# Development of Modern Effective Field Theories 

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Last week, Professor Weinberg discussed the historical development of the theory and his pioneering contributions.

Technical development whose importance should not be underestimated - dimensional regularization in the early 1970s. Respects gauge and chiral symmetries, and dimensional analysis

Weinberg: Phenomenological Lagrangians, Physica 96A (1979) 327.
Power counting formula

$$
\mathcal{A} \sim p^{d} \quad(d-2)=\sum_{i}\left(d_{i}-2\right)+2 L
$$

allows for a systematic expansion in powers of $p$. Non-analytic parts of amplitudes fixed by loops (unitarity $\log (-s)$ has imaginary part).
Li and Pagels

Pion loops studied systematically: Gasser and Leutwyler (1984)

As a graduate student at Harvard, learned EFTs from Howard Georgi's brilliant lecture notes which became his Weak Interactions book.

Weinberg was still there, Mark Wise had done the work with Gilman on $\Delta S=1,2$ weak interactions. EFT ideas were "ancient"

Widely separated scales — $\Lambda_{\mathrm{QCD}}, M_{W}, M_{G} \sim 10^{15} \mathrm{GeV}$
Georgi, Quinn, Weinberg

Leave history to historians

Modern era - loops

Talk is not a review. The subject is vast - also in condensed matter physics, and increasingly in gravity and cosmology

Cover topics I have thought about.

## QFT Lagrangian

$$
L_{D \leq 4}=-\Lambda \mathbb{1}-m^{2} \phi^{\dagger} \phi-\frac{1}{4} F_{\mu \nu}^{A} F^{A \mu \nu}+D_{\mu} \phi^{\dagger} D^{\mu} \phi-\frac{1}{4} \lambda\left(\phi^{\dagger} \phi\right)^{2}+L_{\text {c.t. }}
$$

Can also have topological terms in the action.

$$
[\Lambda]=4 \quad\left[m^{2}\right]=2 \quad[g]=0 \quad[\lambda]=0
$$

If there are multiple $U(1) \mathrm{s}$, can have off-diagonal field strength terms.
Kinetic mixing
AM, P. Galison PLB 136 (1984) 279, later used by Holdom for dark photons.

Implements the symmetries of the theory - Poincaré and internal (flavor) symmetries, gauge invariance.

Locality (cluster decomposition: see Weinberg QFT I)

## QFT

$L$ is not enough — need to have a method of obtaining experimentally measurable results.

Regularization and renormalization - part of the definition of the theory.

With $L_{D \leq 4}$ can compute everything (at least in principle), including non-perturbative effects.

In practice, this involves an expansion in small parameters such as $\alpha$ in QED.

Lattice calculations have numerical precision, finite size effects, etc.
The perturbation series is divergent, and related to non-perturbative effects
Bender and Wu for the anharmonic oscillator

## QFT

The parameters in the Lagrangian are not fundamental - they depend on regularization and renormalization schemes.

Intermediate quantities useful for calculations


## EFT

One does not need to know the exact short-distance theory to be able to compute long-distance quantities with high precision.

You don't need to know about quantum gravity to build a bridge [unless it is an Einstein-Rosen bridge - E. Nardoni]

Some large energy scale $M$, and we want to compute at momenta $p \ll M$.

An EFT can be used to compute in an expansion in $p / M$.
Quantum corrections of the form $\left[\alpha_{s} \log (p / M)\right]^{n}$ can be summed restoring the validity of perturbation theory.

## Reasons for using EFT

- Every theory is an effective theory: Can compute in the standard model, even if there are new interactions at (not much) higher energies.
- Greatly simplifies the calculation by only including the relevant interactions: Gives an explicit power counting estimate for the interactions.
- Deal with only one scale at a time: For example the $B$ meson decay rate depends on $M_{W}, m_{b}$ and $\Lambda_{\mathrm{QCD}}$, and one can get complicated functions of the ratios of these scales. In an EFT, deal with only one scale at a time, so there are no functions, only constants.
- Makes symmetries manifest: Some symmetries are only true for certain limits of QCD, and so are hidden in the QCD Lagrangian.
- QCD has spontaneously broken chiral symmetry, which is manifest in the chiral Lagrangian
- Spin-flavor symmetry in HQET (Isgur-Wise)

$$
b \uparrow, b \downarrow, c \uparrow, c \downarrow
$$

- Sum logs (including IR logs): Use renormalization group improved perturbation theory. The running of constants is not small, e.g.

$$
\alpha_{s}\left(M_{Z}\right) \sim 0.118, \quad \alpha_{s}\left(m_{b}\right) \sim 0.22
$$

Fixed order perturbation theory breaks down. Sum logs of the ratios of scales such as $M_{W} / m_{b}$, which turn into $\mu / m_{b}$ when the $W$ is integrated out.

- Efficient way to characterize new physics: Can include the effects of new physics in terms of higher dimension operators. All the information about the dynamics is encoded in the coefficients. [This also shows it is difficult to discover new physics using low-energy measurements.]
- Include non-perturbative effects in a systematic expansion: Can include $\Lambda_{\mathrm{QCD}} / m$ corrections in a systematic way through matrix elements of higher dimension operators. The perturbative corrections and power corrections are tied together. [Renormalons]
- HQET has non-perturbative corrections that depend on $\lambda_{1,2}$ of order $\Lambda_{\mathrm{QCD}}^{2}$. They have the same value in different calculations, as they are the matrix element of a renormalized local operator.


## EFT Lagrangian

The key concept is locality [cluster decomposition]
The EFT Lagrangian has an expansion in inverse powers of $M$ :

$$
L_{\mathrm{EFT}}=L_{D \leq 4}+\frac{O_{5}}{M}+\frac{O_{6}}{M^{2}}+\ldots+\frac{O_{D}}{M^{D-4}}+\ldots+L_{\mathrm{c} . \mathrm{t}}
$$

[with a regularization and renormalization scheme]. $1 / M$ is the Compton wavelength of particles of mass $M$.

- An EFT is a theory. You can compute using $L_{\text {EFT }}$, including radiative corrections, without any reference to the theory at the scale $M$.
- $L$ is to be treated as an expansion in $1 / M$. If you try and sum terms to all orders, you violate the EFT power counting.


## EFT Lagrangian

Parameters in the EFT Lagrangian are determined by low-energy experiments.

The non-power suppressed effects of high scale dynamics is absorbed in a shift in the low-energy couplings (matching).
Appelquist-Carazzone decoupling theorem

Heavy particles are integrated out in a sequence of EFTs by a procedure known as matching.

## Power Counting

If one works at some typical momentum scale $p$, and neglects terms of dimension $D$ and higher, then the error in amplitudes is of order

$$
\left(\frac{p}{M}\right)^{D-4}
$$

A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a non-zero error.

Only a finite number of coefficients for a given allowable error
Can improve the precision by going to higher powers in $p / M$, and higher order in $\alpha_{s}$.

Usual renormalizable case given by taking $M \rightarrow \infty$. There are still errors from the $\alpha_{s}$ expansion.

## $D>4$ ? Neutrino Masses

The lowest dimension operator in the standard model with $D>4$ is a dimension five operator which gives a neutrino mass

$$
O_{5}=\frac{1}{M} C_{r s} \epsilon^{i j} \epsilon^{k l}\left(l_{i r}^{T} \mathscr{C} l_{k s}\right) H_{j} H_{l} \quad \text { (Weinberg; Wilczek and Zee) }
$$

and violates lepton number. $r, s$ flavor indices, $i, j, k, l$ are $S U(2)$ indices
This gives a Majorana neutrino mass of ( $v \sim 246 \mathrm{GeV}$ )

$$
m_{\nu} \sim C \frac{v^{2}}{M}
$$

or a seesaw scale of $6 \times 10^{15} \mathrm{GeV}$ for $m_{\nu} \sim 10^{-2} \mathrm{eV}$ and $C \sim 1$.
Experiments searching for lepton number violating neutrino masses are very important.

## Proton Decay

The lowest dimension operator in the standard model which violates baryon number is dimension 6 . Natural explanation of baryon number conservation.

$$
L \sim \frac{q q q l}{M_{G}^{2}} \quad \text { (Weinberg; Wilczek and Zee) }
$$

This gives the proton decay rate $p \rightarrow e^{+} \pi^{0}$ as

$$
\Gamma \sim \frac{m_{p}^{5}}{16 \pi M_{G}^{4}}
$$

or

$$
\tau \sim\left(\frac{M_{G}}{10^{15} \mathrm{GeV}}\right)^{4} \times 10^{30} \text { years }
$$

## What about Loops?



Loop integrals depend on external momenta and particle masses, which are all $\ll M$.
dim reg does not introduce powers of $\mu$, only logs

Amplitude is of order

$$
\frac{1}{M^{a}} \frac{1}{M^{b}} \cdots=\frac{1}{M^{a+b+\ldots}}
$$

manifest power counting
Loop graph with insertions of operators $O_{k}$ of dimension $d_{k}$ :

$$
d-4=\sum_{k}\left(d_{k}-4\right)
$$

EFT power counting formula

## Power Counting Formula

Only a finite number of terms to any given order in $1 / M$.
An EFT is just as good as a renormalizable field theory
Order 1/M: one inserttion of $L_{5}$
Order 1/M $M^{2}$ : one insertion of $L_{6}$, or two insertions of $L_{5}$.
General power counting result:

- you can count the powers of $M$ in the denominator
- you can count powers of $p$ in the numerator

Two are equivalent

## Reminder: The higher dimension operators are there

$$
L=L_{S M}+\frac{1}{M^{2}} L_{6}+\left[\frac{1}{M^{4}} L_{8}+\ldots\right]_{\text {drop }}
$$

One-loop graph


$$
A \sim\left[c_{6}^{2}\left(\frac{1}{\epsilon}+\text { finite }_{1}\right)+c_{8}\left(\frac{1}{\epsilon}+\text { finite }_{2}\right)\right]_{\text {drop }}
$$

Need $L_{8}$ to absorb divergences.
Drop both the loop and $L_{8}$ to order $1 / M^{2}$.

## Naive Dimensional Analysis

A.M. and H. Georgi, NPB 234 (1984) 189

A power counting scheme keeping track of $4 \pi$ factors: Lagrangian term has the form

$$
L=f^{2} \Lambda^{2}\left(\frac{\psi}{f \sqrt{\Lambda}}\right)^{a}\left(\frac{\phi}{f}\right)^{b}\left(\frac{D}{\Lambda}\right)^{c}\left(\frac{g F_{\mu \nu}}{\Lambda^{2}}\right)^{d} \quad \Lambda \sim 4 \pi f
$$

$\Lambda$ is the heavy scale, and is what we called $M$ on the previous slides.
This makes a big difference in estimating the coefficients of higher dimension terms.

In SMEFT range $(4 \pi)^{4}-(4 \pi)^{-2} \sim 4 \times 10^{6}$.

## Operators with dimension $<4$

In the SM, there is an operator of dimension two,

$$
L=m^{2} H^{\dagger} H
$$

with $2 m^{2}=m_{h}^{2}$, and one of dimension zero,

$$
L=-\Lambda \mathbb{1}
$$

the cosmological constant.

Unthinking application of power counting:

$$
\begin{array}{lrl}
m^{2} & \sim M^{2} & \Lambda \\
m_{H} & \sim 125 \mathrm{GeV} & \Lambda^{4} \\
1 / 4 & \sim 2.8 \times 10^{-3} \mathrm{eV}
\end{array}
$$

in conflict with $M$ being a scale above the electroweak scale.

## Fine tuning in SMEFT

Treat the SM as an EFT at the electroweak scale, with $M \gg v$.
By assumption, all the particles in the theory have masses at or below the electroweak scale, so $m_{H}$ is of order the electroweak scale

Is this a consistent EFT? Loop effects of dimension two in the EFT are

$$
m_{H}^{2} \lambda^{n}, \quad m_{H}^{4} \frac{1}{M^{2}} \lambda^{n},
$$

[dimensional analysis and the absence of any positive powers of $M$ ] The corrections are at most of order $m_{H}^{2}$, and the EFT is consistent. No fine-tuning of $m_{H}$ in the EFT
No corrections of order $M^{2}$.

## No Quadratic Divergences

The standard argument for the hierarchy problem is that there are quadratic divergences in corrections to the Higgs mass.

But the actual integral gives

$$
\mu^{2 \epsilon} \int d^{d} k \frac{1}{k^{2}-m_{H}^{2}} \sim m_{H}^{2} \log \frac{\mu^{2}}{m_{H}^{2}}
$$

not

$$
\mu^{2 \epsilon} \int d^{d} k \frac{1}{k^{2}-m_{H}^{2}} \sim M^{2}
$$

## Hierarchy Problem

Trying to get rid of quadratic divergences at the electroweak scale is solving the wrong problem.

If you have a more fundamental theory such as a unified theory, how do you generate two widely separated scales $M_{G}$ and $M_{W}$ ?
This is a hierarchy between physical particle masses ( $W, Z$ electroweak bosons vs $X, Y$ GUT bosons)

Fine-tuning of Lagrangian parameters order-by-order in perturbation theory is irrelevant.

- Nature does not use perturbation theory
- Lagrangian parameters are not observables - they are convenient intermediate variables to use
Lattice calculations involve fine-tuning the couplings


## Cosmological Constant Problem

What about $\Lambda$ which is the coefficient of a dimension zero operator?

$$
\Lambda \lambda^{n} \quad \Lambda^{2} \frac{1}{M^{4}} \lambda^{n} \quad \underbrace{m_{H}^{4} \lambda^{n}}_{\text {trouble }}
$$

The low-energy theory induces $\Lambda$ of order $m_{H}^{4}$ or $\Lambda_{\mathrm{QCD}}^{4}$. The only solution we have is anthropic (Weinberg)

## PART II

Widely used EFTs which will be discussed by others later in the seminar series in 1-2 slides each

## Heavy Quark Effective Theory (HQET)

Describes heavy quarks interacting with light degrees of freedom (light quarks and gluons).

$$
p=m_{Q} v+k \quad v \cdot v=1 \quad v=(1,0,0,0)
$$

New feature is fields labelled by a velocity $b_{v}$. Georgi
Makes Isgur-Wise symmetry mainfest
Leads to velocity dependent anomalous dimensions
Falk, Georgi, Grinstein, Wise
Systematic expansion in powers of $\Lambda_{\mathrm{QCD}} / m_{b}$ from $k / m_{b}$ expansion.
Non-perturbative effects included via

$$
-2 \lambda_{1} m_{B}=\langle B| \bar{b}_{v} D_{\perp}^{2} b_{v}|B\rangle \quad-12 \lambda_{2} m_{B}=\langle B| \bar{b}_{v} g \sigma_{\mu \nu} G^{\mu \nu} b_{v}|B\rangle
$$

Used to extract $m_{b}, V_{c b}$, etc.

## Baryon Chiral Perturbation Theory

Jenkins, AM
Use the HQET idea and introduce velocity-dependent baryon fields $B_{v}$.
The chiral expansion is in powers of $p /(4 \pi f)$.
Manifestly no powers of $M_{N} /(4 \pi f)$ which would lead to a breakdown in the chiral expansion.

Using $B_{v}$ ensures that the expansion is in powers of $m_{\pi} /(4 \pi f)$ and in powers of $1 / M_{N}$.

## Dashen,Jenkins, AM

Combined with the $1 / N_{c}$ expansion, explains the spin-flavor structure of baryons. e.g. for the baryon magnetic moments, one can prove that

$$
\frac{F}{D}=\frac{2}{3}+\mathcal{O}\left(\frac{1}{N_{c}^{2}}\right) \Longrightarrow \frac{\mu_{n}}{\mu_{p}}=-\frac{2}{3} \quad-0.68(\exp )
$$

## NRQCD/NRQED

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Bodwin, Braaten, Lepage
Luke, AM, Rothstein

Theory with a non-relativistic quark and antiquark, which orbit around each other (like the Hydrogen atom)


Problem has 3 scales
hard: m
soft: mv
ultrasoft: \(m v^{2}\)

The NR theory has \(m\) integrated out, but still contains \(m v\) and \(m v^{2}\).

The soft and ultrasoft scales talk to each other:
\begin{tabular}{rll} 
quarks: & \(E \sim \frac{1}{2} m v^{2}\), & \(p \sim m v\) \\
soft gluons: & \(E \sim m v\), & \(p \sim m v\) \\
ultrasoft gluons: & \(E \sim m v^{2}\), & \(p \sim m v^{2}\)
\end{tabular}

Luke, AM, Rothstein:
Label fields with a momentum \(\mathbf{p} \sim m v \quad \psi_{\mathbf{p}}(x)\). \(k \sim m v^{2}\) is the Fourier transform of \(x\).

Coulomb interaction:
\[
L=\frac{4 \pi \alpha}{\left(\mathbf{p}^{\prime}-\mathbf{p}\right)^{2}} \psi_{\mathbf{p}^{\prime}}^{\dagger} \psi_{\mathbf{p}} \chi_{-\mathbf{p}^{\prime}}^{\dagger} \chi_{-\mathbf{p}}
\]

\section*{Correlated RGE}

Luke, AM, Rothstein
Renormalize using \(\mu_{S}=m \nu\) and \(\mu_{U}=m \nu^{2}\), and run in \(\nu\) from 1 to \(v\). This method sums all the logs. The conventional method does not work, as one can have logs of
\[
\log \frac{E}{\mu_{U}} \quad \log \frac{\sqrt{2 m E}}{\mu_{S}}
\]

This 2 : 1 running gives a different leading log series than using a single \(\mu\).
AM, Stewart
\(\alpha^{8} \ln ^{3} \alpha\) Lamb shift for the \(n S\) state
\[
\Delta E=\frac{64 m_{R}^{5} \alpha^{8} Z^{6}}{27 \pi^{2} n^{3}}\left(\frac{1}{m_{1}^{2}}+\frac{Z}{4 m_{1} m_{2}}+\frac{Z^{2}}{m_{2}^{2}}\right)^{2} \ln ^{3}(Z \alpha)
\]
\(\alpha^{7} \ln ^{2} \alpha\) hyperfine splitting for muonium and Hydrogen for the \(n S\) state:


Two calculations, by Karshenboim and Yerokhin, for H which differed. Our result agreed with Karshenboim. This has been verified by a more recent calculation. We also obtain the result for Positronium. \(\Delta E_{H} \sim 10 \mathrm{kHz}\), and needed for comparison with experiment.

Hoang, AM, Stewart
QCD Lamb shift. ADM infrared divergence is not present in NRQCD.


Labels and multiple gluon fields leads to a headache.

\section*{NN Potential}

Weinberg, van Kolck
Kaplan, Savage, Wise
Combine pion chiral perturbation theory with \(N N\) interactions to develop a theory of nuclear forces.

Have to combine potential iterations and pion loops


\section*{NRGR}

\section*{Goldberger and Rothstein; Goldberger talk later in this seminar series}

Point particles interacting via gravitational interaction. The point particles in this case are black holes with masses of order \(M_{\text {sun }}\).

Application of QFT to classical physics - extract the classical pieces of loop diagrams

EFT + amplitude methods used to compute the gravitational potential to 3PM

Cheung, Rothstein, Solon; Bern, Cheung, Roiban, Shen, Solon

\section*{Soft-Collinear Effective Field Theory (SCET)}

Bauer, Fleming, Luke, Pirjol, Stewart
Interactions of energetic particles:
null vectors \(n=(1,0,0,1)\) and \(\bar{n}=(1,0,0,-1)\)
\(p=\left(n \cdot p, \bar{n} \cdot p, p_{\perp}\right)\) in light-cone components
Power counting in \(\lambda \ll 1\) :
\[
\begin{array}{rll}
n \text { - collinear: } & Q\left(\lambda^{2}, 1, \lambda\right) & p^{2}=Q^{2} \lambda^{2} \\
\bar{n} \text { - collinear: } & Q\left(1, \lambda^{2}, \lambda\right) & p^{2}=Q^{2} \lambda^{2} \\
\text { ultrasoft: } & Q\left(\lambda^{2}, \lambda^{2}, \lambda^{2}\right) & p^{2}=Q^{2} \lambda^{4}
\end{array}
\]

Have fields with labels, and multiple modes as in NRQCD.
Originally used for \(B\) decays, but now used for LHC processes such as jets, etc.
Next week's talk by Neubert

\section*{PART III}

\author{
Recent work
}

\section*{Extend SM to SMEFT}

\section*{Brivio, Trott for a recent review arXiv:1706.08945}
- Interest is in \(\Delta B=\Delta L=0\) processes at TeV energies
- Leading higher dimension operators are \(d=6\).
- Assuming \(B\) and \(L\) conservation, there are 76 independent hermitian dimension-six operators (not including flavor indices)
- 76 operators divided into eight operator classes.
\[

\]

Buchmuller \& Wyler 1986
Grzadkowski, Iskrzynski, Misiak and Rosiek 2010

\section*{Lots of Operators}

Field redefinitions (equations of motion) used to eliminate operators.
- 76 hermitian \(\Delta B=0\) operators, not including flavor indices.
- 2499 independent hermitian operators for \(n_{g}=3\)
- 1350 CP-even
- 1149 CP-odd terms
- 156 different irreducible flavor representations under
\[
S U\left(n_{g}\right)^{5}=S U\left(n_{g}\right)_{q} \otimes S U\left(n_{g}\right)_{\iota} \otimes S U\left(n_{g}\right)_{u} \otimes S U\left(n_{g}\right)_{d} \otimes S U\left(n_{g}\right)_{e}
\]
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Alonso, Jenkins, AM, Trott arXiv:1312.2014

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\section*{Power Counting for the RGE}

Amplitudes and anomalous dimensions obey power counting:
\[
\mu \frac{\mathrm{d}}{\mathrm{~d} \mu} C^{(6)} \propto C^{(6)}
\]

In the SM, because of the dimension two operator \(H^{\dagger} H\), have
\[
\mu \frac{\mathrm{d}}{\mathrm{~d} \mu} C^{(4)} \propto C^{(4)}+m_{H}^{2} C^{(6)}+\ldots
\]
\(\star\) SM parameter RG evolution affected by dim 6 terms at order \(m_{H}^{2} / M^{2}\). Just as important as dim 6 operators.
Jenkins, AM, Trott: arXiv:1308.2627

\section*{Anomalous Dimension Matrix}

Alonso, Jenkins, AM, Trott: arXiv:1308.2627, 1310.4838, 1312.2014
- Computed the running of the SM dimension-four terms due to dimension-six operators
- Computed the full dimension-six anomalous dimension matrix at one loop, including all Yukawa couplings, for general \(n_{g}\).
- The anomalous dimension matrix has a holomorphic structure Alonso, Jenkins, AM: arXiv:1409.0868
- \(G G G\) and \(G G \widetilde{G}\) have the same anomalous dimension
- Many zeros in the mixing matrix
- Cheung and Shen; Bern, Parra-Martinez, Sawyer

\section*{More General Extensions of the Higgs Sector}
\[
\begin{gathered}
H=\left[\begin{array}{l}
\phi^{+} \\
\phi^{0}
\end{array}\right]=\frac{1}{\sqrt{2}}\left[\begin{array}{c}
i \phi_{1}+\phi_{2} \\
\phi_{4}-i \phi_{3}
\end{array}\right], \quad \phi=\left[\begin{array}{l}
\phi_{1} \\
\phi_{2} \\
\phi_{3} \\
\phi_{4}
\end{array}\right] \\
L=\frac{1}{2} \partial_{\mu} \phi \cdot \partial^{\mu} \phi-\frac{\lambda}{4}\left(\phi \cdot \phi-v^{2}\right)^{2}
\end{gathered}
\]

Custodial symmetry: \(O(4) \sim S U(2) \times S U(2) \supset S U(2) \times U(1)\).
\(O(4) \rightarrow O(3)\) gives \(M_{W}=M_{Z} \cos \theta_{W}\).

\section*{HEFT}

A lot of work on theories with \(S U(2) \times U(1)\) broken symmetry broken by some other mechanism.
strongly coupled theories such as technicolor
An additional light scalar \(h\) added "by hand." Conformally invariant theories where the light scalar is a dilaton.
\(h\) couplings such as \(h \rightarrow Z Z\) not constrained to be their SM values.
SMEFT:
\(\phi=(v+h) \boldsymbol{n}\)
\(\boldsymbol{n} \in S^{3}\)
HEFT:
\(h, \boldsymbol{n}\)

\section*{Scalar Manifold}

Can unify both into a common approach:

\[
L=g_{i j}(\phi) \partial_{\mu} \phi_{i} \partial^{\mu} \phi_{j}
\]
\[
i \in\{a, h\} \quad \phi=\left(\pi^{a}, h\right)
\]

Scalar fields living on some general manifold \(\mathcal{M}\) with \(O(4)\) (custodial) symmetry.

We live at the black dot, and experiments probe the fields near the dot.

\section*{Observables}

In a QFT, the \(S\) matrix is unchanged under field redefinitions.
H.D. Politzer, NPB172 (1980) 349
\[
\phi=f\left(\phi^{\prime}\right) \quad L^{\prime}\left(\phi^{\prime}\right)=L\left(f\left(\phi^{\prime}\right)\right)
\]

Highly non-trivial result which follows from the LSZ reduction formula.
- Green's functions change
- Types of interactions can change, so the diagrams look different
- S-matrix elements are unchanged

The geometrical properties of a manifold are independent of the coordinate parameterization.

Make sure field redefinitions respect the symmetries - gauge, internal, spacetime.

\section*{Geometry}

Alonso, Jenkins, AM

Observables only depend on the geometry of \(\mathcal{M}\) :
\[
S \text { matrix } \longleftrightarrow \text { Geometry }
\]
field redefinitions \(\longleftrightarrow\) coordinate redefinitions

Experimentally measured quantities are given by the curvature of \(\mathcal{M}\).
Riemann is the local coordinate-independent [covariant] object on \(\mathcal{M}\).

\section*{Curvature of \(\mathcal{M}\)}

Sectional curvature:
\[
K(X, Y)=R_{a b c d} X^{a} Y^{b} X^{c} Y^{d}, \quad X \perp Y, \quad X \cdot X=1, \quad Y \cdot Y=1
\]

We have two independent sectional curvatures
\[
K_{\pi \pi}=K\left(X_{\pi}, Y_{\pi}\right) \quad K_{\pi h}=K\left(X_{\pi}, Y_{h}\right)
\]

\section*{Experimental Consequences}

Alonso, Jenkins, AM: arXiv:1511.00724
The scattering amplitude at high energies of longitudinal \(W\)-bosons \(W_{L}\) depends on the curvature (for \(g^{2} \ll \lambda\) ):
\[
\begin{gathered}
\mathcal{A}\left(W_{L} W_{L} \rightarrow W_{L} W_{L}\right)=-4 \lambda+(s+t) K_{\pi \pi} \\
\mathcal{A}\left(W_{L} W_{L} \rightarrow h h\right)=2 \lambda-2 s K_{\pi h}
\end{gathered}
\]

The scale of new physics governing the mass of these resonances is \(\Lambda \sim 4 \pi / \sqrt{K}\)

In composite Higgs models, the sign is fixed; \(K \geq 0\) since the sectional curvatures of \(G / H\) are positive.

Precision Higgs physics can measure the scalar manifold curvature, and test whether it is flat.
Alonso, Jenkins, AM: arXiv:1602.00706. Looked at HEFT on a hyperbolic manifold.
Helset, Martin, Trott: applied to SMEFT

\section*{Unitarity Bounds}

Bounds on EFT coefficients from unitarity: \(\operatorname{Im} \mathcal{A} \propto \sigma \geq 0\).


Inequalities on SMEFT coefficients. Remmen, Rodd

\section*{Coleman-Weinberg Effective Potential}

\author{
AM, Emily Nardoni
}

EFT approach to computing the Coleman-Weinberg effective potential for multi-scale problems.

Sum the leading log series
Approach is a bit different from that used in the literature

\section*{Learn More}

\author{
Effective Field Theories in Particle Physics and Cosmology \\ Les Houches Summer School \\ July 2017 \\ Edited by Sacha Davidson, Paolo Gambino, Mikko Laine, Matthias Neubert, and Christophe \\ Salomon
}

Burgess: Inflationary Cosmology
Baldauf: Large Scale Structure

Future talks in this series```

