Nonfactorizable QCD Effects in Higgs Boson Production via Vector Boson Fusion

Tao Liu, Kirill Melnikov, Alexander Penin

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The 6th China LHC Physics Workshop

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2. Transverse momentum expansion
3. Light-cone and transverse dynamics decoupled

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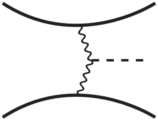
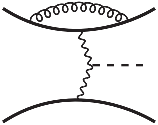
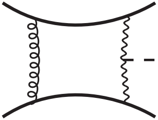
- Summary

VBF Higgs production







Vector Boson fusion(Higgs coupling to vector boson):

second biggest higgs production channel at LHC.

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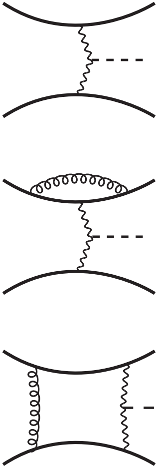

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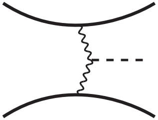

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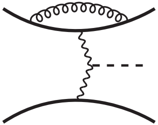

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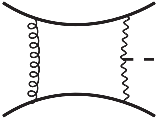

- NLO vanishes by color factor
- NNLO correction is missing before

  estimations: [Figy et al 2008; Bolzoni et al 2012]
- How long does it make sense?

Nonfactorizable NNLO

What we need is one-loop $\otimes$ one-loop and tree $\otimes$ two-loop amplitudes.

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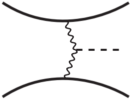
VBF kinematical features:

1. energetic forward quark jets
2. rapidity gap between Higgs and tagging jets

$\Rightarrow$ Regge limit

VBF feature in detail



$$q(p_1) + q'(p2) \to q(p3) + q'(p4) + H(p5)$$

$$q_3 = p3 - p1, \quad q_4 = p4 - p2$$

$p_{\perp,j}$ is a typical transverse momentum of a tagging jet. ($p_{\perp,j} \sim M_{V,H} \sim 100$ GeV)

s is the center-of-mass energy squared of the colliding partons. ($\sqrt{s} \geq 600$ GeV)

$\Rightarrow$ Hierarchy (eikonal approximation).

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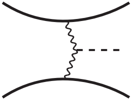

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$e^{|y_H|-|y_j|} \sim 0.05$

$\Rightarrow$ Higgs are produced at central rapidity.

one light-cone components scale as $p_{3,4,\perp}$, the other is suppressed by $\frac{p_\perp}{\sqrt{s}}$

$\Rightarrow$ Glauber vector boson $q_i^2 \approx q_{i,\perp}^2$ at leading power$(p_\perp^2/s)$ approximation.

Leading power approximation in $p_\perp^2/s$

- Hard loop momentum $k \sim \sqrt{s}$ gives subleading power corrections.
- Interested region: $k \ll \sqrt{s}, \quad k_\perp^2 \gg k_+ k_-$.

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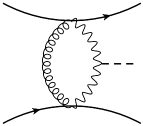
- Sun over permutations $\frac{1}{2k^\pm+i\epsilon} - c.c. = -i\pi\delta(k^\pm)$
- Light-cone components decoupled and Glauber gauge boson propagating in the transverse space.

$\mathcal{M}^{(1)} \quad \propto$



Leading Power in $p_\perp^2/s$

QCD corrections are diagonal in the chiral basis.

$$\mathcal{M}^{(1)} = i\tilde{\alpha}_s \chi^{(1)}(q_3, q_4) M^{(0)}\,,$$

$$\chi^{(1)}(q_3, q_4) = \frac{1}{\pi}\int \frac{\mathrm{d}^2 k}{k^2+\lambda^2} \times \frac{q_3^2+M_V^2}{(k-q_3)^2+M_V^2}\,\frac{q_4^2+M_V^2}{(k+q_4)^2+M_V^2}.$$

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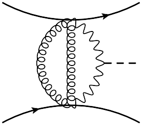
- Glauber phase: $e^{-i\alpha \ln \lambda^2}$

Two-loop amplitude



$$\mathcal{M}^{(2)} = -\frac{\tilde{\alpha}_s^2}{2!}\chi^{(2)}(q_3, q_4)\mathcal{M}^{(0)}\,,$$

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Infrared structure!

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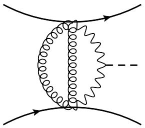

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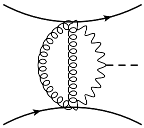

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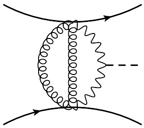

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- $\chi_{\mathrm{nf}} = \left[f^{(1)}\right]^2 - f^{(2)}$
- Glauber phase noncancellation!
- (Nonfactorizable NNLO)/(factorizable NNLO) $\sim \pi^2/N_c^2$!

Results

$$
\begin{aligned}
f^{(1)} &= \int_0^1 \mathrm{d}x \, \frac{\Delta_3 \Delta_4}{r_{12}^2} \left[\ln\left(\frac{r_{12}^2}{r_2 M_V^2} \right) + \frac{r_1 - r_2}{r_2} \right], \\
f^{(2)} &= \int_0^1 \mathrm{d}x \, \frac{\Delta_3 \Delta_4}{r_{12}^2} \left[\left(\ln\left(\frac{r_{12}^2}{r_2 M_V^2} \right) + \frac{r_1 - r_2}{r_2} \right)^2 \right. \\
&\quad - \ln^2\left(\frac{r_{12}}{r_2}\right) - \frac{2 r_{12}}{r_2} \ln\left(\frac{r_{12}}{r_2}\right) - 2\,\mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) \\
&\quad \left. - \left(\frac{r_1 - r_2}{r_2}\right)^2 + \frac{\pi^2}{3} \right],
\end{aligned}
$$

$$
\begin{aligned}
r_1 &= q_3^2 x + q_4^2 (1-x) - q_H^2 x (1-x), \\
r_2 &= q_H^2 \, x(1-x) + M_V^2 \,, \\
r_{12} &= r_1 + r_2 \,, \\
\Delta_i &= q_i^2 + M_V^2 \,.
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$$f^{(1)} = \int_0^1 \mathrm{d}x \frac{\Delta_3\Delta_4}{r_{12}^2}\left[\ln\left(\frac{r_{12}^2}{r_2 M_V^2}\right) + \frac{r_1 - r_2}{r_2}\right],$$

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These one-dimensional integral are suitable for numerical evaluations, so we didn't pursue further analytical expressions.

Results under different limits

scales: q_3, q_4, M_v

1. Forward Higgs production$(x = M_V^2/q_3^2)$:

$$\lim_{q_H \to 0} \chi_{\mathrm{nf}} = \ln^2\left(\frac{1+x}{x}\right) + 2\,\mathrm{Li}_2\left(\frac{1}{1+x}\right) - \frac{\pi^2}{3} + 2\,\frac{1+x}{x}\ln\left(\frac{1+x}{x}\right) + \left(\frac{1-x}{x}\right)^2$$

The coefficient of the quadratic logarithm can be read from one- and two-loop massless amplitudes at zero Higgs momentum.

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The coefficient of the quadratic logarithm can be read from one- and two-loop massless amplitudes at zero Higgs momentum.

2. Forward jet production:

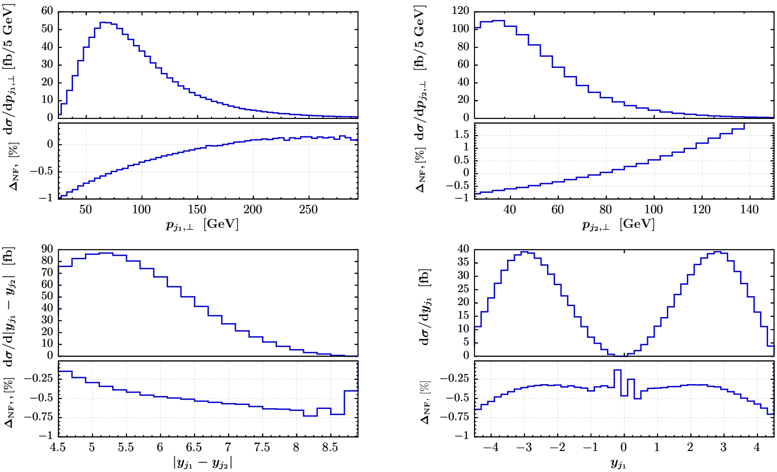
$\lim_{q_3\to 0}\chi_{\rm nf} = \ln^2\left(\frac{1+x}{x}\right) + 2\,{\rm Li}_2\left(\frac{1}{1+x}\right) - \frac{\pi^2}{3}$

3. Forward production:

$\lim_{q_{3,4}\to 0}\chi_{\rm nf} = 1 - \frac{\pi^2}{3}$

Differential distributions

VBF cuts: $p_{\perp,j} > 30\mathrm{GeV}$, $\sqrt{s_{jj}} > 600\mathrm{GeV}$, $|\mathrm{y_{j_1,2}} < 4.5|$, $|\mathrm{y_{j_1}} - \mathrm{y_{j_2}}| > 4.5$



Upper panel display LO contribution.

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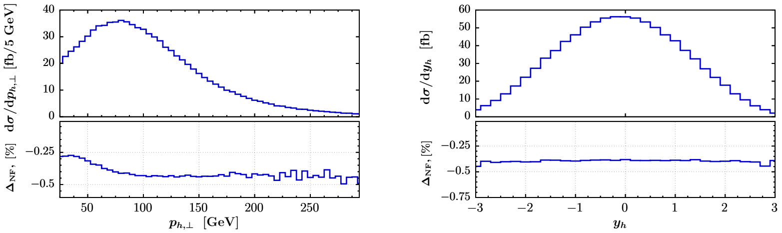

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Comparsion

Cross sections after VBF cuts:

NNLO fact.	NNLO nonfact.	NNNLO fact.
-4%	-0.5%	permill

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The above correction first appears at NNLO and its scale dependence is not compensated.

Summary

- First NNLO nonfactorizable QCD contribution to VBF Higgs production [cited by PDG 2020]

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- Glauber phase π^2 enhanced versus color factor suppression
- Percent correction to LO which is comparable to factorizable counterpart

Thanks for your attention!

Backup

PDG particle data group Reviews, Tables, and Plots

top-mass approximation, see also the exact (two-loop) NLO result [83]). One $2 \to 3$ process is known at NNLO, Higgs production through vector-boson fusion, using an approximation in which the two underlying DIS-like $q \to qV$ scatterings are factorised, the so-called structure function approximation [167, 199]. Corrections beyond the structure function approximation are expected to be small, on the order of a percent or less [200].

[198] D. de Florian and J. Mazzitelli, Phys. Rev. Lett. **111**, 201801 (2013), [arXiv:1309.6594].
[199] J. Cruz-Martinez *et al.*, Phys. Lett. **B781**, 672 (2018), [arXiv:1802.02445].
[200] T. Liu, K. Melnikov and A. A. Penin (2019), [arXiv:1906.10899].

this result is obtained in the DIS/factorised approximation [74] where the fusing gauge bosons are emitted from the two quark legs independently. While, the exact NNLO VBF calculation will remain out-of-reach in the near future, the leading non-factorisable contributions with two forward jets have been estimated [75]. They give some corrections, also of the order of few permill, to inclusive quantities, but they are an order of magnitude larger for differential observables. Full NNLO QCD and NLO EW results

[73] F. A. Dreyer and A. Karlberg, Phys. Rev. Lett. **117**, 7, 072001 (2016), [arXiv:1606.00840].
[74] T. Han, G. Valencia and S. Willenbrock, Phys. Rev. Lett. **69**, 3274 (1992), [hep-ph/9206246].
[75] T. Liu, K. Melnikov and A. Penin (2019), [arXiv:1906.10899].