Sum rules in SMEFT from helicity amplitudes

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Work in collaboration with Jiayin Gu 2008.07551

Work in progress with Jiayin Gu and Cen Zhang

All things EFT seminar. Oct 28. 2020

EFT, amplitude, dispersion

Standard, effective tool of parameterizing the IR effect of new physics.

Example: SMEFT

Connection to UV matching, RGE

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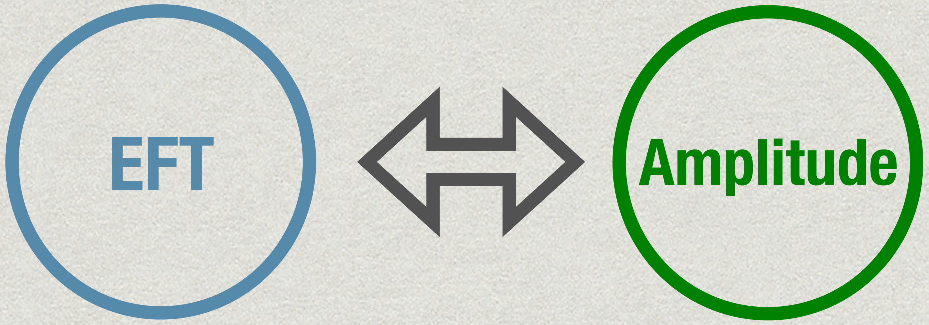

Alternative representation.

More direct connection with observables.

Shadmi, Weiss. 1809.09644
Ma, Shu, Xiao. 1902.06752
Aoude, Machado. 1905.11433
Durieux, Kitahara, Shadmi, Weiss. 1909.10511
Franken, Schwinn. 1910.13407
Falkowski. 1912.07865
Durieux, Machado. 1912.08827
Bachu, Yellespur. 1912.04334
Durieux, Kitahara, Machado, Shadmi, Weiss. 2008.09652

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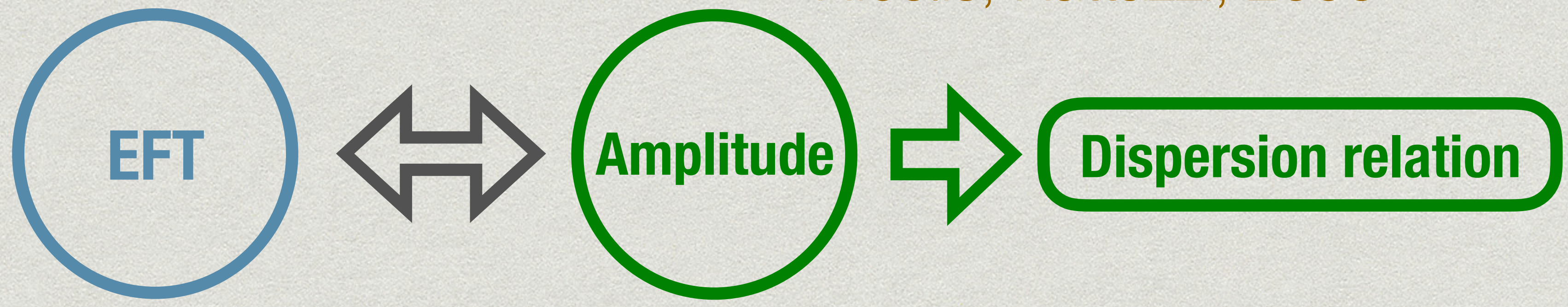
Example: SMEFT

Connection to UV matching, RGE

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006



Follows from general principles of QFT.

Leading to sum rules, positivity bounds.

Connection between IR measurement and UV completion

This talk

* Derive the sum rules for dim-6 SMEFT operators.
* Explore the consequences of sum rules.
* Mostly, work in the limit $E \gg m_W$. SM particles effectively massless.

Earlier work on sum rules:
Low, Rattazzi, Vichi, 0907.5413
Falkowski, Rychkov, Urbano, 1202.1532
Bellazzini, Martucci, Torre, 1405.2906

Sum rules, elastic amplitudes

Forward elastic amplitude: $\tilde{\mathcal{A}}_{ab}(s) \equiv \mathcal{A}(ab \to ab)|_{t=0}$

expand $\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s-\mu^2)^n\,,$ $\qquad c_n = \frac{1}{2\pi i}\oint_{s=\mu^2} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s-\mu^2)^{n+1}}$

deform the contour

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma_{\rm tot}^{ab}}{(s-\mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\rm tot}^{a\bar{b}}}{(s-4m^2+\mu^2)^{n+1}} \right) + c_n^{\infty}$$

$$c_n^{\infty} \equiv \frac{1}{2\pi i}\oint_{s\to\infty} ds \frac{\mathcal{A}(s)}{(s-\mu^2)^{n+1}}\,,$$

$m \sim$ SM particle mass, $\mu \sim$ energy of the experiment, $\Lambda \sim$ scale of NP

$$m \ll \mu \ll \Lambda$$

Sum rules, elastic amplitudes

Forward elastic amplitude: $\tilde{\mathcal{A}}_{ab}(s) \equiv \mathcal{A}(ab \to ab)|_{t=0}$

$$\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s-\mu^2)^n \,, \qquad c_n = \frac{1}{2\pi i} \oint_{s=\mu^2} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s-\mu^2)^{n+1}}$$

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma_{\text{tot}}^{ab}}{(s-\mu^2)^{n+1}} + (-1)^n \frac{\sigma_{\text{tot}}^{a\bar{b}}}{(s-4m^2+\mu^2)^{n+1}} \right) + c_n^\infty$$

dim-6: *n=1*

$$\left. \frac{d\tilde{\mathcal{A}}_{ab}(s)}{ds} \right|_{s=0} = \int_0^\infty \frac{ds}{\pi s} \left(\sigma_{\text{tot}}^{ab} - \sigma_{\text{tot}}^{a\bar{b}} \right) + c_\infty$$

EFT and amplitude

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

Contribution to n-point amplitude can be written as

$$\mathcal{A}_n = \sum_i g_{[i]}\mathcal{A}_n^{[4-n-i]}$$

$\mathcal{A}_n$ n-point amplitude with dimension *4-n*

$g_{[i]}$ coupling with dimension $\boldsymbol{i}$

$\mathcal{A}_n^{[4-n-i]}$ sub-amplitude with coupling stripped

4-point amplitude

$$\mathcal{A}_4 = g_{[0]}\mathcal{A}_4^{[0]} + g_{[-2]}\mathcal{A}_4^{[2]} + g_{[-4]}\mathcal{A}_4^{[4]} + ...$$

$g_{[0]}\mathcal{A}_4^{[0]}$ SM contribution with dimensionless coupling

$g_{[-2]}\mathcal{A}_4^{[2]}$ can come from 1 insertion of dim-6 operator with $g_{[-2]} \propto 1/\Lambda^2$

$g_{[-4]}\mathcal{A}_4^{[4]}$ dim-8

4-point amplitude

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$g_{[0]}\mathcal{A}_4^{[0]}$ SM contribution with dimensionless coupling

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Of the form of 4-point contact interaction

General statement: all SMEFT contribution to elastic scattering is of the contact form.

Two 3-point amplitudes?

$$\mathcal{A}_3 = g_{[0]}\mathcal{A}_3^{[1]} + g_{[-2]}\mathcal{A}_3^{[3]} + \ldots$$

$$\mathcal{A}_3^{[1]}\,\frac{1}{p^2}\,\mathcal{A}_3^{[3]}$$

$$\mathcal{O}_{3W} = \frac{1}{3!} g\epsilon_{abc} W_{\mu}^{a\,\nu} W_{\nu\rho}^{b} W^{c\,\rho\mu}$$

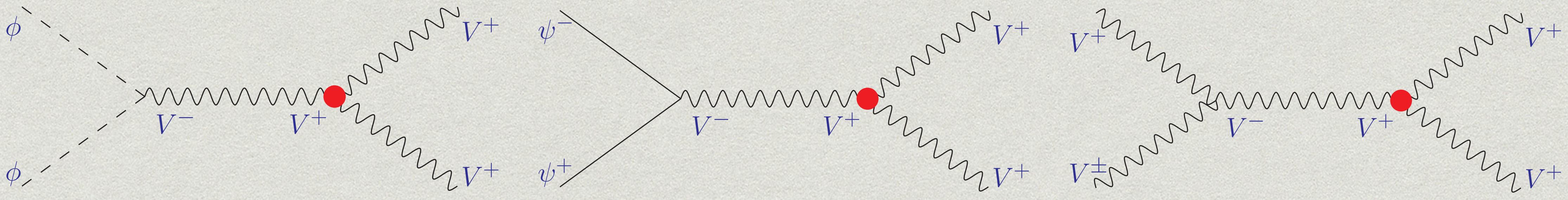
$$\mathcal{O}_{3\widetilde{W}} = \frac{1}{3!} g\epsilon_{abc} \widetilde{W}_{\mu}^{a\,\nu} W_{\nu\rho}^{b} W^{c\,\rho\mu}$$

⇒

$$\mathcal{A}(V^+V^+V^+)$$

$$\mathcal{A}(V^-V^-V^-)$$

Not elastic!



Two 3-point amplitudes?

more possible $\mathcal{A}_3^{[1]} \frac{1}{p^2} \mathcal{A}_3^{[3]}$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}\,, \qquad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$$

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overleftrightarrow{D}_\mu H) D^\nu W^a_{\mu\nu}\,, \qquad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overleftrightarrow{D}_\mu H) \partial^\nu B_{\mu\nu}\,,$$

Can generate **ϕϕ**V type 3 point couplings.
However, $1/p^2$ cancelled by p^2 from this vertex

Still of the form of 4 point contact interaction.

Possible elastic amplitudes

Contact interaction + little group scaling fixes the form of amplitude

elastic 4-point amplitudes $\mathcal{A}(1\,2 \;\to\; 3_{=1}\,4_{=2})$	spinor form of $\mathcal{A}_4^{[2]}$ (d6 operators)	spinor form of $\mathcal{A}_4^{[4]}$ (d8 or d6^2)
$\phi_1\phi_2\phi_1^*\phi_2^*$	s_{ij}	$s_{ij}\times s_{kl}$
$\psi^-\phi\psi^+\phi^*$	$\langle 12\rangle[23]$	$\langle 12\rangle[23]\times s_{ij}$
$\psi_1^-\psi_2^-\psi_1^+\psi_2^+$	$\langle 12\rangle[34]$	$\langle 12\rangle[34]\times s_{ij}$
$V^-\phi V^+\phi^*$	✗	$\langle 12\rangle^2[23]^2$
$V^-\psi^-V^+\psi^+$	✗	$\langle 12\rangle^2[23][34]$
$V_1^-V_2^-V_1^+V_2^+$	✗	$\langle 12\rangle^2[34]^2\,,\;\langle 12\rangle^2[34]^2\frac{t-u}{s}$

In the forward limit:

$$\tilde{\mathcal{A}}_4^{[2]} \equiv \mathcal{A}_4^{[2]}|_{t\to 0} \propto s\,, \qquad \tilde{\mathcal{A}}_4^{[4]} \equiv \mathcal{A}_4^{[4]}|_{t\to 0} \propto s^2\,.$$

Higgs-Higgs(Goldstone) amplitudes

Two independent amplitudes

$$\mathcal{A}(H_i H_j H_i^\dagger H_j^\dagger) = c_s\, s + c_u\, u$$

$$\mathcal{A}(H_i H_i H_i^\dagger H_i^\dagger) = 2c_s\, s + c_u\, t + c_u\, u$$

In the forward limit, gives amplitude proportional to

$$c_s - c_u\,, \qquad 2c_s - c_u$$

Sum rules

$$\mathcal{O}_H = \tfrac{1}{2}(\partial_\mu |H|^2)^2 \qquad \Big| \qquad \mathcal{O}_T = \tfrac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$$

$$\mathcal{A}^{[2]}(\phi^+\phi^- \to \phi^+\phi^-) = \frac{c_H + 3c_T}{\Lambda^2} s\,,$$

$$\mathcal{A}^{[2]}(\phi^+\phi^0 \to \phi^+\phi^0) = -\frac{c_H + c_T}{\Lambda^2} s - \frac{c_H - c_T}{\Lambda^2} u$$

$$c_s \to -\frac{c_H + c_T}{\Lambda^2}\,, \qquad c_u \to -\frac{c_H - c_T}{\Lambda^2}$$

$$\frac{c_H + 3c_T}{\Lambda^2} = \frac{d\tilde{\mathcal{A}}_{\phi^+\phi^-}}{ds}\bigg|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\text{tot}}^{\phi^+\phi^-} - \sigma_{\text{tot}}^{\phi^+\phi^+}\right) + c_\infty$$

$$-\frac{2c_T}{\Lambda^2} = \frac{d\tilde{\mathcal{A}}_{\phi^+\phi^0}}{ds}\bigg|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\text{tot}}^{\phi^+\phi^0} - \sigma_{\text{tot}}^{\phi^+\phi^{0*}}\right) + c_\infty$$

Sum rules

$$\mathcal{O}_H = \frac{1}{2}(\partial_\mu |H|^2)^2 \qquad \Big| \qquad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D_\mu} H)^2$$

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Low, Rattazzi, Vichi, 0907.5413

Higgs-fermion

$\mathcal{O}_{H\ell} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{\ell}_L \gamma^\mu \ell_L$ $\mathcal{O}'_{H\ell} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{\ell}_L \sigma^a \gamma^\mu \ell_L$	$\mathcal{O}_{He} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{e}_R \gamma^\mu e_R$
$\mathcal{O}_{Hq} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{q}_L \gamma^\mu q_L$ $\mathcal{O}'_{Hq} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}_L \sigma^a \gamma^\mu q_L$	$\mathcal{O}_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{u}_R \gamma^\mu u_R$ $\mathcal{O}_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H \bar{d}_R \gamma^\mu d_R$

$$\frac{2(c_{Hq} - c'_{Hq})}{\Lambda^2} = \left.\frac{d\tilde{\mathcal{A}}_{u_L\,\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{u_L\,\phi^0} - \sigma_{\rm tot}^{u_L\,\phi^{0*}}\right) + c_\infty$$

$$\frac{2c_{Hu}}{\Lambda^2} = \left.\frac{d\tilde{\mathcal{A}}_{u_R\,\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{u_R\,\phi^0} - \sigma_{\rm tot}^{u_R\,\phi^{0*}}\right) + c_\infty$$

$$\frac{2(c_{Hq} + c'_{Hq})}{\Lambda^2} = \left.\frac{d\tilde{\mathcal{A}}_{d_L\,\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{d_L\,\phi^0} - \sigma_{\rm tot}^{d_L\,\phi^{0*}}\right) + c_\infty$$

$$\frac{2c_{Hd}}{\Lambda^2} = \left.\frac{d\tilde{\mathcal{A}}_{d_R\,\phi^0}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{d_R\,\phi^0} - \sigma_{\rm tot}^{d_R\,\phi^{0*}}\right) + c_\infty$$

+ similar ones for leptons

4-fermion

Similar classification and counting leads to 20 sum rules, from amplitudes with $\psi^+\psi^+\psi^-\psi^-$ helicity configuration.

For example:

$$\frac{c_{ee}}{\Lambda^2}(\overline{e_R}\gamma_\mu e_R)(\overline{e_R}\gamma^\mu e_R)$$

$$-\frac{2c_{ee}}{\Lambda^2} = \left.\frac{d\tilde{\mathcal{A}}_{e_R\,\overline{e_R}}}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma_{\rm tot}^{e_R\,\overline{e_R}} - \sigma_{\rm tot}^{e_R\,e_R}\right) + c_\infty$$

IR vs UV contribution

* IR contribution from SM: low energy poles, forward divergences, etc.
* Contribute to both sides of the sum rule.
* Known physics, can be computed and subtracted.

Boundary term

$$c_\infty = \frac{1}{2\pi i} \oint_{s\to\infty} ds \frac{\tilde{A}(s)}{s^2}$$

Froissart bound: $\mathscr{A} < s\log^2 s$ **as** $|s| \to \infty \Rightarrow c_n^\infty = 0$ **for** $n > 1.$

n=1, some model dependence

t-channel vector $\quad \tilde{A}(s) \to \left.\frac{-g\,s}{t - M^2}\right|_{t=0} = \frac{g\,s}{M^2} \quad \Rightarrow \quad c_\infty = \frac{g}{M^2}$

Precision vs direct search

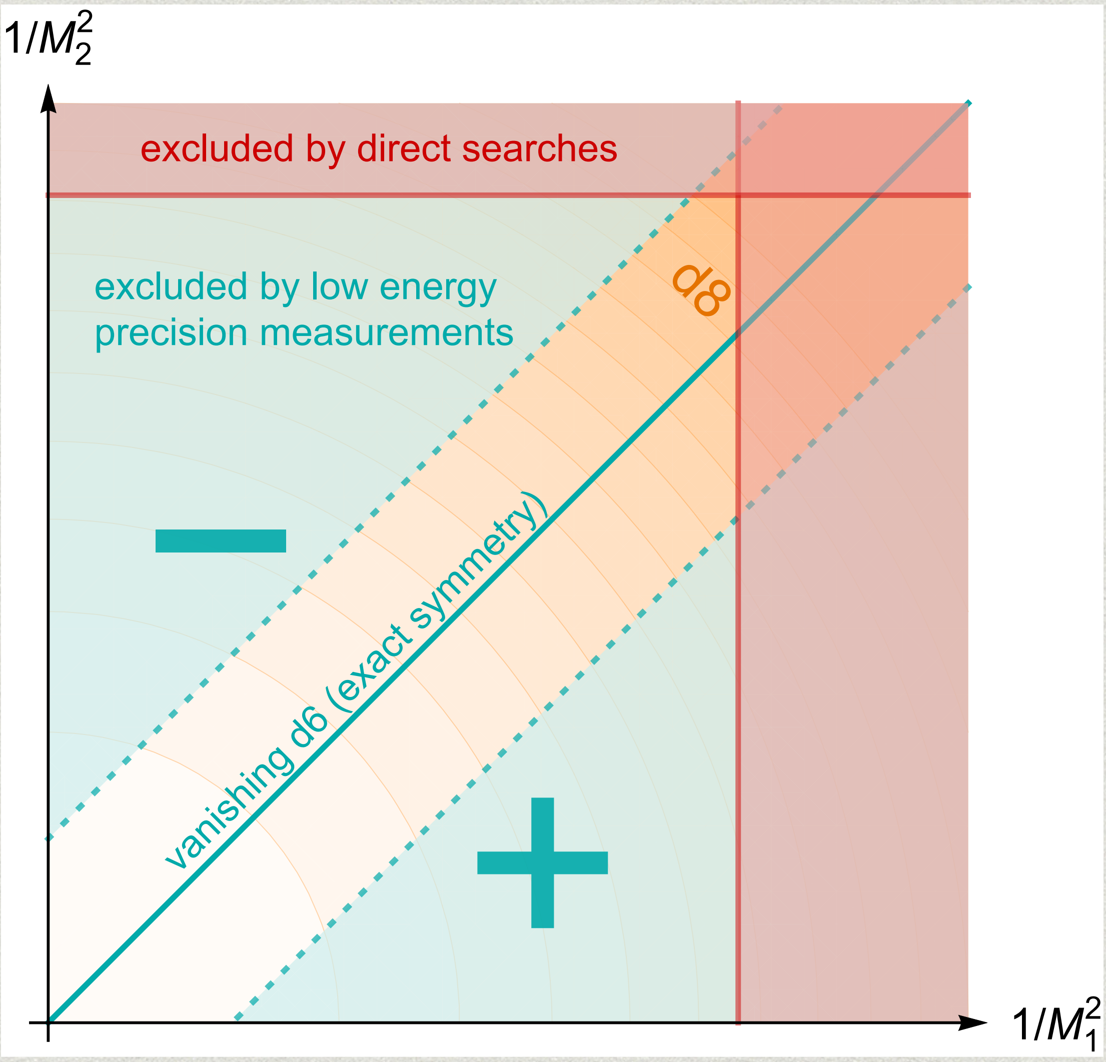
Schematically

$$\delta g \propto \frac{c}{M_{1,2}^2} = \left.\frac{dA}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{s\pi}\left(\sigma(ab \to X_1) - \sigma(a\bar{b} \to X_2)\right) + \ldots$$

δg shift in low energy coupling

$X_{1,2}$ NP particles with masses $M_{1,2}$

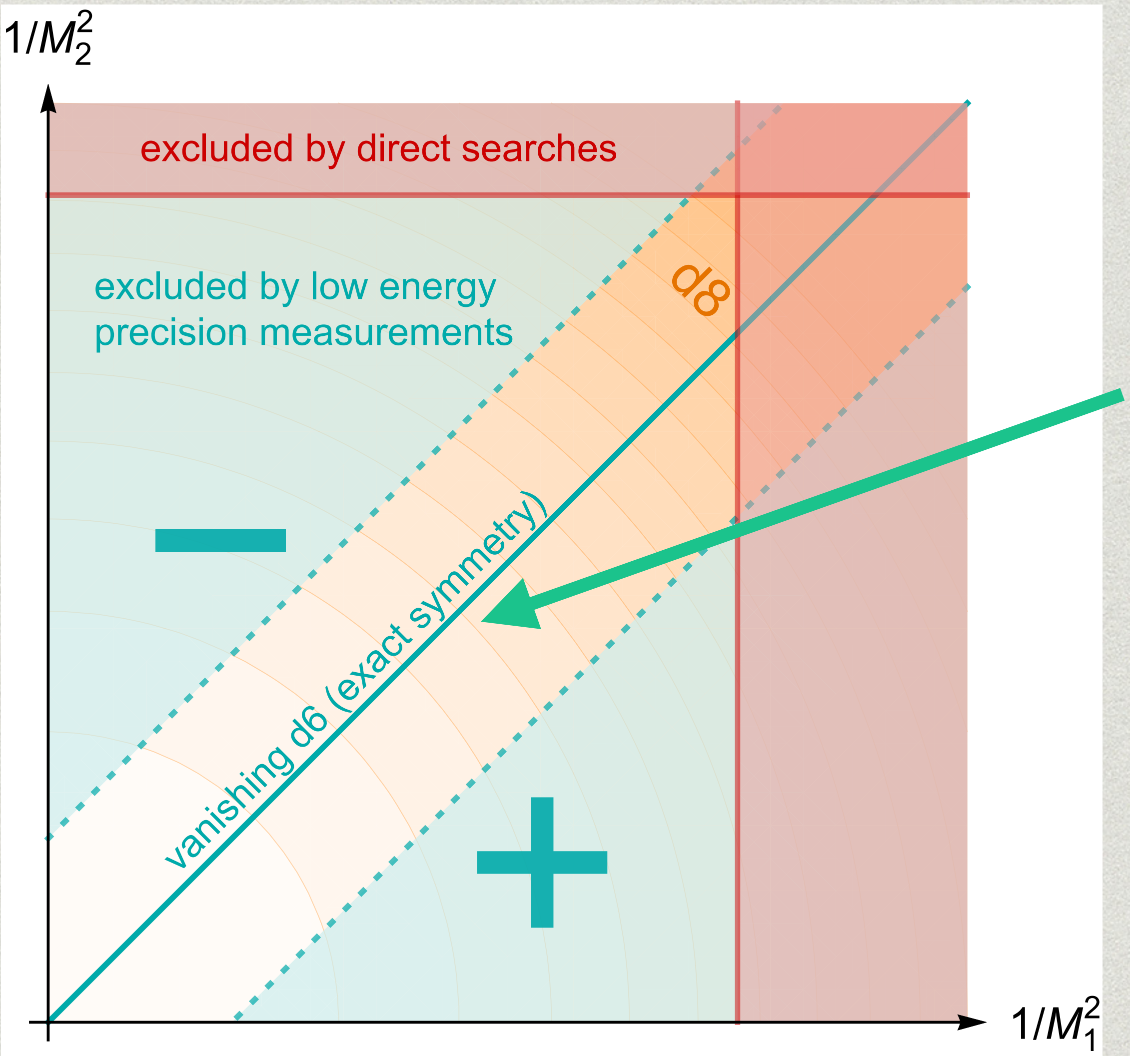
Precision vs direct search



$$\delta g \propto \frac{c}{M_{1,2}^2} = \frac{dA}{ds}\bigg|_{s=0} = \int_0^\infty \frac{ds}{s\pi}\left(\sigma(ab \to X_1) - \sigma(a\bar{b} \to X_2)\right)$$

$X_{1,2}$ mass: $M_{1,2}$

Precision vs direct search



Possible cancellation can occur for dim-6 contribution. Possible to implement a symmetry.

Not possible for dim-8. due to positivity.

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006

$$\delta g \propto \frac{c}{M_{1,2}^2} = \left.\frac{dA}{ds}\right|_{s=0} = \int_0^\infty \frac{ds}{s\pi}\left(\sigma(ab \to X_1) - \sigma(a\bar{b} \to X_2)\right)$$

$X_{1,2}$ mass: $M_{1,2}$

Custodial symmetries

$$\tilde{\mathcal{A}}^{[2]}_{ab}(s) \underset{s\leftrightarrow u}{=} \tilde{\mathcal{A}}^{[2]}_{a\bar{b}}(u) = \tilde{\mathcal{A}}^{[2]}_{a\bar{b}}(-s) = -\tilde{\mathcal{A}}^{[2]}_{a\bar{b}}(s)$$

Vanishes if there is a symmetry *S*:

$$\tilde{\mathcal{A}}^{[2]}_{ab}(s) = \tilde{\mathcal{A}}^{[2]}_{a\bar{b}}(s) \qquad \text{under } \mathcal{S} : a \leftrightarrow a\,, \quad b \leftrightarrow \bar{b}$$

Possibilities of such a symmetry

- a and b share a set of quantum numbers, labelled as i.
  a and b have charge σ_a^i and σ_b^i .

Under symmetry S $\quad \sigma_a^i \to \sigma_a^i\,, \qquad \sigma_b^i \to -\sigma_b^i$

Or, equivalently $\quad \mathcal{S}' : \quad a \to \bar{a}\,, \quad b \to b\,.$

Examples of symmetries

Custodial symmetry of the Higgs:

$$\begin{array}{ccc} t_{3L}\backslash t_{3R} & 1/2 & -1/2 \\ 1/2 & \phi^+ & \phi^{0*} \\ -1/2 & \phi^0 & -\phi^- \end{array}, \qquad \text{where } H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$P_{LR}: \quad \phi^+ \to \phi^+, \quad \phi^0 \to \phi^{0*}$$

Custodial symmetry of fermions

$$P_{LR}: \quad f \to f, \quad \phi^0 \to \phi^{0*}$$

$$T_L^3 = T_R^3 = 0\,, \qquad or \qquad T_L = T_R\,, \quad T_L^3 = T_R^3$$

Agashe, Contino, Da Rold, Pomarol hep/ph/0605341

Matching and RGE

- EFT matching can be carried out at different orders (tree, 1-loop, etc.).
- O_1 is renormalized by O_2 if O_2 gives a divergent to helicity amplitude which can come from a contact contribution of O_1 .

  Cheung, Shen, 1505.01844
  Craig, Jiang, Li, Sutherland, 2001.00017
- Dispersion relation with loop amplitudes should capture some of these information.

Example

Beautiful mirror

Choudhury, Tait, Wagner, hep-ph/0109097

Introducing a set of new fermions

$$\Psi_{L,R} = \begin{pmatrix} B \\ X \end{pmatrix} \sim (3, 2, -5/6)$$

$$\hat{B}_{L,R} \sim (3, 1, -1/3)\,,$$

which mix with the third gen. quarks after EWSB

$$-\mathcal{L} \supset M_1 \bar{\Psi}_L \Psi_R + M_2 \bar{\hat{B}}_L \hat{B}_R + y_L \bar{Q}_L H \hat{B}_R + y_R \bar{\Psi}_L \tilde{H} b_R + \text{h.c.}$$

Sum rules, leading order

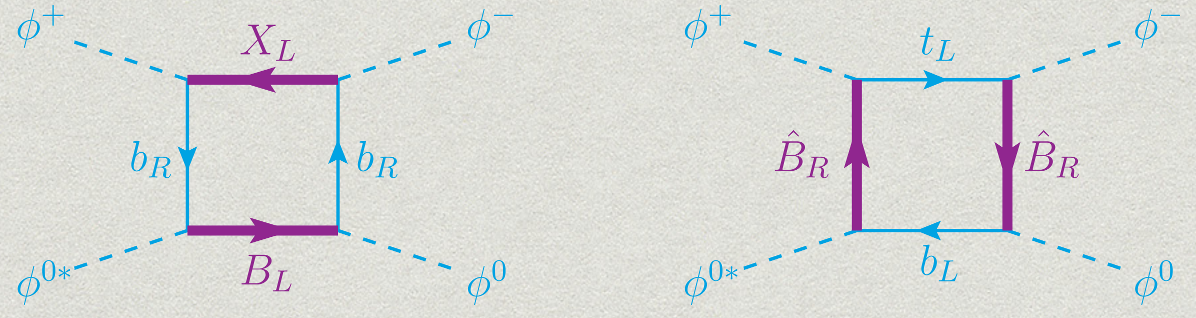
$$\frac{4\,\delta g_{Lb}}{v^2} = -\frac{2(c_{Hq} + c'_{Hq})}{\Lambda^2} = \frac{d\tilde{\mathcal{A}}_{t_L\,\phi^-}}{ds}\bigg|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{t_L\,\phi^- \to F^{-\frac{1}{3}}} - \sigma^{t_L\,\phi^+ \to F^{\frac{5}{3}}}\right)$$

$$\frac{4\,\delta g_{Rb}}{v^2} = -\frac{2c_{Hd}}{\Lambda^2} = \frac{d\tilde{\mathcal{A}}_{b_R\,\phi^-}}{ds}\bigg|_{s=0} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{b_R\,\phi^- \to F^{-\frac{4}{3}}} - \sigma^{b_R\,\phi^+ \to F^{\frac{2}{3}}}\right)$$

Evaluating RHS ⇒ $\delta g_{Lb} = \frac{y_L^2 v^2}{4M_2^2}\,, \qquad \delta g_{Rb} = \frac{y_R^2 v^2}{4M_1^2}$

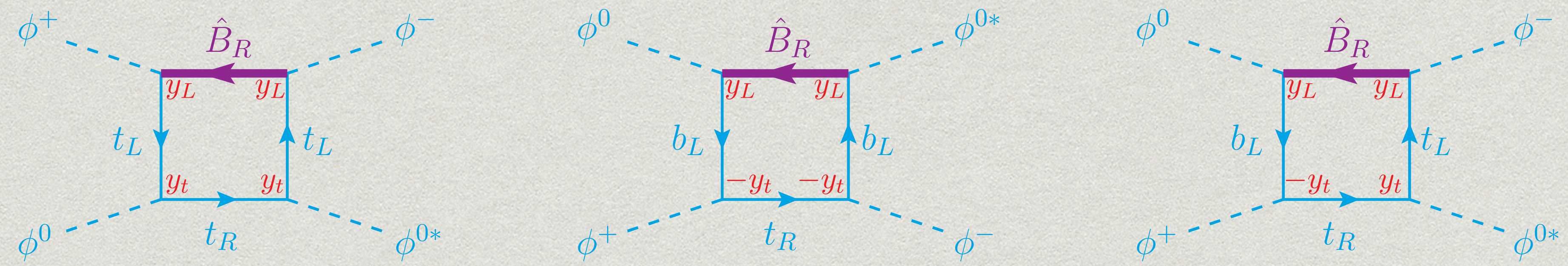
Agreeing with integrating out heavy fermions at tree level.

1-loop $\mathcal{A}^{[2]}(\phi^+\phi^0 \;\to\; \phi^+\phi^0)$



$$
\begin{aligned}
\left.\frac{d\tilde{\mathcal{A}}_{\phi^+\phi^0}}{ds}\right|_{s=0} &= \int_0^\infty \frac{ds}{\pi s}\left(0 - \sigma^{\phi^+\phi^{0*}\to\overline{X}_L B_L} - \sigma^{\phi^+\phi^{0*}\to\bar{t}_L b_L}\right) \\
&= -\,\frac{y_R^4}{8\pi^2\,M_1^2} - \frac{3\,y_L^4}{32\pi^2\,M_2^2}\,.
\end{aligned}
$$

1-loop



$$
\begin{aligned}
\left.\frac{d\tilde{\mathcal{A}}_{\phi^+\phi^0}}{ds}\right|_{s=0} &= \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{\phi^+\phi^0\to\overline{B}_R t_R} - \sigma^{\phi^+\phi^{0*}\to\bar{t}_L b_L}\right) \\
&= \frac{3y_t^2 y_L^2}{16\pi^2 M_2^2}\left[\left(2\log(\frac{M_2^2}{m_t^2}) - \frac{13}{6} + \ldots\right) - \left(\log(\frac{M_2^2}{m_t^2}) - 2 + \ldots\right)\right] \\
&= \frac{3y_t^2 y_L^2}{16\pi^2\, M_2^2}\left[\log\left(\frac{M_2^2}{m_t^2}\right) - \frac{1}{6}\right] + \mathcal{O}(\frac{m_t^2}{M_2^4})\,.
\end{aligned}
$$

T-parameter

$$\left.\frac{d\tilde{A}_{\phi^+\phi^0}}{ds}\right|_{s=0} = -\frac{2c_T}{\Lambda^2} = \frac{-2\alpha T}{v^2}$$

$$T \approx \frac{3}{16\pi^2\alpha v^2}\left[\frac{16}{3}\delta g_{Rb}^2 M_1^2 + 4\delta g_{Lb}^2 M_2^2 - 4\delta g_{Lb}\frac{M_2^2\, m_{\rm top}^2}{M_2^2 - m_{\rm top}^2}\log\left(\frac{M_2^2}{m_{\rm top}^2}\right)\right]$$

$$\delta g_{Lb} = \frac{y_L^2 v^2}{4M_2^2}\,, \qquad \delta g_{Rb} = \frac{y_R^2 v^2}{4M_1^2}$$

Agree with 1-loop calculation in the full theory

The log term: RGE running

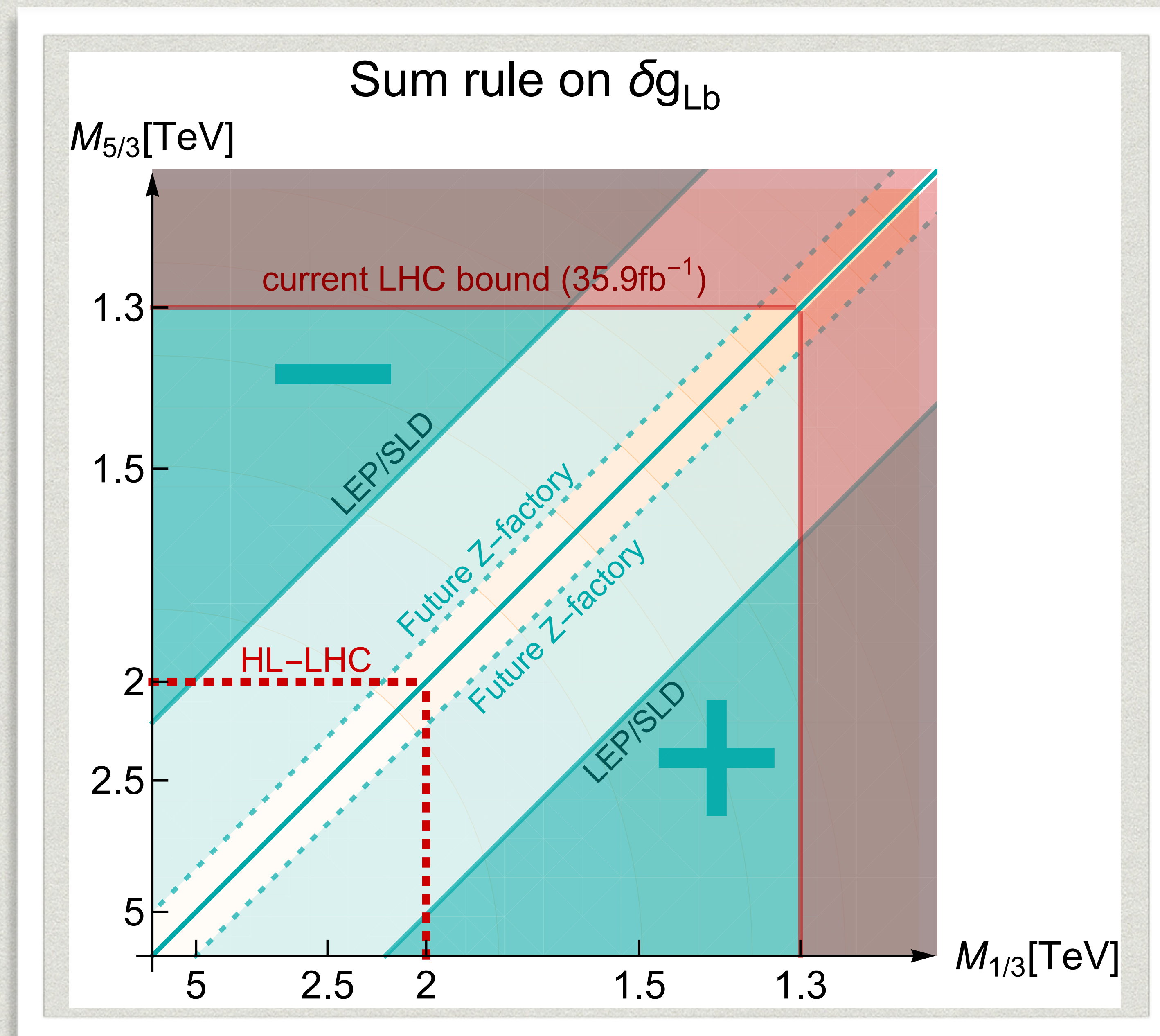
$$c_T(\mu) = c_T(\mu_0) - \frac{3y_t^2}{8\pi^2}(-c'_{Hq} + c_{Hu} + c_T)\log(\frac{\mu_0^2}{\mu^2})$$

In this model $\frac{c'_{Hq}}{\Lambda^2} = -\frac{y_L^2}{4M_2^2}\,, \qquad c_{Hu} = 0$

RGE running generates $\left.\frac{d\tilde{\mathcal{A}}_{\phi^+\phi^0}}{ds}\right|_{s=0} = \frac{-2\left(c_T(\mu) - c_T(\mu_0)\right)}{\Lambda^2} = \frac{3y_t^2 y_L^2}{16\pi^2 M_2^2}\log(\frac{\mu_0^2}{\mu^2})$

RGE effect captured by dispersion relation calculation.

Precision vs direct search



Probing dim-8?

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, 2006. + many.

- Dim-8 operators have positivity bound.
    - Stronger limit on the parameter space.
    - Could be a cleaner test of general properties (Unitarity, locality, analyticity) of UV completion.
- Unfortunately, typically, dim-6 will dominate a process. Hard to see the effect of dim-8.

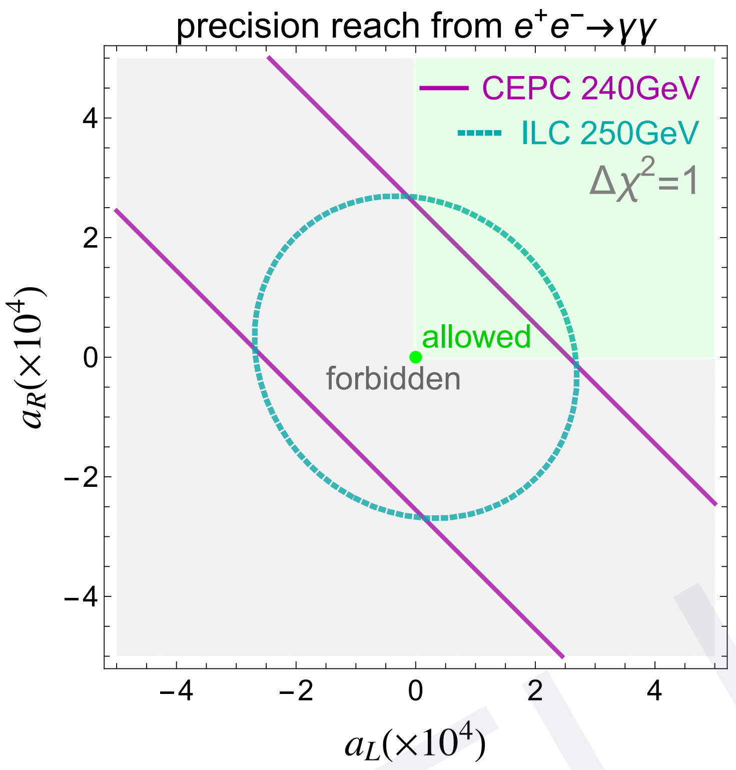
The e^+e^- → γγ channel

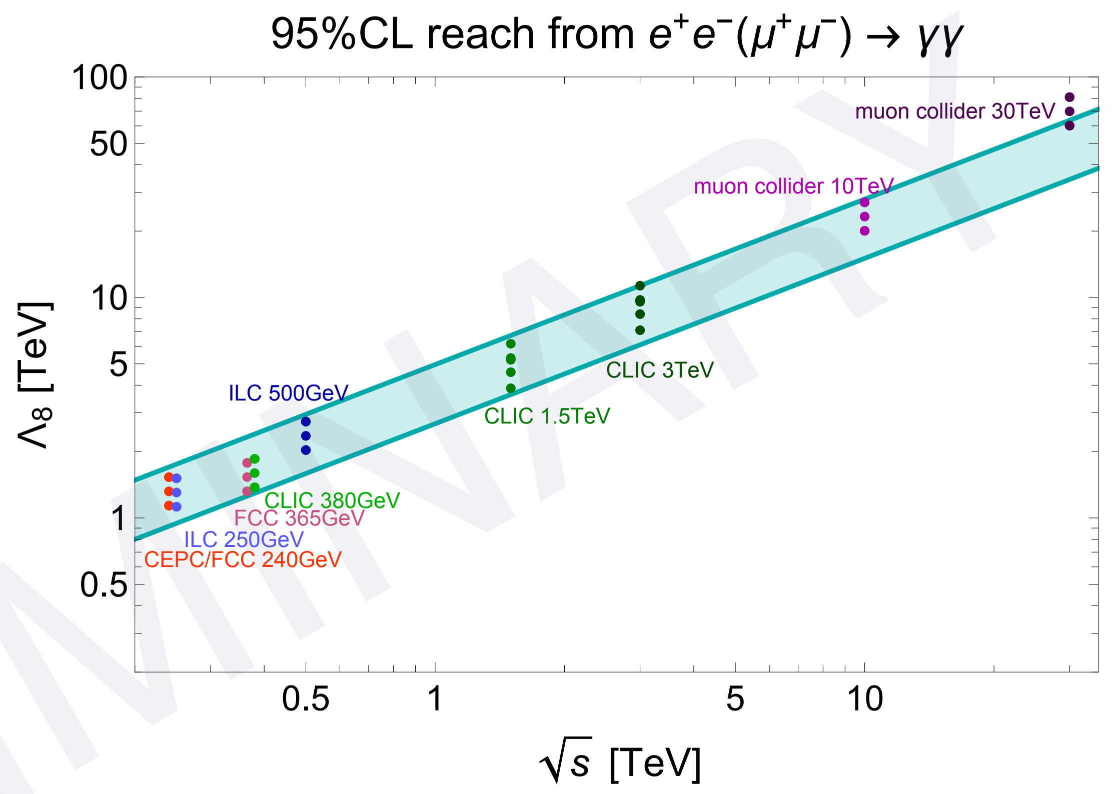
Work in progress with Jiayin Gu and Cen Zhang, 2011.xxxxx

- Effect from dim-6 operator either vanishing or suppressed.
  - Due to the nature of the amplitude and the experimental constraints.
- SM × dim-8 interference is the leading channel.
- Positivity bound on dim-8 leads to prediction

$$\sigma(e^+e^- \to \gamma\gamma) > \sigma_{\rm SM}(e^+e^- \to \gamma\gamma)$$

Dimension 8 operators? (current work, JG, C. Zhang and L.-T. Wang)





- Positivity bounds resolve the flat direction between a_L and a_R for unpolarized beams.
- Best reach still from high energy colliders.

$$\mathcal{O}^{(8)}_{\ell B} = -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \text{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{eB} = -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \text{h.c.})B_{\mu\nu}B^{\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{\ell W} = -\frac{1}{4}(i\bar{\ell}_L\gamma^{\{\rho}D^{\nu\}}\ell_L + \text{h.c.})W^a_{\mu\nu}W^{a\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{eW} = -\frac{1}{4}(i\bar{e}_R\gamma^{\{\rho}D^{\nu\}}e_R + \text{h.c.})W^a_{\mu\nu}W^{a\mu}{}_{\rho},$$
$$\mathcal{O}^{(8)}_{\ell BW} = -\frac{1}{4}(i\bar{\ell}_L\sigma^a\gamma^{\{\rho}D^{\nu\}}\ell_L + \text{h.c.})B_{\mu\nu}W^{a\mu}{}_{\rho},$$

$$a_L = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{\ell B} - \cos\theta_W\sin\theta_W\, c^{(8)}_{\ell BW} + \sin^2\theta_W\, c^{(8)}_{\ell W}\right),$$
$$a_R = \frac{v^4}{\Lambda^4}\left(\cos^2\theta_W\, c^{(8)}_{eB} + \sin^2\theta_W\, c^{(8)}_{eW}\right),$$

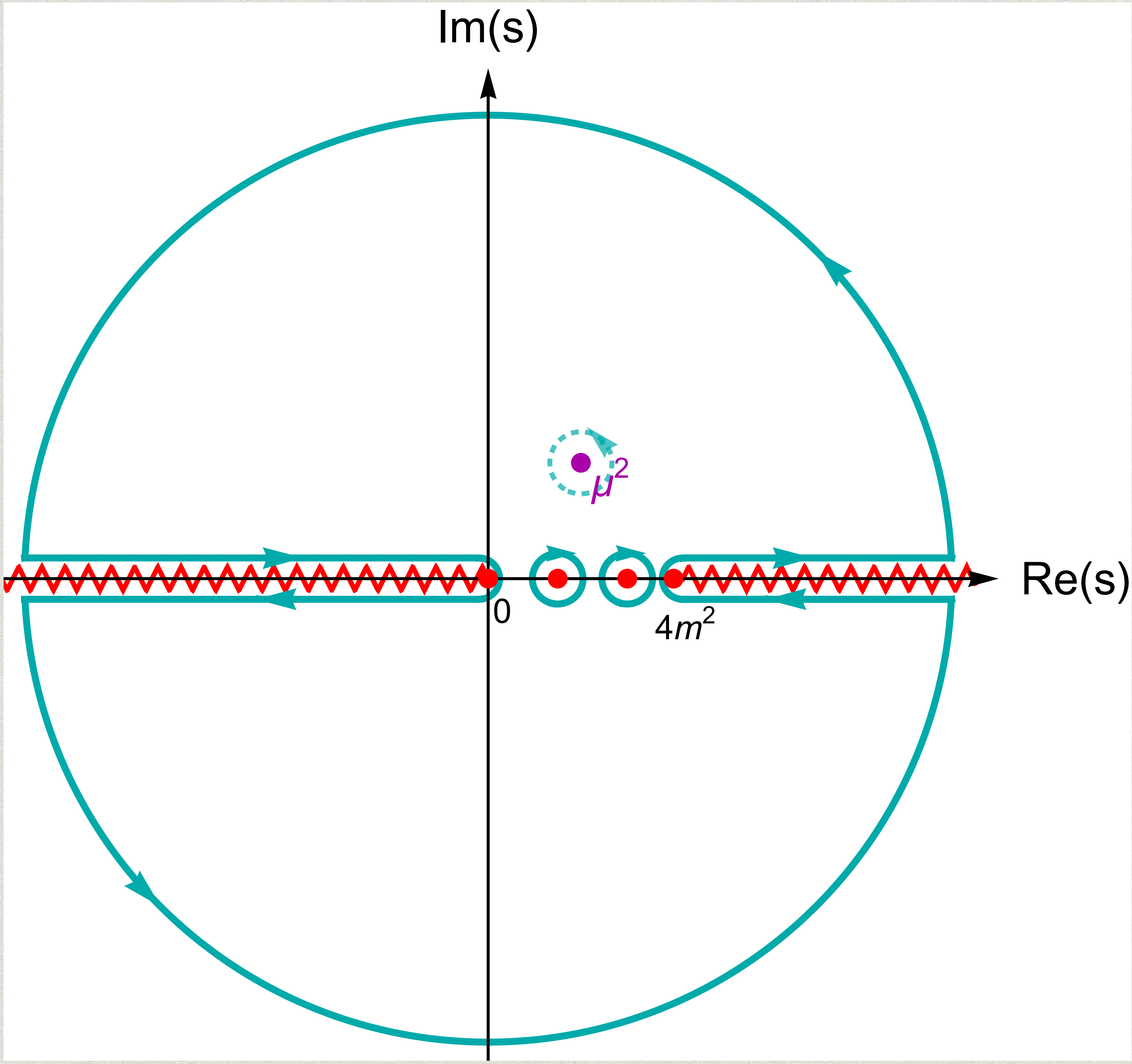
Conclusions

- General principles of QFT ⇒ dispersion relation and sum rule.
  - An interesting angle in connecting IR precision measurement and UV completion.
- Interesting to investigate more general lessons by going beyond forward elastic scattering.

Arkani-Hamed, Huang: "EFT-hedra"
Remmen, Rodd, 2010.04723

- Interesting to identify unambiguous exp tests, such as positivity of dim-8.

extra


Im(s)
Re(s)
μ^2
0
$4m^2$

Georgi-Machacek

$$\mathcal{L}_{\rm int} = \kappa_\xi H^\dagger \sigma^a H \xi_a + \frac{\kappa_\chi}{\sqrt{2}}(\widetilde{H}^\dagger \sigma^a H \chi_a + \text{h.c.})$$

$\xi_a : 3_0, \quad \chi_a : 3_{-1}$

$$\frac{c_H + 3c_T}{\Lambda^2} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{\phi^+\phi^-\to\xi^0} - \sigma^{\phi^+\phi^+\to\chi^{++}}\right) = \frac{\kappa_\xi^2}{m_\xi^4} - \frac{4\kappa_\chi^2}{m_\chi^4}$$

$$-\frac{2c_T}{\Lambda^2} = \int_0^\infty \frac{ds}{\pi s}\left(\sigma^{\phi^+\phi^0\to\chi^+} - \sigma^{\phi^+\phi^{0*}\to\xi^+}\right) = \frac{2\kappa_\chi^2}{m_\chi^4} - \frac{2\kappa_\xi^2}{m_\xi^4}$$

$c_T = 0$ if $\kappa_\xi = \kappa_\chi$, $m_\xi = m_\chi$