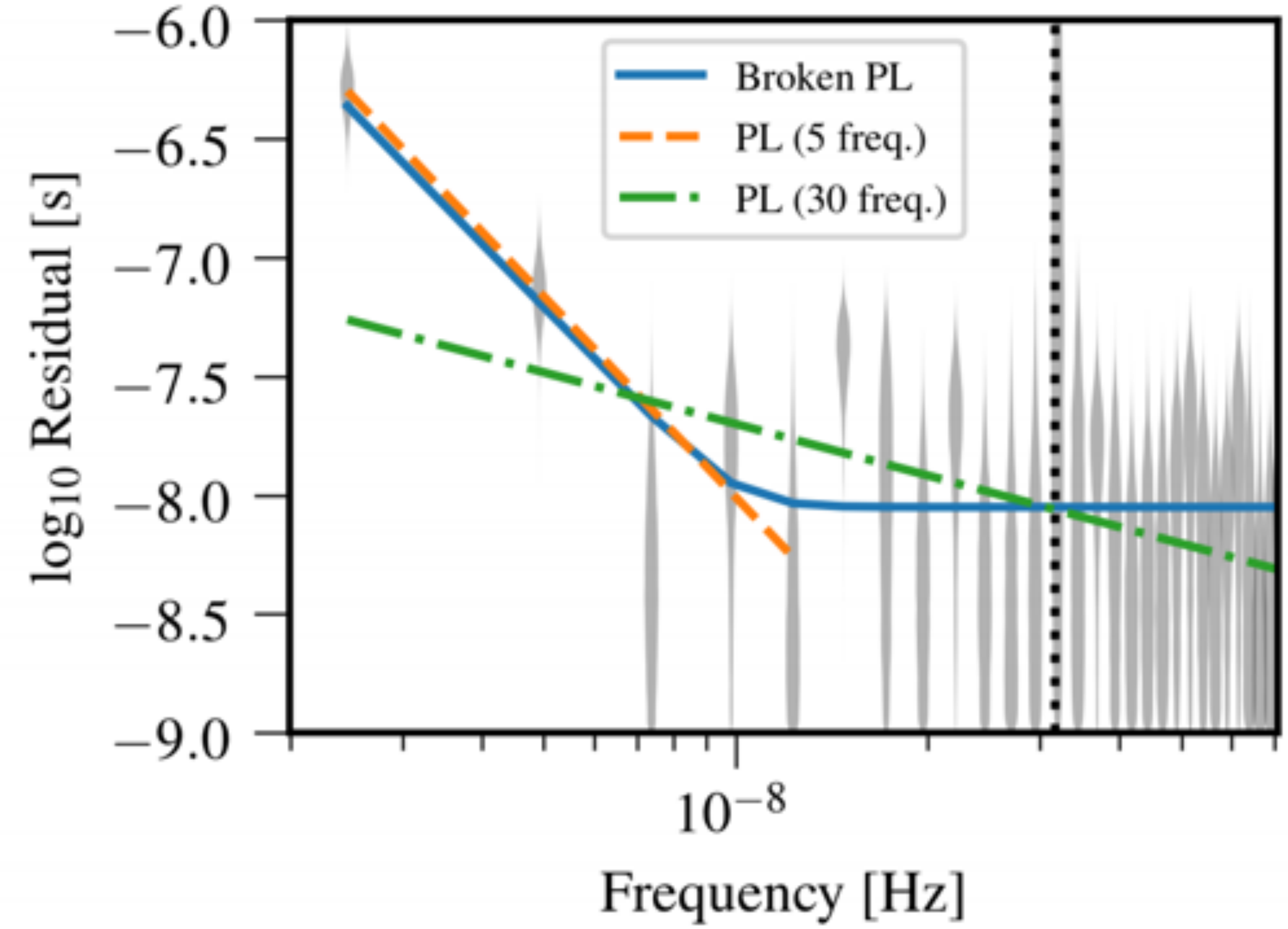
$$h_c(f) = A_{\rm GWB} \left(\frac{f}{f_{\rm yr}}\right)^{\alpha}, \qquad (1)$$

with $\alpha = -2/3$ for a population of inspiraling SMBHBs in circular orbits whose evolution is dominated by GW emission (Phinney 2001). We performed our analysis in terms of the timing-residual cross-power spectral density

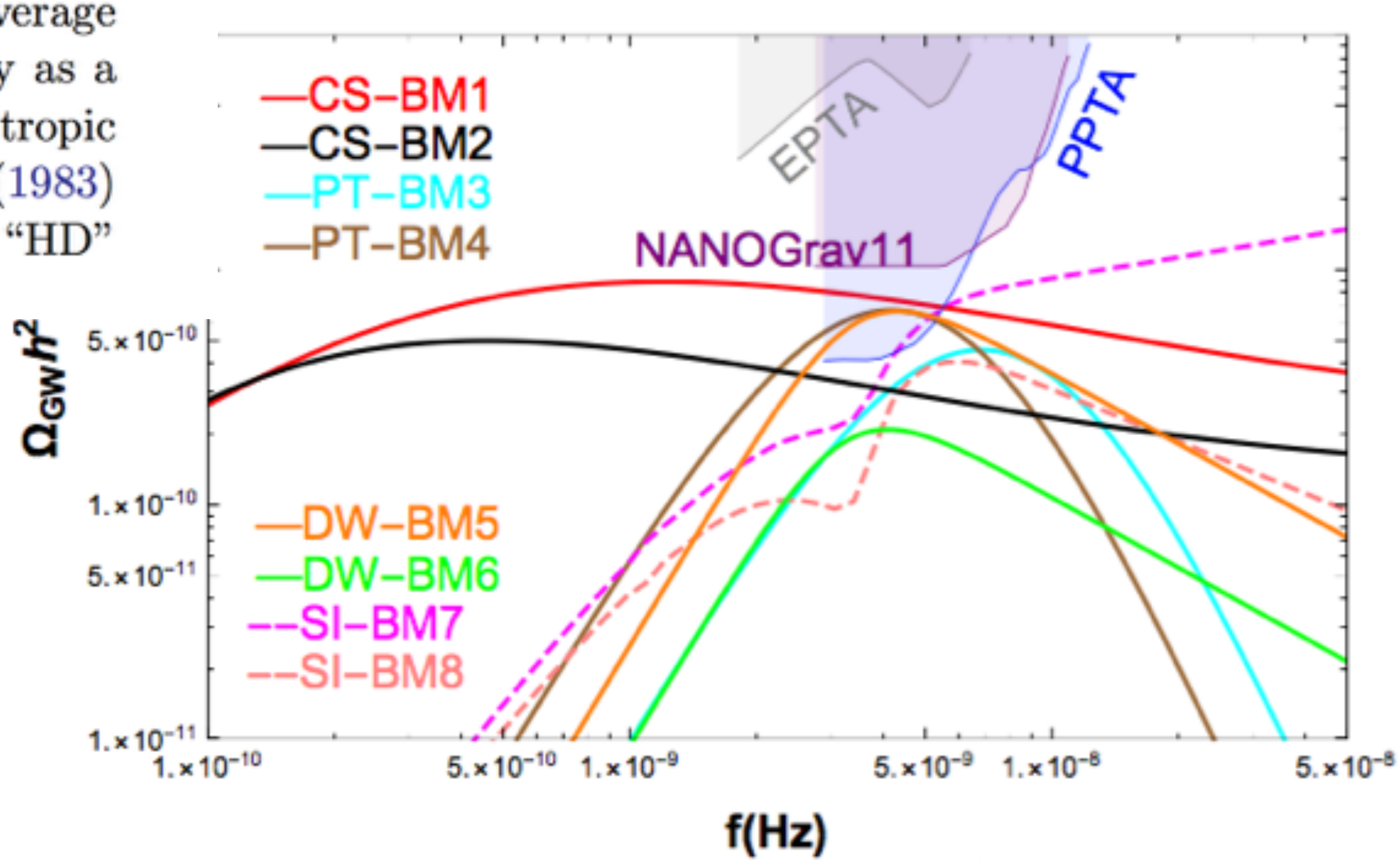
$$S_{ab}(f) = \Gamma_{ab} \frac{{A_{\rm GWB}}^2}{12\pi^2} \left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}. \qquad (2)$$

where $\gamma = 3 - 2\alpha$ (so the fiducial SMBHB $\alpha = -2/3$ corresponds to $\gamma = 13/3$), and where Γ_{ab} is the overlap reduction function (ORF), which describes average correlations between pulsars a and b in the array as a function of the angle between them. For an isotropic GWB, the ORF is given by Hellings & Downs (1983) and we refer to it casually as "quadrupolar" or "HD" correlations.

2009.04496



GW for NanoGrav ???



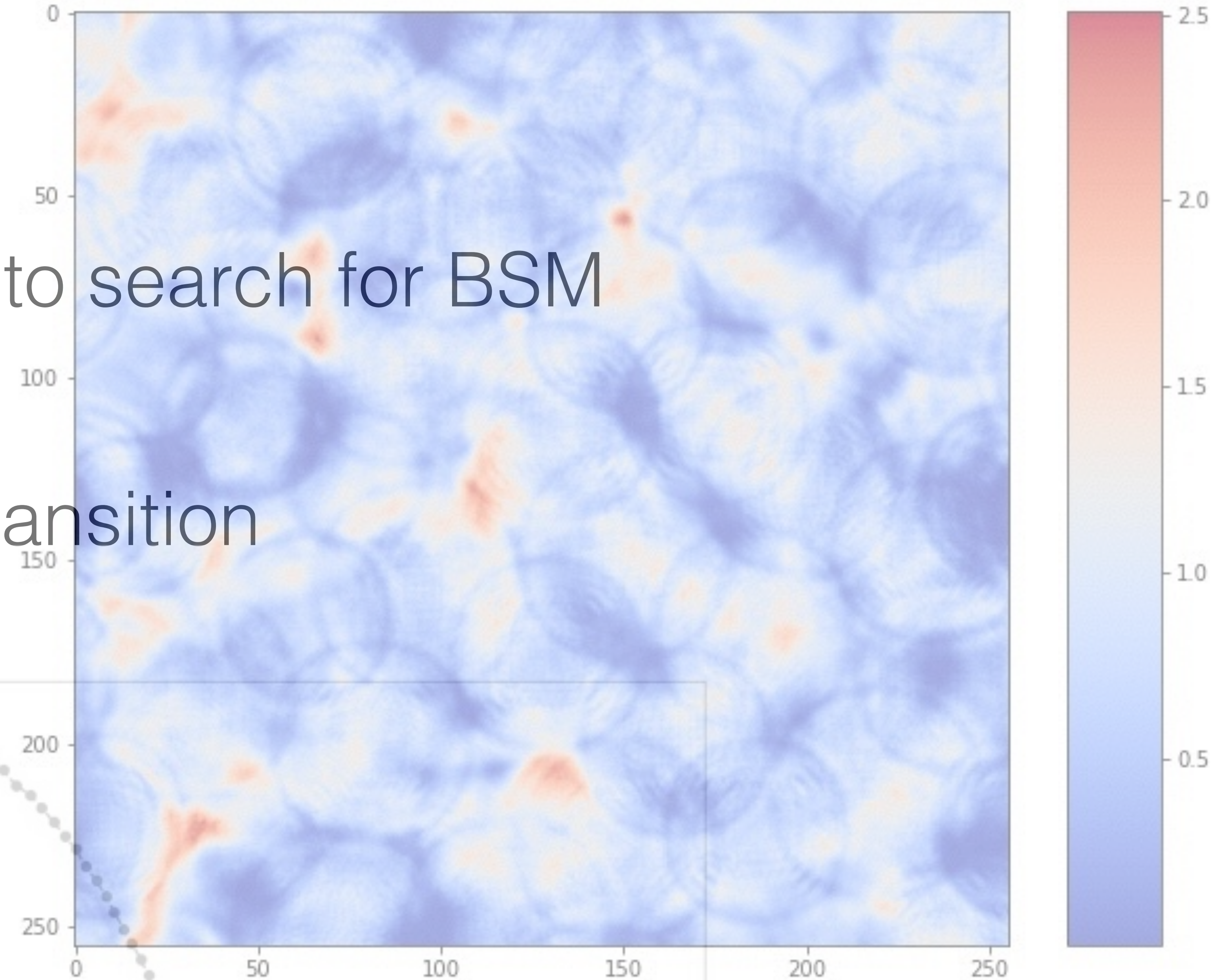
LGB, R.G. Cai, JL, RYZ,2009.13893

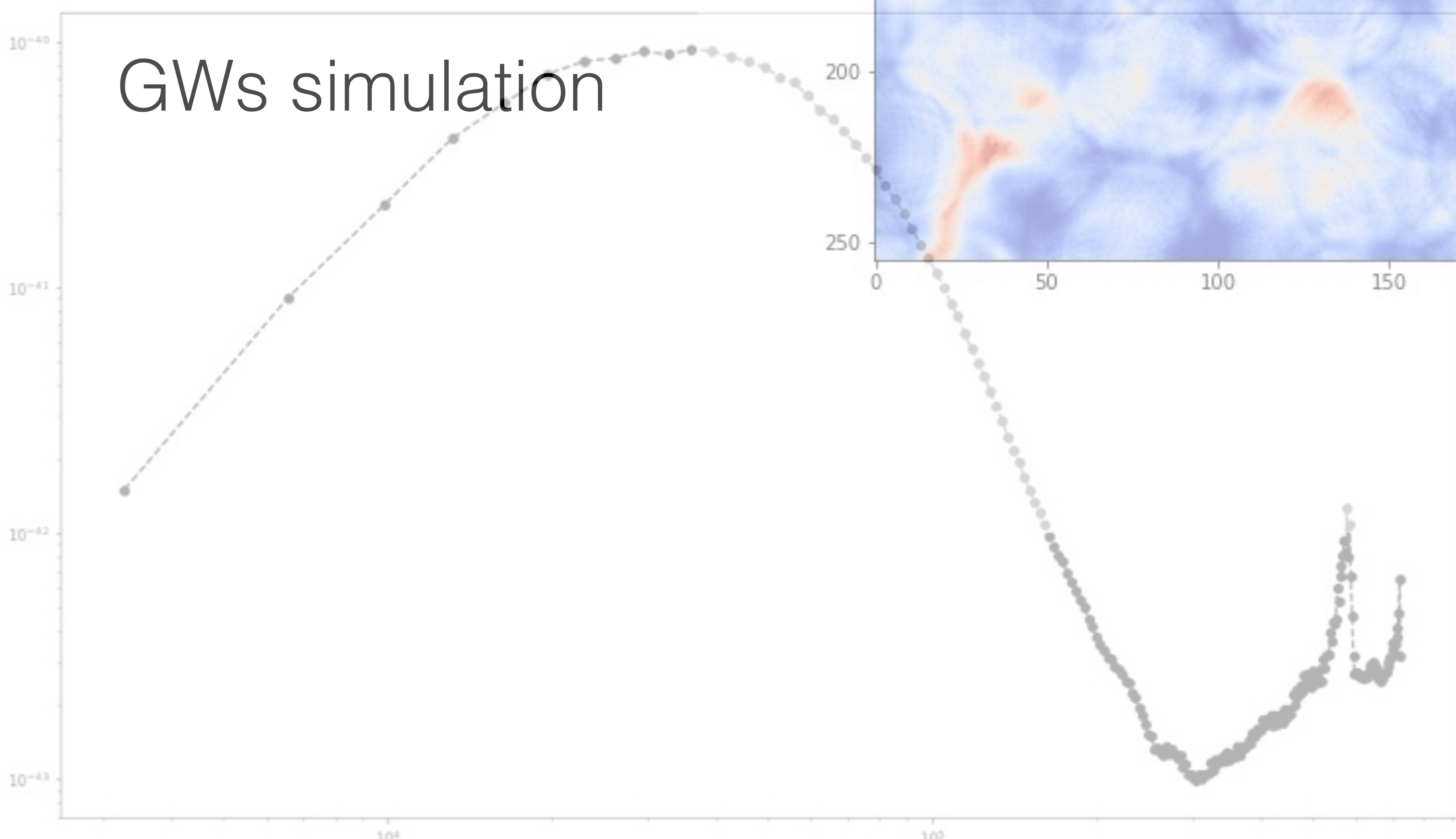
Summary

GW as a new window to search for BSM

Simulation of phase transition

GWs simulation





Thanks

谢谢 ！

GW sources

$$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW1}} & \text{for } f < f_*, \\ \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW2}} & \text{for } f > f_*, \end{cases}$$

1807.00786

Table 1. Cosmological GW sources

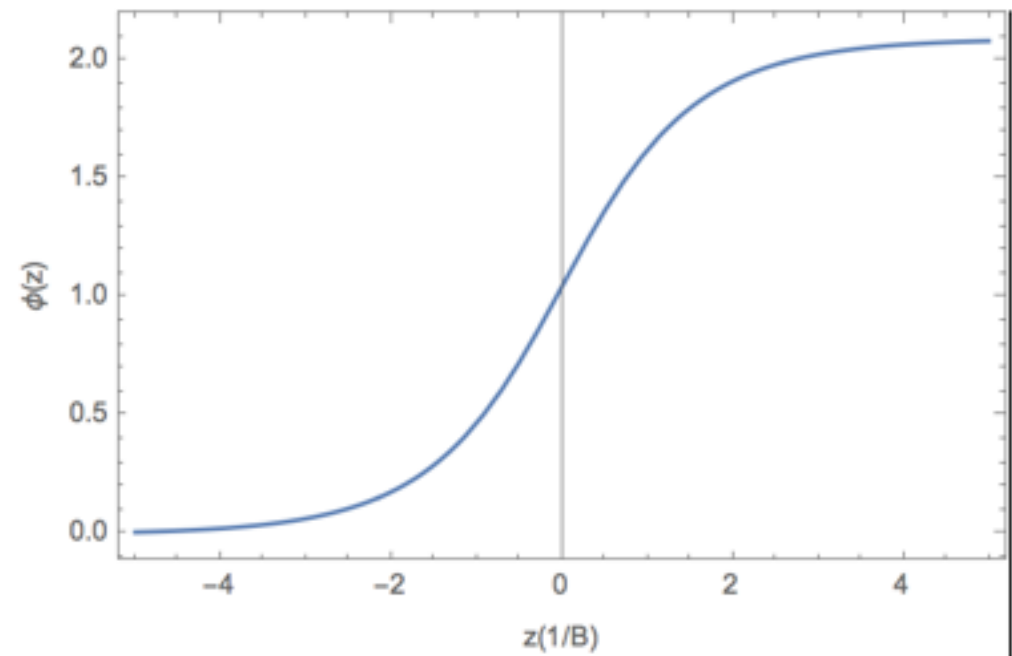
source	$n_{\rm GW1}$	$n_{\rm GW2}$	f_* [Hz]	$\Omega_{\rm GW}$
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{\rm PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times 10^{-4}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times 10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times 10^{-6}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{\rm pl}}\right)^2$
Cosmic strings (loops 1)	$[1,2]$	$[-1,-0.1]$	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	$[-1,-0.1]$	0	$\sim 3\times 10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	$[0,0.2]$	$[0,0.2]$	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1{\rm TeV}^3}\right)^2\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Magnetic fields	3	α_B+1	$\sim 10^{-6}\left(\frac{T_*}{10^2{\rm GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}{\rm G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi-6)(\epsilon-\eta)$	—	$\sim 2\times 10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times 10^5\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{1/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{-4/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times 10^{-2}\left(\frac{M_{\rm PBH}}{10^{20}\ {\rm g}}\right)^{-1/2}$	$\sim 7\times 10^{-9}\left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3-2\mu$	—	$\sim 1.4\times 10^{-6}\left(\frac{H_s}{0.15M_{\rm pl}}\right)^4$

DW solution:

$$V = \frac{\mu_H^2}{2}v^2 + \frac{\lambda_H}{4}v^4 + \frac{\mu_S^2}{2}v_s^2 + \frac{\lambda_S}{4}v_s^4 + \frac{\lambda_{SH}}{4}v_s^2v^2 + \frac{\mu_3}{2\sqrt{2}}v_s^3\cos(3\phi).$$

With $\eta^2 = v_s^2/2$, the kinetic term of ϕ can be obtained as,

$$\mathcal{L}_{\text{kinetic}}\left(\phi\right) = \eta^2\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) \ .$$



The field equation,

$$\partial_\mu \frac{\partial \mathcal{L}_{\text{kinetic}}}{\partial_\mu(\partial\phi)} + \frac{\partial V}{\partial\phi} = 0 \ ,$$

yields

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}z^2} - \frac{1}{3B^2}\sin(3\phi) = 0 \ ,$$

$$\frac{1}{B^2} = -\frac{9}{4}\mu_3 v_s \ , \phi = \frac{4}{3}\arctan(e^{\frac{z}{B}})$$

Tension: $$\sigma = \int dz \rho_{\text{wall}}\left(z\right) = \int \left(\left|\frac{dS}{dz}\right|^2 + V\left(\frac{S(z)}{\sqrt{2}}, \frac{v}{\sqrt{2}}\right) - V\left(\frac{v_s}{\sqrt{2}}, \frac{v}{\sqrt{2}}\right)\right)dz$$

Back Up

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}(\frac{d\phi_b}{dr})^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0\,, \qquad \frac{d\phi_b}{dr}|_{r=0} = 0$$

Bubble nucleation:

$$\Gamma \approx A(T)e^{-S_3/T} \sim 1$$

Latent heat:

$$\alpha = \frac{1}{\rho_R}\left[-(V_{\rm EW} - V_f) + T\left(\frac{dV_{\rm EW}}{dT} - \frac{dV_f}{dT}\right)\right]\Big|_{T=T_*}$$

phase transition inverse duration:

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}|_{T=T_n}$$

GW from FOPT

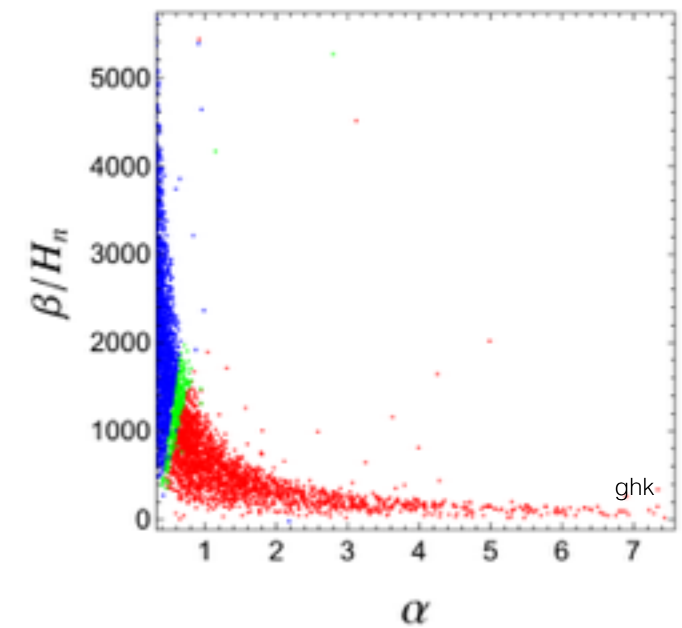
$$\Omega_{\rm GW}(f)h^2 \approx \Omega_{\rm sw}(f)h^2 + \Omega_{\rm turb}(f)h^2$$

Sound Wave: $$\Omega h^2_{\rm sw}(f) = 2.65\times10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1} v_b \left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\rm sw}}\right)^3 \left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

phase transition duration: $$\tau_{sw} = min\left[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma $$\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$$

$$f_{\rm sw} = 1.9\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}} {\rm Hz}$$



MHD turbulence: $$\Omega h^2_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b \frac{(f/f_{\rm turb})^3(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

$$f_{\rm turb} = 2.7\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}} {\rm Hz}$$

GW from DW

Amplitude: $$\Omega_{\mathrm{GW}}^{dw} h^2 (t_0)_{\mathrm{peak}} \simeq 5.20 \times 10^{-20} \times \tilde{\epsilon}_{\mathrm{gw}} \mathcal{A}^4 \left(\frac{10.75}{g_*}\right)^{1/3} \left(\frac{\sigma_{\mathrm{wall}}}{1\mathrm{TeV}^3}\right)^4 \left(\frac{1\mathrm{MeV}^4}{\Delta V}\right)^2$$

peak frequency: $$f^{dw}(t_0)_{\mathrm{peak}} = \frac{a(t_{\mathrm{dec}})}{a(t_0)} H(t_{\mathrm{dec}}) \simeq 3.99 \times 10^{-9} \mathrm{Hz} \mathcal{A}^{-1/2} \left(\frac{1\mathrm{TeV}^3}{\sigma_{\mathrm{wall}}}\right)^{1/2} \left(\frac{\Delta V}{1\mathrm{MeV}^4}\right)^{1/2}$$

Slope: $\Omega_{GW}^{dw} h^2 \propto f^3$ when $f < f_{peak}$, and $\Omega_{GW}^{dw} h^2 \propto f^{-1}$ when $f \geqslant f_{peak}$

Domain wall decay before they overclose Universe

$$\sigma_{wall} < 2.93 \times 10^4 \mathrm{TeV}^3 \mathcal{A}^{-1} \left(\frac{0.1 sec}{t_{dec}}\right)$$

$$t_{dec} \approx \mathcal{A}\sigma_{wall}/(\Delta V)$$

Domain wall decay before the BBN

$$\Delta V \gtrsim 6.6 \times 10^{-2} \mathrm{MeV}^4 \mathcal{A} \left(\frac{\sigma_{wall}}{1\mathrm{TeV}^3}\right)$$

GW from CS

Nambu-Goto cosmic strings characterized solely by the string tension μ, with string tension $\mu \approx 2\pi v_S^2\, n$ with n being winding number

Kibble mechanism:

$$\mu \approx \frac{10^{-15}}{\mathrm{G}} \left(\frac{T_p}{10^{11}\ \mathrm{GeV}}\right)^2$$

G is Newton's constant

GW energy density spectrum from cosmic string networks

$$\Omega_{\mathrm{GW}}(f) = \sum_k \Omega_{\mathrm{GW}}^{(k)}(f) \ ,$$

with k-mode being

$$\Omega_{\mathrm{GW}}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k}{f} \frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha\,(\alpha + \Gamma G\mu)} \int_{t_F}^{t_0} d\tilde{t}\, \frac{C_{eff}(t_i^{(k)})}{t_i^{(k)\,4}} \left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 \left[\frac{a(t_i^{(k)})}{a(\tilde{t})}\right]^3 \Theta(t_i^{(k)} - t_F)$$

F_α : fraction of the energy released by long strings

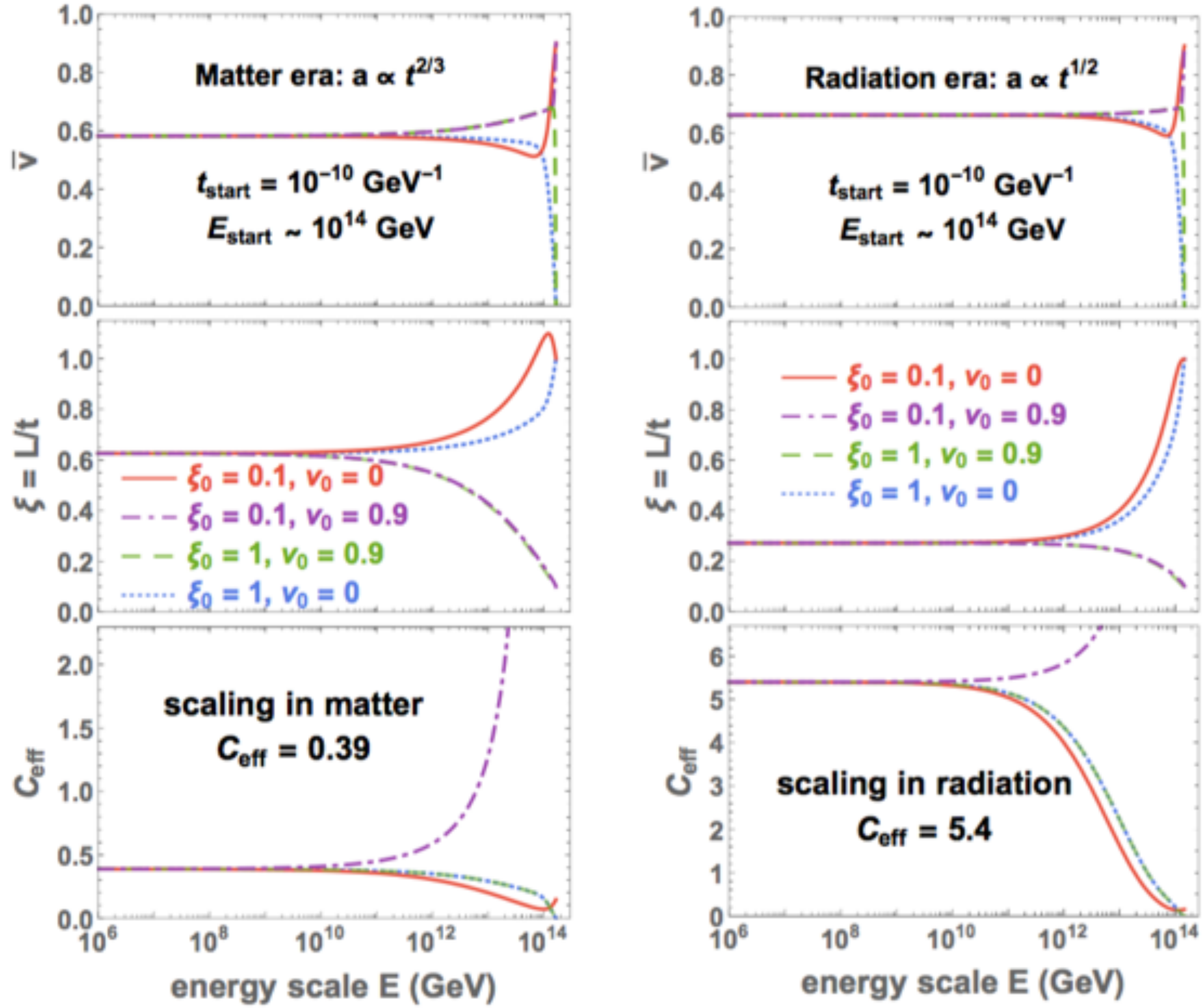
C_{eff} : loop production efficiency

The cosmic string network reaches scaling after formation at time t_F , which connect with the phase transition through $\sqrt{\rho_{tot}(t_F)} \equiv \mu$

formation time of loops of the k mode

$$t_i^{(k)}(\tilde{t}, f) = \frac{1}{\alpha + \Gamma G\mu} \left[\frac{2k}{f} \frac{a(\tilde{t})}{a(t_0)} + \Gamma G\mu\, \tilde{t}\right]$$

$\tilde{t}$: GW emission time



Nambu-Goto string VOS equation to obtain the scaling regime

$$\frac{dL}{dt} = HL\,(1+\bar{v}^2) + \frac{1}{2}\tilde{c}\,\bar{v},$$
$$\frac{d\bar{v}}{dt} = (1-\bar{v}^2)\left[\frac{k(\bar{v})}{L} - 2H\bar{v}\right],$$

correlation length ξ = L/t

mean velocity $\bar{v}$

$$k(\bar{v}) = \frac{2\sqrt{2}}{\pi}(1-\bar{v}^2)(1+2\sqrt{2}\bar{v}^3)\frac{1-8\bar{v}^6}{1+8\bar{v}^6},$$

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Sphaleron details

SM, one higgs

$$
E_{\mathrm{sph},T} = \frac{4\pi v[T]}{g}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 s_\mu^2 + \frac{8}{\xi^2} f^2(1-f)^2 s_\mu^4 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 s_\mu^2 + s_\mu^2((1-f)^2 h^2 \right.
$$
$$
\left. - 2fh(1-f)(1-h)c_\mu^2 + f^2(1-h)^2 c_\mu^2) + \frac{\xi^2}{g^2 v[T]^4}(V_{eff}[\mu, h, T])\right] \quad (
$$

$$
\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f)s_\mu^2 - \frac{1}{8}(2h^2(1-f) - 2h(1-h)(1-2f)c_\mu^2 + 2f(1-h)^2 c_\mu^2),
$$
$$
\frac{d}{d\xi}\left(\xi^2 \frac{dh}{d\xi}\right) = 2h(1-f)^2 - 2f(1-f)(1-2h)c_\mu^2 - 2f^2(1-h)c_\mu^2 + \frac{\xi^2}{g_2^2}\frac{1}{v[T]^4}\frac{\partial V_{\mathrm{eff}}}{\partial h},
$$

SM+S

$$
E_{\mathrm{sph}}[f,h,k] = \frac{4\pi v}{g_2}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f-f^2)^2 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 + h^2(1-f)^2 \right.
$$
$$
\left. + \frac{\xi^2}{2}\frac{v_S^2}{v^2}\left(\frac{dk}{d\xi}\right)^2 + \frac{\xi^2}{g_2^2 v^4} V_{\mathrm{eff}}(h,k,T)\right].
$$

For the xSM model, we consider the following sphaleron field ansatz [1]:

$$
A_i(\mu, r, \theta, \phi) = -\frac{i}{g} f(r)\partial_i U(\mu, \theta, \phi) U^{-1}(\mu, \theta, \phi), \quad (1)
$$
$$
H(\mu, r, \theta, \phi) = \frac{v(T)}{\sqrt{2}}\left[(1-h(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h(r) U(\mu, \theta, \phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right], \quad (2)
$$
$$
S(\mu, r, \theta, \phi) = v_S(T) k(r), \quad (3)
$$

where A_i are SU(2) gauge fields, and the matrix U is defined as

$$
U(\mu, \theta, \phi) = \begin{pmatrix} e^{i\mu}(c_\mu - i s_\mu c_\theta) & e^{i\phi} s_\mu s_\theta \\ -e^{-i\phi} s_\mu s_\theta & e^{-i\mu}(c_\mu + i s_\mu c_\theta) \end{pmatrix}, \quad (4)
$$

where the $s_{\mu(\theta)} = \sin\mu(\theta)$ and $c_{\mu(\theta)} = \cos\mu(\theta)$. The sphaleron energy is obtained for $\mu = \pi/2$ [2]. From the sphaleron energy in the main body of this paper, the equations of motion can be found:

$$
\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f) - \frac{v[T]^2 h^2}{4\Omega[T]^2}(1-f), \quad (5)
$$
$$
\frac{d}{d\xi}\left(\xi^2 \frac{dh}{d\xi}\right) = 2h(1-f)^2 + \frac{\xi^2}{g^2}\frac{1}{v[T]^2\Omega[T]^2}\frac{\partial V_{\mathrm{eff}}(h,k,T)}{\partial h}, \quad (6)
$$
$$
\frac{d}{d\xi}\left(\xi^2 \frac{dk}{d\xi}\right) = \frac{\xi^2}{g^2}\frac{1}{v_S[T]^2\Omega[T]^2}\frac{\partial V_{\mathrm{eff}}(h,k,T)}{\partial k}. \quad (7)
$$

The sphaleron solutions can be obtained with the following boundary conditions,

$$
\lim_{\xi\to 0} f(\xi) = 0, \lim_{\xi\to 0} h(\xi) = 0, \quad \lim_{\xi\to 0} k'(\xi) = 0,
$$
$$
\lim_{\xi\to\infty} f(\xi) = 1, \lim_{\xi\to\infty} h(\xi) = 1, \lim_{\xi\to\infty} k(\xi) = 1. \quad (8)
$$

Sphaleron details

$$A_i(r,\theta,\phi) = -\tfrac{i}{g} f(gvr)\partial_i U^\infty (U^\infty)^{-1}, \quad (23)$$

$$\Phi_1(r,\theta,\phi) = \tfrac{v_1}{\sqrt{2}} h_1(gvr) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (24)$$

$$\Phi_2(r,\theta,\phi) = \tfrac{v_2}{\sqrt{2}} h_2(gvr) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (25)$$

where A_i are SU(2) gauge fields, $A_i = \frac{1}{2} A_i^a \tau^a$, $v = \sqrt{v_1^2 + v_2^2}$, and U is defined as

$$U^\infty = \frac{1}{r} \begin{pmatrix} z & x+iy \\ -x+iy & z \end{pmatrix}, \quad (26)$$

$$\begin{aligned} E_{\rm sph}[f,h_1,h_2] = \tfrac{4\pi v}{g} \int_0^\infty d\xi \Bigg[& 4\left(\tfrac{df}{d\xi}\right)^2 + \tfrac{8}{\xi^2}(f-f^2)^2 \\ & + \tfrac{\xi^2}{2}\tfrac{v_1^2}{v^2}\left(\tfrac{dh_1}{d\xi}\right)^2 + \tfrac{\xi^2}{2}\tfrac{v_2^2}{v^2}\left(\tfrac{dh_2}{d\xi}\right)^2 \\ & + \left(\tfrac{v_1^2}{v^2}h_1^2 + \tfrac{v_2^2}{v^2}h_2^2\right)(1-f)^2 \\ & + \tfrac{\xi^2}{g^2 v^4} V(h_1,h_2)\Bigg], \quad (\end{aligned}$$

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f) - \left(\frac{v_1^2}{4v^2}h_1^2 + \frac{v_2^2}{4v^2}h_2^2\right) \times (1-f), \quad (29)$$

$$\frac{d}{d\xi}\left(\xi^2 \frac{dh_1}{d\xi}\right) = h_1(1-f)^2 + \frac{\xi^2}{g^2 v_1^4}\frac{\partial V(h_1,h_2)}{\partial h_1}, \quad (30)$$

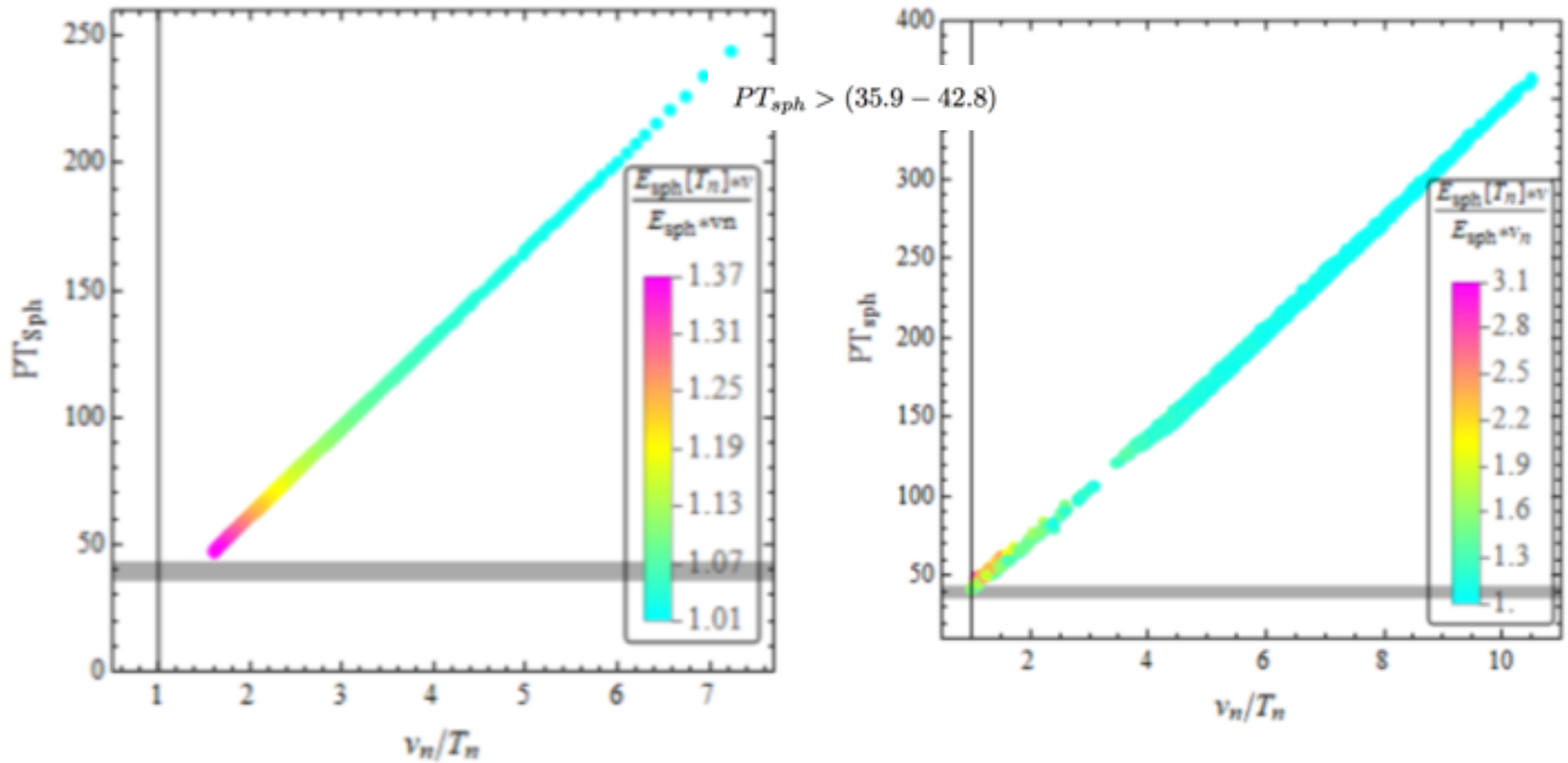
$$\frac{d}{d\xi}\left(\xi^2 \frac{dh_2}{d\xi}\right) = h_2(1-f)^2 + \frac{\xi^2}{g^2 v_2^4}\frac{\partial V(h_1,h_2)}{\partial h_2}. \quad (31)$$

We solve the above equations with the following boundary conditions

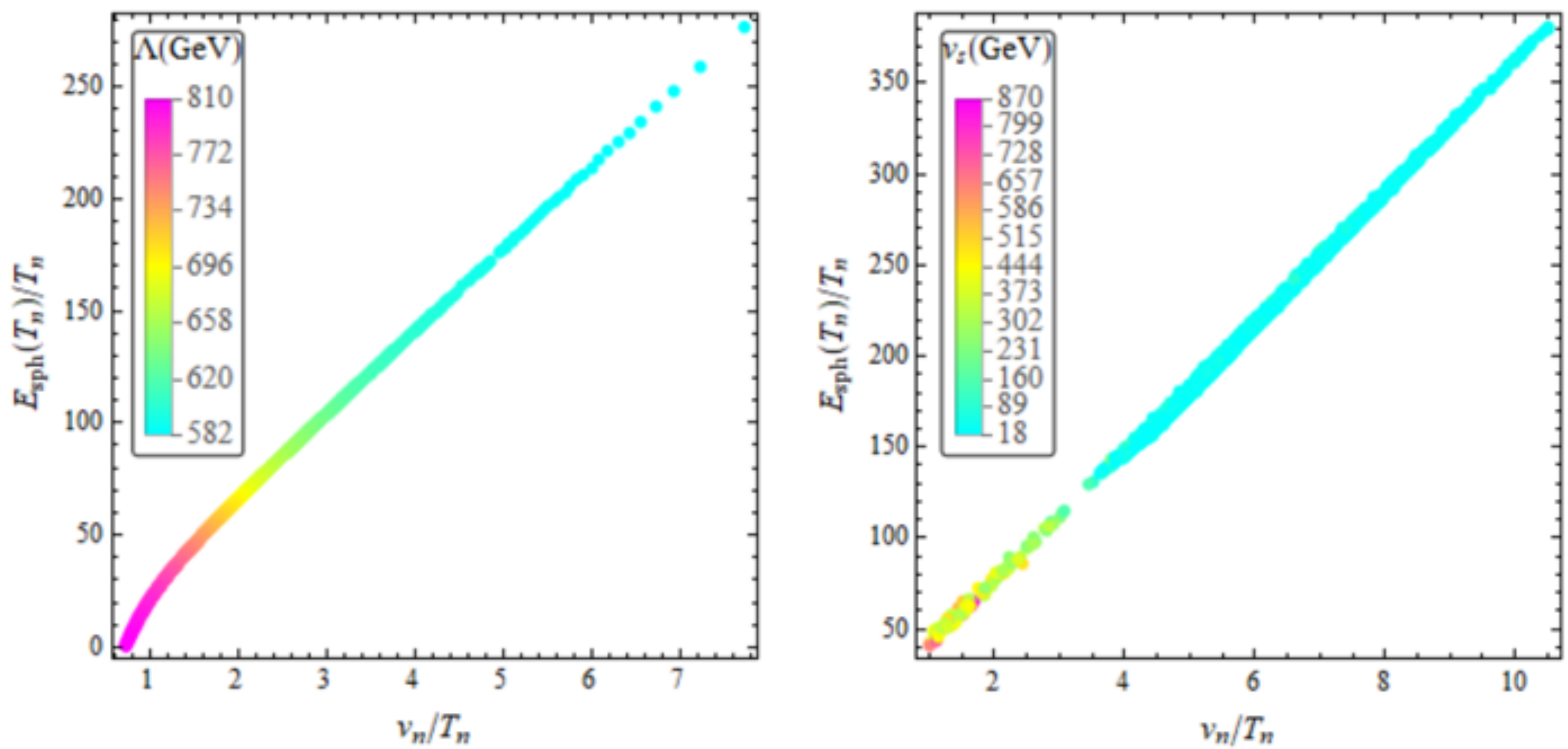
$$\lim_{\xi\to 0} f(\xi) = 0, \ \lim_{\xi\to 0} h_1(\xi) = 0, \ \lim_{\xi\to 0} h_2(\xi) = 0, \quad (32)$$

$$\lim_{\xi\to \infty} f(\xi) = 1, \lim_{\xi\to \infty} h_1(\xi) = 1, \lim_{\xi\to \infty} h_2(\xi) = 1. \quad (33)$$

Sphaleron energy and SFOEWPT condition



SMEFT xSM



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