ALL *THINGS* EFT...

Non-local actions in EFTs
Adventures in non-locality and non-linearity

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Most work with Basem El-Menoufi (Manchester)
1402.3252, 1503.06099, 1507.06321
and ongoing research with Basem and solo

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Comments:

1) Tool is perturbative calculations at weak field and low energy
- this is surprisingly useful

2) Work is open-ended
– hopefully there is still something interesting here for you

3) Please email me with comments, paper suggestions etc.
- donoghue @umass.edu

4) I am also working on "ineffective field theory" aka Quadratic Gravity (with Gabriel Menezes)
There is a continuum – see recent conference **Quantum gravity, higher derivatives and non-locality** . Talks by Woodard, Tomboulis, Holdom, Shapiro, de Rham (+'t Hooft and Penrose)….. (videos and slide available)

Outline

1) **Motivations**

2) **Test cases and analysis**

   QED, non-local and anomalies
   Adding gravity
   QCD – what provides gauge invariance?
   Wilson lines
   General covariance

3) **Simple applications**

   - hints of a bounce
   - non-local partner of cosmological constant

Teaser:

$$\mathcal{L} = -\frac{m^4}{120\pi^2}\left[R_{\lambda\sigma}\frac{\log((\Box + m^2)/m^2)}{\Box^2}R^{\lambda\sigma} - \frac{1}{8}R\frac{\log((\Box + m^2)/m^2)}{\Box^2}R\right]$$

- with explanations to come by the end of the talk

Motivations

1) **Gravitational EFT beyond scattering amplitude**
   - Barvinsky – Vilkovisky
   - also Gasser and Leutwyler ChPTh

2) **Anomalies from an EFT perspective**
   -Deser, Duff, Isham vs Riegert

3) **Applications with gravity**
   - Mottola and anomaly driven cosmology
   - Deser Woodard – nonlocal

4) **Non-local terms in inflation**
   - Miao and Woodard

Quantum GR as an Effective Field Theory

- This is ideal application for EFT
- Unknown high energy completion yields **local** operators
    - uncertainty principle

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

- Low energy propagation known from GR

- Low energy – long distance propagation in position space
    - non-analytic in momentum space

Sample Predictions:

Potential:

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

JFD, Bjerrum Bohr, Holstein
Kriplovich, Kirilin

Universal
Holstein, Ross
JFD, Bjerrum Bohr, Vanhove

Light bending

$$\theta \simeq \frac{4G_NM}{b} + \frac{15}{4}\frac{G_N^2M^2\pi}{b^2} + \left(8c^S + 9 - 48\log\frac{b}{2b_0}\right)\frac{\hbar G_N^2 M}{\pi b^3} + \dots$$

with $c^s = \frac{371}{120}, \frac{113}{120}, -\frac{29}{8}$ for scalar, photon, graviton

Not universal – non-geodesic
JFD, Bjerrum Bohr, Holstein,
Plante, Vanhove
Bai, Huang
Chi
Light cones ill-defined in QG

What are the quantum predictions?

Not the divergences

- they come from the Planck scale
- unreliable part of theory

Not the parameters

- local terms in L
- we would have to measure them

$$Amp \sim q^2 \ln(-q^2) \quad , \quad \sqrt{-q^2}$$

Low energy propagation

- not the same as terms in the Lagrangian
- most always non-analytic dependence in momentum space
- can't be Taylor expanded – can't be part of a local Lagrangian
- long distance in coordinate space

Non-local and non-analytic:

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

Classical expansion parameter

Quantum expansion parameter

Short range

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2\ln(-q^2) + c'Gq^2\right]$$

Classical

quantum

short range

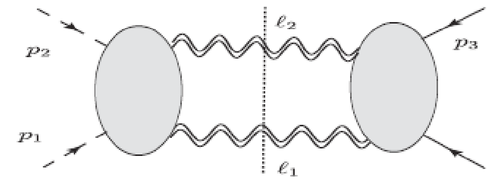
Non-analytic

analytic

Also visible in light bending calculation

Gravitational cut:

$$iM^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2\left[\frac{\kappa^2}{t} + \kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}} + \hbar\kappa^4\frac{15}{512\pi^2} \times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right) + \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right) + \kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\right], \quad (11)$$



$bu^\eta \to c^S$ in previous slide

Again, square roots reproduce classical behavior, and **logs give quantum effects**

Amplitude turned into bending angle via eikonal approximation

$$\mathcal{M}^{0+1}(\boldsymbol{\Delta}^\perp) = 2(s - M_\sigma^2)\int d^2\mathbf{b}^\perp e^{-i\boldsymbol{\Delta}^\perp\cdot\mathbf{b}^\perp}\left[e^{i(\chi_0 - i\ln[1+i\chi_2])} - 1\right]$$

Beyond scattering amplitudes

GR more than scattering
- but QFT techniques less developed

Non-local effective actions:
- most work done by Barvinsky, Vilkovisky and collab.
- covariant
- "expansion in curvature"

Others:
Avramidi
Starobinsky

Note: This is a different expansion from EFT derivative expansion

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$ EFT

$$S_{curv} \sim \int d^4x \sqrt{-g} \; ... + c(\mu) R^2 + dR \log(\Box/\mu^2) R + R^2 \frac{1}{\Box} R + ... + R^{n+1} \frac{1}{\Box^n} R + ...$$ BV

What is this expansion? First term:

We are used to the local derivative/energy expansion in GR

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots \right\}$$

and we know that quantum corrections generate R^2 terms

$$\Delta \mathcal{L}_0^{(1)} = \frac{1}{8\pi^2} \frac{1}{\epsilon} \left\{ \frac{1}{120} R^2 + \frac{7}{20} R_{\mu\nu} R^{\mu\nu} \right\}$$

but we can include quantum content in a **non-local** action:

$$\begin{aligned} S_{tot} = \int d^4x \sqrt{g} \, \frac{2}{\kappa^2} R & \\ & + \left[\bar{\alpha} R \log\left(\nabla^2/\Lambda_1^2\right) R + \bar{\beta} C_{\mu\nu\alpha\beta} \log(\nabla^2/\Lambda_2^2) C^{\mu\nu\alpha\beta} \right. \\ & \left. + \bar{\gamma} \left(R_{\mu\nu\alpha\beta} \log\left(\nabla^2\right) R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} \log\left(\nabla^2\right) R^{\mu\nu} + R \log\left(\nabla^2\right) R \right) \right] + \ldots. \end{aligned}$$

Logs are tied to divergences

	α	β	γ	$\bar{\alpha}$	β	$\bar{\gamma}$
Scalar	$5(6\xi-1)^2$	-2	2	$5(6\xi-1)^2$	3	-1
Fermion	-5	8	7	0	18	-11
Vector	-50	176	-26	0	36	-62
Graviton	430	-1444	424	90	126	298

Coefficients of different fields. All numbers should be divided by $11520\pi^2$

Starting to decode the action: Look at $log\,\nabla^2$

Everyone agrees on the flat space limit:

$$\langle x|\ln\left(\frac{\Box}{\mu^2}\right)|y\rangle \equiv L(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \ln\left(\frac{-q^2}{\mu^2}\right) .$$

Although written in quasi-local form, this is non-local

$$\int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}\; R_{\mu\nu}(x)\langle x|\log\Box|y\rangle^{\mu\nu,\alpha\beta}\; R_{\alpha\beta}(y)$$

One of our themes here: What is "Log Box"?

- i.e. beyond flat space

Proper time representation:

$$
\begin{aligned}
L(x-y) &= \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)} \log -(k^2+i\epsilon)/\mu^2 \\
&= \int \frac{d^4k}{(2\pi)^4} \int_0^\infty \frac{ds}{s} \left[e^{i(k^2+i\epsilon)s} - e^{i\mu^2 s} \right] e^{-ik\cdot(x-y)} \\
&= \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-i\frac{(x-y)^2-i\epsilon}{4s}} + \text{ local}
\end{aligned}
$$

- calculated by completing the square in exponent

- return to this later

Lets see the same expansion in ChPTh

Review: **Gasser and Leutwyler**

- enhance QCD to **local** chiral symmetry with external sources

$$\mathcal{L} = \mathcal{L}^{0}_{\rm QCD} + \bar{q}\gamma^{\mu}\{v_{\mu}(x) + \gamma_5 a_{\mu}(x)\}q - \bar{q}\{s(x) - i\gamma_5 p(x)\}q$$

With

$$v'_{\mu} + a'_{\mu} = V_{\rm R}(v_{\mu} + a_{\mu})V^{+}_{\rm R} + iV_{\rm R}\partial_{\mu}V^{+}_{\rm R},$$

$$v'_{\mu} - a'_{\mu} = V_{\rm L}(v_{\mu} - a_{\mu})V^{+}_{\rm L} + iV_{\rm L}\partial_{\mu}V^{+}_{\rm L},$$

$$s' + ip' = V_{\rm R}(s + ip)V^{+}_{\rm L}.$$

Form effective Lagrangian in energy expansion
After renormalization consider finite effects (“unitarity effects”

$$U = u(1 + i\xi - \tfrac{1}{2}\xi^2 + \cdots)u,$$

$$\Gamma_{\mu} = \tfrac{1}{2}[u^{+}, \partial_{\mu}u] - \tfrac{1}{2}iu^{+}F^{\rm R}_{\mu}u - \tfrac{1}{2}iuF^{\rm L}_{\mu}u^{+}$$

$$\hat{\Gamma}_{\mu\nu} = \partial_{\mu}\hat{\Gamma}_{\nu} - \partial_{\nu}\hat{\Gamma}_{\mu} + [\hat{\Gamma}_{\mu}, \hat{\Gamma}_{\nu}].$$

$$\hat{\Gamma}^{ab}_{\mu\nu} = -\tfrac{1}{2}\,{\rm tr}\,([\lambda^{a}, \lambda^{b}]\Gamma_{\mu\nu})$$

Then integrate out the quantum fluctuation ξ

$$D = D_0 + \delta\,,$$

$$\delta = \{\hat{\Gamma}^\mu, \partial_\mu\} + \hat{\Gamma}^\mu \hat{\Gamma}_\mu + \bar{\sigma}\,,$$

Expand using tadpoles, bubbles, etc

$$Z_{\text{one loop}} = \tfrac{1}{2} i \ln \det D_0 + \tfrac{1}{4} i \operatorname{Tr}(D_0^{-1}\delta) - \tfrac{1}{4} i \operatorname{Tr}(D_0^{-1}\delta D_0^{-1}\delta) + \cdots$$

Perform renormalization

$$J(q^2) = \frac{1}{i}\int d^d z\, e^{iqz}\, \Delta^2(z)$$

$$= \frac{1}{i}(2\pi)^{-d}\int d^d k (M^2 - k^2)^{-1}(M^2 - (q-k)^2)^{-1}$$

$$J(0) = -2\lambda - \frac{1}{16\pi^2}\left(\ln\frac{M^2}{\mu^2} + 1\right)$$

$$\lambda = \frac{1}{16\pi^2}\mu^{d-4}\left\{\frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1)\right\}.$$

Residual is “unitarity effect”

$$\bar{J}(q^2) = J(q^2) - J(0)$$

Collect terms:

$$\bar{J}(q^2) = \int_0^1 dx \log\left[\frac{m^2 - x(1-x)q^2}{\mu^2}\right]$$

$$\bar{M}(q^2) = \frac{1}{12}(1 - \frac{4m^2}{q^2})\bar{J}(q^2)$$

$$\bar{J}(x-y) = F.T.\ \bar{J}(q^2) = < x| \text{``}\log\frac{(\Box + m^2)}{\mu^2}\text{''} |y > \rightarrow_{m\to 0} < x| \log\frac{(\Box)}{\mu^2}|y >$$

Get non-local effective Lagrangian

$$Z_u = \int dx\, dy \{\tfrac{1}{2}\bar{M}(x-y)\, \mathrm{tr}\, \Gamma_{\mu\nu}(x)\, \Gamma^{\mu\nu}(y) + \tfrac{1}{4}\bar{J}(x-y)\, \mathrm{tr}\, \hat{\sigma}(x)\, \hat{\sigma}(y)\}$$

Beautiful result:

“ALL” amplitudes contained here

Just take trace and read of amplitudes

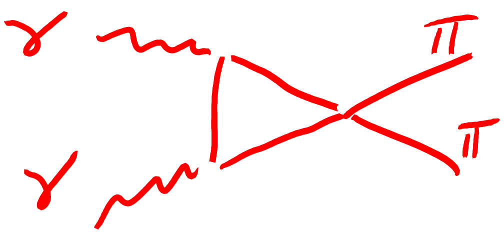
This is the equivalent of the BV action

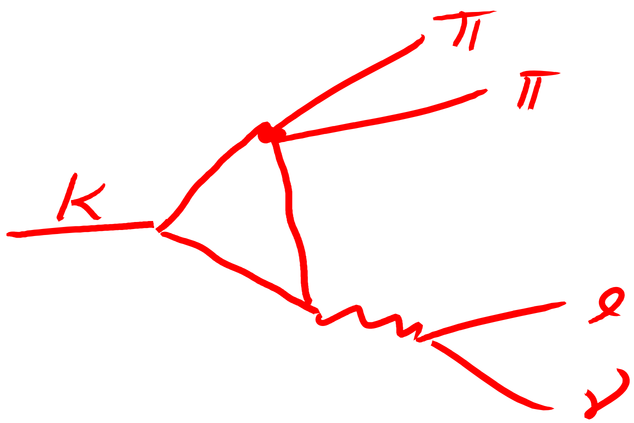
What is missing?

Triangle diagrams

$$Z_{\text{one loop}} = \tfrac{1}{2} i \ln \det D_0 + \tfrac{1}{4} i \,\mathrm{Tr}\,(D_0^{-1}\delta) - \tfrac{1}{4} i \,\mathrm{Tr}\,(D_0^{-1}\delta D_0^{-1}\delta) + \cdots$$

Occurs first for $\gamma\gamma \rightarrow \pi\pi$ and $K_{\ell 4}$ decay





Back to gravity and Barvinsky Vilkovisky

What are the higher order terms in this curvature expansion?

Again it is triangle diagrams

-three vertices for $RR\ (1/\nabla^{2)}R$

"Third order in the curvature"

- too complicated to be practical in general

194 pages of dense results, such as these:

$$
\begin{aligned}
&\int dx\, g^{1/2}\,\mathrm{tr}\,\hat{a}_4(x,x) = \int dx\, g^{1/2}\,\mathrm{tr}\,\Big\{\frac{\Box_2{}^2}{120}\hat{P}_1\hat{P}_2 + \frac{\Box_2{}^2}{1260}\hat{P}_1 R_2 + \frac{\Box_2{}^2}{1680}\hat{\mathcal{R}}_{1\mu\nu}\hat{\mathcal{R}}_2^{\mu\nu} \\
&+ \frac{\Box_2{}^2}{15120}R_{1\,\mu\nu}R_2^{\mu\nu}\hat{1} + \frac{\Box_3}{24}\hat{P}_1\hat{P}_2\hat{P}_3 - \frac{\Box_3}{630}\hat{\mathcal{R}}_1{}^{\mu}{}_{\alpha}\hat{\mathcal{R}}_2{}^{\alpha}{}_{\beta}\hat{\mathcal{R}}_3{}^{\beta}{}_{\mu} \\
&+ \Big(\frac{\Box_1}{180} + \frac{\Box_2}{180} + \frac{\Box_3}{90}\Big)\hat{\mathcal{R}}_1^{\mu\nu}\hat{\mathcal{R}}_{2\,\mu\nu}\hat{P}_3 + \Big(\frac{\Box_1}{7560} - \frac{\Box_3}{15120}\Big)R_1R_2\hat{P}_3 \\
&+ \Big(\frac{\Box_1}{1680} + \frac{\Box_1{}^2}{1680\Box_2} + \frac{\Box_3}{2520} + \frac{\Box_1\Box_3}{1680\Box_2} - \frac{\Box_3{}^2}{336\Box_2} + \frac{\Box_3{}^3}{1120\Box_1\Box_2}\Big)R_1^{\mu\nu}R_{2\,\mu\nu}\hat{P}_3 \\
&+ \frac{\Box_3}{720}\hat{P}_1\hat{P}_2R_3 + \Big(\frac{13\Box_1}{30240} - \frac{\Box_3}{15120}\Big)R_1\hat{\mathcal{R}}_2^{\mu\nu}\hat{\mathcal{R}}_{3\,\mu\nu} \\
&+ \Big(\frac{\Box_1}{840} + \frac{\Box_3}{210} + \frac{\Box_2\Box_3}{210\Box_1} + \frac{\Box_3{}^2}{210\Box_1}\Big)R_1^{\alpha\beta}\hat{\mathcal{R}}_{2\,\alpha}{}^{\mu}\hat{\mathcal{R}}_{3\,\beta\mu} \\
&+ \Big(\frac{\Box_1{}^2}{25200\Box_3} + \frac{\Box_1\Box_2}{50400\Box_3} - \frac{\Box_3}{25200} + \frac{\Box_3{}^3}{50400\Box_1\Box_2}\Big)R_1R_2R_3\hat{1} \\
&+ \Big(-\frac{\Box_1{}^2}{9450\Box_3} - \frac{\Box_1\Box_2}{18900\Box_3} - \frac{\Box_3}{12600} + \frac{\Box_3{}^3}{12600\Box_1\Box_2}\Big)R^{\mu}_{1\,\alpha}R^{\alpha}_{2\,\beta}R^{\beta}_{3\,\mu}\hat{1} \\
&+ \Big(\frac{\Box_1}{151200} - \frac{\Box_1{}^2}{151200\Box_2} + \frac{\Box_3}{25200} + \frac{\Box_1\Box_3}{18900\Box_2} - \frac{13\Box_3{}^2}{151200\Box_2} \\
&+ \frac{\Box_3{}^3}{50400\Box_1\Box_2}\Big)R_1^{\mu\nu}R_{2\,\mu\nu}R_3\hat{1} + \frac{1}{252}\hat{\mathcal{R}}_1^{\alpha\beta}\nabla^{\mu}\hat{\mathcal{R}}_{2\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{3\nu\beta} \\
&+ \frac{1}{60}\hat{\mathcal{R}}_1^{\mu\nu}\nabla_{\mu}\hat{P}_2\nabla_{\nu}\hat{P}_3 + \frac{1}{180}\nabla_{\mu}\hat{\mathcal{R}}_1^{\mu\alpha}\nabla^{\nu}\hat{\mathcal{R}}_{2\,\nu\alpha}\hat{P}_3 - \frac{1}{1890}R_1^{\mu\nu}\nabla_{\mu}R_2\nabla_{\nu}\hat{P}_3 \\
&+ \Big(\frac{1}{630} + \frac{\Box_1}{420\Box_2} + \frac{\Box_3}{210\Box_2} - \frac{\Box_3{}^2}{280\Box_1\Box_2}\Big)\nabla^{\mu}R_1^{\nu\alpha}\nabla_{\nu}R_{2\,\mu\alpha}\hat{P}_3 \\
&+ \frac{1}{180}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\hat{P}_2\hat{P}_3 + \Big(\frac{1}{1260} + \frac{\Box_3}{105\Box_1}\Big)R_{1\,\alpha\beta}\nabla_{\mu}\hat{\mathcal{R}}_2^{\mu\alpha}\nabla_{\nu}\hat{\mathcal{R}}_3^{\nu\beta} \\
&+ \Big(-\frac{1}{1260} - \frac{\Box_3}{210\Box_1}\Big)R_1^{\alpha\beta}\nabla_{\alpha}\hat{\mathcal{R}}_2^{\mu\nu}\nabla_{\beta}\hat{\mathcal{R}}_{3\,\mu\nu} - \frac{1}{7560}R_1\nabla_{\alpha}\hat{\mathcal{R}}_2^{\alpha\mu}\nabla^{\beta}\hat{\mathcal{R}}_{3\,\beta\mu} \\
&+ \Big(\frac{1}{630} + \frac{\Box_2}{105\Box_1} + \frac{\Box_3}{105\Box_1}\Big)R_1^{\mu\nu}\nabla_{\mu}\nabla_{\lambda}\hat{\mathcal{R}}_2^{\lambda\alpha}\hat{\mathcal{R}}_{3\,\alpha\nu} + \Big(\frac{1}{226800} - \frac{\Box_1}{8400\Box_2} \\
&- \frac{\Box_1{}^2}{10080\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1} - \frac{\Box_3}{25200\Box_2} - \frac{\Box_3{}^2}{25200\Box_1\Box_2}\Big)R_1^{\alpha\beta}\nabla_{\alpha}R_2\nabla_{\beta}R_3\hat{1} \\
&+ \Big(-\frac{\Box_1}{37800\Box_2} + \frac{\Box_3}{5400\Box_2} - \frac{\Box_3{}^2}{12600\Box_1\Box_2}\Big)\nabla^{\mu}R_1^{\nu\alpha}\nabla_{\nu}R_{2\,\mu\alpha}R_3\hat{1} \\
&+ \Big(-\frac{1}{9450} - \frac{\Box_1}{12600\Box_2} + \frac{\Box_1{}^2}{8400\Box_2\Box_3} - \frac{\Box_3}{6300\Box_2}\Big)R_1^{\mu\nu}\nabla_{\mu}R_2^{\alpha\beta}\nabla_{\nu}R_{3\,\alpha\beta}\hat{1} \\
&+ \Big(-\frac{1}{3150} - \frac{\Box_1}{9450\Box_2} - \frac{\Box_1{}^2}{6300\Box_2\Box_3} - \frac{\Box_3}{3150\Box_1} + \frac{\Box_3}{9450\Box_2} + \frac{\Box_3{}^2}{3150\Box_1\Box_2}\Big) \\
&\times R_1^{\mu\nu}\nabla_{\alpha}R_{2\,\beta\mu}\nabla^{\beta}R^{\alpha}_{3\,\nu}\hat{1} + \Big(\frac{1}{420\Box_2} + \frac{\Box_3}{280\Box_1\Box_2}\Big)\nabla_{\alpha}\nabla_{\beta}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}R_2^{\alpha\beta}\hat{P}_3 \\
&+ \Big(-\frac{1}{3780\Box_2} - \frac{1}{6300\Box_3} - \frac{\Box_1}{4200\Box_2\Box_3} + \frac{\Box_3}{25200\Box_1\Box_2}\Big)\nabla_{\alpha}\nabla_{\beta}R_1^{\mu\nu}\nabla_{\mu}\nabla_{\nu}R_2^{\alpha\beta}R_3\hat{1}
\end{aligned}
$$

37

$$
\begin{aligned}
&\Gamma_{12}(-\Box_1,-\Box_2,-\Box_3) = \Gamma(-\Box_1,-\Box_2,-\Box_3)\frac{1}{D^3}\Big(-2\Box_1{}^5 \\
&+ 4\Box_1{}^4\Box_2 - 4\Box_1{}^2\Box_2{}^3 + 2\Box_1\Box_2{}^4 + 4\Box_1{}^4\Box_3 \\
&- 24\Box_1{}^3\Box_2\Box_3 + 12\Box_1{}^2\Box_2{}^2\Box_3 + 8\Box_1\Box_2{}^3\Box_3 + 12\Box_1{}^2\Box_2\Box_3{}^2 \\
&- 20\Box_1\Box_2{}^2\Box_3{}^2 - 4\Box_1{}^2\Box_3{}^3 + 8\Box_1\Box_2\Box_3{}^3 + 2\Box_1\Box_3{}^4\Big) \\
&+ \frac{\ln(\Box_1/\Box_2)}{9D^3\Box_2\Box_3}\Big(-2\Box_1{}^5\Box_2 + 10\Box_1{}^4\Box_2{}^2 - 20\Box_1{}^3\Box_2{}^3 \\
&+ 20\Box_1{}^2\Box_2{}^4 - 10\Box_1\Box_2{}^5 + 2\Box_2{}^6 - \Box_1{}^5\Box_3 - 21\Box_1{}^4\Box_2\Box_3 \\
&- 6\Box_1{}^3\Box_2{}^2\Box_3 + 66\Box_1{}^2\Box_2{}^3\Box_3 - 25\Box_1\Box_2{}^4\Box_3 - 13\Box_2{}^5\Box_3 \\
&+ 5\Box_1{}^4\Box_3{}^2 + 36\Box_1{}^3\Box_2\Box_3{}^2 - 162\Box_1{}^2\Box_2{}^2\Box_3{}^2 - 36\Box_1\Box_2{}^3\Box_3{}^2
\end{aligned}
$$

51

$$
\begin{aligned}
&+ 29\Box_2{}^4\Box_3{}^2 - 10\Box_1{}^3\Box_3{}^3 - 6\Box_1{}^2\Box_2\Box_3{}^3 + 78\Box_1\Box_2{}^2\Box_3{}^3 \\
&- 30\Box_2{}^3\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4 - 2\Box_1\Box_2\Box_3{}^4 + 16\Box_2{}^2\Box_3{}^4 \\
&- 5\Box_1\Box_3{}^5 - 5\Box_2\Box_3{}^5 + \Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_1/\Box_3)}{9D^3\Box_2\Box_3}\Big(-\Box_1{}^5\Box_2 + 5\Box_1{}^4\Box_2{}^2 - 10\Box_1{}^3\Box_2{}^3 \\
&+ 10\Box_1{}^2\Box_2{}^4 - 5\Box_1\Box_2{}^5 + \Box_2{}^6 - 2\Box_1{}^5\Box_3 \\
&- 21\Box_1{}^4\Box_2\Box_3 + 36\Box_1{}^3\Box_2{}^2\Box_3 - 6\Box_1{}^2\Box_2{}^3\Box_3 - 2\Box_1\Box_2{}^4\Box_3 \\
&- 5\Box_2{}^5\Box_3 + 10\Box_1{}^4\Box_3{}^2 - 6\Box_1{}^3\Box_2\Box_3{}^2 - 162\Box_1{}^2\Box_2{}^2\Box_3{}^2 \\
&+ 78\Box_1\Box_2{}^3\Box_3{}^2 + 16\Box_2{}^4\Box_3{}^2 - 20\Box_1{}^3\Box_3{}^3 + 66\Box_1{}^2\Box_2\Box_3{}^3 \\
&- 36\Box_1\Box_2{}^2\Box_3{}^3 - 30\Box_2{}^3\Box_3{}^3 + 20\Box_1{}^2\Box_3{}^4 - 25\Box_1\Box_2\Box_3{}^4 \\
&+ 29\Box_2{}^2\Box_3{}^4 - 10\Box_1\Box_3{}^5 - 13\Box_2\Box_3{}^5 + 2\Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_2/\Box_3)}{9D^3\Box_2\Box_3}\Big(\Box_1{}^5\Box_2 - 5\Box_1{}^4\Box_2{}^2 + 10\Box_1{}^3\Box_2{}^3 \\
&- 10\Box_1{}^2\Box_2{}^4 + 5\Box_1\Box_2{}^5 - \Box_2{}^6 - \Box_1{}^5\Box_3 \\
&+ 42\Box_1{}^3\Box_2{}^2\Box_3 - 72\Box_1{}^2\Box_2{}^3\Box_3 + 23\Box_1\Box_2{}^4\Box_3 + 8\Box_2{}^5\Box_3 \\
&+ 5\Box_1{}^4\Box_3{}^2 - 42\Box_1{}^3\Box_2\Box_3{}^2 + 114\Box_1\Box_2{}^3\Box_3{}^2 - 13\Box_2{}^4\Box_3{}^2 \\
&- 10\Box_1{}^3\Box_3{}^3 + 72\Box_1{}^2\Box_2\Box_3{}^3 - 114\Box_1\Box_2{}^2\Box_3{}^3 + 10\Box_1{}^2\Box_3{}^4 \\
&- 23\Box_1\Box_2\Box_3{}^4 + 13\Box_2{}^2\Box_3{}^4 - 5\Box_1\Box_3{}^5 - 8\Box_2\Box_3{}^5 + \Box_3{}^6\Big) \\
&+ \frac{\ln(\Box_1/\Box_2)}{(\Box_1-\Box_2)}\frac{1}{3\Box_3} \\
&+ \frac{\ln(\Box_1/\Box_3)}{(\Box_1-\Box_3)}\frac{1}{3\Box_2} \\
&+ \frac{1}{3D^2}\Big(16\Box_1{}^2 - 12\Box_1\Box_2 - 4\Box_2{}^2 - 12\Box_1\Box_3 + 8\Box_2\Box_3 - 4\Box_3{}^2\Big),
\end{aligned}
$$

<u>This is a weak field expansion</u>

Example: Schwarzchild

$$R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{48G^2M^2}{r^6}$$

Next order terms brings in extra factor of GM

$$\frac{1}{\Box}R^{\cdots\cdot} \sim \frac{GM}{r}$$

Expansion breaks down near horizon

Or, another relevant expansion:

$$\partial^2 h + h\partial^2 h + ... = R \qquad \to \partial^2 h = R - R\frac{1}{\partial^2}R \; + ...$$

Now return to the question of log Box

We wish to use some covariant definition:

But $\ln \nabla^2$ is not uniquely defined!

1) Single propagator version:

Barvinsky
Vilkovisky

$$\ln\left(\nabla^2/\mu^2\right) = -\int_0^\infty dm^2 \left[\frac{1}{\nabla^2+m^2} - \frac{1}{\mu^2+m^2}\right]$$

2) Double propagator version:

Osborn
Erdmenger

$$\frac{i}{16\pi^2} < x|\log \nabla^2|y >= \Delta_F^2(x-y) - \frac{i}{16\pi^2}\left[\frac{1}{\epsilon} - \gamma - \log 4\pi\right]\delta^4(x-y)$$

-in some settings this is clearly the right answer

- in background field various flat space relations are not valid

3) Wilson lines and geodetic distance

- introduced later

4) Also what to do with tensor indices $R_{\mu\nu} \log \nabla^2 R^{\mu\nu}$?

Does it matter?

A) No – can be corrected for at next order

- differences are next order in gravitational field
- can shift the difference to the next order in expansion
- BV do this

B) **Yes – some choices introduce spurious IR effects**

- when not including full third order terms this is problem
- want choice which matches real IR behavior

EFT prescription – don't want spurious IR behavior

"First, do no harm"

So one set of themes here:

Understanding the expansion in curvature

How to make non-local terms generally covariant (gauge invariant)

How to define Log Box covariantly in a useful way

Anomalies:

These are IR properties also

Argument – Deser Duff Isham vs Reigert

1) **Deser Duff Isham – anomalies are in logarithms**
   - accompanying renormalization come logs

 i.e. gauge fields

$$(\omega - 2)\int d^4x\,\sqrt{-g}\,(e^2F^2) + \int d^4x\,\sqrt{-g}\,(e^2F^2)\ln(\Box + R)$$

Or conformal anomaly

$$\int d^4x\,\sqrt{-g}\,C^2(2)\ln(\Box + \acute{R})\,.$$

Of course, hard to define "Log (Box +R)" in these cases

2) Riegert (1983)

- direct integration of conformal anomaly
- non-logarithmic but not local

$$\Gamma_A = \frac{1}{(4\pi)^2}\int d^4x\, d^4y\, \{\sqrt{-g}\,[aC^2 + eF^2 + \tfrac{1}{2}b(E + \tfrac{2}{3}\nabla^2 R)]\}_x G(x,y)\{\tfrac{1}{4}\sqrt{-g}(E + \tfrac{2}{3}\nabla^2 R)\}_y$$

Here $G(x, y)$is the inverse of fourth order Paneitz operator

$$\Delta_4 \equiv \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu$$

Non-local phenomenology:

1) **Mottola - anomaly driven dynamical dark energy ~(2010)**
   - uses auxiliary field to make the Riegert action local

$$S_{anom}[g;\varphi,\psi] = \frac{b'}{2}\int d^4x\,\sqrt{-g}\,\left\{-\varphi\Delta_4\varphi + \left(E - \frac{2}{3}\Box R\right)\varphi\right\}$$
$$+\frac{b}{2}\int d^4x\,\sqrt{-g}\,\left\{-2\varphi\Delta_4\psi + \left(F + \frac{c}{b}H\right)\varphi + \left(E - \frac{2}{3}\Box R\right)\psi\right\}$$

- this becomes a new dynamical gravitational d.o.f.
  - unusual kinetic operator

- produces effects of dynamical dark energy

- non-speculative – just uses Riegert action

2) **Deser and Woodard – non-local cosmology**

Motivated speculation:

$$\Delta\mathcal{L} \equiv \frac{1}{16\pi G} R\sqrt{-g} \times f\left(\frac{1}{\Box} R\right)$$

Can be used to drive present accelerated expansion

Non-local effects in inflation

Miao-Woodard "Fine-tuning may not be enough" (2015)

Inflaton potentials need to be pretty flat
Couplings to other fields needed for reheating
- these will produce (divergent) shifts in inflaton potential

Can be fine-tuned to produce flat potential

But, non-local terms come at same time – cannot be fine-tuned!

Example: $\mathcal{L} = -g\chi^2\phi^2$

$$\Delta\mathcal{L} = -\frac{g^2}{2\pi^2}\phi^4\left[\frac{1}{\epsilon} + ...\right] + \frac{g^2}{2\pi^2}\phi^2 \ \log\Box \ \phi^2$$

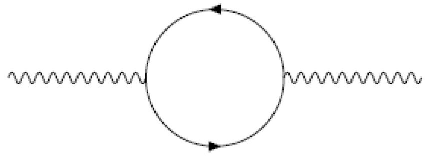
Non-local piece cannot be removed

Here, double propagator version is correct
- M&W do this in dS

End of motivations

Now to calculations

Example 1 : QED with massless fields



Obtain photon effective action by integrating out charged particles

Derivation is exactly the same as G&L derivation above

$$S = \int d^4x \; -\frac{1}{4} F_{\rho\sigma} \left[\frac{1}{e^2(\mu)} + b_i \ln\left(\Box/\mu^2\right) \right] F^{\rho\sigma} \qquad + \mathcal{O}(F^4)$$

This is the log q^2 from the vacuum polarization
- running coupling

$$b = \frac{1}{12\pi}$$

JFD and BEl-M 2015

QED trace anomaly for effective field theorists:

QED Lagrangian has no scale

$$A_\mu(x) \to \lambda A_\mu(\lambda x),\ \psi(x) \to \lambda^{3/2}\psi(\lambda x),\ \phi(x) \to \lambda\phi(\lambda x)$$

Such that $J^\mu_{\rm scale} = x_\nu \theta^{\mu\nu}$, $\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = 0$

But loops introduce scale dependence in the derivatives

$$S = \int d^4x \ -\frac{1}{4}F_{\rho\sigma}\left[\frac{1}{e^2(\mu)} - b\log\left(\nabla^2/\mu^2\right)\right]F^{\rho\sigma}$$

Now: $L(x-y) \to \lambda^{-4}\left(L(x-y) - \ln\lambda^2\delta^4(x-y)\right)$

$$\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = \frac{\partial\hat{\mathcal{L}}_\lambda}{\delta\lambda}|_{\lambda=1} = \frac{b}{4}F_{\rho\sigma}F^{\rho\sigma} \qquad \hat{\mathcal{L}}_\lambda = \lambda^{-4}\mathcal{L}[\lambda A(\lambda x)]$$

Anomaly not derivable from any local Lagrangian,

-but does come from a non-local action

- IR property, independent of any renormalization scheme

Now add gravity to QED:

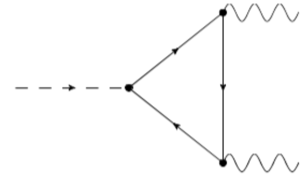


FIG. 1: Triangle diagram.

FIG. 2: Bubble diagrams.

Note: for concise results use **regions**

- right now I am using on-shell photons – off-shell graviton

Result for scalar:

$$\Gamma^{\rm ren}[g,A] = \frac{1}{4}\int_p\int_{p'} \tilde{h}^{\mu\nu}(-q)\,\tilde{A}^\alpha(p)\,\tilde{A}^\beta(-p')\left[\left(\frac{1}{e^2(\mu)} - \frac{1}{48\pi^2}\ln\left(\frac{-q^2}{\mu^2}\right)\right)\mathcal{M}^0_{\mu\nu,\alpha\beta} + \mathcal{M}^s_{\mu\nu,\alpha\beta}\right]$$

with

$$\mathcal{M}^s_{\mu\nu,\alpha\beta}(\xi) = \frac{1}{48\pi^2 q^2}\left(Q_\mu Q_\nu - (5-24\xi)(q_\mu q_\nu - q^2\eta_{\mu\nu})\right)\left(p'_\alpha p_\beta - p\cdot p'\eta_{\alpha\beta}\right)$$

↖ *

$$Q_\mu = (p+p')_\mu$$

i.e. logs and $1/\,q^2$ effects

Write a covariant effective action:

- matching
- return to log terms soon

The residual terms are

$$\Gamma_{anom.}[g,A] = \int d^4x \sqrt{g} \left[n_R F_{\rho\sigma} F^{\rho\sigma} \frac{1}{\Box} R + n_C F^{\rho\sigma} F^{\gamma}_{\lambda} \frac{1}{\Box} C_{\rho\sigma\gamma}{}^{\lambda} \right]$$

where for scalar $(\xi = \frac{1}{6})$ and fermions

$$n_R^{(s,f)} = -\frac{\beta^{(s,f)}}{12e}, \quad n_C^s = -\frac{e^2}{96\pi^2}, \quad n_C^f = \frac{e^2}{48\pi^2}$$

This is “third order in the curvature” in BV expansion

The first term **exactly matches** the equivalent Riegert action

$$\Gamma_{Riegert} = \frac{b}{4} \int d^4x \sqrt{g}\, F^2 \frac{1}{\Delta_4} \left(E - \frac{2}{3} \Box R \right)$$

Another point of reference:

Drummond – Hathrell integrating out a massive charged particle
- local effective Lagrangian

$$\Gamma_{local}[g,A] = \frac{e^2}{m^2}\int d^4x\,\sqrt{g}\left[l_1\,F_{\mu\nu}F^{\mu\nu}R + l_2\,F_{\mu\sigma}F_\nu{}^\sigma R^{\mu\nu} + l_3\,F^{\mu\nu}F^\alpha_\beta R_{\mu\nu\alpha}{}^\beta + l_4\,\nabla_\mu F^{\mu\nu}\nabla_\alpha F^\alpha_\nu\right]$$

$$l_1 = -\frac{1}{576\pi^2},\quad l_2 = \frac{13}{1440\pi^2},\quad l_3 = -\frac{1}{1440\pi^2},\quad l_4 = -\frac{1}{120\pi^2}$$

As $m \to 0$ get nonlocal form

$$\Gamma_{anom.}[g,A] = \int d^4x\,\sqrt{g}\left[n_R F_{\rho\sigma}F^{\rho\sigma}\frac{1}{\Box}R + n_C F^{\rho\sigma}F^\gamma_\lambda\frac{1}{\Box}C_{\rho\sigma\gamma}{}^\lambda\right]$$

$$n_R^{(s,f)} = -\frac{\beta^{(s,f)}}{12e},\quad n_C^s = -\frac{e^2}{96\pi^2},\quad n_C^f = \frac{e^2}{48\pi^2}$$

<u>Look again at anomalies (in presence of gravity)</u>:

1) **Scale anomaly (as above)**

- comes from logs $$\Gamma^{(1)}[A,h] \to \Gamma^{(1)}[A,h] + \frac{b_i}{2}\int d^4x\, h^{\mu\nu}\left[\log \lambda^2 T^{cl}_{\mu\nu}\right]$$

-obtains anomaly with first term of covariant trace relation

$$T^{\mu}_{\mu} = \frac{b_i}{2}\left(\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + 2h^{\mu\nu}T^{cl}_{\mu\nu}\right)$$

2) **Conformal rescaling of fields**

$$g_{\mu\nu} \to (1+2\sigma)g_{\mu\nu} \qquad h_{\mu\nu} \to h_{\mu\nu} + 2\sigma\eta_{\mu\nu} \qquad \phi \to (1-\sigma)\phi.$$

-here we need the Riegert part of action

$$\Gamma^{(1)}[A,h] \to \Gamma^{(1)}[A,h] - b_i\int d^4x\, \sigma\frac{1}{\Box}\left(\partial_\lambda F_{\mu\nu}\partial^\lambda F^{\mu\nu}\right)$$

- again recover trace relation using $\partial_\lambda F_{\alpha\beta}\partial^\lambda F^{\alpha\beta} = \frac{1}{2}\Box\left(F_{\mu\nu}F^{\mu\nu}\right)$

$$T^{\mu}_{\mu} = \frac{b_i}{2}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

Need <u>both</u> logs and Riegert action

The problem of covariant $\ln \nabla^2$

Expect

$$\frac{b_i}{4}\int d^4x\ \eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}\ln\left(\Box/\mu^2\right)F_{\alpha\beta} \to \frac{b_i}{4}\int d^4x\sqrt{-g}\ g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}\ln\left(\nabla^2/\mu^2\right)F_{\alpha\beta}$$

Recall single propagator version

$$\ln\left(\nabla^2/\mu^2\right) = -\int_0^\infty dm^2\left[\frac{1}{\nabla^2+m^2}-\frac{1}{\mu^2+m^2}\right]$$

and double propagator version

$$\frac{i}{16\pi^2} < x|\log\nabla^2|y >= \Delta_F^2(x-y) - \frac{i}{16\pi^2}\left[\frac{1}{\epsilon}-\gamma-\log 4\pi\right]\delta^4(x-y)$$

Both versions have IR singularities not found in direct calculation

#1 is $1/\lambda^2$ and #2 is $\ln\lambda$

For example, with single propagator version:

$$\int d^4x\sqrt{g}\,F^{\alpha\beta}\ln\left(\frac{\nabla^2}{\mu^2}\right)F_{\alpha\beta}=\int d^4x\,\left[F^{\alpha\beta}\ln\left(\Box/\mu^2\right)F_{\alpha\beta}+h_{\mu\nu}\left(\mathcal{O}_1^{\mu\nu}+\mathcal{O}_2^{\mu\nu}\right)\right]$$

where

$$\mathcal{O}_1^{\mu\nu}=\frac{1}{2}\eta^{\mu\nu}F^{\alpha\beta}\ln(\Box/\mu^2)F_{\alpha\beta}-2F^{\mu}_{\ \alpha}\log(\Box/\mu^2)F^{\nu\alpha}$$

$$\mathcal{O}_2^{\mu\nu}=\partial^\mu\partial^\nu F_{\alpha\beta}\frac{1}{\Box}F^{\alpha\beta}+\partial^\mu\partial^\nu F^{\alpha\beta}\frac{1}{\Box}F_{\alpha\beta}-\eta^{\mu\nu}\partial_\lambda F_{\alpha\beta}\frac{1}{\Box}\partial^\lambda F^{\alpha\beta}$$

Unphysical
- 1/(photon mass/momentum)

These terms show no relation to what was found by calculation

<u>Quick partial summary</u>:

Easy to see non-local effects in perturbation theory

See the BV expansion in curvatures in action
- matching

Anomalies found as non-local terms in effective action
- both log and Riegert forms needed

We have not solved the covariant Log Box issue yet
- IR mismatch

Example 2 – QCD and gauge invariance

Start with the same calculation (flat space):

$$S = \int d^4x \; -\frac{1}{4} F^a_{\rho\sigma} \left[\frac{1}{e^2(\mu)} - b \ln\left(\Box/\mu^2\right) \right] F^{a\rho\sigma}$$

But now the log term is not gauge invariant:

$$\mathbf{F}_{\mu\nu}(x) \to U(x)\mathbf{F}_{\mu\nu}(x)U^\dagger(x) \qquad\qquad \mathbf{F}_{\mu\nu} = \frac{\lambda^a}{2} F^a_{\mu\nu}$$

The gauge transformations are different at different points

$$\mathrm{Tr}\left[\mathbf{F}_{\mu\nu}(x)\; L(x-y)\; \mathbf{F}^{\mu\nu}(y)\right] \to \mathrm{Tr}\left[U(x)\mathbf{F}_{\mu\nu}(x)U^\dagger(x)\; L(x-y)\; U(y)\mathbf{F}^{\mu\nu}(y)U^\dagger(y)\right]$$

So here is a simpler setting to work on "Log Box"

Note: **G&L non-local action also violates underlying local symmetry**

Proposed solution – Wilson lines

$$\mathbf{W}(y-x) = P \ \exp\left[\int_x^y d\ell^\mu \mathbf{A}_\mu(\ell)\right]$$

This transforms as:

$$\mathbf{W}(y-x) \to U(y)\mathbf{W}(y-x)U^\dagger(x)$$

We could work with the invariant:

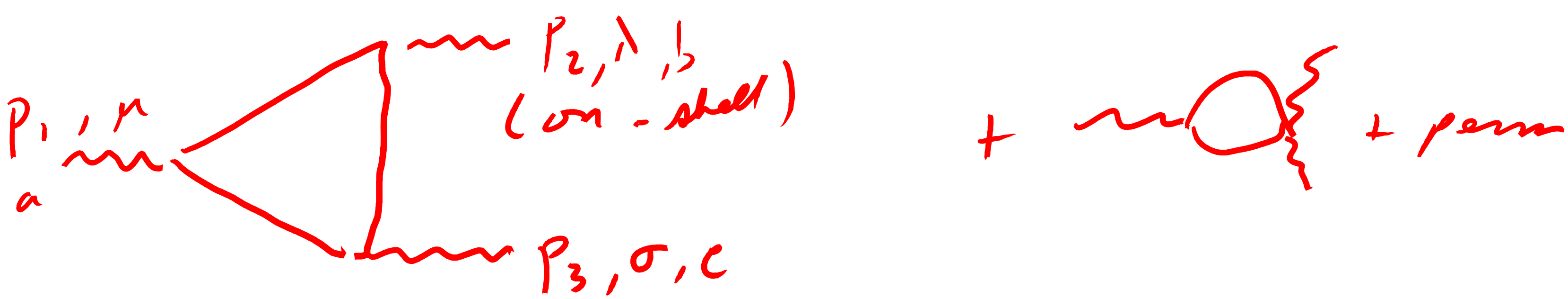
$$\mathrm{Tr}\left[\mathbf{F}_{\mu\nu}(x)\ \mathbf{W}(x-y)\ \mathbf{F}^{\mu\nu}(y)\mathbf{W}(y-x)\right]\ L(x-y)$$

Question:

- does this cause any trouble? (No)
- is it positively indicated in perturbation theory? (Yes)

Verification – direct calculation

Calculated using two hard gluons (off-shell $\sim q^2$) and one on-shell



Find local divergences in F^2 - charge renormalization
Find completion of non-local $F = \partial A + A^2$ as expected
Also find some new non-local terms, including

$$(p_1 - p_3)^\lambda \ (p_3^\mu p_1^\sigma - p_1 \cdot p_3 \eta^{\mu\sigma}) \ \frac{1}{p_3^2 - p_1^2} \left[\log p_3^2 - \log p_1^2\right]$$

↑ $\sim F_{\alpha\beta} F^{\alpha\beta}$

Evaluating the Wilson line matrix element

To first order: $$\int d^4x d^4y \frac{1}{4} \mathrm{Tr} \left[\mathbf{F}_{\mu\nu}(y) \int_x^y d\ell^\mu \, \mathbf{A}_\mu(\ell) \; \mathbf{F}^{\mu\nu}(x) \right] \; L(x-y) + \mathrm{perm}$$

Parameterize: $$\ell^\mu = x^\mu + \lambda(y-x)^\mu \qquad 0 \le \lambda \le 1$$

Remove overall delta functions

The residual integrals is $$\int d^4z e^{ip_1\cdot z} \left[\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot z} \log k^2 \right] \int_0^1 d\lambda \; z^\sigma \epsilon_\sigma e^{i\lambda p_2 \cdot z}$$

Can evaluate this using $$e^{i\lambda p_2\cdot z} z_\sigma = \frac{\partial}{\partial(\lambda p_2^\sigma)} e^{i\lambda p_2 \cdot z}$$

Resulting matrix element:

$$\epsilon \cdot (p_1 - p_3) \, \frac{1}{p_3^2 - p_1^2} \left[\log p_3^2 - \log p_1^2 \right]$$

This is exactly what is found in the direct calculation

Quick partial summary

Wilson lines can restore gauge invariance
- "does no harm"
- positively seen in direct calculation

Could reformulate PT in terms of covariant propagators
- with explicit Wilson lines

Schwinger
DeWitt
Latosinski

For gravity, this can help with $R_{\mu\nu} \log \nabla^2 R^{\mu\nu}$ terms

$$\mathcal{U}^{\alpha}_{\beta}(x, x') = \mathrm{P}\ \exp \int_x^{x'} dy^\mu\, \Gamma^\alpha_{\mu\beta}(y),$$

I have evaluated this to first order in similar kinematics
- does no harm
- but not yet checked direct calculation

Back to gravity – still do not fully understand Log Box

- here is a proposal

Proper time representation of Minkowski scalar propagator

$$D_F(x-y) = \frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^2} e^{-im^2 s} e^{-i\frac{(x-y)^2 - i\epsilon}{4s}}$$

Schwinger- DeWitt adiabatic expansion of propagator

$$(-g(x))^{\frac{1}{4}}\, D(x,y)\, (-g(y))^{\frac{1}{4}} = \frac{\Delta^{\frac{1}{2}}}{16\pi^2}\int_0^\infty \frac{ds}{s^2}\left[1 + \sum_{n=1}^{\infty} a_n (is)^n\right] e^{-im^2 s} e^{-i\frac{\sigma - i\epsilon}{2s}}$$

where $\sigma(x,y)$ is the **geodetic distance** between x and y

- in flat space $\sigma = \frac{1}{2}(x-y)^2$

And Δ is the Van-Vleck Morette determinant

$$\Delta(x,x') = -det[-\sigma_{;\mu\nu'}]$$

Proposal for Log Box

- perhaps we are mislead by focusing on propagator form
  - calling it "Log Box"
- it is a function of the distance apart – recall:

$$L(x-y) = \frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-i\frac{(x-y)^2 - i\epsilon}{4s}},$$

We can make this covariant using the geodetic distance

$$\frac{1}{2}(x-y)^2 \to \sigma^2$$

$$L(x-y) \to L(\sigma)$$

This result in a simple covariant expression for Log Box

$$L(\sigma) = \frac{1}{16\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-i\frac{\sigma - i\epsilon}{2s}},$$

Perturbative evaluation of geodetic distance

Systematic Post-Minkowski expansion of geodetic distance

Le Poncin-Lafitte
Linet
Teyssandier

$$\sigma = \frac{1}{2}(x-y)^2 + \sigma_1$$

$$\sigma_1 = \frac{1}{2}(x-y)^\mu (x-y)^\nu \int_0^1 d\lambda h_{\mu\nu}(x(\lambda))$$

This leads to

$$L(\sigma) = L_0(x-y) - i\frac{1}{32\pi^2}\int_0^\infty \frac{ds}{s^4} e^{-i\frac{(x-y)^2 - i\epsilon}{4s}} \frac{1}{2}(x-y)^\mu (x-y)^\nu \int_0^1 d\lambda h_{\mu\nu}(x(\lambda))$$

End of development section:

We have seen:

Relation of non-local terms to loop diagrams

Construction of covariant actions

Anomalies for EFT (both logs and Riegert)

Use of Wilson lines for gauge invariance/covariance

Proposal of geodetic distance for Log Box $\rightarrow L(\sigma)$

Simple applications:

1) Hints of cosmic bounce

2) Nonlocal partner of cosmological constant

Hints of a cosmic bounce:

JFD and BEl-M 2014

In FLRW cosmology
- spatially uniform, but temporarily varying

Use “in-in’ B.C. – time evolution, not scattering

Keep the Log Box terms (not third order in curvature)
- but work in P.T. to first order only

Log Box as free L(x-y) .

Expanding phase behaves normally – effect dies off

Contracting phase has some new features

<u>**Non-local FLRW equations to first order**</u>:

$$\frac{3a\dot{a}^2}{8\pi} + N_s \left[6(\sqrt{a}\,\ddot{a})_t \int dt'\, L(t-t')\mathcal{R}_1 + 6\left(\frac{\dot{a}^2}{\sqrt{a}}\right) \int dt'\, L(t-t')\mathcal{R}_2 + 12(\sqrt{a}\dot{a})_t \int dt'\, L(t-t')\frac{d\mathcal{R}_3}{dt'} \right] = a^3\rho$$

with

$$\mathcal{R}_1 = -\sqrt{a}\ddot{a}(6\alpha + 2\beta + 2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(6\alpha + \beta)$$

$$\mathcal{R}_2 = -\sqrt{a}\ddot{a}(12\alpha + \beta - 2\gamma) - \frac{\dot{a}^2}{\sqrt{a}}(12\alpha + 5\beta - 6\gamma)$$

$$\mathcal{R}_3 = \sqrt{a}\ddot{a}(6\alpha + 2\beta + 2\gamma) + \frac{\dot{a}^2}{\sqrt{a}}(6\alpha + \beta)$$

and the <u>**time-dependent weight:**</u>

$$L(t-t') = \lim_{\epsilon \to 0} \left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')\log(\mu_R\,\epsilon) \right]$$

For scalars: $\alpha = \frac{1}{2304\pi^2} \quad \beta = \frac{-1}{5760\pi^2}, \quad \gamma = \frac{1}{5760\pi^2}$

Collapsing universe – singularity avoidance?

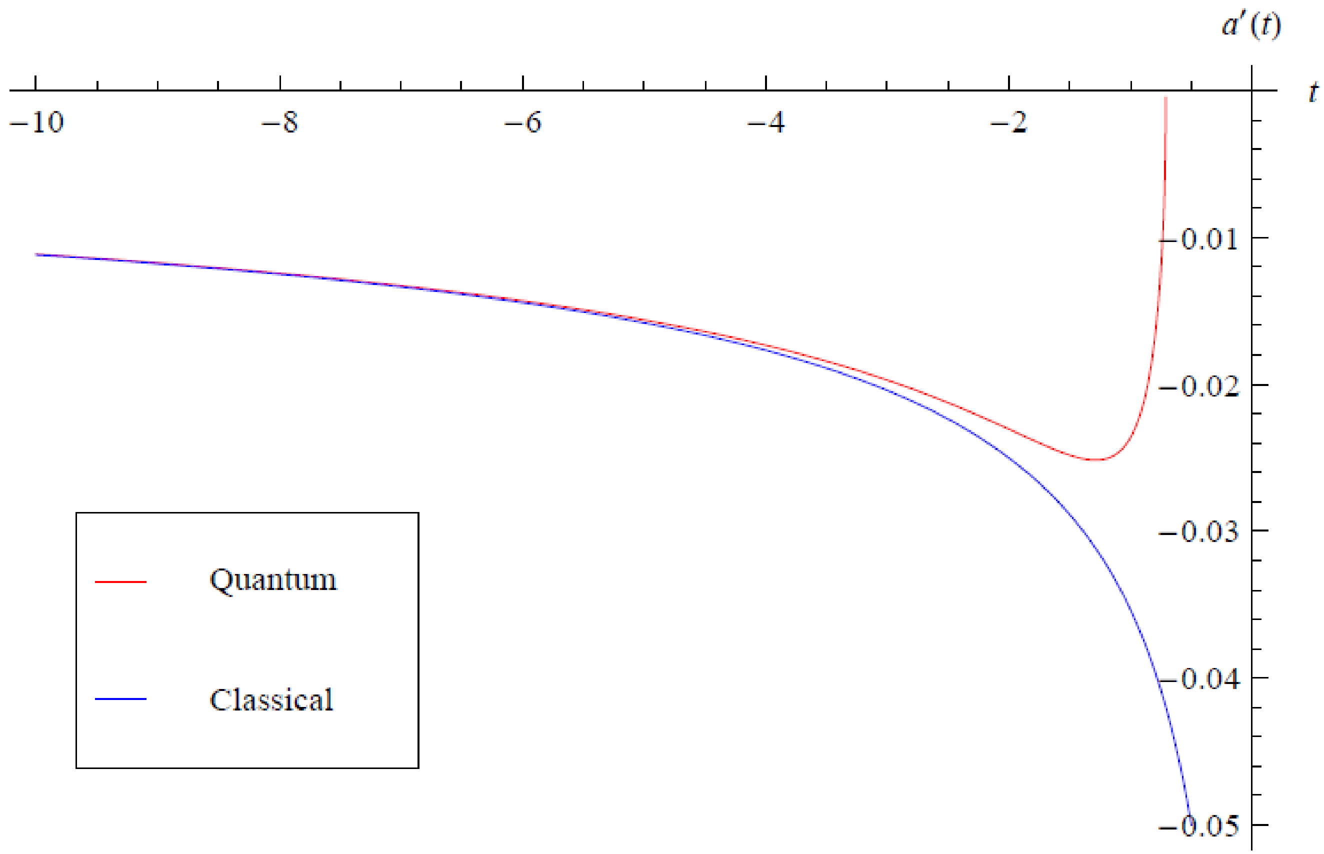


FIG. 12: Collapsing radiation-filled universe with gravitons only considered.

No free parameters in this result

For caveats, etc. see paper

<u>Nonlocal "partner" to cosmological constant</u>

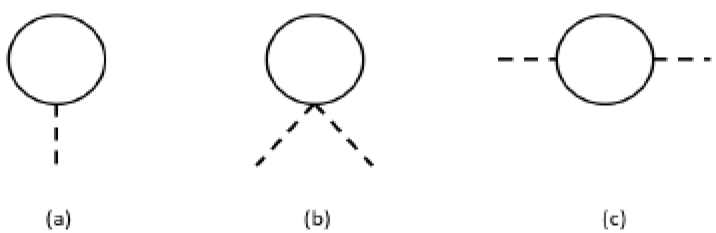
This is follow-up to my recent C.C. and cutoffs paper arXiv: 2009.00728

- PI measure contribution removes cutoff^4 effect in self energies

$$\Delta\mathcal{L} = -i\frac{1}{8}i\delta^4(0)\ \log(-g)$$

- important for cutoff regularization, vanishes in dim-reg

To discuss renormalization of Λ "right", use dim-reg



$$\begin{aligned} S_{grav} &= \int d^4x\ \sqrt{-g}\left[-\Lambda_{cc} + \frac{2}{\kappa^2}R + ...\right] \\ &= \int d^4x\ \left[-\Lambda_{cc}\left(1 + \frac{1}{2}\eta^{\mu\nu}h_{\mu\nu}\right) + ...\right] \end{aligned}$$

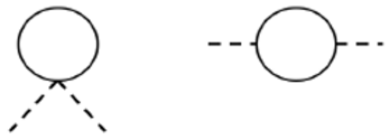
Coupling to the gravitational field

$$\Delta\mathcal{L} \ = \ -i\frac{1}{2}h_{\mu\nu} \times \int \frac{d^4p}{(2\pi)^4}\frac{2p^\mu p^\nu - \eta^{\mu\nu}(p^2 - m^2)}{p^2 - m^2 + i\epsilon}$$

$$\Delta\Lambda \sim \frac{m^2}{d}\int \frac{d^dk}{(2\pi)^2}\frac{i}{k^2 - m^2} \sim m^4\left[\frac{1}{\epsilon} + ...\right]$$

Second order in the field:

$$\left(1+\frac{1}{2}h^{\alpha}_{\alpha}-\frac{1}{4}h^{\alpha}_{\beta}h^{\beta}_{\alpha}+\frac{1}{8}\left[h^{\alpha}_{\alpha}\right]^2+O(h^3)\right)$$



- obtain exactly the same divergent terms
- adjust renormalized Λ to match experimental value

But diagram (c) has non-local content

- Some is renormalization of R^2 terms

$$\mathcal{M}\sim h^{\mu\nu}h^{\alpha\beta}(q_\mu q_\nu q_\alpha q_\beta+\ldots)m^0\left[\frac{1}{\epsilon}+\log\right]\to R^2+R\log\Box R$$

- Some is renormalization of Einstein action

$$\mathcal{M}\sim h^{\mu\nu}h^{\alpha\beta}(q_\mu \eta_{\nu\alpha} q_\beta+\ldots)m^2\left[\frac{1}{\epsilon}+\log\right]\to R+?$$

- Some is nonlocal effect in cosmological constant

$$\mathcal{M}\sim h^{\mu\nu}h^{\alpha\beta}(\eta_{\mu\beta}\eta_{\nu\alpha}+\ldots)m^4\left[\frac{1}{\epsilon}+\log\right]\to \Lambda+?$$

Lets calculate this

Ingredients:

$$Q_{\mu\nu} = q^2\eta_{\mu\nu} - q_\mu q_\nu$$

$$\mathrm{Log} = \int_0^1 dx \log\left[\frac{m^2 - x(1-x)q^2}{m^2}\right]$$

Result:

$$\begin{aligned} \mathcal{M} &= h^{\mu\nu}h^{\alpha\beta}\left[Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\beta}\right] \\ &\times \left[\frac{m^4}{480\pi^2}\frac{\mathrm{Log}}{q^4} + \frac{m^2}{2880\pi^2}\frac{1}{q^2}\right] \end{aligned}$$

Transform to effective action

$$R_{\lambda\sigma}(x)R^{\lambda\sigma}(y) - \frac{1}{8}R(x)R(y) \to \frac{1}{4}h^{\mu\nu}h^{\alpha\beta}\left[Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\beta}\right]$$

$$\log\left[\frac{\Box + m^2}{m^2}\right] \to \mathrm{Log}$$

Effective action

- quasi-local notation

$$\mathcal{L} = -\frac{m^4}{120\pi^2}\left[R_{\lambda\sigma}\frac{\log((\Box+m^2)/m^2)}{\Box^2}R^{\lambda\sigma} - \frac{1}{8}R\frac{\log((\Box+m^2)/m^2)}{\Box^2}R\right]$$
$$+ \frac{m^2}{720\pi^2}\left[R_{\lambda\sigma}\frac{1}{\Box}R^{\lambda\sigma} - \frac{1}{8}R\frac{1}{\Box}R\right]$$

First line is zeroth-order in derivative expansion (caveat next slide), second line is second order

This is for scalar field

- fermion result is -2 times this

Also find other effects such as:

$$m^2 R\frac{\log((\Box+m^2)/m^2)}{\Box}R$$

<u>Decoupling:</u>

- but effects of heavy mass should be local at low energy!!
- Appelquist – Carrazone / uncertainty principle

This does work:

Recall:

$$\begin{aligned}\mathcal{M} &= h^{\mu\nu}h^{\alpha\beta}\left[Q_{\mu\nu}Q_{\alpha\beta}+Q_{\mu\alpha}Q_{\nu\beta}+Q_{\mu\beta}Q_{\nu\beta}\right] \\ &\times \left[\frac{m^4}{480\pi^2}\frac{\text{Log}}{q^4}+\frac{m^2}{2880\pi^2}\frac{1}{q^2}\right]\end{aligned}$$

But for large mass:

$$\text{Log} = \int_0^1 dx \log\left[\frac{m^2 - x(1-x)q^2}{m^2}\right] = -\frac{1}{6}\frac{q^2}{m^2} + \ldots$$

Non-localities vanish when far below the mass

Fine tuning is not enough

Cosmological constant is highly fine-tuned

But nonlocal partner cannot be fine-tuned

$$\begin{aligned} \Lambda_{NL} &\sim m^4 \, R \frac{\log \Box}{\Box^2} R &\quad \text{above m} \\ &\sim 0 &\quad \text{below m} \end{aligned}$$

Like effective scale dependent cosmological constant
Effect occurs sequentially as the universe evolves past masses

Relative effect potentially large $\frac{m_e^4}{\Lambda_{expt}} \sim 10^{35}$

Multiple inflations?

Caution: This is a weak field expansion

<u>Lots here for future research:</u>

EFT side:
- import lessons of IR properties of scattering amplitudes
- gauge invariant perturbation theory
- utility in EFT applications

GR side:
- verifications of suggestions for non-local functions
- useful techniques
- applications
- any approximations for strong field region?

Cosmology side:
- effects of non-local actions
- model building

Summary:

Quantum effects can be packaged in non-local effective actions

For gravity the BV expansion in curvature is a weak field expansion

Anomalies are found in non-local actions in the IR

Wilson loops can be used to aid in covariant answers

Proposal for Log Box using geodetic distance

Non-local partner of cosmological constant
- cannot be fine-tuned away