Unnuclear Physics

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All Things EFT seminar

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Ref.: H.-W. Hammer and DTS, arXiv:2103.12610

Very short summary

- Unnucleus = field in norelativistic CFT
- Unnuclear physics = nonrelativistic version of Georgi's "unparticle physics"
- Realized in nuclear reaction with final-state neutrons

Plan

- Georgi's unparticle
- Nonrelativistic conformal field theory (NRCFT)
  Nishida, DTS arXiv:0706.3746
- Rates of processes involving unnuclei
- Few-neutron systems as unnuclei and consequences of nuclear reactions

Georgi's unparticle

H. Georgi, 2007

- Unparticle = field in a CFT
- In CFT: $\langle \mathscr{U}(x)\mathscr{U}(0)\rangle = \dfrac{c}{|x|^{2\Delta_{\mathscr{U}}}}$
- In momentum space $G_{\mathscr{U}}(p) \sim p^{2\Delta_{\mathscr{U}}-4}$
- Particle corresponds to free field: $\Delta_\phi = 1,\ G_\phi(p) \sim p^{-2}$
- otherwise the propagator has cuts, not poles

Example of unparticles

- Simplest example: $\mathcal{U} = \phi^n, \ \Delta = n$
- More sophisticated: Bank-Zaks fixed point in gauge theory at sufficiently large N_f
  - $N_c = 3$: $? \leq N_f \leq 16$

Processes involving unparticles

- Imagine the SM particles are coupled to a unparticle sector
- $A_1 + A_2 \to B + \mathcal{U}$
- Energy spectrum of B is continuous
- If $\mathcal{U} = \phi^n$: $A_1 + A_2 \to B + n\phi$
- Near end point: differential cross section depends on the n-particle phase space
- Unparticle of dimension Δ is equivalent to Δ massless particles

Nonrelativistic QFT

- Second-quantized formulation of QM
  - $S = \int dt\, d\mathbf{x}\, \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi - \frac{1}{2} \int dt\, d\mathbf{x}\, d\mathbf{y}\, V(\mathbf{x} - \mathbf{y}) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x)$
- Galilean symmetry, including
  - space and time translation
  - Galilean boosts

Schrödinger symmetry

- The free NR field theory $V(\mathbf{x}-\mathbf{y})=0$ has extra symmetries
- Scale invariance: $\mathbf{x} \to \lambda \mathbf{x},\ t \to \lambda^2 t,\ \psi \to \lambda^{-\Delta}\psi$
  - $\Delta = d/2$ is the dimension of ψ
- "Proper conformal transformation"
  - $t \to \dfrac{t}{1+ct},\ \mathbf{x} \to \dfrac{\mathbf{x}}{1+ct},\ \psi \to \psi' = \cdots$

Schrödinger algebra

- Spatial translation P_i , time translation H
- Galilean boost $K^i = \int d\mathbf{x}\, m x^i \psi^\dagger \psi$
- Dilatation $D = \int d\mathbf{x}\, \mathbf{x} \cdot \left(-\frac{i}{2}\psi^\dagger \overleftrightarrow{\nabla} \psi\right)$
- "Proper conformal transformation" $C = \int d\mathbf{x}\, m x^2 \psi^\dagger \psi$
- Angular momentum $M_{ij} = -\frac{i}{2} \int d\mathbf{x}\, \psi^\dagger (x_i \overleftrightarrow{\nabla}_j - x_j \overleftrightarrow{\nabla}_i)\, \psi$
- Mass $M = \int d\mathbf{x}\, m \psi^\dagger \psi$

Schrödinger algebra

$X \setminus Y$	P_j	K_j	D	C	H
P_i	0	$-i\delta_{ij}M$	$-iP_i$	$-iK_i$	0
K_i	$i\delta_{ij}M$	0	iK_i	0	iP_i
D	iP_j	$-iK_j$	0	$-2iC$	$2iH$
C	iK_j	0	$2iC$	0	iD
H	0	$-iP_j$	$-2iH$	$-iD$	0

$$[J_{ij}, N] = [J_{ij}, D] = [J_{ij}, C] = [J_{ij}, H] = 0,$$
$$[J_{ij}, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i), \qquad [J_{ij}, K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i),$$
$$[J_{ij}, J_{kl}] = i(\delta_{ik}J_{jl} + \delta_{jl}J_{ik} - \delta_{il}J_{jk} - \delta_{jk}J_{il}).$$

Schrödinger algebra

- Commutator of D with an operator is determined by the operator's dimension:
  - $[D, O] = i\Delta_O O$

O	H	P	M	K	C
Δ_O	2	1	0	-1	-2

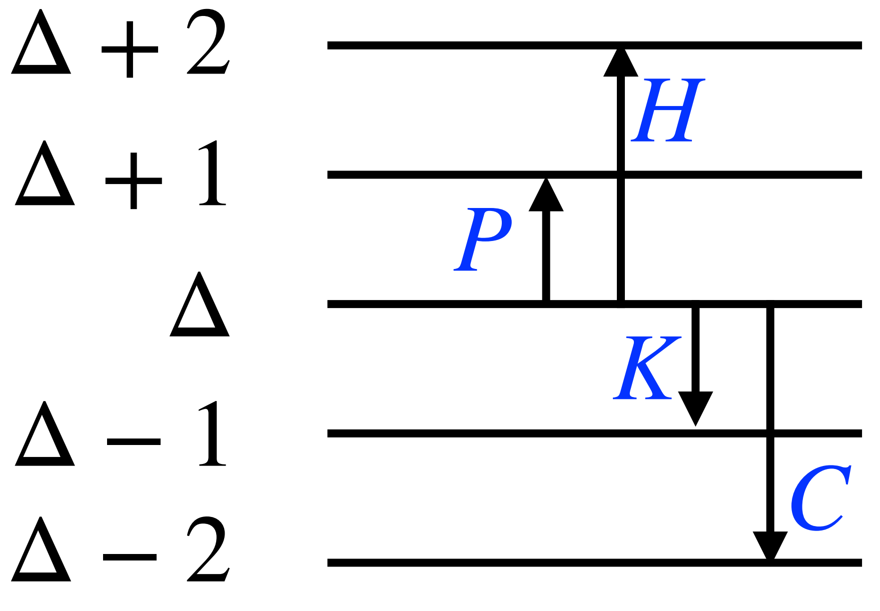
- $H, C,$ and D form a SO(2,1) subalgebra

Local operators

- Local operators are classified by mass and dimension
  - $[M, O(x)] = -M_O O(x)$
  - $[D, O(0)] = i\Delta_O O(0)$
- Commuting with P and H increases the dimension by 1 and 2, commuting with K and C by -1 and -2
- Representation theory for operators with $M \neq 0$ is simple

Raising and lowering dimensions

- Operators with $M \neq 0$ are organized in towers
- Dimension raised by P and H, lowered by K and C
- Primary operators: $[K, O(0)] = [C, O(0)] = 0$



Operator-state correspondence

- Dimension of a primary operator = energy of a state in a harmonic potential

- Example: in free theory $[\psi] = \frac{d}{2}$, ground state of 1 particle in harmonic potential: $E = \frac{d}{2}\hbar\omega$

Two-point function

- Let $\mathcal{U}$ be a primary operator in a NRCFT
- Characterized by mass M and dimension Δ
- Propagator
  - $G_{\mathcal{U}}(t, \mathbf{x}) = -i\langle T\mathcal{U}(t, \mathbf{x})\mathcal{U}^\dagger(0,\mathbf{0})\rangle = C\frac{\theta(t)}{(it)^\Delta}\exp\left(\frac{iMx^2}{2t}\right)$
  - $G_{\mathcal{U}}(\omega, \mathbf{p}) \sim \left(\frac{p^2}{2M} - \omega\right)^{\Delta - \frac{5}{2}}$

$\omega - \frac{p^2}{2M}$ is the energy of the in the CM frame

OPE in NRCFT

S. Golkar and DTS, 2014

- In contrast to relativistic CFT, OPE coefficients in NRCFT are functions

$$O_1(x)O_2(0) = \sum \frac{1}{|\mathbf{x}|^{\Delta_1+\Delta_2-\Delta_n}} c_n\left(\frac{x^2}{t}\right) O_n(0)$$

- The function c_n can be determined if one of the operator is a free field operator with $\Delta = d/2$

Fermions at unitarity

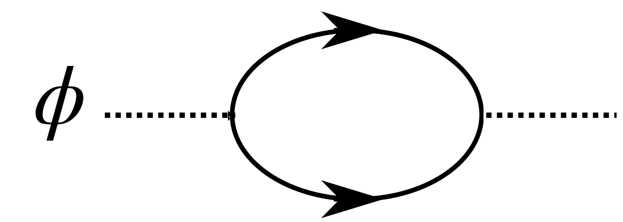
- $\psi = \psi(\mathbf{x}_i, \mathbf{y}_j)$ $\mathbf{x}_i$: spin-up, $\mathbf{y}_j$: spin-down
- When $\mathbf{x}_i \to \mathbf{y}_j : \psi(\mathbf{x}, \mathbf{y}) = \frac{1}{|\mathbf{x} - \mathbf{y}|} f\left(\frac{\mathbf{x} + \mathbf{y}}{2}\right) + O(\mathbf{x} - \mathbf{y})$
- For example, ground state of 2 particles in harmonic potential:
  - $\psi(\mathbf{x}, \mathbf{y}) \sim \frac{e^{-(x^2+y^2)/2}}{|\mathbf{x} - \mathbf{y}|}$ $E = 2$
  - An operator with $M = 2m$ and $\Delta = 2$

Fermions at unitarity as a NRCFT

- $L = i\psi^\dagger(\partial_t + \frac{\nabla^2}{2m})\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$
- Introducing auxiliary field ϕ
- $L = i\psi^\dagger(\partial_t + \frac{\nabla^2}{2m})\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger\phi - \phi^\dagger\psi_\downarrow\psi_\uparrow + \frac{\phi^\dagger\phi}{c_0}$
    - Propagator of ϕ

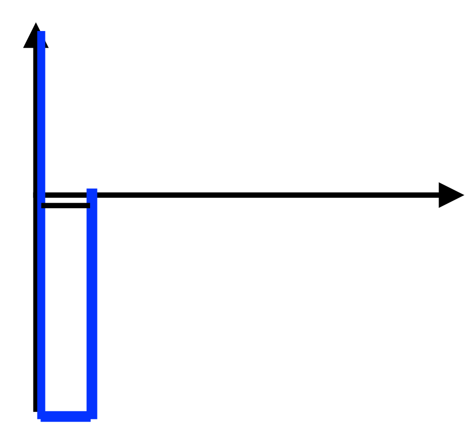
Renormalization

- $G_\phi^{-1}(\omega,\mathbf{p}) = c_0^{-1}$ + one-loop integral



- $= c_0^{-1} + \Lambda + \left(\dfrac{p^2}{4m} - \omega\right)^{1/2}$
- Unitarity: fine-tuning so that $c_0 + \Lambda = 0$
- (scattering length: $c_0 + \Lambda = \dfrac{1}{a}$)
- Physically: fine-tune the attractive short-range potential to have a bound state at threshold

$$G_\phi(\omega,\mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$



Operator dimensions for fermions at unitarity

- Dimensions of operators can be obtained either by field theory or quantum mechanical calculation in a harmonic trap
- Lowest dimension operators

N	S	L	O	Δ
2	0	0	$\psi_\uparrow\psi_\downarrow$	2
3	1/2	1	$\psi_\downarrow\psi_\uparrow\boldsymbol{\nabla}\psi_\uparrow$	4.273
3	1/2	0	$\psi_\downarrow\boldsymbol{\nabla}\psi_\uparrow\cdot\boldsymbol{\nabla}\psi_\uparrow$	4.666
4	0	0	$\psi_\downarrow\psi_\uparrow\boldsymbol{\nabla}\psi_\downarrow\cdot\boldsymbol{\nabla}\psi_\uparrow$	5.0–5.1

N (l)	$\mathcal{O}_N^{(l)}$	$\Delta_{\mathcal{O}}$	$E/\hbar\omega$ in $d=3$
2 $(l=0)$	$\psi_\uparrow\psi_\downarrow$	2	2 [30]
3 $(l=0)$	$\psi_\uparrow\psi_\downarrow(\partial_t\psi_\uparrow)$	$5+O(\bar{\epsilon}^2)$	4.66622 [26]
3 $(l=1)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow)$	$4+O(\bar{\epsilon}^2)$	4.27272 [26]
4 $(l=0)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)$	$6-\bar{\epsilon}+(\bar{\epsilon}^2)$	≈ 5.028 [33]
5 $(l=0)$	$(*)$	$9-\frac{11\pm\sqrt{105}}{16}\bar{\epsilon}+O(\bar{\epsilon}^2)$	≈ 8.03 [33]
5 $(l=1)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\boldsymbol{\partial}\psi_\uparrow$	$8-\bar{\epsilon}+O(\bar{\epsilon}^2)$	≈ 7.53 [33]
6 $(l=0)$	$\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)^2$	$10-2\bar{\epsilon}+(\bar{\epsilon}^2)$	≈ 8.48 [33]

$(*) = a\,\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\partial^2\psi_\uparrow + b\,\psi_\uparrow\partial_i\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)\partial_i\psi_\uparrow + c\,\psi_\uparrow\psi_\downarrow((\partial_i\boldsymbol{\partial}\psi_\uparrow)\cdot\boldsymbol{\partial}\psi_\downarrow)\partial_i\psi_\uparrow - d\,\psi_\uparrow\psi_\downarrow(\boldsymbol{\partial}\psi_\uparrow\cdot\boldsymbol{\partial}\psi_\downarrow)i\partial_t\psi_\uparrow$ with $(a,b,c,d)=(\pm 19\sqrt{3}-5\sqrt{35}, \mp 16\sqrt{3}, -6\sqrt{35}\mp 6\sqrt{3}, 16\sqrt{35})$.

Y. Nishida, DTS, arXiv:1004.3597

Systems with large scattering length

- α-particles
  - Coulomb interaction complicates the picture
- D^0-$\bar{D}^{*0}$ [X(3872)]
- He-4 *atoms*, $a \sim 100$ Å
- Trapped ultra-cold atoms (Feshbach resonance)
- Neutrons

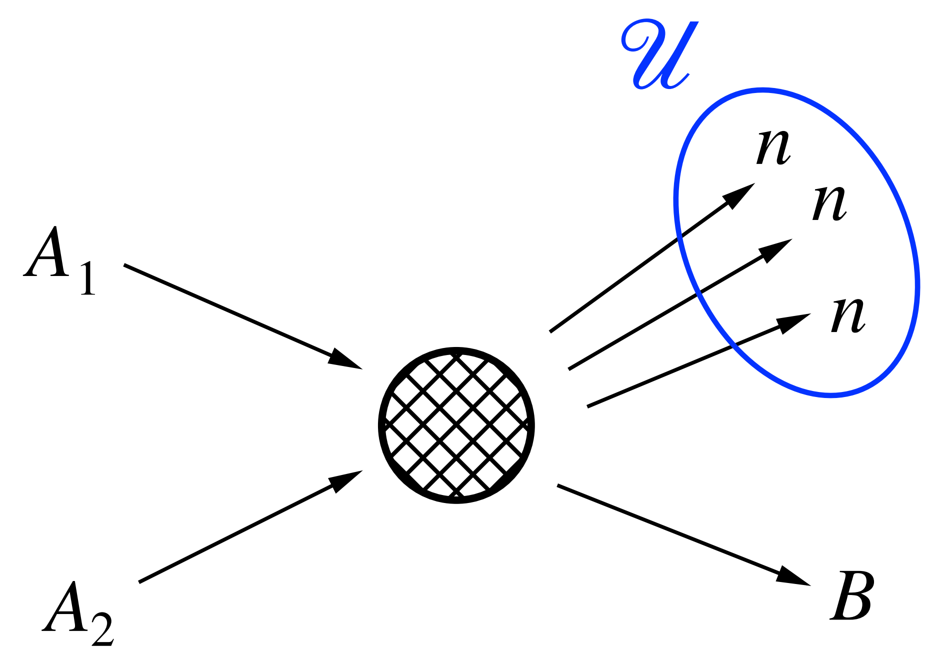
Few-neutron systems as unnuclei

- Neutrons have large scattering length: $a_{nn} \approx -19$ fm
- vs effective range $r_0 \approx 2.8$ fm
- Idealized regime: $a = \infty, r_0 = 0$: "unitarity fermion"
- Physics of fermions at unitarity is described by a NRCFT

Nuclear reactions

- Many nuclear reactions with emissions of neutrons:
  - $^3H + {}^3H \to {}^4He + 2n$
  - $^7Li + {}^7Li \to {}^{11}C + 3n$
  - $^4He + {}^8He \to {}^8Be + 4n$
- Final-state neutrons can be considered as forming an "unnucleus" - a field in NRCFT
  - Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV

Few-neutron systems as unnuclei



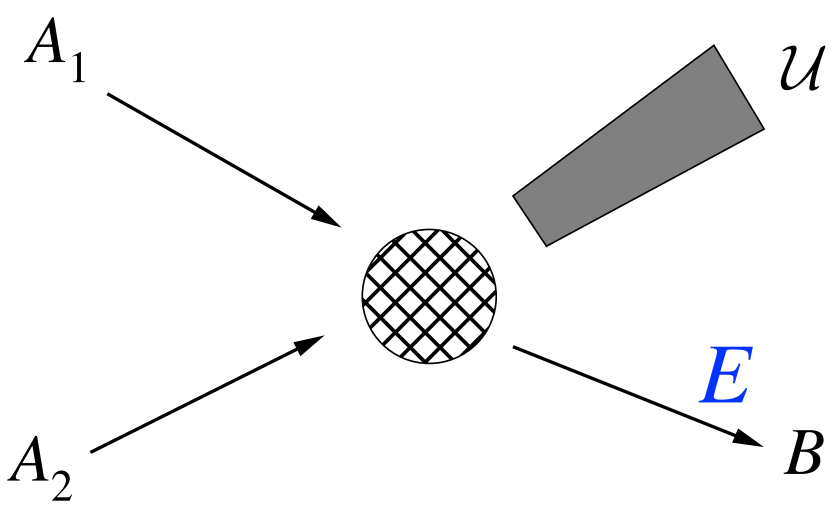
Factorization:

$$\frac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E_B} \quad \text{Im}\, G_{\mathscr{U}}(E_{\mathscr{U}}, \mathbf{p})$$

primary reaction has larger energy than final-state interaction

For 2 neutrons: Watson and Migdal ~ 1950s

Rates of processes involving an unnucleus



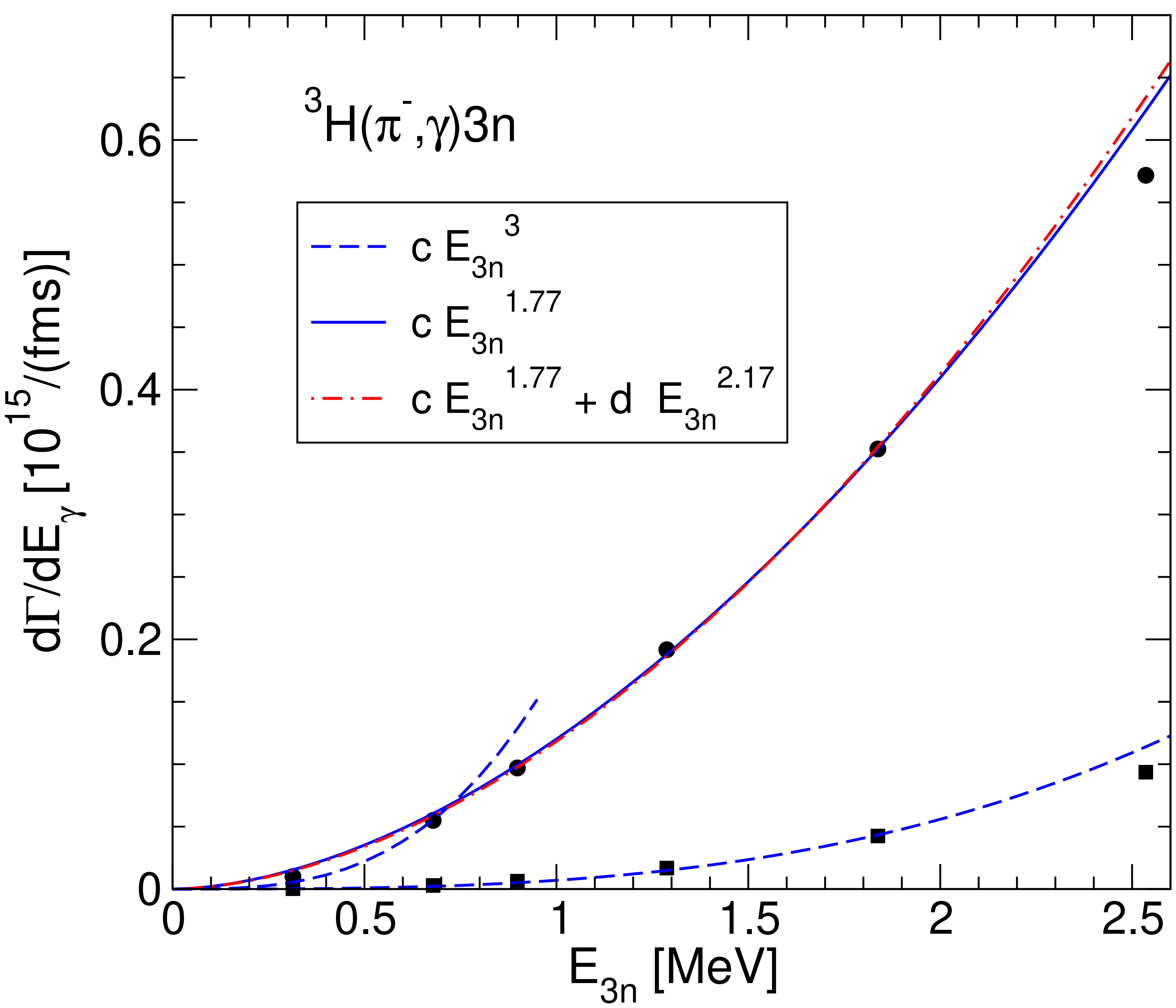
$$E_{\rm kin} = E + E_{\mathscr{U}}$$

- $\dfrac{d\sigma}{dE} \sim |\mathscr{M}|^2 \sqrt{E}\,\mathrm{Im}\,G_{\mathscr{U}}(E_{\rm kin} - E, \mathbf{p})$
- $\mathrm{Im}\,G_{\mathscr{U}}(E_{\rm kin} - E, \mathbf{p}) \sim \left(E_{\rm kin} - E - \dfrac{p^2}{2M_{\mathscr{U}}}\right)^{\Delta - \frac{5}{2}}$
- Near end point: $\dfrac{d\sigma}{dE} \sim (E_0 - E)^{\Delta - \frac{5}{2}}$

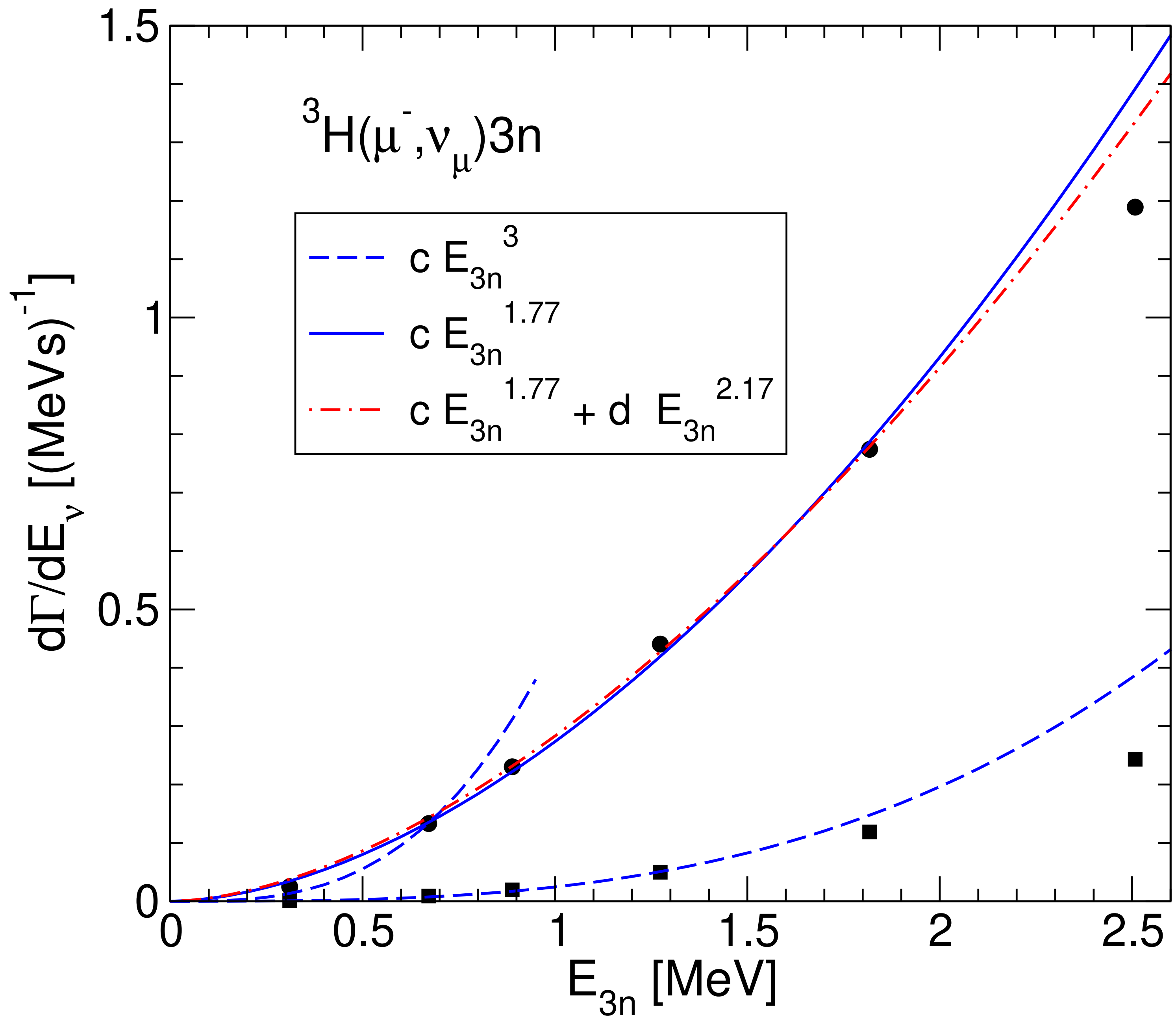
Nuclear reactions

- $^3H + ^3H \to ^4He + 2n$ $\alpha = -0.5$
- $^7Li + ^7Li \to ^{11}C + 3n$ $\alpha = 1.77$
- $^4He + ^8He \to ^8Be + 4n$ $\alpha = 2.5 - 2.6$

- Prediction:
  - $\frac{d\sigma}{dE} \sim (E_0 - E)^\alpha$
  - Regime of validity: kinetic energy of neutrons in their c.o.m. frame between $\hbar^2/ma^2 \sim 0.1$ MeV $\hbar^2/mr_0^2 \sim 5$ MeV



$^3H(\pi^-,\gamma)3n$
$c\,E_{3n}^{3}$
$c\,E_{3n}^{1.77}$
$c\,E_{3n}^{1.77} + d\ E_{3n}^{2.17}$
$d\Gamma/dE_\gamma$ [10^{15}/(fms)]
E_{3n} [MeV]
0
0.2
0.4
0.6
0
0.5
1
1.5
2
2.5



$^3H(\mu^-,\nu_\mu)3n$
$c\,E_{3n}^{\ 3}$
$c\,E_{3n}^{\ 1.77}$
$c\,E_{3n}^{\ 1.77} + d\ E_{3n}^{\ 2.17}$
$d\Gamma/dE_\nu$ [(MeVs)$^{-1}$]
E_{3n} [MeV]
0
0.5
1
1.5
2
2.5

- In both reactions the unnuclear scaling regime can be seen from “data”
- Deviation from power-law scaling starts at around 2.5 MeV
- somewhat smaller than the naive estimate of 5 MeV because ^{3}He nucleus is extended?

Conclusion

- Viewing final-state neutrons as forming an unparticle allow one to predict power-law behavior of the diff cross section near end point
- More details calculation: cross-over to free particle behavior below 0.1 MeV
  - RG flow from unitary fermions to free fermions
- Unparticle behavior in other systems?

Thank you