# Two-loop anomalous dimensions of higher dimensional QCD operators and Higgs EFT amplitudes

Ke Ren · November 7, 2020

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Qingjun Jin, Ke Ren, Gang Yang, arXiv: 2011.02494

# Content

- Background
- Overview
- Details
- Conclusion

# Higgs → 3g amplitude

Higgs EFT: integrate out top quark

$$\mathcal{L}_{eff} = \hat{C}_0 H \mathcal{O}_{4;0} + \sum_{k=1}^{\infty} \frac{1}{m_t^{2k}} \sum_i \hat{C}_i H \mathcal{O}_{4+2k;i}$$

$\mathcal{O}_{4;0}$ → $\mathrm{tr}(F^2)$

$\mathcal{O}_{4+2k;i}$ → $\mathrm{tr}(F^3)$, $\mathrm{tr}(F\bar{\psi}\not{D}\psi)$, etc.

Higgs+1-jet production: $\langle hg|\hat{C}_i H\mathcal{O}_{\Delta;i}|gg\rangle \sim \langle 0|\mathcal{O}_{\Delta;i}|ggg\rangle$

$\hat{C}_i$ → from matching

$\langle 0|\mathcal{O}_{\Delta;i}|ggg\rangle$ → dominant process

# Higgs → 3g amplitude

Higgs EFT: integrate out top quark

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$\mathrm{tr}(F^2)$ ← $\hat{C}_0 H \mathcal{O}_{4;0}$

$\hat{C}_i H \mathcal{O}_{4+2k;i}$ → $\mathrm{tr}(F^3)$, $\mathrm{tr}(F\bar{\psi}\not{D}\psi)$, etc.

Higgs+1-jet production: $\langle hg|\hat{C}_i H \mathcal{O}_{\Delta;i}|gg\rangle \sim \langle 0|\mathcal{O}_{\Delta;i}|ggg\rangle$

$\hat{C}_i$ → from matching

$\sim$ → dominant process

problem changes to: 3-gluon form factor of composite operators

# Speactrum of anomalous dim

1. Phenomenological: higher order to splitting kernel
2. Theoretical: Integrability

Integrability of $\mathcal{N} = 4$ sYM:

Minahan JHEP, 0303, 013.
Berenstein JHEP, 0204, 013
Belitsky Nucl.phys.B, 768:116

Dilatation operator → Hamiltonian of solvable spin-chain

Anomalous dimensions → Thermal Bethe Ansatz

→ String spectrum under BMN limit

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Dilatation operator → Hamiltonian of solvable spin-chain

Anomalous dimensions → Thermal Bethe Ansatz

→ String spectrum under BMN limit

Integrability of QCD:

Belitsky Int.J.Mod.Phy.A, 2004,19:4715
Belitsky Phys.Rev.Lett. 94,151603
Lipatov Nucl.Phys.B.Proc.Suppl,245,188

broken except for special kinematic limit (light-front, Regge)

# Overview

Computing target: higher-dim QCD operators

1. 2-loop anomalous dimensions and possible pattern

2. analytical structure of 2-loop form factors (e.g.,MTP)

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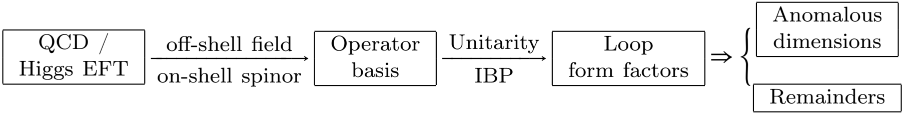
0. operator basis construction




work flow

# Operator considered

Consider gauge invariant scalar with even $CP$:

$$c(a_1,...,a_n)\left(D_{\mu_{11}}...D_{\mu_{1m_1}}F_{\nu_1\rho_1}\right)^{a_1}\cdots\left(D_{\mu_{n1}}...D_{\mu_{nm_n}}F_{\nu_n\rho_n}\right)^{a_n}\mathcal{T}(g_{\mu\nu})$$

only contains $g_{\mu\nu}$, no $\epsilon_{\mu\nu\rho\sigma}$

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only contains $g_{\mu\nu}$, no $\epsilon_{\mu\nu\rho\sigma}$

1. Due to Higgs + 1-jet production, mainly consider twist $n=3$.
2. Classification strategy also applies to $CP$ odd operator.
3. Color basis $c(a,b)=\delta^{ab}$,

   $c(a,b,c)$: $f^{abc}$, $d^{abc}$

   $c(a,b,c,d)$: 6 single traces, 3 double traces

# Classification: field theory method

D.o.F redundancy:

1. Equation of motion: $D_\mu F^{\mu\nu} = 0$
2. Bianchi identity: $D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0$
3. equivalence at length $l$: $\mathcal{O}_l - \mathcal{O}'_l = \Xi_{L>l}$ $\Rightarrow$ $\mathcal{O}_l \sim \mathcal{O}'_l$

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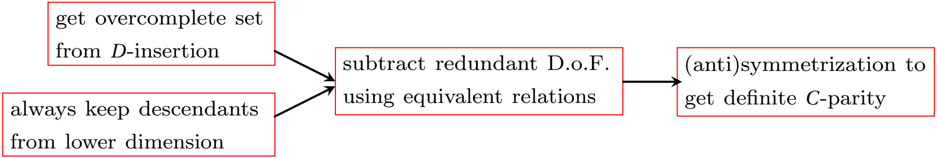

# Classification: on-shell method

Using operator-spinor dictionary:

| operator | $s_{ij}$ | $D_{\dot\alpha\alpha}$ | $f_{\alpha\beta}$ | $\bar{f}_{\dot\alpha\dot\beta}$ |
|---|---|---|---|---|
| spinor | $\langle ij\rangle[ji]$ | $\tilde\lambda_{\dot\alpha}\lambda_\alpha$ | $\lambda_\alpha\lambda_\beta$ | $-\tilde\lambda_{\dot\alpha}\tilde\lambda_{\dot\beta}$ |

- Helicity sector: structure under $f, \bar{f}$-decomposition

$$1^-2^-3^+ \to \ f f \bar{f}, \qquad 1^-2^-3^- \to \ f f f$$

- Descendants: overall $s_{123} \ \to \ \partial^2\mathcal{O}$

- D.o.F redundancy easy to clear →
  1. E.o.M: on-shell condition
  2. Bianchi Id: Schounten Id
  3. higher length excluded

# Classification: on-shell method

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Fix helicity, enumerate independent tree-level minimal form factors

descendants always kept by keeping $s_{123}$

distinguish $C$-parity from $s,t,u$-symmetry

→ Under spinor-helicity, write form factor back to projected operator, then full operator

→ one form factor creates various operators, but they are equivalent

# Classification: on-shell method

| operator | $s_{ij}$ | $D_{\dot{\alpha}\alpha}$ | $f_{\alpha\beta}$ | $\bar{f}_{\dot{\alpha}\dot{\beta}}$ |
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Example:

$$\begin{aligned} s_{12}\langle 12\rangle^3[13][23] &= (\lambda_{1\alpha}\lambda_2{}^{\alpha})(\tilde{\lambda}_{1\dot{\alpha}}\tilde{\lambda}_2{}^{\dot{\alpha}})(\lambda_1{}^{\beta}\lambda_{2\beta})(\lambda_{1\gamma}\lambda_2{}^{\gamma})(\lambda_{1\delta}\lambda_2{}^{\delta})(\tilde{\lambda}_{1\dot{\beta}}\tilde{\lambda}_3{}^{\dot{\beta}})(\tilde{\lambda}_{2\dot{\sigma}}\tilde{\lambda}_3{}^{\dot{\sigma}}) \\ &= (\lambda_{1\alpha}\tilde{\lambda}_{1\dot{\alpha}})(\lambda_1{}^{\beta}\tilde{\lambda}_{1\dot{\beta}})(\lambda_{1\gamma}\lambda_{1\delta})(\lambda_2{}^{\alpha}\tilde{\lambda}_2{}^{\dot{\alpha}})(\lambda_{2\beta}\tilde{\lambda}_{2\dot{\sigma}})(\lambda_2{}^{\gamma}\lambda_2{}^{\delta})(\tilde{\lambda}_3{}^{\dot{\beta}}\tilde{\lambda}_3{}^{\dot{\sigma}}) \\ &\sim D_{\alpha\dot{\alpha}}D^{\beta}{}_{\dot{\beta}}f_{\gamma\delta}D^{\alpha\dot{\alpha}}D_{\beta\dot{\sigma}}f^{\gamma\delta}\tilde{f}^{\dot{\beta}\dot{\sigma}} \sim \mathrm{Tr}(D_{12}F_{34}D_{15}F_{34}F_{25}) \end{aligned}$$

Also can give $\mathrm{Tr}(D_{12}F_{34}D_{13}F_{45}F_{25})$ in the similarly way, while:

$$\mathrm{Tr}(D_{12}F_{34}D_{13}F_{45}F_{25}) \overset{\text{Bianchi}}{=\!=\!=} -\frac{1}{2}\mathrm{Tr}(D_{12}F_{34}D_{15}F_{34}F_{25})$$

# Operator basis

see also: Lehman, JHEP 02:081
Henning, JHEP 08:016

Length-2: only one independent operator at each dim $\Delta$

Length-3 counting:

$$N(\Delta) = \begin{cases} \frac{(\Delta-4)(\Delta-5)}{6}, & \Delta (\mathrm{mod}\ 6) \neq 0, \\ \frac{\Delta^2-9\Delta+24}{6}, & \Delta (\mathrm{mod}\ 6) \equiv 0. \end{cases}$$

| dim $\Delta$ | 6 | 8 | 10 | 12 | 14 | 16 |
|---|---|---|---|---|---|---|
| # of basis | 1 | 2 | 5 | 10 | 15 | 22 |

Length-4 counting: more complicated but in analogy

# Operator basis

2-loop: Gehrmann, JHEP 1202,056

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2-loop: recently 2011.02494
Jin,Ren,Yang

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Length-4 counting: more complicated but in analogy

# 2-loop computation for $\Delta \leq 16$

Work flow of our 2-loop computation:

for a selected cut: $\prod(\text{tree blocks}) \xrightarrow{\text{channel sum}} \text{cut integrand}$

$$\xrightarrow{\text{IBP with cuts}} \sum_{\text{cut permitted}} c_i I_i$$

$$\xrightarrow{\text{collect all cut channels}} \sum_{\text{complete}} c_i I_i = \mathscr{F}^{(l)}$$

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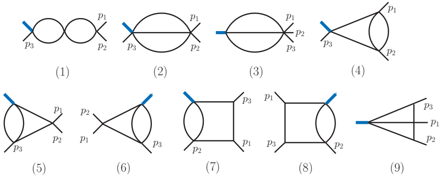
$\downarrow$

1+3-pt MI basis: hep-ph/0008287,hep-ph/0101124



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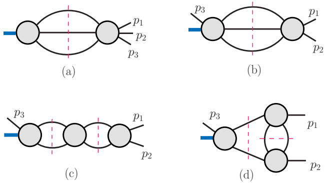
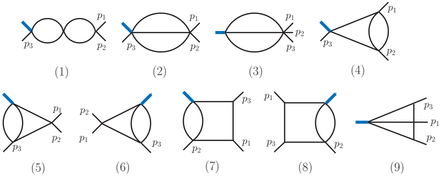
complete cuts for $\{I_i\}$ →



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factorize $\{e_i\}$ before IBP:
projector method, 1710.10208

$\xrightarrow{\text{IBP with cuts}} \sum_{\text{cut permitted}} c_i I_i$

$\xrightarrow{\text{collect all cut channels}} \sum_{\text{complete}} c_i I_i = \mathscr{F}^{(l)}$

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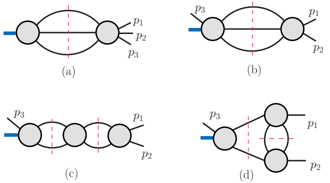


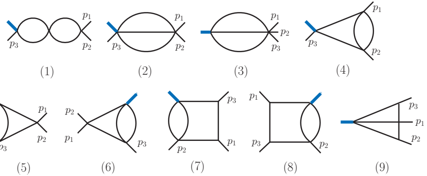

# Dilatation operator

Dilatation operator: $\mathbb{D} = -\frac{d\log Z}{d\log\mu} = \sum_{n=1}^{\infty}\left[\left(\frac{\alpha_s}{4\pi}\right)^n \mathbb{D}^{(n)} + \left(\frac{\alpha_s}{4\pi}\right)^{n+\frac{1}{2}} \mathbb{D}^{(n+\frac{1}{2})}\right]$

First three orders:

odd orders result from length-changed mixing

$$\mathbb{D}^{(1)} = 2\epsilon Z^{(1)}_{L\to L}, \qquad \mathbb{D}^{(\frac{3}{2})} = 3\epsilon\left(Z^{(1)}_{L\to(L+1)} + Z^{(2)}_{L\to(L-1)}\right),$$

$$\mathbb{D}^{(2)} = 4\epsilon\left(Z^{(2)}_{L\to L} - \frac{1}{2}(Z^{(1)}_{L\to L})^2 + \frac{1}{2\epsilon}\beta_0 Z^{(1)}_{L\to L} + Z^{(1)}_{L\to(L+2)} + Z^{(3)}_{L\to(L-2)}\right)$$

$$= 4\epsilon\left(Z^{(2)}_{L\to L}\Big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(1)}_{L\to(L+2)} + Z^{(3)}_{L\to(L-2)}\right)$$

Eigenvalues of $\mathbb{D}$: anomalous dimensions

To get UV divergence: taking IR subtraction Catani Phys.Lett.B 427:161

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Truncate length> 3 first:

$$\mathbb{D}^{(1)} = 2\epsilon\left(Z^{(1)}_{2\to2} + Z^{(1)}_{3\to3}\right),$$

$$\mathbb{D}^{(\frac{3}{2})} = 3\epsilon\left(Z^{(1)}_{2\to3} + Z^{(2)}_{3\to2}\right) = 3\epsilon Z^{(2)}_{3\to2},$$

$$\mathbb{D}^{(2)} = 4\epsilon\left(Z^{(2)}_{2\to2}\Big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(2)}_{3\to3}\Big|_{\frac{1}{\epsilon}-\text{part}}\right).$$

2-loop processes $2\to2$, $3\to3$, $3\to2$ are enough to get all these $Z$

# Dilatation operator

As an example, basis operators at dimension 8 $(L \leq 3)$

$$L=2: \quad \mathcal{O}_{8;0} = \frac{1}{2}\partial^4 \mathrm{tr} F^2,$$

$$L=3: \quad \mathcal{O}_{8;\alpha;f;1} = \mathrm{tr}(D_1 F_{23} D_4 F_{23} F_{14}) - \mathcal{O}_{8;\beta;f;1}, \ \mathcal{O}_{8;\beta;f;1} = \frac{1}{6}\partial^2 \mathrm{tr}(F_{12}F_{13}F_{23})$$

Dilatation operator: $\hat{\lambda} = N_c \frac{\alpha_s}{4\pi}, \quad \hat{g} = \frac{g}{4\pi}$

$$\mathbb{D}_{\mathcal{O}_8} = \begin{pmatrix} -\frac{22}{3}\hat{\lambda} - \frac{136}{3}\hat{\lambda}^2 & 0 & 0 \\ -\frac{\hat{\lambda}^2}{\hat{g}} & \frac{7}{3}\hat{\lambda} + \frac{269}{18}\hat{\lambda}^2 & 10\hat{\lambda}^2 \\ -3\frac{\hat{\lambda}^2}{\hat{g}} & 0 & \hat{\lambda} + \frac{25}{3}\hat{\lambda}^2 \end{pmatrix}$$

Anomalous dim: $\hat{\gamma}^{(1)}_{\mathcal{O}_8} = \{-\frac{22}{3}; 1; \frac{7}{3}\}, \quad \hat{\gamma}^{(2)}_{\mathcal{O}_8} = \{-\frac{136}{3}; \frac{25}{3}; \frac{269}{18}\}$

# Dilatation operator

Consider full basis of dimension 8

$$L=2: \quad \mathcal{O}_{8;0} = \frac{1}{2}\partial^4 \mathrm{tr}F^2,$$

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$$L=4: \quad g\Xi_1, \ g\Xi_2, \ g\Xi_3, \ g\Xi_4$$

add length-4 truncated before

$$\mathbb{D}^{(1)} = 2\epsilon\left(Z^{(1)}_{2\to2} + Z^{(1)}_{3\to3} + Z^{(1)}_{4\to4} + Z^{(1)}_{3\to4} + \frac{\beta_0}{2\epsilon}Z^{(0)}_{4\to4}\right),$$

$$\mathbb{D}^{(\frac{3}{2})} = 3\epsilon\left(Z^{(2)}_{3\to2}\right),$$

$$\mathbb{D}^{(2)} = 4\epsilon\left(Z^{(2)}_{2\to2}\big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(2)}_{3\to3}\big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(2)}_{4\to4}\big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(2)}_{3\to4}\big|_{\frac{1}{\epsilon}-\text{part}} + Z^{(2)}_{4\to3} + \frac{\beta_1}{4\epsilon}Z^{(0)}_{4\to4}\right)$$

not calculated, not correcting truncated result

1-loop $3\to4\&4\to4$, 2-loop $4\to3$ processes are enough to determine 3,4-mixing.

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$$L=4: \quad g\Xi_1, \; g\Xi_2, \; g\Xi_3, \; g\Xi_4 \qquad \text{add length-4 truncated before}$$

$$\mathbb{D}_{\mathcal{O}_{8;I}} = \begin{pmatrix} -\frac{22\hat\lambda}{3} - \frac{136\hat\lambda^2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\hat\lambda^2}{\hat g} & \frac{7\hat\lambda}{3} + \frac{269\hat\lambda^2}{18} & 10\hat\lambda^2 & -\frac{10\hat\lambda}{3} + z_{24}\hat\lambda^2 & -\frac{14\hat\lambda}{3} + z_{25}\hat\lambda^2 & 3\hat\lambda + z_{26}\hat\lambda^2 & \frac{13\hat\lambda}{3} + z_{27}\hat\lambda^2 \\ -3\frac{\hat\lambda^2}{\hat g} & 0 & \hat\lambda + \frac{25\hat\lambda^2}{3} & z_{34}\hat\lambda^2 & z_{35}\hat\lambda^2 & z_{36}\hat\lambda^2 & z_{37}\hat\lambda^2 \\ 0 & 0 & -4\hat\lambda^2 & -5\hat\lambda + z_{44}\hat\lambda^2 & 2\hat\lambda + z_{45}\hat\lambda^2 & z_{46}\hat\lambda^2 & z_{47}\hat\lambda^2 \\ 0 & 0 & -8\hat\lambda^2 & -4\hat\lambda + z_{54}\hat\lambda^2 & 9\hat\lambda + z_{55}\hat\lambda^2 & z_{56}\hat\lambda^2 & z_{57}\hat\lambda^2 \\ 0 & \hat\lambda^2 & 0 & z_{64}\hat\lambda^2 & z_{65}\hat\lambda^2 & -\frac{8\hat\lambda}{3} + z_{66}\hat\lambda^2 & -\hat\lambda + z_{67}\hat\lambda^2 \\ 0 & \frac{26\hat\lambda^2}{9} & 0 & z_{74}\hat\lambda^2 & z_{75}\hat\lambda^2 & -2\hat\lambda + z_{76}\hat\lambda^2 & \frac{17\hat\lambda}{3} + z_{77}\hat\lambda^2 \end{pmatrix}$$

Anomalous dim: $\hat\gamma^{(1)}_{\mathcal{O}_8} = \{-\frac{22}{3}; 1; \frac{7}{3}\}$, $\quad \hat\gamma^{(2)}_{\mathcal{O}_8} = \{-\frac{136}{3}; \frac{25}{3}; \frac{235}{18}\}$ $\qquad \frac{269}{18} \to \frac{235}{18}$

# Finite remainders

1. transcendental functions appeared: 2d Harmonic polylog, polylog

$$\mathscr{R}_{\mathcal{O}}^{(2),\pm} = \sum_{n=0}^{4} \mathscr{R}_{\mathcal{O}}^{(2),\pm}\Big|_{\text{deg-}n} + \mathscr{R}_{\mathcal{O}}^{(2),\pm}\Big|_{\log^2(-q^2)} + \mathscr{R}_{\mathcal{O}}^{(2),\pm}\Big|_{\log(-q^2)}$$

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2. Universal degree-4: maximal transcendental principle holds

↓

Brandhuber Phys.Rev.Lett.119,161601
Jin&Yang Commun.Theor.Phhys.72 065201

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2. Universal degree-4: maximal transcendental principle holds

not for quark cases: Jin&Yang JHEP02,169 ↑

↓ Brandhuber Phys.Rev.Lett.119,161601
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3. Universal building blocks for degree-3,2:

$$\mathcal{R}_{\mathcal{O}}^{(2),\pm}\Big|_{\text{deg-3}} = \sum_a \xi_a T_{3,a}, \quad \mathcal{R}_{\mathcal{O}}^{(2),\pm}\Big|_{\text{deg-2}} = \sum_a \eta_a T_{2,a}$$

$\xi_a$, $\eta_a$: rational function of Mandelstam

# Looking back to integrability

Inspired from integrability: analytical expression of dilatation operator

Appearance of $S^{(i)}$: an example in $\mathcal{N}=4$: Loebbert et al. JHEP,2016(12):90

$$(\mathfrak{D}^{(1)})^{m_1,m_2}_{n_1,n_2} = 2\left(S^{(1)}_{n_1}\delta^{m_1}_{n_1}\delta^{m_2}_{n_2} - \frac{\theta_{n_2 m_2}}{n_2-m_2}\delta^{m_1+m_2}_{n_1+n_2} + \left\{ {n_1\leftrightarrow n_2 \atop m_1\leftrightarrow m_2} \right\}\right).$$

$$(\tilde{\mathfrak{D}}^{(2)}_i)^{m_1,m_2}_{n_1,n_2} = -2\left[2S^{(1)}_{n_1}S^{(2)}_{n_1} + S^{(3)}_{n_1}\right]\delta^{m_1}_{n_1}\delta^{m_2}_{n_2} + \frac{\theta_{n_2 m_2}}{n_2-m_2}\delta^{m_1+m_2}_{n_1+n_2}\left[S^{(2)}_{n_1} + 3S^{(2)}_{m_1} + \left(S^{(1)}_{m_1} - S^{(1)}_{n_1}\right)\left(S^{(1)}_{n_1} + 3S^{(1)}_{m_1} - 4S^{(1)}_{n_2-m_2-1}\right)\right] + \left\{ {n_1\leftrightarrow n_2 \atop m_1\leftrightarrow m_2} \right\},$$

How about QCD ?

$$S^{(i)}_{\Delta} = \sum_{k=1}^{\Delta}\frac{1}{k^i}$$

factors in denominator of $\mathbb{D}$ no larger than $\frac{\Delta}{2}$ for 1-loop, $(\frac{\Delta}{2})^3$ for 2-loop

# Looking back to integrability

Inspired from integrability: analytical expression of dilatation operator

Appearance of $S^{(i)}$: an example in $\mathcal{N}=4$: Loebbert et al. JHEP,2016(12):90

$$(\mathfrak{D}^{(1)})^{m_1,m_2}_{n_1,n_2} = 2\left(S^{(1)}_{n_1}\delta^{m_1}_{n_1}\delta^{m_2}_{n_2} - \frac{\theta_{n_2 m_2}}{n_2-m_2}\delta^{m_1+m_2}_{n_1+n_2} + \{ {}^{n_1\leftrightarrow n_2}_{m_1\leftrightarrow m_2} \}\right).$$

$$(\tilde{\mathfrak{D}}^{(2)}_i)^{m_1,m_2}_{n_1,n_2} = -2\left[2S^{(1)}_{n_1}S^{(2)}_{n_1} + S^{(3)}_{n_1}\right]\delta^{m_1}_{n_1}\delta^{m_2}_{n_2} + \frac{\theta_{n_2 m_2}}{n_2-m_2}\delta^{m_1+m_2}_{n_1+n_2}\Big[S^{(2)}_{n_1} + 3S^{(2)}_{m_1} + \left(S^{(1)}_{m_1} - S^{(1)}_{n_1}\right)\left(S^{(1)}_{n_1} + 3S^{(1)}_{m_1} - 4S^{(1)}_{n_2-m_2-1}\right)\Big] + \{ {}^{n_1\leftrightarrow n_2}_{m_1\leftrightarrow m_2} \},$$

How about QCD ?

$$S^{(i)}_{\Delta} = \sum_{k=1}^{\Delta}\frac{1}{k^i}$$

?

factors in denominator of $\mathbb{D}$ no larger than $\frac{\Delta}{2}$ for 1-loop, $(\frac{\Delta}{2})^3$ for 2-loop

√

# Conclusion

1. We construct good operator basis for QCD operators (Yang-Mills sector) using spin-helicity formalism.
2. We calculate the two-loop minimal form factors for length-3 operators up to dimension 16 using Unitarity-IBP method.
3. We calculate the anomalous dimension spectrum of length-3 operators and estimate the error of length truncation.
4. We explore the analytic strcuture of finite remainder functions, such as maximal transcendentality, universal building blocks.
5. So far no obvious sign of integrability, but there still exists some comparable properties between QCD and $\mathcal{N} = 4$.

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Thank you