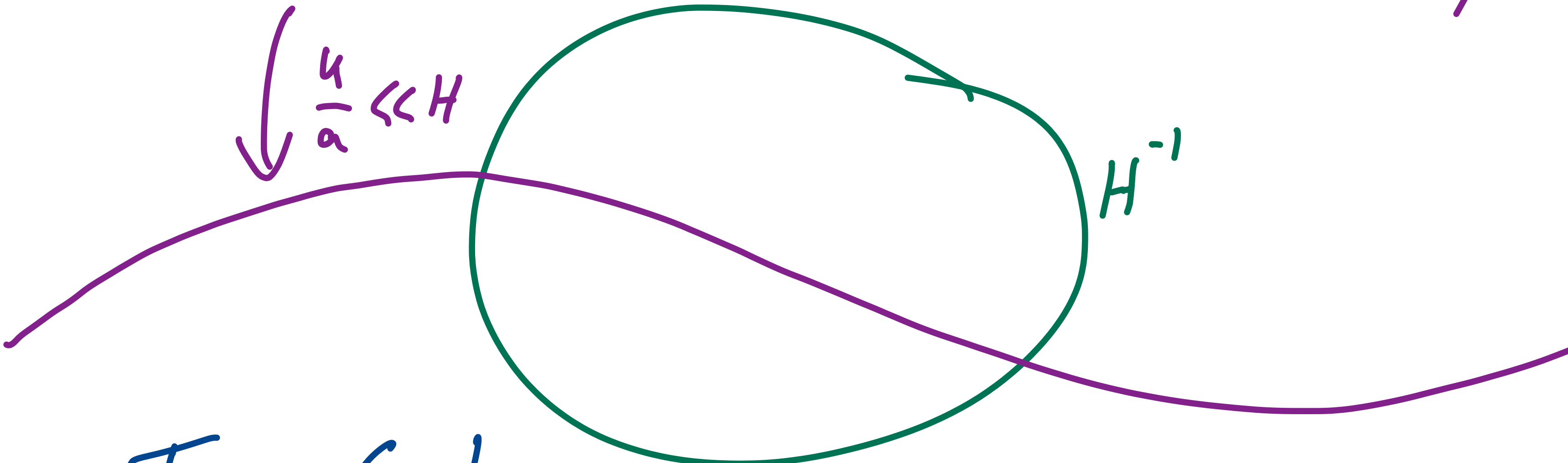
Soft de Sitter Effective Theory



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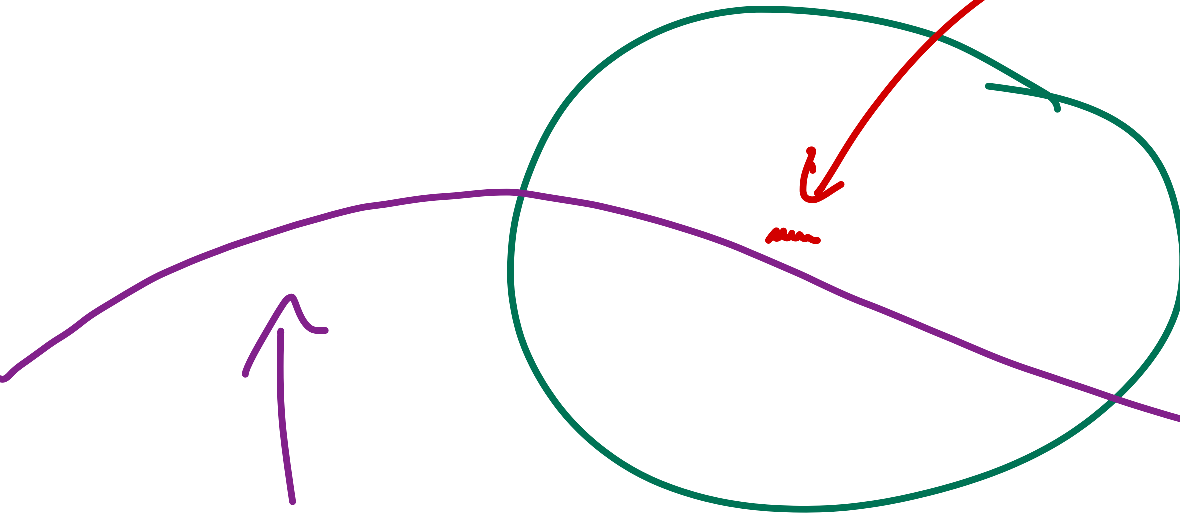
All Things EFT, Nov 18

$$k_{physical} = \frac{k}{a(t)}$$

UV modes

$$\frac{k}{a} \gg H$$



Modes: φ_+ , φ_-

SdSET

$$\frac{k}{a} \ll H$$

$\sim H^{-1}$

Symmetries:
Spacetime +
reparametrization

* Initial Conditions

The IR of QFT in de Sitter

Conceptual
- What DOFs emerge?
- What governs their dynamics?
- What symmetries persist?

Practicle
- How are operators organized?
- Can divergent integrals be tamed?
- Can IR logs be systematically summed?

Confusions Abound

- Want to expose late time and long wavelength behavior of (in-in) correlation functions

- Calculate in a frame
  $\Rightarrow$ space + time treated independently

- Full theory calculations use hard cutoffs

Soft de Sitter Effective Theory

Full theory

$$S_\phi = \int d^3x \, d\underline{t} \, \frac{\left(a(\underline{t})H\right)^3}{H^4}\left[-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda_\phi}{4!}\phi^4\right]$$

SdSET

$$S_\pm = \int d^3x \, d\underline{t}\left[-\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right) - \sum_{n\geq 2}(aH)^{3-n\alpha-\beta}\frac{c_{n,1}}{n!}\varphi_+^n\varphi_-\right]$$

Applications of SdSET

1) Correlators for massive scalars in dS

"Physics beyond the horizon is irrelevant"

2) Starobinsky's stochastic inflation

"Resum marginal operators using RG"

3) Metric fluctuations during inflation

"Power counting $\Rightarrow$ superhorizon modes freeze out"

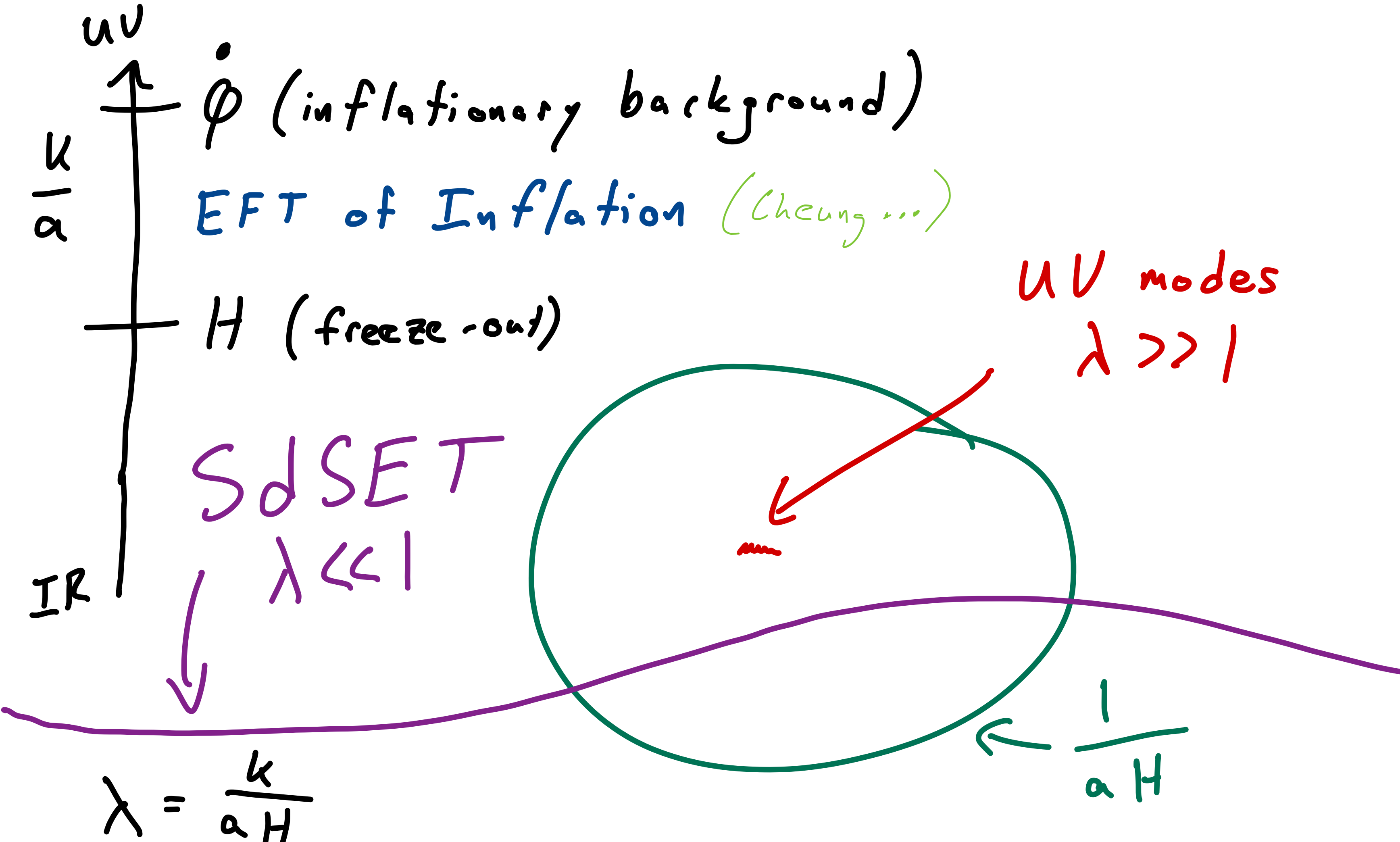
4) Eternal inflation

"Tower of relevant operators appear $\Rightarrow$ novel phase"

Why EFT?

- dS provides natural "ruler": the inverse comoving horizon $\Lambda_{uv} = aH$
- Interested in long wavelengths $k \ll aH$
- Large seperation of scales begs for EFT with power counting

$$\lambda \sim k/(aH)$$


UV
$\frac{k}{a}$
$\dot{\phi}$ (inflationary background)
EFT of Inflation (Cheung ...)
H (freeze-out)
SdSET
$\lambda \ll 1$
IR
UV modes
$\lambda \gg 1$
$\frac{1}{aH}$
$\lambda = \frac{k}{aH}$

(Continuum) EFT

Conceptual

- Isolate propagating DOFs
- Quadratic action $\Rightarrow$ dynamics
- Expose symmetries

Practical

- Power counting $\equiv$ dim analysis
- Regulate integrals w/o breaking symmetries
- RG sums full theory IR logs

One-to-many Mode Expansion

Factorize into soft and hard modes

$$\phi(\vec{x},t) = \phi_S(\vec{x},t) + \Phi_H(\vec{x},t)$$

Integrate out hard modes

$\Rightarrow$ Local operator expansion

Observables reconstructed order-by-order in λ

Isolating the Soft Modes

dS Spacetime

dS metric: $ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$

proper time

Notation: $\underline{t} \equiv Ht$

& $\tau \equiv -\exp(-\underline{t})/H$

conformal time

Scalar fields in dS

EOM $\ddot{\phi} + 3\dot{\phi} + \frac{k^2}{(aH)^2}\phi + \frac{m^2}{H^2}\phi = 0$

Soft limit $\phi_s = (aH)^{-3/2+\nu}\varphi_s$

w/ $\nu = \pm\sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$

or $\alpha = \frac{3}{2} - \nu \quad \beta = \frac{3}{2} + \nu$ s.t. $\alpha + \beta = 3$

WLOG $\alpha < \beta$

SdSET Fields

Two IR degrees of freedom

- "Growing" mode φ_+ ← Correlators of interest
- "Decaying" mode φ_-

w/ $\phi_s = H\left((aH)^{-\alpha}\varphi_+ + (aH)^{-\beta}\varphi_-\right)$

Stochastic Initial Conditions

Canonical Quantization

Full theory quantum field $\phi = \int \left(\bar{\phi} a^\dagger + \bar{\phi}^* a \right)$

w/ $\left[a^\dagger_{\vec{k}}, a_{\vec{k}'} \right] = (2\pi)^3 \delta(\vec{k} - \vec{k}')$

Bunch-Davies $\bar{\phi} = -i e^{i(\nu + \frac{1}{2})\frac{\pi}{2}} \frac{\sqrt{\pi}}{2} H(-\tau)^{3/2} H_\nu^{(1)}(-k\tau)$

Take Soft Limit

Expand ϕ for $k\tau \ll 1$

$$\phi_s \simeq \int \Big[(aH)^{-\alpha}\, \bar{\varphi}_+ \overbrace{\left(e^{i\delta_r} a^\dagger + e^{-i\delta_r} a\right)}^{\tilde{a}} + (aH)^{-\beta}\, \bar{\varphi}_- \underbrace{\left(i e^{-i\delta_v} a^\dagger - i e^{i\delta_v} a\right)}_{\tilde{b}} \Big]$$

w/ $\bar{\varphi}_+ \sim \dfrac{1}{k^{3/2-\alpha}}$ + $\bar{\varphi}_- \sim \dfrac{1}{k^{3/2-\beta}}$

Stocastic Random Variables

$\tilde{a}_{\vec{k}}$ and $\tilde{b}_{\vec{k}}$ are real

Satisfy $\left[\tilde{a}^\dagger, \tilde{a}\right] = \left[\tilde{b}^\dagger, \tilde{b}\right] = 0$

$$\langle \tilde{a}_{\vec{k}} \, \tilde{a}_{\vec{k}'} \rangle = \langle \tilde{b}_{\vec{k}} \, \tilde{b}_{\vec{k}'} \rangle = (2\pi)^3 \, \delta(\vec{k} + \vec{k}')$$

$a_{\vec{k}} |0\rangle$

Initial Conditions

Identify operators

$$\varphi_+ = \int \bar{\varphi}_+ \, \tilde{a}$$

$$\varphi_- = \int \bar{\varphi}_- \, \tilde{b}$$

Endowed with classical power spectra

$$\left\langle \varphi_+(\vec{k}) \, \varphi_+(\vec{k}') \right\rangle \sim \frac{1}{k^{3-2\alpha}} (2\pi)^3 \delta(\vec{k} + \vec{k}')$$

Augmented by non-Gaussian corrections from UV

SdSET

Defining SdSET

- DOF φ_+ and φ_-
- Power counting $\lambda \sim \frac{k}{\Lambda_{uv}}$ w/ $\Lambda_{uv} = aH$
- Symmetries (1) "space time"
  (2) "reparametrization"
- Initial conditions

* Very close analogy w/ Heavy Quark EFT

Power Counting

$$\lambda \sim \frac{k}{aH}$$

$+$

$$S \sim \lambda^0$$

$$\Longrightarrow$$

$$t \sim 1$$

$$\vec{x} \sim 1/\lambda$$

$$\vec{k} \sim \lambda$$

$$\varphi_+ \sim \lambda^\alpha$$

$$\varphi_- \sim \lambda^\beta$$

Symmetries

(1) Spacetime

$$t \to t \qquad \vec{k} \to \eta \vec{k}$$

$$\vec{x} \to \frac{1}{\eta} \vec{x} \qquad \varphi_+ \to \eta^{\alpha} \varphi_+$$

$$a \to \eta\, a \qquad \varphi_- \to \eta^{\beta} \varphi_-$$

(+ additional isometry transformation for static dS)

(2) Reparametrization invariance (RPI)

$$\varphi_+ \to \varphi_+ + (aH)^{\alpha-\beta} \varphi_-$$

$$\varphi_- \to (1-\varepsilon)\, \varphi_-$$

Free SdSET from Top Down

Plug $\phi_s = H\left(a^{-\alpha}\varphi_+ + a^{-\beta}\varphi_-\right)$ into S'_ϕ

Combine terms using int by parts and e.g. $H^2(\alpha^2 - 3\alpha) + m^2 = 0$, $\alpha + \beta = 3$, ...

$$\Rightarrow \quad S_{2,\pm} = \int \mathrm{d}^3x\, \mathrm{d}\underset{\sim}{t}\, \frac{1}{2}\Big[[aH]^{2\nu}\dot{\varphi}_+^2 + [aH]^{-2\nu}\dot{\varphi}_-^2 + 2\dot{\varphi}_+\dot{\varphi}_- - 2\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right)$$
$$- [aH]^{2\nu-2}\partial_i\varphi_+\partial^i\varphi_+ - [aH]^{-2\nu-2}\partial_i\varphi_-\partial^i\varphi_- - 2[aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

$\underline{t}$

Free SdSET from Top Down

Drop subleading terms

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}t\,\frac{1}{2}\Big[[aH]^{2\nu}\dot\varphi_+^2 + \cancel{[aH]^{-2\nu}\dot\varphi_-^2} + 2\dot\varphi_+\dot\varphi_- - 2\nu\left(\dot\varphi_+\varphi_- - \varphi_+\dot\varphi_-\right)$$
$$- [aH]^{2\nu-2}\partial_i\varphi_+\partial^i\varphi_+ - \cancel{[aH]^{-2\nu-2}\partial_i\varphi_-\partial^i\varphi_-} - 2[aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

Int by parts and let $\varphi_- \to \varphi_- + \frac{1}{2}(aH)^{2\nu}\varphi_+$

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}t\,\Big[\dot\varphi_+\dot\varphi_- - \nu\left(\dot\varphi_+\varphi_- - \varphi_+\dot\varphi_-\right) - [aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

Treat as interaction $\Rightarrow \mathcal{O}(\lambda^4)$

(Weinberg)

Free SdSET

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{dt}\left[-\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right) - \frac{1}{[aH]^2}\partial_i\varphi_+\partial^i\varphi_-\right] + \mathcal{O}(\lambda^4)$$

$$\Downarrow$$

$$\dot{\varphi}_+ = \frac{1}{2\nu[aH]^2}\partial^2\varphi_+ \qquad\qquad \dot{\varphi}_- = -\frac{1}{2\nu[aH]^2}\partial^2\varphi_-$$

Leading power solutions are constant

Locality

- $\varphi_\pm$ & Φ_H are momentum eigenstates
- Momentum conservation $\Rightarrow$ $\cancel{\mathcal{L} \supset \varphi_\pm^3 \Phi_H}$
- Leading effect of integrating out Φ_H occurs at 1-loop

$$\delta S \sim \int \Phi_H^2(\vec{x}, \tau)\, \Phi_H^2(\vec{y}, \tau') \underset{\substack{\uparrow \\ FT + k \ll p}}{\sim} e^{-p|\tau - \tau'|}$$

$\Rightarrow$ Local interaction

Leading Interactions

$$S_{int} \supset -\int (aH)^{3-n\alpha} \frac{c_n}{n!} \varphi_+^n$$

can be removed with

$$\varphi_- \to \varphi_- + \frac{n c_n}{3\nu(3-n\alpha)n!} (aH)^{3-n\alpha} \varphi_+^{n-1}$$

Powercounting Interactions

$$S_{int} \supset - \int (aH)^{3-n\alpha-m\beta} \frac{c_{n,m}}{n!\,m!} \varphi_+^n \varphi_-^m$$

$$\sim \lambda^{n\alpha+m\beta-3} \quad \text{w/} \quad m \geq 1$$

relevant $\quad n\alpha + m\beta < 3$

marginal $\quad n\alpha + m\beta = 3$

irrelevant $\quad n\alpha + m\beta > 3$

Powercounting Interactions

$$S_{int} \supset -\int (aH)^{3-n\alpha-m\beta} \frac{c_{n,m}}{n!\,m!} \varphi_+^n \varphi_-^m$$

$$\sim \lambda^{(n-1)\alpha + (m-1)\beta} \quad \text{w/} \quad m \geq 1$$

$$m^2 \neq 0 \iff 0 < \alpha < 3/2 \text{ and } 3/2 < \beta < 3$$

Interactions are $\mathcal{O}(\lambda) \Rightarrow$ irrelevant

$$m^2 \to 0 \Rightarrow \alpha \to 0 \ \ldots$$

Dynamical Dimensional Regularization

Often encounter $\displaystyle\int \frac{d^d p}{p^d}$

$\Rightarrow$ dim reg fails

Introduce "dyn dim reg"

Trick: analytically continue in α

$$\Rightarrow \int \frac{d^d p}{p^d} \longrightarrow \int \frac{d^d p}{p^{d+\delta\alpha}}$$

Applications

Massive Scalars in dS

- Massive Scalars are "free"

  Interactions are irrelevant

- See paper for variety of calculations

  (Green and Premkumar)

- Non-trivial role of initial conditions

Light Scalars in dS

As $m^2 \to 0$, $\alpha \to 0$

$$S_{int} = -\int \sum_{n>1} \frac{c_n}{n!} (aH)^{(1-n)\alpha} \varphi_+^n \varphi_-$$

$$\sim \lambda^{(n-1)\alpha} \to \lambda^0$$

$\Rightarrow$ Tower of marginal interactions

$$\text{EOM: } 2\nu \dot{\varphi}_+ = -\sum_{n>1} (aH)^{(2-n)\alpha} \frac{c_n}{n!} \varphi_+^n$$

Light Scalars in dS

Composite operators

$$\mathcal{O}_n = \varphi_+^n \sim \lambda^{n\alpha} \to \lambda^0$$

$\Rightarrow$ RG mixing expected

Contract any two legs

$$\langle \mathcal{O}_n \cdots \rangle \supset \langle \mathcal{O}_{n-2} \cdots \rangle \binom{n}{2} \frac{C_\alpha^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$

Light Scalars in dS

$$\int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$ is scaleless and diverges as $\alpha \to 0$

Isolate UV divergence

$$p^2 \to p^2 + k_{IR}^2$$

$$\langle \mathcal{O}_n \ldots \rangle \supset \langle \mathcal{O}_{n-2} \cdots \rangle \binom{n}{2} \frac{C_\alpha^2}{4\pi^2} \left(-\frac{1}{2\alpha} - \gamma_E - \log \frac{aH}{k_{IR}} \right)$$

Dynamical RG $\Leftrightarrow$ Stocastic Inflation

Resum time dependent logs:

$$\frac{\partial}{\partial \underline{t}} \langle \mathcal{O}_n \ldots \rangle = -\frac{n}{3} \sum_{m \geq 1} \frac{c_m}{m!} \langle \mathcal{O}_{n-1} \, \mathcal{O}_m \ldots \rangle + \frac{n(n-1)}{8\pi^2} \langle \mathcal{O}_{n-2} \ldots \rangle$$

(Starobinsky; Starobinsky, Yokoyama)

Is equivalent to a Fokker-Planck eq

for $P(\varphi, t)$ w/ $\langle \varphi^n \rangle = \int d\varphi \, P(\varphi, t) \varphi^n$ (Baumgart & Sundrum)

Metric Fluctuations During Inflation

Power counting + symmetries

$\Rightarrow$ no time dependance

$$ds^2 = -N^2 dt^2 + a^2(t)\, e^{2\zeta(\vec{x},t)} \left(e^{2\gamma(\vec{x},t)}\right)_{ij} \left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

ζ is adiabatic scalar fluctuation

γ_{ij} is tensor fluctuation

N + N^i are Lagrange multipliers

(Arnowitt Deser Misner)

Metric Fluctuations During Inflation

Full theory quadratic Lagrangian

$$\mathcal{L}_{2,\zeta} = -\frac{M_{\rm pl}^2 \partial_t H}{H^2 c_s^2} \left(\partial_t \zeta^2 - a^{-2} c_s^2 \partial_i \zeta \partial^i \zeta\right)$$

Many non-linear symmetries, e.g.

$$\vec{x} \to e^{-\eta} \vec{x} \quad \Rightarrow \quad \zeta \to \zeta(e^{-\eta}\vec{x}) - \eta$$

Can take SdSET limit w/ $\zeta \to \zeta_+ + \ldots$

Metric Fluctuations During Inflation

What types of operators could cause time evolution?

$$\mathcal{L}_{int} \supset \cancel{\zeta_+^{n-1} \zeta_-} + \ldots$$

Violate shift symmetries

$\Rightarrow$ Only allowed terms involve $\frac{\partial_i}{aH} \zeta_\pm$

$\Rightarrow$ Power suppressed!

* Also applies to tensor modes!

Full theory argument: Salopek + Bond; Maldacena (tree)
Assassi, Baumann, Green; Senatore, Zaldarriaga (all orders)

Eternal Inflation

Metric dynamics are important

$$\Rightarrow \quad S_{int} \supset \int \sqrt{-g}\,(aH)^{-n\alpha} \frac{c_n}{n!} \varphi_+^n$$

Can not be removed by field redef
(int by parts generates gravitional contributions)

As $\alpha \to 0$, $\varphi_+^n \sim 1/\lambda^3$ $\Rightarrow$ relevant operators

$\Rightarrow$ Non-perturbative!

Summary

Summary

It works!!

$$S_\pm = \int d^3x \, d\underline{t} \left[-\nu \left(\dot\varphi_+ \varphi_- - \varphi_+ \dot\varphi_- \right) - \sum_{n \geq 2} (aH)^{3-n\alpha-\beta} \frac{c_{n,1}}{n!} \varphi_+^n \varphi_- \right]$$

Summary

1) Correlators for massive scalars in dS

"Physics beyond the horizon is irrelevant"

2) Starobinsky's stochastic inflation

"Resum marginal operators using RG"

3) Metric fluctuations during inflation

"Power counting ⇒ superhorizon modes freeze out"

4) Eternal inflation

"Tower of relevant operators appear ⇒ novel phase"